Time Series Analysis & Forecasting Using R

7. Exponential smoothing

tsibble feasts fable

bit.ly/tidyts2023

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Outline

1. Exponential smoothing
2. Trend methods
3. Lab Session 14
4. Seasonal methods
5. ETS taxonomy
6. Lab Session 15
7. Non-Gaussian forecast distributions

Outline

Pharmaceutical Benefits Scheme

Pharmaceutical Benefits Scheme

The Pharmaceutical Benefits Scheme (PBS) is the Australian government drugs subsidy scheme.

- Many drugs bought from pharmacies are subsidised to allow more equitable access to modern drugs.
- The cost to government is determined by the number and types of drugs purchased. Currently nearly 1% of GDP.
- The total cost is budgeted based on forecasts of drug usage.

Pharmaceutical Benefits Scheme

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Opp demands drug price restriction after PBS budget blow-out

The Federal Opposition has called for tighter controls on drug prices after the Pharmaceutical Benefits Scheme (PBS) budget blew out by almost $800 million.

The money was spent on two new drugs including the controversial anti-smoking aid Zyban, which dropped in price from $220 to $22 after it was listed on the PBS.

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FEATURES

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Pharmaceutical Benefits Scheme

- In 2001: $4.5 billion budget, under-forecasted by $800 million.
- Thousands of products. Seasonal demand.
- Subject to covert marketing, volatile products, uncontrollable expenditure.
- Although monthly data available for 10 years, data are aggregated to annual values, and only the first three years are used in estimating the forecasts.
- All forecasts being done with the FORECAST function in MS-Excel!

Historical perspective

- Developed in the 1950s and 1960s as methods (algorithms) to produce point forecasts.
- Combine a “level”, “trend” (slope) and “seasonal” component to describe a time series.
- The rate of change of the components are controlled by “smoothing parameters”: α, β and γ respectively.
- Need to choose best values for the smoothing parameters (and initial states).
- Equivalent ETS state space models developed in the 1990s and 2000s.

A model for levels, trends, and seasonalities

We want a model that captures the level (ℓ_t), trend (b_t) and seasonality (s_t).

How do we combine these elements?

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Multiplicatively?

$$y_t = \ell_{t-1} b_{t-1} s_{t-m} (1 + \varepsilon_t)$$

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Perhaps a mix of both?

$$y_t = (\ell_{t-1} + b_{t-1}) s_{t-m} + \varepsilon_t$$

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Perhaps a mix of both?

$$y_t = (\ell_{t-1} + b_{t-1}) s_{t-m} + \varepsilon_t$$

How do the level, trend and seasonal components evolve over time?

ETS models

General notation $\quad$ E T S : **E**xponen**T**ial **S**moothing

$\nearrow \quad \uparrow \quad \nwarrow$

Error **T**rend **S**eason

Error: Additive (`"A"`) or multiplicative (`"M"`)

ETS models

General notation E T S : **E**xponen**T**ial **S**moothing

Error **T**rend **S**eason

Error: Additive (`"A"`) or multiplicative (`"M"`)

Trend: None (`"N"`), additive (`"A"`), multiplicative (`"M"`), or damped (`"Ad"` or `"Md"`).

ETS models

General notation E T S : **E**xponen**T**ial **S**moothing

Error **T**rend **S**eason

Error: Additive (`"A"`) or multiplicative (`"M"`)

Trend: None (`"N"`), additive (`"A"`), multiplicative (`"M"`), or damped (`"Ad"` or `"Md"`).

Seasonality: None (`"N"`), additive (`"A"`) or multiplicative (`"M"`)

ETS(A,N,N): SES with additive errors

$$
\begin{aligned}
\text{Forecast equation} \quad & \hat{y}_{T+h|T} = \ell_T \\
\text{Measurement equation} \quad & y_t = \ell_{t-1} + \varepsilon_t \\
\text{State equation} \quad & \ell_t = \ell_{t-1} + \alpha\varepsilon_t
\end{aligned}
$$

where $\varepsilon_t \sim \text{NID}(0, \sigma^2)$.

ETS(A,N,N): SES with additive errors

$$\begin{aligned}
\text{Forecast equation} \quad & \hat{y}_{T+h|T} = \ell_T \\
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\text{State equation} \quad & \ell_t = \ell_{t-1} + \alpha\varepsilon_t
\end{aligned}$$

where $\varepsilon_t \sim \text{NID}(0, \sigma^2)$.

- “innovations” or “single source of error” because equations have the same error process, ε_t.
- Measurement equation: relationship between observations and states.
- Transition/state equation(s): evolution of state(s) over time.

ETS(M,N,N): SES with multiplicative errors

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T$
Measurement equation	$y_t = \ell_{t-1}(1 + \varepsilon_t)$
State equation	$\ell_t = \ell_{t-1}(1 + \alpha\varepsilon_t)$

where $\varepsilon_t \sim \text{NID}(0, \sigma^2)$.

ETS(M,N,N): SES with multiplicative errors

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T$
Measurement equation	$y_t = \ell_{t-1}(1 + \varepsilon_t)$
State equation	$\ell_t = \ell_{t-1}(1 + \alpha\varepsilon_t)$

where $\varepsilon_t \sim \text{NID}(0, \sigma^2)$.

- Models with additive and multiplicative errors with the same parameters generate the same point forecasts but different prediction intervals.

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3. Lab Session 14
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5. ETS taxonomy
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7. Non-Gaussian forecast distributions

Holt's linear trend

Additive errors: ETS(A,A,N)

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T + hb_T$
Measurement equation	$y_t = \ell_{t-1} + b_{t-1} + \varepsilon_t$
State equations	$\ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t$
	$b_t = b_{t-1} + \beta\varepsilon_t$

Holt's linear trend

Additive errors: ETS(A,A,N)

Forecast equation $\quad \hat{y}_{T+h|T} = \ell_T + hb_T$

Measurement equation $\quad y_t = \ell_{t-1} + b_{t-1} + \varepsilon_t$

State equations
$$\begin{aligned} \ell_t &= \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t \\ b_t &= b_{t-1} + \beta\varepsilon_t \end{aligned}$$

Multiplicative errors: ETS(M,A,N)

Forecast equation $\quad \hat{y}_{T+h|T} = \ell_T + hb_T$

Measurement equation $\quad y_t = (\ell_{t-1} + b_{t-1})(1 + \varepsilon_t)$

State equations
$$\begin{aligned} \ell_t &= (\ell_{t-1} + b_{t-1})(1 + \alpha\varepsilon_t) \\ b_t &= b_{t-1} + \beta(\ell_{t-1} + b_{t-1})\varepsilon_t \end{aligned}$$

Example: Australian population

```r
aus_economy <- global_economy |>
  filter(Country == "Australia") |>
  mutate(Pop = Population / 1e6)
fit <- aus_economy |> model(AAN = ETS(Pop))
report(fit)
```

```
Series: Pop
Model: ETS(A,A,N)
  Smoothing parameters:
    alpha = 1
    beta  = 0.327

  Initial states:
 l[0]  b[0]
 10.1 0.222

  sigma^2:  0.0041

  AIC  AICc   BIC
-77.0 -75.8 -66.7
```

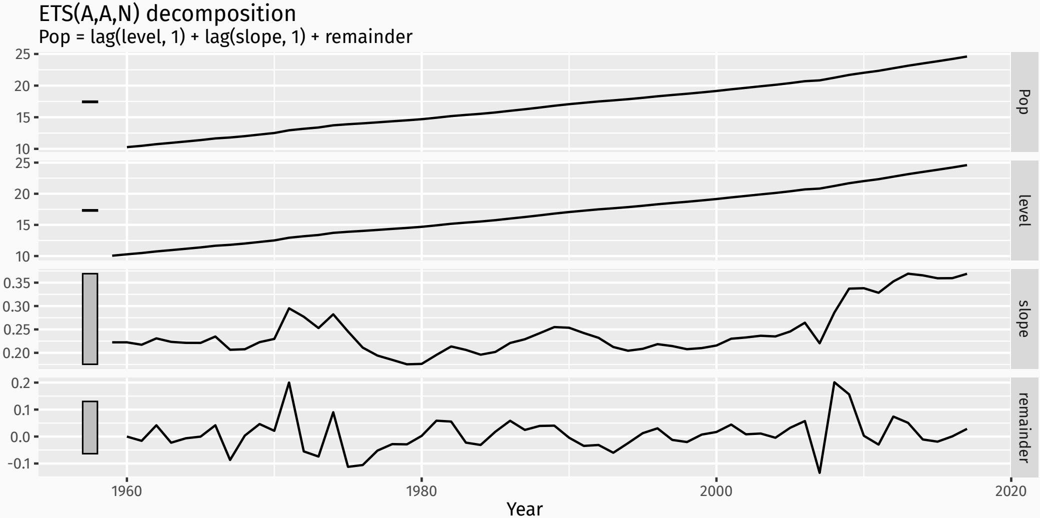
Example: Australian population

```r
components(fit)
```

```
# A dable: 59 x 7 [1Y]
# Key:      Country, .model [1]
# :         Pop = lag(level, 1) + lag(slope, 1) + remainder
   Country   .model  Year   Pop level slope remainder
   <fct>     <chr>  <dbl> <dbl> <dbl> <dbl>     <dbl>
 1 Australia AAN     1959  NA    10.1 0.222 NA
 2 Australia AAN     1960  10.3  10.3 0.222 -0.000145
 3 Australia AAN     1961  10.5  10.5 0.217 -0.0159
 4 Australia AAN     1962  10.7  10.7 0.231  0.0418
 5 Australia AAN     1963  11.0  11.0 0.223 -0.0229
 6 Australia AAN     1964  11.2  11.2 0.221 -0.00641
 7 Australia AAN     1965  11.4  11.4 0.221 -0.000314
 8 Australia AAN     1966  11.7  11.7 0.235  0.0418
 9 Australia AAN     1967  11.8  11.8 0.206 -0.0869
10 Australia AAN     1968  12.0  12.0 0.208  0.00350
```

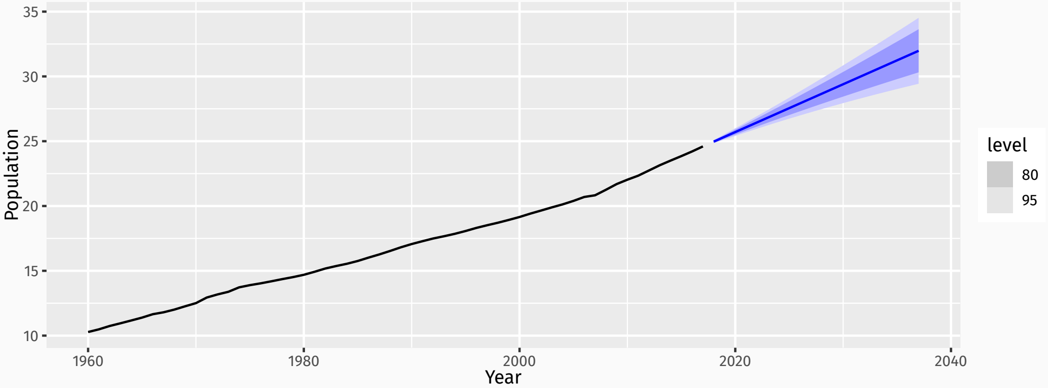
Example: Australian population

```r
components(fit) |> autoplot()
```



Example: Australian population

```
fit |>
  forecast(h = 20) |>
  autoplot(aus_economy) +
  labs(y = "Population", x = "Year")
```



ETS(A,Ad,N): Damped trend method

Additive errors

$$
\begin{aligned}
\text{Forecast equation} \quad & \hat{y}_{T+h|T} = \ell_T + (\phi + \cdots + \phi^{h-1})b_T \\
\text{Measurement equation} \quad & y_t = (\ell_{t-1} + \phi b_{t-1}) + \varepsilon_t \\
\text{State equations} \quad & \ell_t = (\ell_{t-1} + \phi b_{t-1}) + \alpha\varepsilon_t \\
& b_t = \phi b_{t-1} + \beta\varepsilon_t
\end{aligned}
$$

ETS(A,Ad,N): Damped trend method

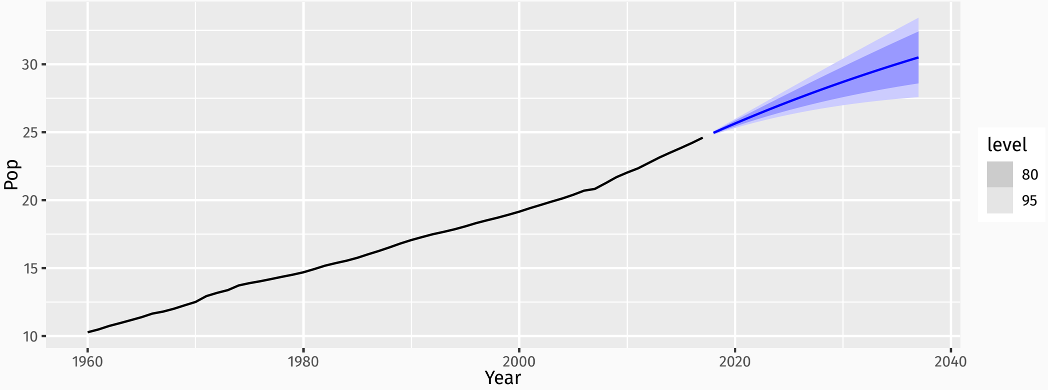
Additive errors

$$
\begin{aligned}
\text{Forecast equation} \quad & \hat{y}_{T+h|T} = \ell_T + (\phi + \cdots + \phi^{h-1})b_T \\
\text{Measurement equation} \quad & y_t = (\ell_{t-1} + \phi b_{t-1}) + \varepsilon_t \\
\text{State equations} \quad & \ell_t = (\ell_{t-1} + \phi b_{t-1}) + \alpha\varepsilon_t \\
& b_t = \phi b_{t-1} + \beta\varepsilon_t
\end{aligned}
$$

- Damping parameter $0 < \phi < 1$.
- If $\phi = 1$, identical to Holt's linear trend.
- As $h \to \infty$, $\hat{y}_{T+h|T} \to \ell_T + \phi b_T/(1-\phi)$.
- Short-run forecasts trended, long-run forecasts constant.

Example: Australian population

```
aus_economy |>
  model(holt = ETS(Pop ~ trend("Ad"))) |>
  forecast(h = 20) |>
  autoplot(aus_economy)
```



Example: National populations

```
fit <- global_economy |>
  mutate(Pop = Population / 1e6) |>
  model(ets = ETS(Pop))
fit
```

```
# A mable: 263 x 2
# Key:     Country [263]
   Country                      ets
   <fct>                    <model>
 1 Afghanistan         <ETS(A,A,N)>
 2 Albania             <ETS(M,A,N)>
 3 Algeria             <ETS(M,A,N)>
 4 American Samoa      <ETS(M,A,N)>
 5 Andorra             <ETS(M,A,N)>
 6 Angola              <ETS(M,A,N)>
 7 Antigua and Barbuda <ETS(M,A,N)>
 8 Arab World          <ETS(M,A,N)>
 9 Argentina           <ETS(A,A,N)>
10 Armenia             <ETS(M,A,N)>
# i 253 more rows
```

Example: National populations

```
fit |>
  forecast(h = 5)
```

```
# A fable: 1,315 x 5 [1Y]
# Key:     Country, .model [263]
   Country     .model  Year             Pop .mean
   <fct>       <chr>  <dbl>          <dist> <dbl>
 1 Afghanistan ets     2018    N(36, 0.012) 36.4
 2 Afghanistan ets     2019    N(37, 0.059) 37.3
 3 Afghanistan ets     2020     N(38, 0.16) 38.2
 4 Afghanistan ets     2021     N(39, 0.35) 39.0
 5 Afghanistan ets     2022     N(40, 0.64) 39.9
 6 Albania     ets     2018 N(2.9, 0.00012)  2.87
 7 Albania     ets     2019   N(2.9, 6e-04)  2.87
 8 Albania     ets     2020  N(2.9, 0.0017)  2.87
 9 Albania     ets     2021  N(2.9, 0.0036)  2.86
10 Albania     ets     2022  N(2.9, 0.0066)  2.86
# i 1,305 more rows
```

Outline

Lab Session 14

Try forecasting the Chinese GDP from the `global_economy` data set using an ETS model.

Experiment with the various options in the `ETS()` function to see how much the forecasts change with damped trend, or with a Box-Cox transformation. Try to develop an intuition of what each is doing to the forecasts.

[Hint: use `h=20` when forecasting, so you can clearly see the differences between the various options when plotting the forecasts.]

Outline

ETS(A,A,A): Holt-Winters additive method

Forecast equation	$\hat{y}_{t+h\|t} = \ell_t + hb_t + s_{t+h-m(k+1)}$
Observation equation	$y_t = \ell_{t-1} + b_{t-1} + s_{t-m} + \varepsilon_t$
State equations	$\ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t$
	$b_t = b_{t-1} + \beta\varepsilon_t$
	$s_t = s_{t-m} + \gamma\varepsilon_t$

- k = integer part of $(h-1)/m$.
- $\sum_i s_i \approx 0$.
- Parameters: $0 \le \alpha \le 1$, $0 \le \beta^* \le 1$, $0 \le \gamma \le 1-\alpha$ and m = period of seasonality (e.g. $m = 4$ for quarterly data).

ETS(M,A,M): Holt-Winters multiplicative method

$$
\begin{aligned}
\text{Forecast equation} \qquad \hat{y}_{t+h|t} &= (\ell_t + hb_t)s_{t+h-m(k+1)} \\
\text{Observation equation} \qquad y_t &= (\ell_{t-1} + b_{t-1})s_{t-m}(1 + \varepsilon_t) \\
\text{State equations} \qquad \ell_t &= (\ell_{t-1} + b_{t-1})(1 + \alpha\varepsilon_t) \\
b_t &= b_{t-1}(1 + \beta\varepsilon_t) \\
s_t &= s_{t-m}(1 + \gamma\varepsilon_t)
\end{aligned}
$$

- k is integer part of $(h-1)/m$.
- $\sum_i s_i \approx m$.
- Parameters: $0 \le \alpha \le 1$, $0 \le \beta^* \le 1$, $0 \le \gamma \le 1-\alpha$ and m = period of seasonality (e.g. $m = 4$ for quarterly data).

Example: Australian holiday tourism

```r
holidays <- tourism |>
  filter(Purpose == "Holiday")
fit <- holidays |> model(ets = ETS(Trips))
fit
```

```
# A mable: 76 x 4
# Key:     Region, State, Purpose [76]
   Region          State Purpose           ets
   <chr>           <chr> <chr>         <model>
 1 Adelaide        SA    Holiday <ETS(A,N,A)>
 2 Adelaide Hills  SA    Holiday <ETS(A,A,N)>
 3 Alice Springs   NT    Holiday <ETS(M,N,A)>
 4 Ballarat        VIC   Holiday <ETS(M,N,A)>
 5 Barkly          NT    Holiday <ETS(A,N,A)>
 6 Barossa         SA    Holiday <ETS(A,N,N)>
 7 Bendigo Loddon  VIC   Holiday <ETS(M,N,N)>
 8 Blue Mountains  NSW   Holiday <ETS(M,N,M)>
 9 Brisbane        QLD   Holiday <ETS(A,A,N)>
10 Bundaberg       QLD   Holiday <ETS(A,N,A)>
```

Example: Australian holiday tourism

```
fit |>
  filter(Region == "Snowy Mountains") |>
  report()
```

```
Series: Trips
Model: ETS(M,N,A)
  Smoothing parameters:
    alpha = 0.157
    gamma = 1e-04

  Initial states:
 l[0] s[0] s[-1] s[-2] s[-3]
  142  -61   131 -42.2 -27.7

  sigma^2:  0.0388

 AIC AICc  BIC
 852  854  869
```

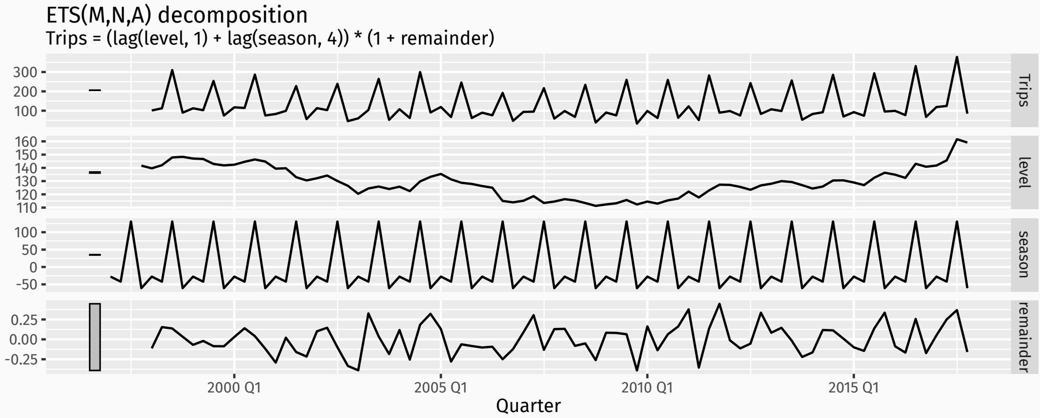
Example: Australian holiday tourism

```r
fit |>
  filter(Region == "Snowy Mountains") |>
  components(fit)
```

```
# A dable: 84 x 9 [1Q]
# Key:     Region, State, Purpose, .model [1]
# :        Trips = (lag(level, 1) + lag(season, 4)) * (1 + remainder)
   Region          State Purpose .model Quarter Trips level season remainder
   <chr>           <chr> <chr>   <chr>    <qtr> <dbl> <dbl>  <dbl>     <dbl>
 1 Snowy Mountains NSW   Holiday ets    1997 Q1  NA      NA  -27.7   NA
 2 Snowy Mountains NSW   Holiday ets    1997 Q2  NA      NA  -42.2   NA
 3 Snowy Mountains NSW   Holiday ets    1997 Q3  NA      NA  131.    NA
 4 Snowy Mountains NSW   Holiday ets    1997 Q4  NA     142. -61.0   NA
 5 Snowy Mountains NSW   Holiday ets    1998 Q1 101.    140. -27.7   -0.113
 6 Snowy Mountains NSW   Holiday ets    1998 Q2 112.    142. -42.2    0.154
 7 Snowy Mountains NSW   Holiday ets    1998 Q3 310.    148. 131.     0.137
 8 Snowy Mountains NSW   Holiday ets    1998 Q4  89.8  148. -61.0    0.0335
 9 Snowy Mountains NSW   Holiday ets    1999 Q1 112.    147. -27.7   -0.0687
10 Snowy Mountains NSW   Holiday ets    1999 Q2 103.    147. -42.2   -0.0199
```

Example: Australian holiday tourism

```r
fit |>
  filter(Region == "Snowy Mountains") |>
  components(fit) |>
  autoplot()
```



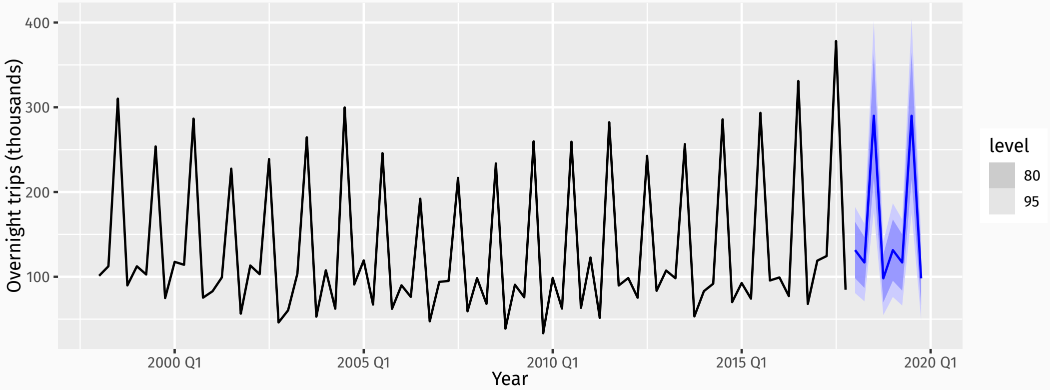
Example: Australian holiday tourism

```r
fit |> forecast()
```

```
# A fable: 608 x 7 [1Q]
# Key:     Region, State, Purpose, .model [76]
   Region          State Purpose .model Quarter        Trips .mean
   <chr>           <chr> <chr>   <chr>    <qtr>       <dist> <dbl>
 1 Adelaide        SA    Holiday ets    2018 Q1 N(210, 457) 210.
 2 Adelaide        SA    Holiday ets    2018 Q2 N(173, 473) 173.
 3 Adelaide        SA    Holiday ets    2018 Q3 N(169, 489) 169.
 4 Adelaide        SA    Holiday ets    2018 Q4 N(186, 505) 186.
 5 Adelaide        SA    Holiday ets    2019 Q1 N(210, 521) 210.
 6 Adelaide        SA    Holiday ets    2019 Q2 N(173, 537) 173.
 7 Adelaide        SA    Holiday ets    2019 Q3 N(169, 553) 169.
 8 Adelaide        SA    Holiday ets    2019 Q4 N(186, 569) 186.
 9 Adelaide Hills  SA    Holiday ets    2018 Q1   N(19, 36)  19.4
10 Adelaide Hills  SA    Holiday ets    2018 Q2   N(20, 36)  19.6
# i 598 more rows
```

Example: Australian holiday tourism

```r
fit |>
  forecast() |>
  filter(Region == "Snowy Mountains") |>
  autoplot(holidays) +
  labs(x = "Year", y = "Overnight trips (thousands)")
```



Outline

Exponential smoothing models

Additive Error

Trend Component		Seasonal Component: N (None)	A (Additive)	M (Multiplicative)
N	(None)	A,N,N	A,N,A	~~A,N,M~~
A	(Additive)	A,A,N	A,A,A	~~A,A,M~~
A_d	(Additive damped)	A,A_d,N	A,A_d,A	~~A,A_d,M~~

Multiplicative Error

Trend Component		Seasonal Component: N (None)	A (Additive)	M (Multiplicative)
N	(None)	M,N,N	M,N,A	M,N,M
A	(Additive)	M,A,N	M,A,A	M,A,M
A_d	(Additive damped)	M,A_d,N	M,A_d,A	M,A_d,M

Estimating ETS models

- Smoothing parameters α, β, γ and ϕ, and the initial states ℓ_0, b_0, $s_0, s_{-1}, \dots, s_{-m+1}$ are estimated by maximising the "likelihood" = the probability of the data arising from the specified model.
- For models with additive errors equivalent to minimising SSE.
- For models with multiplicative errors, **not** equivalent to minimising SSE.

Model selection

Akaike's Information Criterion

$$\mathrm{AIC} = -2\log(L) + 2k$$

where L is the likelihood and k is the number of parameters & initial states estimated in the model.

Model selection

Akaike's Information Criterion

$$\text{AIC} = -2\log(\text{L}) + 2k$$

where L is the likelihood and k is the number of parameters & initial states estimated in the model.

Corrected AIC

$$\text{AIC}_\text{c} = \text{AIC} + \frac{2k(k+1)}{T-k-1}$$

which is the AIC corrected (for small sample bias).

Model selection

Akaike's Information Criterion

$$\text{AIC} = -2\log(\text{L}) + 2k$$

where L is the likelihood and k is the number of parameters & initial states estimated in the model.

Corrected AIC

$$\text{AIC}_{\text{c}} = \text{AIC} + \frac{2k(k+1)}{T-k-1}$$

which is the AIC corrected (for small sample bias).

Bayesian Information Criterion

$$\text{BIC} = \text{AIC} + k(\log(T) - 2).$$

AIC and cross-validation

Minimizing the AIC assuming Gaussian residuals is asymptotically equivalent to minimizing one-step time series cross validation MSE.

Automatic forecasting

From Hyndman et al. (IJF, 2002):

1. Apply each model that is appropriate to the data. Optimize parameters and initial values using MLE.
2. Select best method using AICc.
3. Produce forecasts using best method.
4. Obtain forecast intervals using underlying state space model.

- Method performed very well in M3 competition.
- Used as a benchmark in the M4 competition.

Outline

Lab Session 15

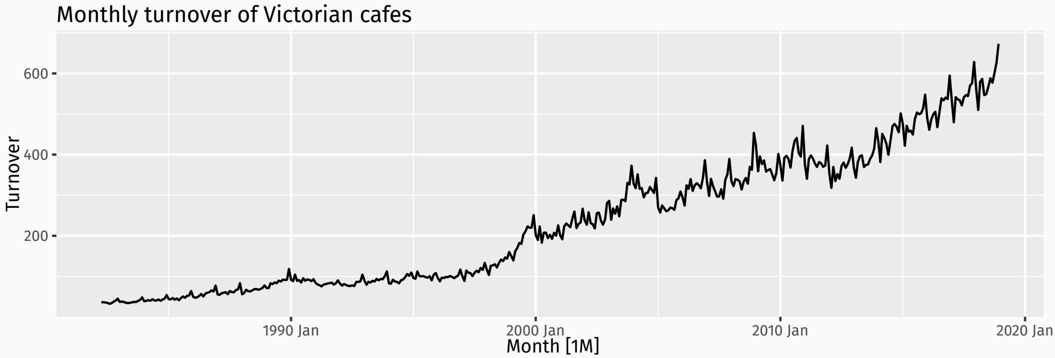
Find an ETS model for the Gas data from `aus_production`.

- Why is multiplicative seasonality necessary here?
- Experiment with making the trend damped. Does it improve the forecasts?

Outline

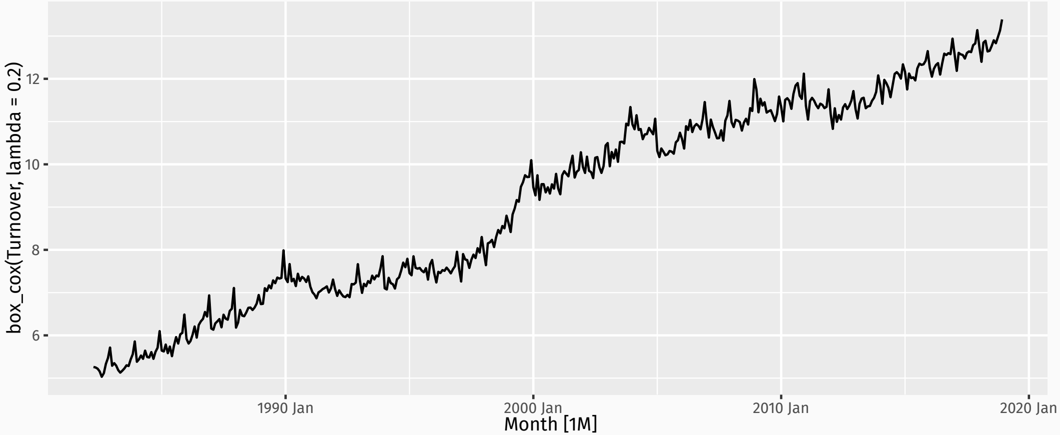
Non-Gaussian forecast distributions

```r
vic_cafe <- tsibbledata::aus_retail |>
  filter(State == "Victoria",
         Industry == "Cafes, restaurants and catering services") |>
  select(Month, Turnover)
vic_cafe |>
  autoplot(Turnover) + labs(title = "Monthly turnover of Victorian cafes")
```



Forecasting with transformations

```
vic_cafe |> autoplot(box_cox(Turnover, lambda = 0.2))
```



Forecasting with transformations

```
fit <- vic_cafe |>
  model(ets = ETS(box_cox(Turnover, 0.2)))
fit
```

```
# A mable: 1 x 1
           ets
       <model>
1 <ETS(A,A,A)>
```

```
(fc <- fit |> forecast(h = "3 years"))
```

```
# A fable: 36 x 4 [1M]
# Key:     .model [1]
   .model    Month        Turnover .mean
   <chr>     <mth>          <dist> <dbl>
 1 ets    2019 Jan  t(N(13, 0.02))  608.
 2 ets    2019 Feb t(N(13, 0.028))  563.
 3 ets    2019 Mar t(N(13, 0.036))  629.
 4 ets    2019 Apr t(N(13, 0.044))  615.
 5 ets    2019 May t(N(13, 0.052))  613.
 6 ets    2019 Jun t(N(13, 0.061))  593.
```

Forecasting with transformations

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fit <- vic_cafe |>
  model(ets = ETS(box_cox(Turnover, 0.2)))
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(fc <- fit |> forecast(h = "3 years"))
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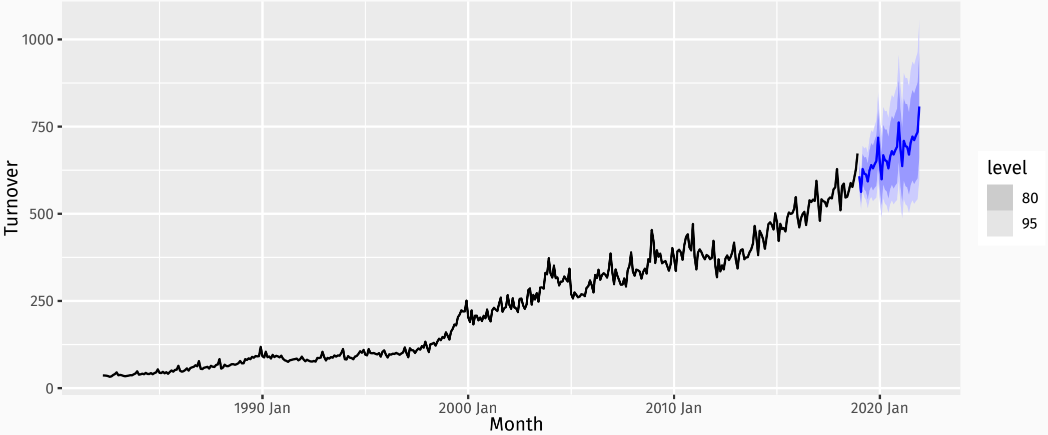
```
# A fable: 36 x 4 [1M]
# Key:     .model [1]
   .model    Month        Turnover .mean
   <chr>     <mth>          <dist> <dbl>
 1 ets    2019 Jan  t(N(13, 0.02))  608.
 2 ets    2019 Feb t(N(13, 0.028))  563.
 3 ets    2019 Mar t(N(13, 0.036))  629.
 4 ets    2019 Apr t(N(13, 0.044))  615.
 5 ets    2019 May t(N(13, 0.052))  613.
 6 ets    2019 Jun t(N(13, 0.061))  593.
```

- `t(N)` denotes a transformed normal distribution.
- back-transformation and bias adjustment is done automatically.

Forecasting with transformations

```r
fc |> autoplot(vic_cafe)
```



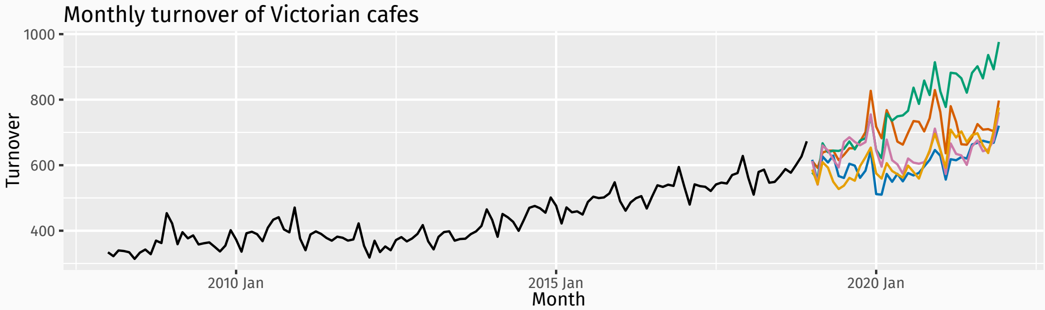
Bootstrapped forecast distributions

```r
sim <- fit |> generate(h = "3 years", times = 5, bootstrap = TRUE)
sim
```

```
# A tsibble: 180 x 5 [1M]
# Key:       .model, .rep [5]
   .model .rep     Month  .innov  .sim
   <chr>  <chr>    <mth>   <dbl> <dbl>
 1 ets    1     2019 Jan  0.0320  613.
 2 ets    1     2019 Feb  0.163   592.
 3 ets    1     2019 Mar -0.0616  639.
 4 ets    1     2019 Apr  0.0903  644.
 5 ets    1     2019 May  0.0488  645.
 6 ets    1     2019 Jun -0.0307  615.
 7 ets    1     2019 Jul -0.101   631.
 8 ets    1     2019 Aug -0.0125  652.
 9 ets    1     2019 Sep  0.0469  651.
10 ets    1     2019 Oct  0.0657  672.
# i 170 more rows
```

Bootstrapped forecast distributions

```
vic_cafe |>
  filter(year(Month) >= 2008) |>
  ggplot(aes(x = Month)) +
  geom_line(aes(y = Turnover)) +
  geom_line(aes(y = .sim, colour = as.factor(.rep)), data = sim) +
  labs(title = "Monthly turnover of Victorian cafes") +
  guides(col = FALSE)
```



Bootstrapped forecast distributions

```
fc <- fit |> forecast(h = "3 years", bootstrap = TRUE)
fc
```

```
# A fable: 36 x 4 [1M]
# Key:     .model [1]
   .model    Month      Turnover .mean
   <chr>     <mth>        <dist> <dbl>
 1 ets    2019 Jan sample[5000]   608.
 2 ets    2019 Feb sample[5000]   563.
 3 ets    2019 Mar sample[5000]   629.
 4 ets    2019 Apr sample[5000]   615.
 5 ets    2019 May sample[5000]   613.
 6 ets    2019 Jun sample[5000]   593.
 7 ets    2019 Jul sample[5000]   624.
 8 ets    2019 Aug sample[5000]   640.
 9 ets    2019 Sep sample[5000]   630.
10 ets    2019 Oct sample[5000]   642.
# i 26 more rows
```

Bootstrapped forecast distributions

```r
fc |> autoplot(vic_cafe) +
  labs(title = "Monthly turnover of Victorian cafes")
```