Time Series Analysis & Forecasting Using R

8. ARIMA models

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Outline

1. ARIMA models
2. Lab Session 16
3. Seasonal ARIMA models
4. Lab Session 17
5. Forecast ensembles

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1 ARIMA models

2 Lab Session 16

3 Seasonal ARIMA models

4 Lab Session 17

5 Forecast ensembles

ARIMA models

AR: autoregressive (lagged observations as inputs)

I: integrated (differencing to make series stationary)

MA: moving average (lagged errors as inputs)

ARIMA models

AR: autoregressive (lagged observations as inputs)

I: integrated (differencing to make series stationary)

MA: moving average (lagged errors as inputs)

An ARIMA model is rarely interpretable in terms of visible data structures like trend and seasonality. But it can capture a huge range of time series patterns.

Stationarity

Definition

If $\{y_t\}$ is a stationary time series, then for all s, the distribution of $(y_t, \dots, y_{t+s})$ does not depend on t.

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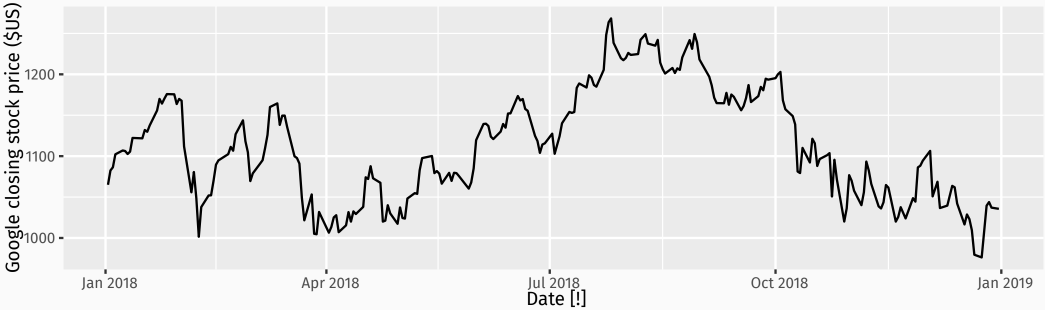
A **stationary series** is:

- roughly horizontal
- constant variance
- no patterns predictable in the long-term

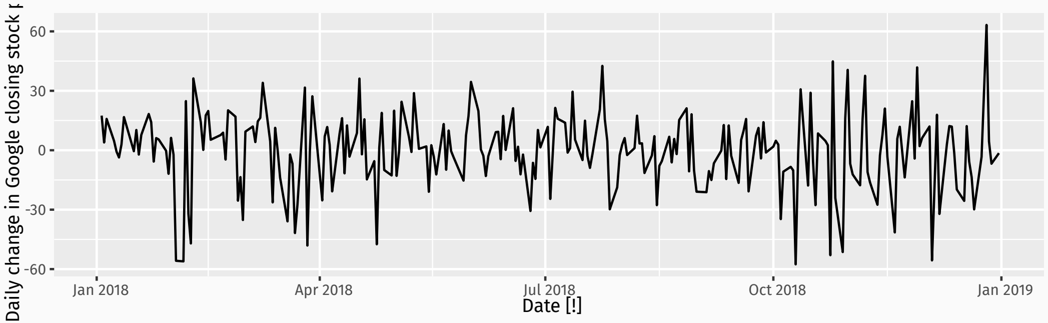
Stationary?

```r
gafa_stock |>
  filter(Symbol == "GOOG", year(Date) == 2018) |>
  autoplot(Close) +
  labs(y = "Google closing stock price ($US)")
```



Stationary?

```
gafa_stock |>
  filter(Symbol == "GOOG", year(Date) == 2018) |>
  autoplot(difference(Close)) +
  labs(y = "Daily change in Google closing stock price")
```



Differencing

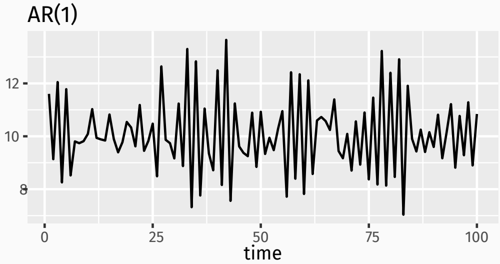
- Differencing helps to **stabilize the mean**.
- The differenced series is the *change* between each observation in the original series.
- Occasionally the differenced data will not appear stationary and it may be necessary to difference the data a second time.
- In practice, it is almost never necessary to go beyond second-order differences.

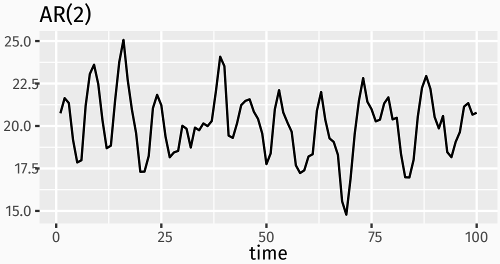
Autoregressive models

Autoregressive (AR) models:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t,$$

where ε_t is white noise. A multiple regression with **lagged values** of y_t as predictors.





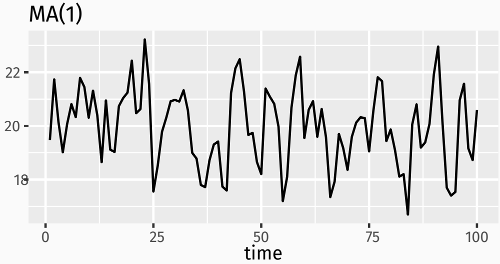
- Cyclic behaviour is possible when $p \geq 2$.

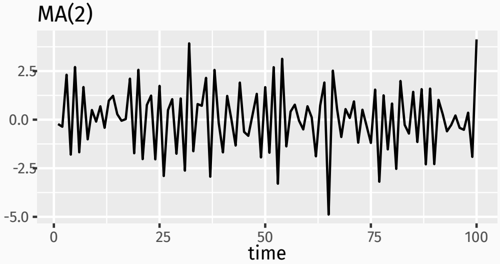
Moving Average (MA) models

Moving Average (MA) models:

$$y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q},$$

where ε_t is white noise. A multiple regression with **lagged *errors*** as predictors. *Don't confuse with moving average smoothing!*





ARIMA models

Autoregressive Moving Average models:

$$\begin{aligned} y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} \\ + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t. \end{aligned}$$

ARIMA models

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- Predictors include both **lagged values of y_t and lagged errors.**

ARIMA models

Autoregressive Moving Average models:

$$
\begin{aligned}
y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} \\
+ \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t.
\end{aligned}
$$

- Predictors include both **lagged values of y_t and lagged errors.**

Autoregressive Integrated Moving Average models

- Combine ARMA model with **differencing**.
- d-differenced series follows an ARMA model.
- Need to choose p, d, q and whether or not to include c.

ARIMA models

ARIMA(p, d, q) model

AR: p = order of the autoregressive part
I: d = degree of first differencing involved
MA: q = order of the moving average part.

- White noise model: ARIMA(0,0,0)
- Random walk: ARIMA(0,1,0) with no constant
- Random walk with drift: ARIMA(0,1,0) with const.
- AR(p): ARIMA(p,0,0)
- MA(q): ARIMA(0,0,q)

Example: National populations

```r
fit <- global_economy |>
  model(arima = ARIMA(Population))
fit
```

```
# A mable: 263 x 2
# Key:     Country [263]
   Country                                        arima
   <fct>                                        <model>
 1 Afghanistan                           <ARIMA(4,2,1)>
 2 Albania                               <ARIMA(0,2,2)>
 3 Algeria                               <ARIMA(2,2,2)>
 4 American Samoa                        <ARIMA(2,2,2)>
 5 Andorra              <ARIMA(2,1,2) w/ drift>
 6 Angola                                <ARIMA(4,2,1)>
 7 Antigua and Barbuda  <ARIMA(2,1,2) w/ drift>
 8 Arab World                            <ARIMA(0,2,1)>
 9 Argentina                             <ARIMA(2,2,2)>
```

Example: National populations

```
fit |>
  filter(Country == "Australia") |>
  report()
```

```
Series: Population
Model: ARIMA(0,2,1)

Coefficients:
         ma1
      -0.661
s.e.   0.107

sigma^2 estimated as 4.063e+09:  log likelihood=-699
AIC=1401   AICc=1402   BIC=1405
```

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```

$$y_t = 2y_{t-1} - y_{t-2} - 0.7\varepsilon_{t-1} + \varepsilon_t$$
$$\varepsilon_t \sim \text{NID}(0, 4 \times 10^9)$$

Understanding ARIMA models

- If $c = 0$ and $d = 0$, the long-term forecasts will go to zero.
- If $c = 0$ and $d = 1$, the long-term forecasts will go to a non-zero constant.
- If $c = 0$ and $d = 2$, the long-term forecasts will follow a straight line.
- If $c \neq 0$ and $d = 0$, the long-term forecasts will go to the mean of the data.
- If $c \neq 0$ and $d = 1$, the long-term forecasts will follow a straight line.
- If $c \neq 0$ and $d = 2$, the long-term forecasts will follow a quadratic trend.

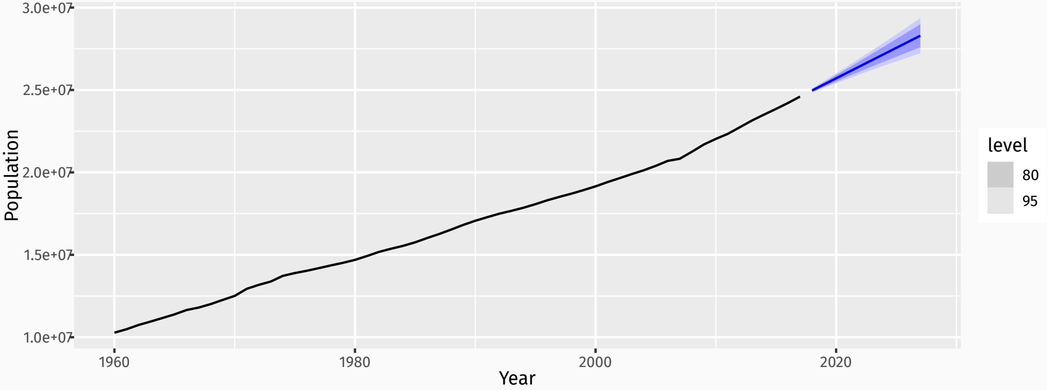
Understanding ARIMA models

Forecast variance and d

- The higher the value of d, the more rapidly the prediction intervals increase in size.
- For $d = 0$, the long-term forecast standard deviation will go to the standard deviation of the historical data.

Example: National populations

```
fit |>
  forecast(h = 10) |>
  filter(Country == "Australia") |>
  autoplot(global_economy)
```



How does ARIMA() work?

Hyndman and Khandakar (JSS, 2008) algorithm:

- Select no. differences d via KPSS test.
- Select p, q and inclusion of c by minimising AICc.
- Use stepwise search to traverse model space.

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- Use stepwise search to traverse model space.

$$\text{AICc} = -2\log(L) + 2(p+q+k+1)\left[1 + \frac{(p+q+k+2)}{T-p-q-k-2}\right]$$

where L is the maximised likelihood fitted to the *differenced* data, $k = 1$ if $c \neq 0$ and $k = 0$ otherwise.

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where L is the maximised likelihood fitted to the *differenced* data, $k = 1$ if $c \neq 0$ and $k = 0$ otherwise.

Note: Can’t compare AICc for different values of d.

How does ARIMA() work?

Step1: Select current model (with smallest AICc) from:

ARIMA$(2, d, 2)$

ARIMA$(0, d, 0)$

ARIMA$(1, d, 0)$

ARIMA$(0, d, 1)$

How does ARIMA() work?

Step1: Select current model (with smallest AICc) from:

$\text{ARIMA}(2, d, 2)$

$\text{ARIMA}(0, d, 0)$

$\text{ARIMA}(1, d, 0)$

$\text{ARIMA}(0, d, 1)$

Step 2: Consider variations of current model:

- vary one of $p, q,$ from current model by ± 1;
- p, q both vary from current model by ± 1;
- Include/exclude c from current model.

Model with lowest AICc becomes current model.

How does ARIMA() work?

Step1: Select current model (with smallest AICc) from:

$\text{ARIMA}(2, d, 2)$

$\text{ARIMA}(0, d, 0)$

$\text{ARIMA}(1, d, 0)$

$\text{ARIMA}(0, d, 1)$

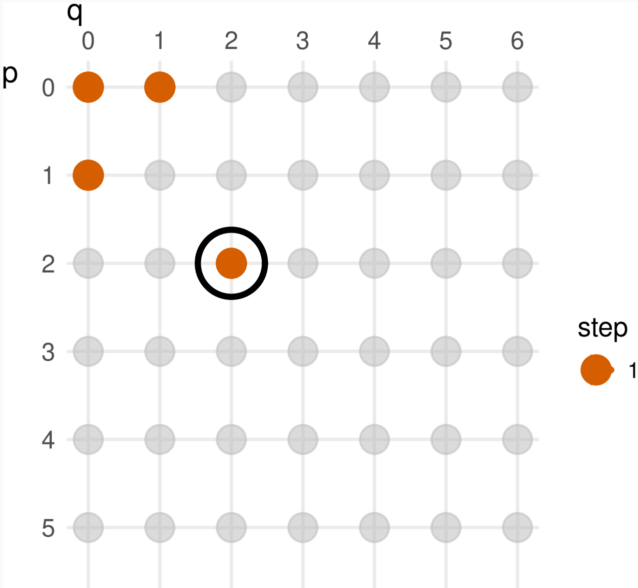
Step 2: Consider variations of current model:

- vary one of $p, q,$ from current model by ± 1;
- p, q both vary from current model by ± 1;
- Include/exclude c from current model.

Model with lowest AICc becomes current model.

Repeat Step 2 until no lower AICc can be found.

How does ARIMA() work?



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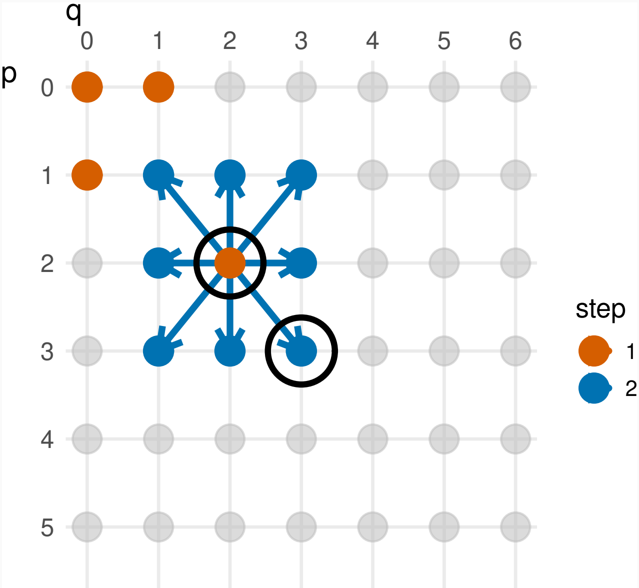

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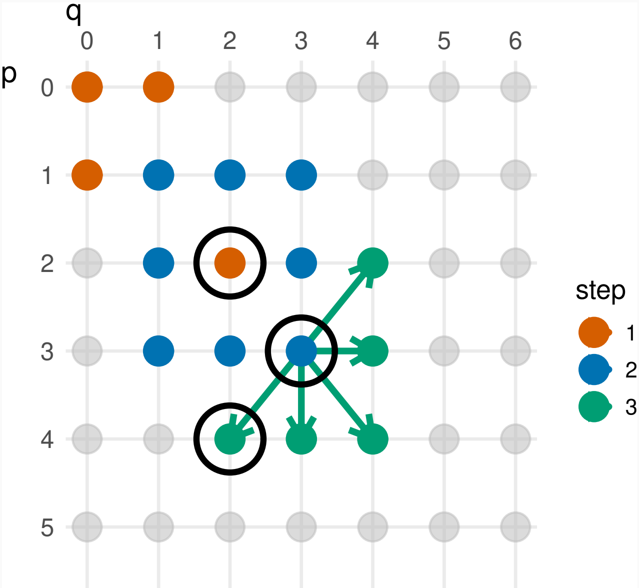

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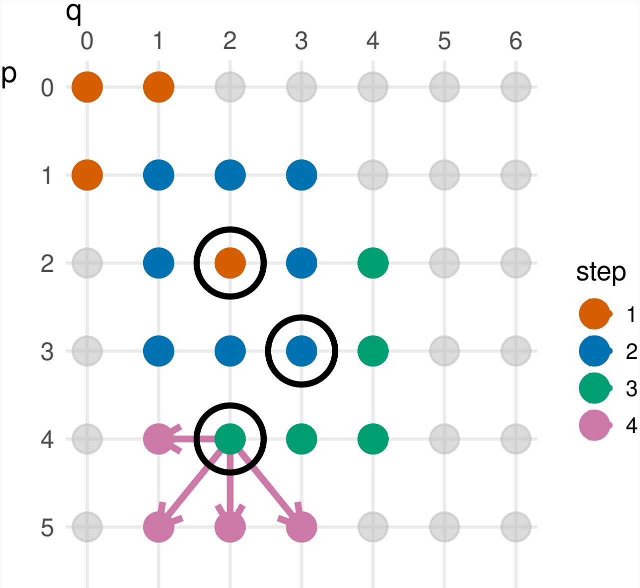

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Lab Session 16

For the United States GDP data (from `global_economy`):

- Fit a suitable ARIMA model for the logged data.
- Produce forecasts of your fitted model. Do the forecasts look reasonable?

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Seasonal ARIMA models

ARIMA	$\underbrace{(p, d, q)}$	$\underbrace{(P, D, Q)_m}$
	↑ Non-seasonal part of the model	↑ Seasonal part of of the model

- m = number of observations per year.
- d first differences, D seasonal differences
- p AR lags, q MA lags
- P seasonal AR lags, Q seasonal MA lags

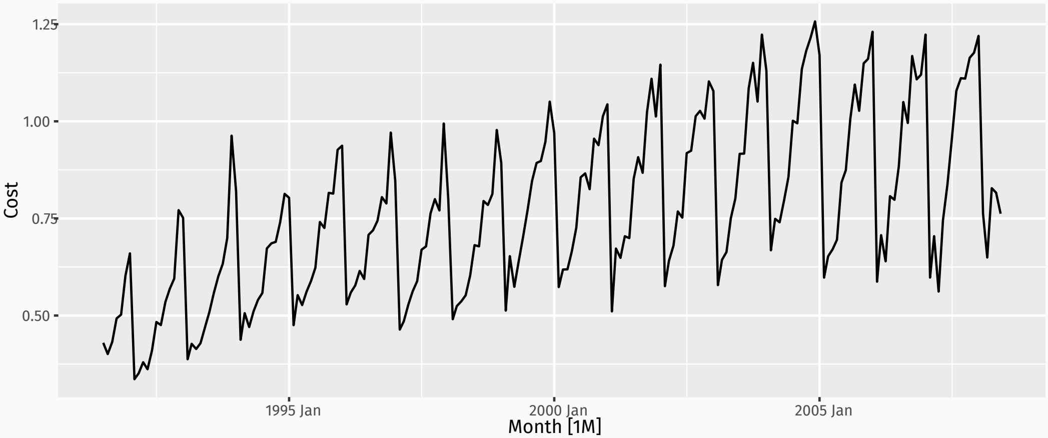
Seasonal and non-seasonal terms combine multiplicatively

Cortecosteroid drug sales

```
h02 <- PBS |>
  filter(ATC2 == "H02") |>
  summarise(Cost = sum(Cost) / 1e6)
```

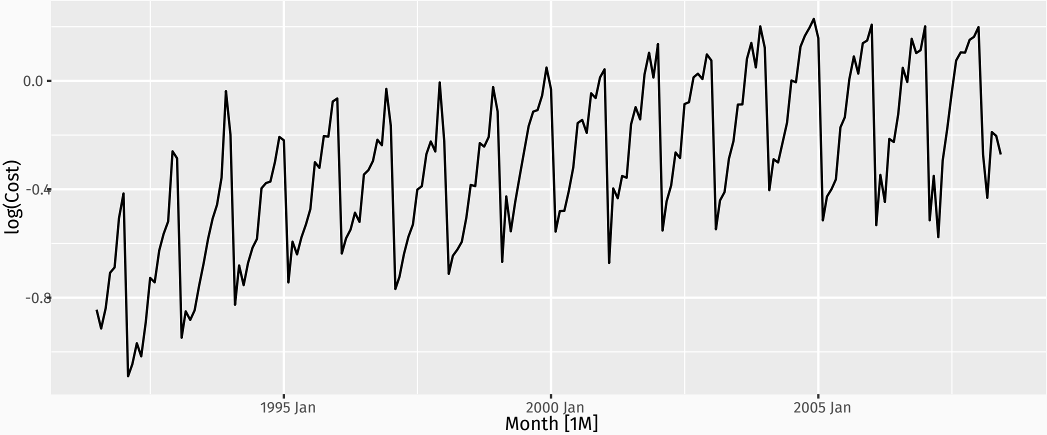
Cortecosteroid drug sales

```
h02 |> autoplot(
  Cost
)
```



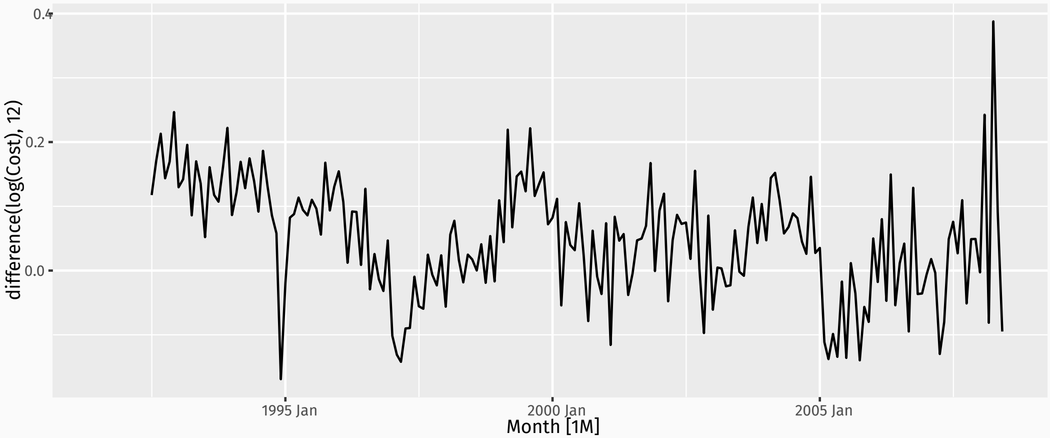
Cortecosteroid drug sales

```
h02 |> autoplot(
  log(Cost)
)
```



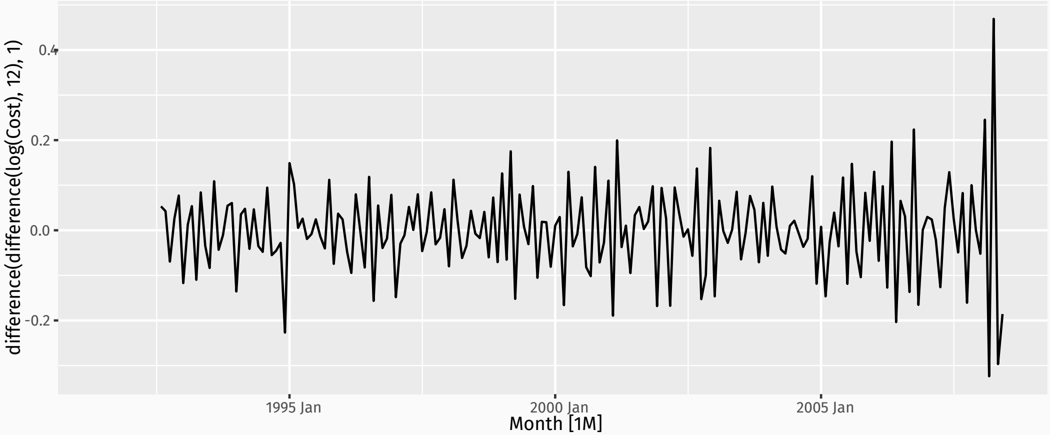
Cortecosteroid drug sales

```
h02 |> autoplot(
  log(Cost) |> difference(12)
)
```



Cortecosteroid drug sales

```r
h02 |> autoplot(
  log(Cost) |> difference(12) |> difference(1)
)
```



Example: US electricity production

```
h02 |>
  model(arima = ARIMA(log(Cost))) |>
  report()
```

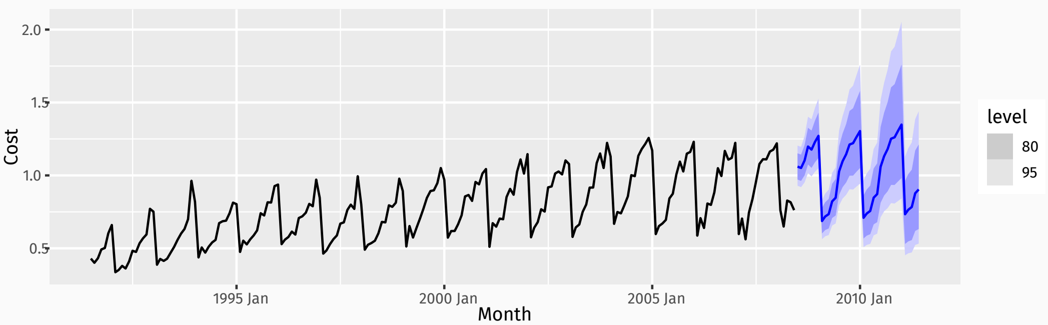
```
Series: Cost
Model: ARIMA(2,1,0)(0,1,1)[12]
Transformation: log(Cost)

Coefficients:
          ar1      ar2     sma1
      -0.8491  -0.4207  -0.6401
s.e.   0.0712   0.0714   0.0694

sigma^2 estimated as 0.004387:  log likelihood=245
AIC=-483   AICc=-483   BIC=-470
```

Example: US electricity production

```
h02 |>
  model(arima = ARIMA(log(Cost))) |>
  forecast(h = "3 years") |>
  autoplot(h02)
```



Cortecosteroid drug sales

```r
fit <- h02 |>
  model(best = ARIMA(log(Cost),
    stepwise = FALSE,
    approximation = FALSE,
    order_constraint = p + q + P + Q <= 9
  ))
report(fit)
```

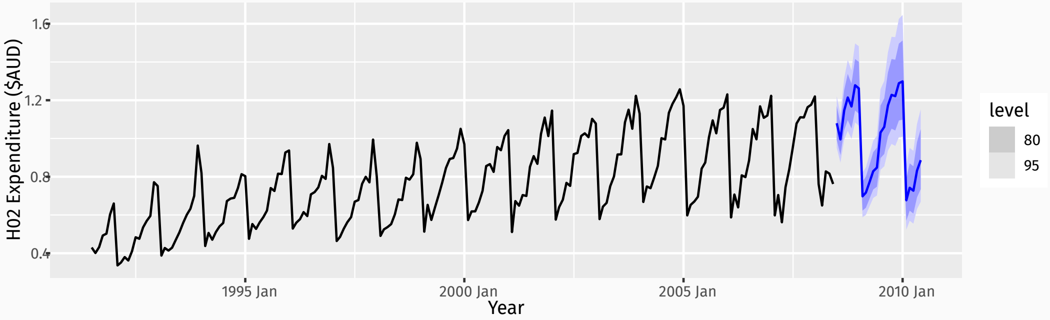
```
Series: Cost
Model: ARIMA(4,1,1)(2,1,2)[12]
Transformation: log(Cost)

Coefficients:
          ar1    ar2    ar3     ar4     ma1   sar1    sar2    sma1   sma2
      -0.0425  0.210  0.202  -0.227  -0.742  0.621  -0.383  -1.202  0.496
s.e.   0.2167  0.181  0.114   0.081   0.207  0.242   0.118   0.249  0.213

sigma^2 estimated as 0.004049:  log likelihood=254
AIC=-489   AICc=-487   BIC=-456
```

Cortecosteroid drug sales

```r
fit |>
  forecast() |>
  autoplot(h02) +
  labs(y = "H02 Expenditure ($AUD)", x = "Year")
```



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Lab Session 17

For the Australian tourism data (from `tourism`):

- Fit a suitable ARIMA model for all data.
- Produce forecasts of your fitted models.
- Check the forecasts for the “Snowy Mountains” and “Melbourne” regions. Do they look reasonable?

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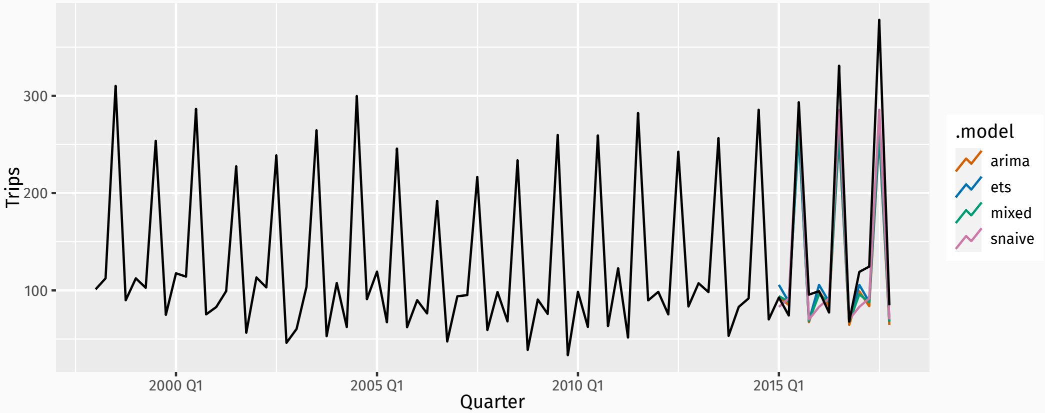
Forecast ensembles

```
train <- tourism |>
  filter(year(Quarter) <= 2014)
fit <- train |>
  model(
    ets = ETS(Trips),
    arima = ARIMA(Trips),
    snaive = SNAIVE(Trips)
  ) |>
  mutate(mixed = (ets + arima + snaive) / 3)
```

- Ensemble forecast `mixed` is a simple average of the three fitted models.
- `forecast()` will produce distributional forecasts taking into account the correlations between the forecast errors of the component models.

Forecast ensembles

```r
fc <- fit |> forecast(h = "3 years")
fc |>
  filter(Region == "Snowy Mountains", Purpose == "Holiday") |>
  autoplot(tourism, level = NULL)
```



Forecast ensembles

```r
accuracy(fc, tourism) |>
  group_by(.model) |>
  summarise(
    RMSE = mean(RMSE),
    MAE = mean(MAE),
    MASE = mean(MASE)
  ) |>
  arrange(RMSE)
```

```
# A tibble: 4 x 4
  .model  RMSE   MAE  MASE
  <chr>  <dbl> <dbl> <dbl>
1 mixed   19.8  16.0 0.997
2 ets     20.2  16.4 1.00
3 snaive  21.5  17.3 1.17
4 arima   21.9  17.8 1.06
```

Forecast ensembles

Can we do better than equal weights?

Forecast ensembles

Can we do better than equal weights?

- Hard to find weights that improve forecast accuracy.
- Known as the “forecast combination puzzle”.
- Solution: FFORMA

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- Solution: FFORMA

FFORMA (Feature-based FORecast Model Averaging)

- Vector of time series features used to predict best weights.
- A modification of xgboost is used.
- Method came 2nd in the 2018 M4 international forecasting competition.
- Main author: Pablo Montero-Manso (now Uni Sydney)
- Not (yet) available for fable.