

## ✓ ¡Felicitaciones! ¡Aprobaste!

Calificación recibida 100 % Para Aprobar 80 % o más

Ir al siguiente elemento

### Problem Set #2

Calificación de la entrega más reciente: 100 %



1. Consider a directed graph with distinct and nonnegative edge lengths and a source vertex  $s$ . Fix a destination vertex  $t$ , and assume that the graph contains at least one  $s$ - $t$  path. Which of the following statements are true? [Check all that apply.]

1 / 1 punto

- ☐ The shortest  $s$ - $t$  path must include the minimum-length edge of  $G$ .
- ☐ The shortest  $s$ - $t$  path must exclude the maximum-length edge of  $G$ .
- ☒ The shortest (i.e., minimum-length)  $s$ - $t$  path might have as many as  $n - 1$  edges, where  $n$  is the number of vertices.

✓ Correcto

- ☒ There is a shortest  $s$ - $t$  path with no repeated vertices (i.e., a "simple" or "loopless" such path).

✓ Correcto



2. Consider a directed graph  $G$  with a source vertex  $s$ , a destination  $t$ , and nonnegative edge lengths. Under what conditions is the shortest  $s$ - $t$  path guaranteed to be unique?

1 / 1 punto

- ☐ None of the other options are correct.
- ☐ When all edges lengths are distinct positive integers and the graph  $G$  contains no directed cycles.
- ☒ When all edge lengths are distinct powers of 2.
- ☐ When all edge lengths are distinct positive integers.

✓ Correcto

Two sums of distinct powers of two cannot be the same (imagine the numbers are written in binary).

3. Consider a directed graph  $G = (V, E)$  and a source vertex  $s$  with the following properties: edges that leave the source vertex  $s$  have arbitrary (possibly negative) lengths; all other edge lengths are nonnegative; and there are no edges from any other vertex to the source  $s$ . Does Dijkstra's shortest-path algorithm correctly compute shortest-path distances (from  $s$ ) in this graph?



1 / 1 punto

- ☒ Always
- ☐ Only if we add the assumption that  $G$  contains no directed cycles with negative total weight.
- ☐ Never
- ☐ Maybe, maybe not (depends on the graph)

✓ Correcto

One approach is to see that the proof of correctness from the videos still works. A slicker solution is to notice that adding a positive constant  $M$  to all edges incident to  $s$  increases the length of every  $s$ - $v$  path by exactly  $M$ , and thus preserves the shortest path.

4. Consider a directed graph  $G$  and a source vertex  $s$ . Suppose  $G$  has some negative edge lengths but no negative cycles, meaning  $G$  does not have a directed cycle in which the sum of the edge lengths is negative. Suppose you run Dijkstra's algorithm on  $G$  (with source  $s$ ). Which of the following statements are true? [Check all that apply.]



1 / 1 punto

- ☐ Dijkstra's algorithm might loop forever.
- ☒ Dijkstra's algorithm always terminates, and in some cases the paths it computes will be the correct shortest paths from  $s$  to all other vertices.

✓ **Correcto**  
See Question 3.

- ☐ It's impossible to run Dijkstra's algorithm on a graph with negative edge lengths.
- ☒ Dijkstra's algorithm always terminates, but in some cases the paths it computes will not be the shortest paths from  $s$  to all other vertices.

✓ **Correcto**  
Nonnegativity of the edge lengths was used in the correctness proof for Dijkstra's algorithm; with negative edge lengths, the algorithm is no longer correct in general.



5. Consider a directed graph  $G$  and a source vertex  $s$ . Suppose  $G$  contains a negative cycle (a directed cycle in which the sum of the edge lengths is negative) and also a path from  $s$  to this cycle. Suppose you run Dijkstra's algorithm on  $G$  (with source  $s$ ). Which of the following statements are true? [Check all that apply.]



1 / 1 punto

- ☐ Dijkstra's algorithm might loop forever.
- ☒ Dijkstra's algorithm always terminates, but in some cases the paths it computes will not be the shortest paths from  $s$  to all other vertices.

✓ **Correcto**

- ☐ It's impossible to run Dijkstra's algorithm on a graph with a negative cycle.
- ☐ Dijkstra's algorithm always terminates, and in some cases the paths it computes will be the correct shortest paths from  $s$  to all other vertices.