6

Driveline Dynamics

6.1 Introduction

The vehicle driveline is a dynamic system consisting of inertia and elastic components and when it is subjected to excitations, mechanical resonances occur. These span a large range of frequencies and are generally referred to as noise, vibration and harshness (NVH). Vehicle drivelines with manual transmissions are lightly damped and tend to be more prone to vibration. Driveline torsional frequencies are in general excited by changes in torque which arise from engine or driver inputs.

There are many sources of driveline excitation and in general the disturbing inputs can be categorized into two types: sudden/discrete and persistent. The first category includes throttle inputs, tip-in/tip-out and sudden changes in the engine torque. The second type may arise from engine torsional/torque fluctuations or from worn or misaligned components in the driveline such as Hooke's joints. The driveline vibrations are sometimes categorized as engine speed, vehicle speed or acceleration related. Free play between the components or gear lash will often exacerbate driveline torsional oscillations.

The lowest frequency driveline oscillation (longitudinal vehicle oscillation) is called vehicle shunt. This phenomenon arises from high rate of pedal application to change the driveline torque instantly. The existence of a large gear backlash in the driveline will amplify this type of oscillation starting with an initial jerk. The driver can, therefore, distinguish the longitudinal oscillations and this reduces the driver's subjective perception of the vehicle's performance. Vehicle shunt is commonly followed by torsional vibrations of the entire powertrain, referred to as shuffle. These oscillations that usually are below 10 Hz correspond to the fundamental driveline resonance frequencies and are mainly caused by the compliances of drivetrain.

Throttle tip-in is defined as a fast depression of throttle and tip-out as a sudden release of throttle; these are two common excitations of vehicle driveline oscillations. They often occur on hill climbing or in downhill motion and are typically more problematic for vehicles with manual transmissions since, unlike automatic transmissions, they do not have viscous damping within the driveline. The vibration induced by the impacts of gear pairs is another mechanism of high frequency driveline vibration. In certain conditions, it is called gear rattle and is an impulsive phenomenon in which the unloaded gears knock each other and the result is an undesirable rattle noise.

The aim of this chapter is to analyse the driveline dynamics and understand the design issues relating to driveline NVH.

6.2 Modelling Driveline Dynamics

The transient response of vehicle driveline has a significant influence on the driver's subjective impression of the vehicle performance. It is very difficult to define objective parameters, which describe the driveline

Vehicle Powertrain Systems, First Edition. Behrooz Mashadi and David Crolla.

dynamics and which therefore strongly influence the driveability of vehicles. Hence, although the analysis of vehicle driveline dynamics is fairly straightforward in that it is similar to many other examples of engineering multibody systems, the interpretation of the results poses difficulties. In order to better understand the dynamic behaviour of the powertrain and reduce unwanted oscillations, analysis and modelling are required. The aim is that computer simulations should produce practical design information for the designers for the refinement and tuning of the driveline parameters.

6.2.1 Modelling Methods

Various methods and techniques are available for modelling physical systems. For mechanical systems, the direct application of Newton's laws of motion is the most common approach and leads to the governing equations of motion. The ability to draw accurate free body diagrams (FBD) is a crucial part of this traditional approach. On the other hand, Kane's approach to dynamics analysis [1] is based on system energies; it is advantageous in that it does not involve unnecessary unknown internal forces and moments, and the governing equations of motion are generated in the form of a set of first order differential equations ready for numerical solution.

Other modelling tools like the block diagram (BD) method or signal flow graphs (SFG) are graphical methods used in modelling dynamic systems. In fact, all such methods are derivations from the law of conservation of energy and can be applied to any physical systems. The graphical methods, BD and SFG, are based on the flow of signals in a dynamic system. An alternative approach is called the bond graph method, in which the flow of energy is taken into consideration.

In this approach, a physical system is represented by simple symbols that indicate the power flow paths or *bonds*. There are basic elements in bond graphs, like other modelling methods, that are connected to one another and a network structure results. Application of the basic laws to the bond graph construction generates the governing equations of the system. Like Kane dynamics, the bond graph approach also generates the equations of motion in the form of first order differential equations that can be readily solved. One main advantage of bond graph modelling is its ability to model multiple energy systems such as electromechanical systems.

In this chapter, the modelling of a driveline system is carried out by the application of the bond graph technique. For those not familiar with the subject, a brief introduction is provided for the bond graph technique in Appendix A for quick reference. This will suffice for the simple models that are developed here. Interested readers should refer to specialist books or materials for more information [2–4].

Example 6.2.1

For a torsional vibrating system consisting of a rotating inertia J, a torsional spring K_T and a torsional damper B_T shown in Figure 6.1, derive the equations of motion:

(a) Apply Newton's law.
(b) Use the bond graph method.
(c) Compare the results.

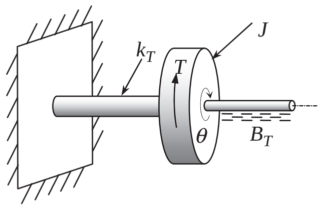

Figure 6.1 A torsional vibrating system

Solution

This system is a torsional equivalent of the well-known mass-spring damper system.

(a) For the displacement θ of the inertia J, the governing equation of motion according to the FBD of Figure 6.2 is obtained by combining the individual equations:

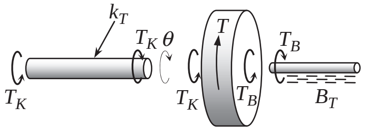

Figure 6.2 The free body diagram of a torsional vibrating system

$$J\ddot{\theta} = T(t) - T_K - T_B, T_K = K_T\theta, T_B = B_T\dot{\theta}$$

The result is:

$$J\ddot{\theta} + B_T\dot{\theta} + K_T\theta = T(t)$$

(b) The details of applying the bond graph technique to this system are available in Appendix A (Example A.4.3). The result is a set of equations given below:

$$\frac{dp_1}{dt} = S_e - k_2 q_2 - R_3 \frac{p_1}{I_1}$$

$$\frac{dq_2}{dt} = \frac{p_1}{I_1}$$

in which:

$$S_e = T,\ I_1 = J,\ k_2 = k_T,\ R_3 = B_T,\ p_1 = J\dot{\theta} \text{ and } q_2 = \theta$$

(c) The equations derived from bond graph are already in state-space form and suitable for solution using several techniques. The equation resulting from the direct application of Newton's law is a second order differential equation. This can be transformed to two first order equations by using the definitions: $x_1 = J\theta$ and $x_2 = J\dot{\theta}$. The result is:

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = T - \frac{k_T}{J} x_1 - \frac{B_T}{J} x_2$$

Substituting $p_1 = x_2$ and $q_2 = x_1/J$, the two sets become identical.

6.2.2 Linear Versus Non-linear Models

In reality, all physical systems are non-linear systems to different degrees and obtaining their accurate responses to input excitations over a wide range requires the development of non-linear models. The vehicle driveline is a complex dynamical system comprising many non-linear sub-systems and elements. As a result, the governing equations of motion of the vehicle driveline, in general, are highly non-linear and comprise a set of non-linear differential equations. Closed form solutions are in general not available for such equations and only numerical integration techniques are workable. Results obtainable from non-linear models are mainly restricted to time history solutions from which it is often difficult to provide useful design information. Non-linearities in the driveline dynamics include:

- non-linearities of component behaviour, for example, stiffness (spring), non-linearity of shafts and dry friction between two rotating elements;
- free plays and backlash;
- non-linearities of engine torque variation with throttle inputs and speed.

Linear analysis offers the analyst a wide range of outputs which potentially provide a design insight into vehicle dynamics problems. Much of the knowledge relating to the understanding of dynamic behaviour of systems is based on linear properties that allow powerful linear analysis tools such as eigenvalue extraction, frequency response analysis, etc. to be applied to the system model. When large deformations, speeds or accelerations are involved, it becomes more likely that non-linearities are significant and the application of linear tools is no longer valid. Nevertheless, situations exist in which the behaviour of the non-linear elements can be regarded as linear. For instance, if the relative displacements of the subsystems remain small, the mechanical characteristics can be rather well described by linear functions. Therefore, in such conditions it is possible to define linear models for which linear equations of motion can be derived. The general advantages of treating the system as a linear system are listed below:

- Linear models are easy to work with (i.e., equations of motion can easily be derived and solved).
- Closed form solutions for the equations of motion are possible.
- Linear analytical tools such as frequency response, eigenvalue solution, steady-state/transient analyses, and linear control theory can be employed for the interpretation of system behaviour.

The general assumptions involved in the linear description of driveline system models may be itemized as follows:

- The behaviour of system components is linear.
- The inputs or excitations are small.
- All resulting output displacements, speeds and accelerations are small.

In many practical working conditions these assumptions are true and thus the linear models are reliable.

The modelling techniques for both linear and non-linear systems are similar but working with non-linear models is difficult. Linear models, in contrast, are easy to deal with and the designer is able to apply a wide range of linear analysis tools to extract design information from them. In this chapter the modelling and discussions will be limited only to linear models as dealing with non-linear behaviour of components is beyond the scope of this book.

6.2.3 Software Use

There are now several multi-body systems dynamics packages available with which the equations of motion of vehicle driveline models for any complexity of problem can be generated and solved. These codes usually perform static solutions, kinematic and dynamic analyses, and the outputs are time histories and/or animations.

For vehicle driveline modelling, software packages include: Adams/Driveline [5], AMESim [6], Dymola [7], GT-Suite [8], Modelica [9], SimDrive [10] and VALDYN [11]. These packages offer standard libraries for typical elements in the vehicle driveline and the user can employ their graphical interfaces to develop the driveline model by putting together the elements of a driveline system.

Although the usefulness and the power of these packages have been well proven, nevertheless the appropriateness of using them in students' learning phase is debatable. Two main disadvantages are, first, they need a considerable level of expertise, as the development of the model and extraction of results can often be time-consuming, and second, the user is unaware of the relations between the system components as no parametric relations are presented. The main advantage of purpose-built driveline simulations, on the other hand, is that they are easier to use and allow quick studies to be undertaken. Students on the driveline-related courses can learn more through building their own models and obtaining their solutions. The more sophisticated packages can be used for commercial design purposes and final tuning of driveline parameters.

6.3 Bond Graph Models of Driveline Components

A *word bond graph* is a simple representation of a system by using the bond graph *arrows* to demonstrate the relations of the system components. For the vehicle driveline system, such a representation is illustrated in Figure 6.3 for a typical FWD vehicle. This word bond graph shows at a glance the transfer of the energy through the components of a driveline system. If the bond graph of each subsystem or component of the driveline is replaced in the word bond graph, the actual bond graph of the system obtained. Thus, this simple representation of the system can be used as the starting point for preparation of the complete bond graph of the driveline.

6.3.1 The Engine

The engine excitations resulting from the torque pulses caused during the power strokes are sources of driveline oscillations. The engine is a complicated system with a large number of mechanisms and elements so that constructing a full bond graph for the engine, therefore, is a complicated task.

A simplified model of an engine, however, can be considered by assuming it to be a source of mechanical power with quasi-steady characteristics. In this form, the internal dynamics of the engine are

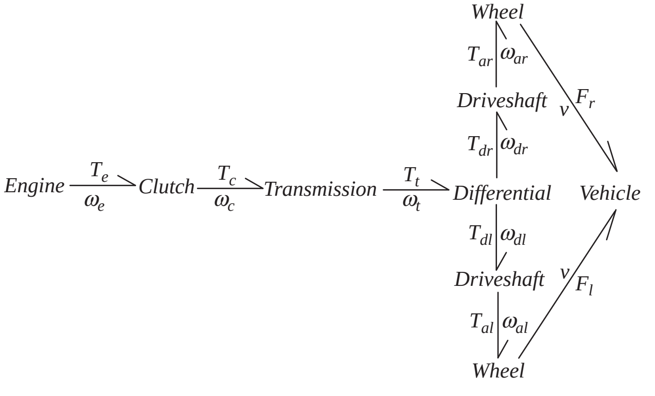

Figure 6.3 A word bond graph for vehicle driveline

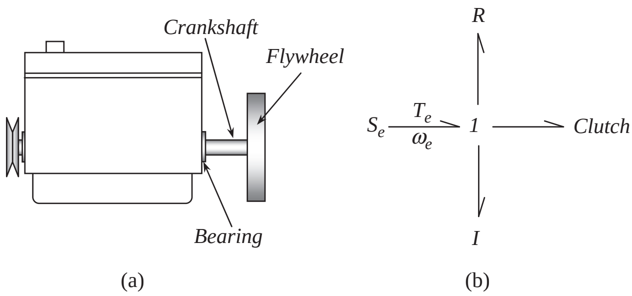

Figure 6.4 Schematic model (a) and simple bond graph of engine (b)

not taken into consideration and only its steady torque-speed characteristic is given as input to the driveline system. This simplified engine model consists of a crankshaft mounted on a bearing and connected to a flywheel (see Figure 6.4a). The total inertia connected to the engine includes the flywheel, crankshaft and the clutch assembly that rotate with the engine speed. Bearing friction can also be included in the model. The bond graph of such a simple engine is depicted in Figure 6.4b, inertia by the *I* element and the bearing friction by the *R* element.

It was assumed that the crankshaft was a rigid element; if an elasticity is to be included for the crankshaft between the bearing and the flywheel (i.e. the shaft can twist between these two points), the bond graph model can be modified to that shown in Figure 6.5 with the *C* element as the elasticity of the crankshaft.

6.3.2 The Clutch

The clutch plays an important role in transmission torsional vibrations. It typically consists of dry friction damping and torsional spring effects. The clutch works in two phases: fully engaged and transient phases (see Chapter 4). In the fully engaged phase, the dry friction does not have any influence except in extreme cases with large torques above the capacity of clutch so that slip occurs. In the engaged case the circumferential (torsional) clutch spring stiffness (see Figure 6.6) is influential in driveline vibrations.

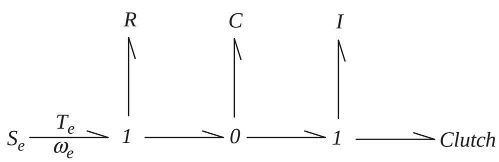

Figure 6.5 Engine bond graph with elasticity

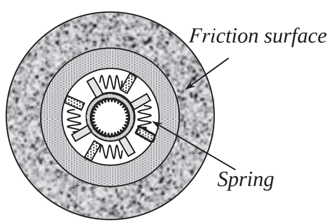

Figure 6.6 Schematic of a clutch plate and its springs

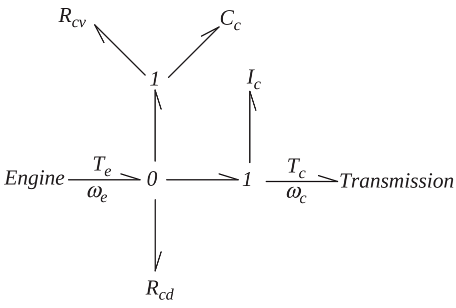

Figure 6.7 Bond graph of the clutch

The transient release phase, in which the power flow from the engine to the transmission takes place gradually, involves both dry friction and spring stiffness effects.

In the bond graph of Figure 6.7, the *C* element is to account for the circumferential springs and the upper *R* element (R_{cv}) represents a viscous damping for the springs. The lower *R* element (R_{cd}) represents the damping of friction surface which should be removed for the fully engaged phase. Finally, the *I* element is to model an equivalent rotating inertia in the clutch output (including spline and attachments). In fact, the springs in reality have stiffness, inertia and damping all combined. For modelling purposes, these three properties are treated separately as ideal elements.

6.3.3 The Transmission

The transmission contains several elements including shafts, gears and bearings. Constructing a bond graph for a gearbox, therefore, requires a detailed analysis of the system components. It is, however, possible to consider the gearbox as a simple system with rigid elements having only inertia and friction. In this way, the inertia can be assumed to act only at the input or only at the output of the gearbox. Figure 6.8 clarifies this concept. In Figure 6.8a a simple gearbox including an input shaft, a layshaft and an output shaft is considered. The inertias of the three shafts (including the gears) are I_1, I_2 and I_3 respectively, and n_1 and n_2 are the gear ratios between the two mating gears. With the assumption of rigid shafts and gears, the two cases of Figure 6.8b and c are possible. The lumped inertias I_B and I_C are:

$$I_B = \frac{1}{n^2}\left(n^2 I_1 + n_2^2 I_2 + I_3\right) \tag{6.1}$$

$$I_C = n^2 I_B \tag{6.2}$$

In only one gear ratio, a transmission acts like a transformer. Since other gear ratios are also selectable, a *Modulated* transformer (MTF) is used to indicate the changing character of the gear ratio. The bond graphs

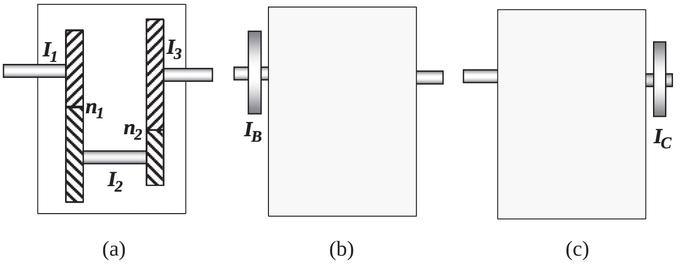

Figure 6.8 Lumped inertia models for a gearbox

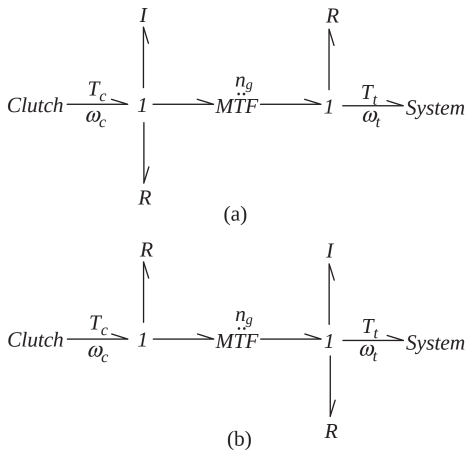

Figure 6.9 Bond graphs of gearbox with lumped inertia at (a) input shaft and (b) output shaft

for the simplified models of a gearbox are shown in Figure 6.9. The R elements are the bearings at the input and output shafts of the transmission and n_g is the transmission gear ratio.

6.3.4 Propeller and Drive Shafts

A propeller shaft (rear-wheel drive (RWD) layout only) or a driveshaft can be considered as an elastic element with lumped inertia and friction elements. The inertia and the friction elements can be included at one or two ends. In Figure 6.10, the inertia and the friction elements are considered to be present at both input and output ends. In order to include an internal damping in the elastic element, a common flow joint is used.

6.3.5 The Differential

A differential is a device to allow the two wheels on an axle to assume different speeds while the vehicle is turning. An open differential has the components shown in Figure 6.11. It has an input shaft with torque T_p and speed ω_p, and two output shafts with torques and speeds of T_L, T_R, ω_L and ω_R.

A differential including six gears, shafts and bearings, is a complicated system for modelling. A complete bond graph of the differential can be constructed by considering the details of the system.

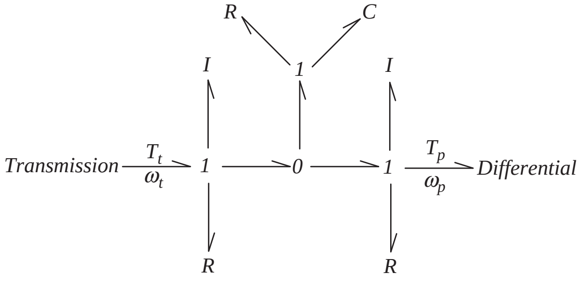

Figure 6.10 Bond graph of a propeller or drive shaft

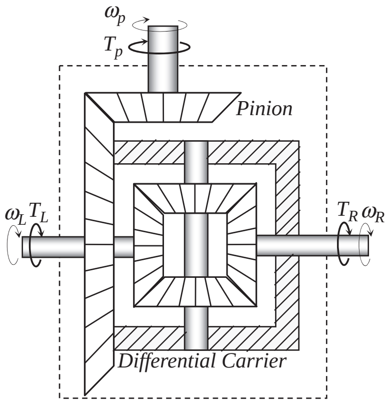

Figure 6.11 Main components of the vehicle differential

The resulting bond graph, however, will not be useful for driveline modelling. In order to use a simple differential model, the relations between input and outputs must be taken into consideration. In an open differential, the torques at output shafts are equal to half the input torque, while the speeds are different but related. Assuming no power loss in the differential, the output power must be equal to the input power:

$$P_L + P_R = P_i \tag{6.3}$$

In terms of the torques and speeds:

$$T_L\omega_L + T_R\omega_R = T_p\omega_p \tag{6.4}$$

Taking into account the torque relation $T_L = T_R = \frac{1}{2}T_p$ we obtain:

$$\omega_L + \omega_R = 2\omega_p \tag{6.5}$$

In fact, a differential provides half the input torque at each shaft but does not double the speeds, which makes it different from a transformer. This phenomenon is not one that can be modelled by any of the basic components available for bond graphs. Thus a new element must be defined for a differential which has the property of halving the effort but the flows must comply with Equation 6.5. Since this property resembles that of the transformer, the element will be called a Differential Transformer or DTF for short. The bond graph for a DTF is depicted in Figure 6.12.

The overall bond graph of a differential including the inertia properties and viscous friction in the bearings is depicted in Figure 6.13 by using a DTF element. Note that the TF element used at the input is

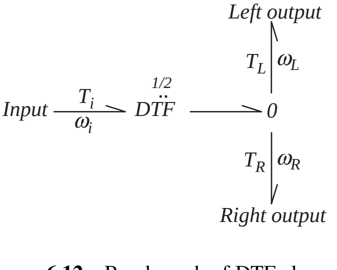

Figure 6.12 Bond graph of DTF element

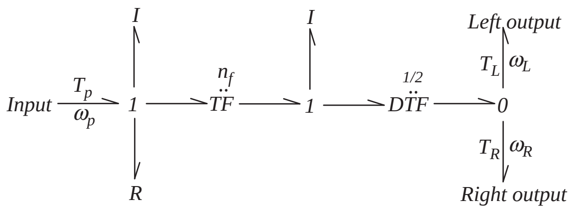

Figure 6.13 Bond graph of a differential

for the final drive ratio between the pinion and crownwheel gears. The first I element belongs to the input shaft whereas the second one is for the crownwheel and carrier. Inertias and frictions for output shafts can also be included in the bond graph.

6.3.6 The Wheel

The wheel component includes the rim, rigid rotating parts and the tyre. A full dynamic model of the wheel accounts for the tyre deformations, wheel slip and road interactions. These are complicated phenomena due to the complex structure of the tyre and force generation mechanism. A simple model that includes the rigid body wheel properties and tyre deformation is schematically shown in Figure 6.14. F_x, W and T_W are the tractive force, wheel load and wheel torque respectively. The wheel has effective radius r_W, mass m_W and inertia I_W. It is assumed that the deformation of the tyre takes place at the contact surface and since the tyre material has a visco-elastic property, a combined elasticity and damping are considered to account for its deformation and internal damping.

The bond graph of the wheel with properties explained above is produced and depicted in Figure 6.15. Note that the input energy first causes the rigid parts to turn, then transforms to a force at the road level

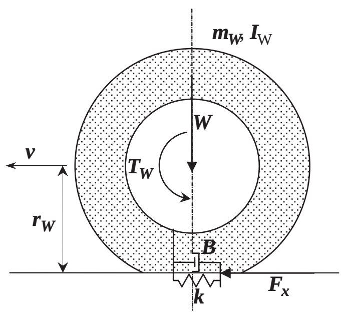

Figure 6.14 Model of wheel and tyre deformation

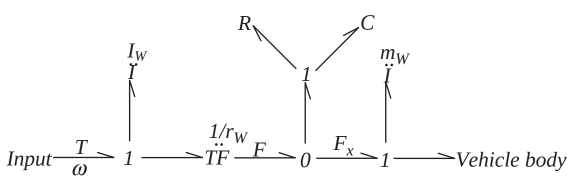

Figure 6.15 Bond graph of wheel and tyre

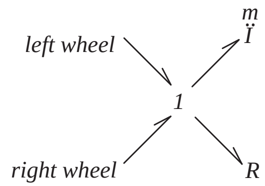

Figure 6.16 Bond graph of vehicle body

where it deforms the tyre. Then, the remaining energy results in the tractive force which moves the wheel mass and the remaining effort is passed to the vehicle body.

6.3.7 Vehicle

For a conventional two wheel drive (2WD) vehicle, the motive force results from those forces generated by the two tyres. The vehicle mass imposes an inertia force against the motive force together with the resistive forces acting in the same direction. The bond graph of the vehicle body, therefore, is very simple as illustrated in Figure 6.16.

6.4 Driveline Models

Once the bond graph models of different components are available, the full driveline bond graph can easily be constructed by putting together the bond graphs of the components. The resulting bond graph involves details of the vehicle driveline components and it is expected that it can provide an account of the accurate behaviour of the system. Nonetheless, working with such model could be difficult due to complicated equations and because the detailed information required is difficult to obtain in practice. On the other hand, not all of parameters have critical effects on the behaviour of the driveline and sometimes simple models can demonstrate the fundamental properties of the system. Therefore, in addition to the full driveline model, it is also intended to develop simplified driveline models with less complexity but at acceptable levels.

6.4.1 Full Driveline Model

In the process of developing a full driveline model, some simplifications can be performed by merging similar properties of two adjacent elements (e.g. their inertias). The result will look like the bond graph of Figure 6.17. In this model, the elasticity of engine crankshaft has been ignored.

6.4.2 Straight-Ahead Motion

The motion of a vehicle in the longitudinal direction is of major importance when the driveline vibrations are studied. Thus if it is assumed that the vehicle moves only straight ahead, then the differential plays no other role than to produce the final drive ratio and the two left and right wheels can be combined. The resulting bond graph for such motions will be of the form shown in Figure 6.18. In the following sections only straight-ahead motion will be considered.

6.4.3 Rigid Body Model

Assuming all the driveline components are rigid elements with no elasticity, then all the *C* elements in the bond graph vanish. The bond graph of Figure 6.19 results from removing the *0* nodes with *C* elements in the bond graph of Figure 6.18, and combining any two adjacent *1* nodes resulting from this process.

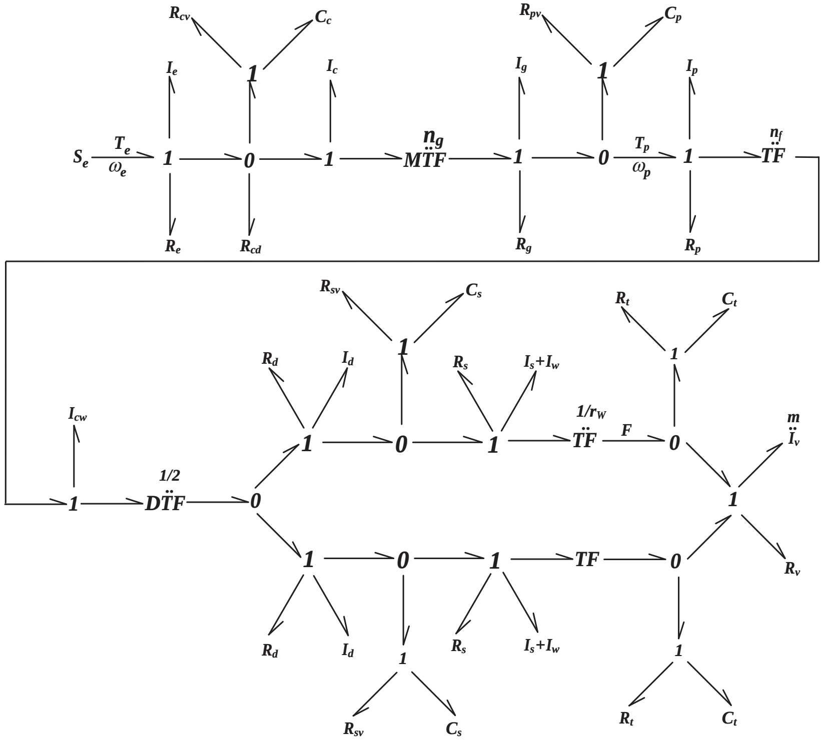

Figure 6.17 Bond graph of vehicle driveline system

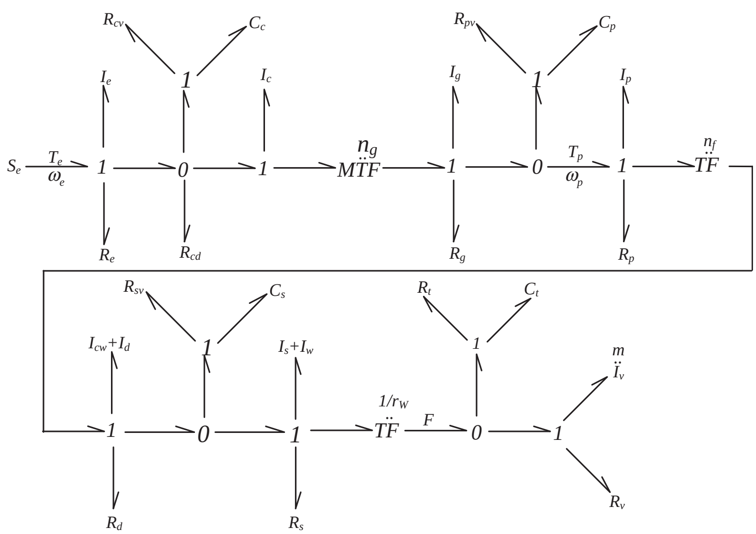

Figure 6.18 Bond graph of driveline system in straight-ahead motion

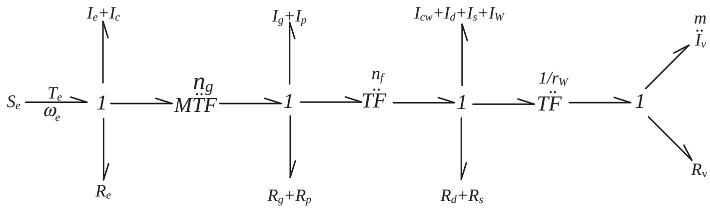

Figure 6.19 Bond graph of rigid body driveline

The bond graph of Figure 6.19 can be simplified by using the equivalent I (and R) rule for transformers (see Appendix, Section A.4.5). Note that two sequential transformers can be replaced by one transformer by simple multiplication of the modules of the two transformers. The resulting bond graph is shown in Figure 6.20. I_{eq} and R_{eq} in the bond graph are obtained from the following equations:

$$I_{eq} = I_v + \frac{1}{r_W^2}\left[I_{cw} + I_d + I_s + I_W + n_f^2(I_g + I_p) + n^2(I_e + I_c)\right] \tag{6.6}$$

$$R_{eq} = R_v + \frac{1}{r_W^2}\left[R_d + R_s + n_f^2(R_g + R_p) + n^2 R_e\right] \tag{6.7}$$

Equation 6.6 represents the effect of rotating inertia discussed in Section 3.9. From the simple bond graph of Figure 6.21 the equation of motion is:

$$\frac{dp_{eq}}{dt} = e_{eq} = \frac{n_g n_f}{r_W} T_e - R_{eq}\frac{p_{eq}}{I_{eq}} \tag{6.8}$$

Recalling that p_{eq} is vI_{eq}, Equation 6.8 can be written as:

$$I_{eq}\frac{dv}{dt} = \frac{n_g n_f}{r_W} T_e - R_{eq} v \tag{6.9}$$

which is in the form of $F_T - F_R = m_{eq}a$ of Equation 3.129 for vehicle longitudinal motion, including the effect of rotating masses.

6.4.4 Driveline with Clutch Compliance

If all the components are considered as rigid parts, but only clutch compliance is taken into consideration, the bond graph of driveline in a straight-ahead motion becomes that shown in Figure 6.21.

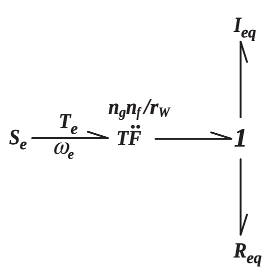

Figure 6.20 Simplified bond graph of rigid body driveline in straight-ahead motion

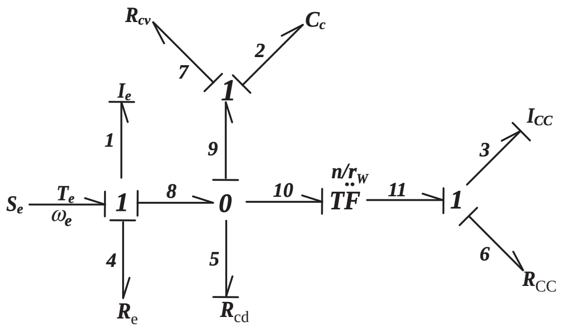

Figure 6.21 Bond graph of driveline with only clutch compliance

The transformers are combined and the two end elements I_{CC} and R_{CC} can be determined from the following equations with the same terminology as those of Figure 6.18:

$$I_{CC} = I_v + \frac{1}{r_W^2}\left[I_{cw} + I_d + I_s + I_w + n_f^2(I_g + I_p) + n^2 I_c\right] \tag{6.10}$$

$$R_{CC} = R_v + \frac{1}{r_W^2}\left[R_d + R_s + n_f^2(R_g + R_p)\right] \tag{6.11}$$

The equations of driveline motion according to the bond graph (with causal strokes inserted) of Figure 6.21 are as follows:

$$\frac{dp_1}{dt} = T_e - \frac{p_1}{I_1}R_e - T_1 \tag{6.12}$$

$$\frac{dq_2}{dt} = \frac{p_1}{I_1} - \frac{k_2}{R_{CV}}q_2 - \frac{n}{r_W}\frac{p_3}{I_3} \tag{6.13}$$

$$\frac{dp_3}{dt} = \frac{n}{r_W}T_1 - \frac{p_3}{I_3}R_{CC} \tag{6.14}$$

in which:

$$T_1 = k_2 q_2 + R_{CV}\dot{q}_2 \tag{6.15}$$

6.4.5 *Driveline with Driveshaft Compliance*

If rigid components are considered for all parts except the driveshafts which are taken as elastic elements, the overall bond graph of Figure 6.18 in straight-ahead motion will reduce to the bond graph of Figure 6.22.

$$I_{Eq} = I_e + I_c + \frac{1}{n^2}(I_{cw} + I_d) + \frac{1}{n_g^2}(I_g + I_p) \tag{6.16}$$

$$R_{Eq} = R_e + \frac{1}{n^2}R_d + \frac{1}{n_g^2}(R_g + R_p) \tag{6.17}$$

$$I_{DC} = I_v + \frac{1}{r_W^2}(I_s + I_w) \tag{6.18}$$

$$R_{DC} = R_v + \frac{1}{r_W^2}R_s \tag{6.19}$$

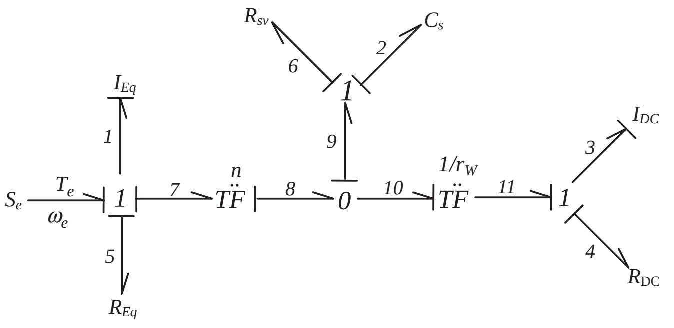

Figure 6.22 Bond graph of driveline with only driveshaft compliance

The equations of motion of the driveline according to Figure 6.22 are:

$$\frac{dp_1}{dt} = T_e - \frac{p_1}{I_1} R_{Eq} - T_1 \tag{6.20}$$

$$\frac{dq_2}{dt} = \frac{1}{n}\frac{p_1}{I_1} - \frac{1}{r_W}\frac{p_3}{I_3} \tag{6.21}$$

$$\frac{dp_3}{dt} = \frac{1}{r_W} T_1 - \frac{p_3}{I_3} R_{DC} \tag{6.22}$$

In which:

$$T_1 = \frac{1}{n}(k_2 q_2 + R_{Sv}\dot{q}_2) \tag{6.23}$$

6.4.6 *Driveline with Clutch and Driveshaft Compliances*

In the two preceding sections, the compliances of the clutch and driveshaft alone were considered to be unaccompanied by the other. In this section both compliances will be included in a single driveline model, as shown in Figure 6.23.

The two end elements I_{CDC} and R_{CDC} as well as two equivalent inertia and damping I_{deq} and R_{deq} can be determined from the following equations with the same terminology of Figure 6.18:

$$I_{CDC} = I_v + \frac{1}{r_W^2}(I_s + I_w) \tag{6.24}$$

$$R_{CDC} = R_v + \frac{1}{r_W^2} R_s \tag{6.25}$$

$$I_{deq} = I_d + I_{cw} + n_f^2(I_g + I_p) + n^2 I_c \tag{6.26}$$

$$R_{deq} = R_d + n_f^2(R_g + R_p) \tag{6.27}$$

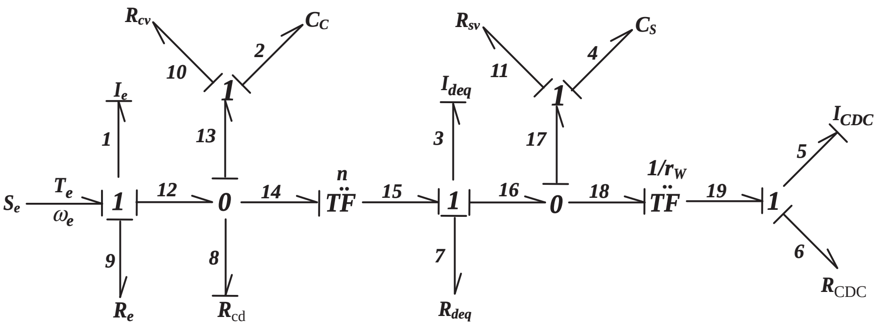

Figure 6.23 Bond graph of driveline with clutch and driveshaft compliances

The equations of motion of the driveline according to Figure 6.23 are:

$$\frac{dp_1}{dt} = T_e - \frac{p_1}{I_1}R_e - T_1 \tag{6.28}$$

$$\frac{dq_2}{dt} = \frac{p_1}{I_1} - \frac{T_1}{R_{Cd}} - n\frac{p_3}{I_3} \tag{6.29}$$

$$\frac{dp_3}{dt} = nT_1 - \frac{p_3}{I_3}R_{deq} - T_2 \tag{6.30}$$

$$\frac{dq_4}{dt} = \frac{p_3}{I_3} - \frac{1}{r_W}\frac{p_5}{I_5} \tag{6.31}$$

$$\frac{dp_5}{dt} = \frac{1}{r_W}T_2 - \frac{p_5}{I_5}R_{CDC} \tag{6.32}$$

in which:

$$T_1 = k_2q_2 + R_{Cv}\dot{q}_2 \tag{6.33}$$

$$T_2 = k_4q_4 + R_{Sv}\dot{q}_4 \tag{6.34}$$

6.5 Analysis

Driveline models with different elastic elements were constructed and the governing equations of motions were derived in the preceding sections. In this section, the equations of the system will be solved to observe the effects of compliances in the driveline on the system oscillations.

It is assumed that changes in engine torque due to throttle variations provide inputs to cause driveline oscillations. Sudden throttle changes such as tip-in (fast depression of throttle) and tip-out (sudden release of throttle) are two common excitations of vehicle driveline leading to large changes in vehicle acceleration. In the following sections for different driveline models, it will be assumed that the vehicle is at a steady motion and suddenly the throttle is changed and the resulting oscillations will be observed.

6.5.1 *Effect of Clutch Compliance*

Equations of motion of the vehicle driveline can be written in more familiar form by substituting into Equations 6.12–6.15 the equivalent values $p_1 = I_e\omega_e$, $p_3 = m_{eq}v$ and $q_2 = \theta_C$. The results are:

$$I_e\frac{d\omega_e}{dt} = T_e - R_e\omega_e - T_1 \tag{6.35}$$

$$\frac{d\theta_C}{dt} = \omega_e - \frac{n}{r_W}v \tag{6.36}$$

$$m_{CC}\frac{dv}{dt} = \frac{n}{r_W}T_1 - R_{CC}v \tag{6.37}$$

$$T_1 = k_C\theta_C + R_{Cv}\dot{\theta}_C \tag{6.38}$$

Solution of above equations can be obtained either by a MATLAB® or a Simulink® program. The MATLAB program will be similar to those used in previous chapters to simulate systems with of differential equations. The following example is solved by such a program written in the MATLAB environment.

Example 6.5.1

For a vehicle with driveline parameter values given in Table 6.1 and an engine MT formula parameters of Example 5.5.2, use a driveline model with only clutch compliance in the fully engaged phase:

Table 6.1 Numerical values for driveline parameters

Element	Name	Value	Units
I_e	Flywheel inertia	0.30	kgm^2
I_c	Clutch inertia	0.04	kgm^2
I_g	Transmission output inertia	0.05	kgm^2
I_p	Inertia at differential input	0.01	kgm^2
I_{cw}	Crownwheel inertia	0.10	kgm^2
I_d	Differential output inertia	0.10	kgm^2
I_s	Driveshaft output inertia	0.50	kgm^2
I_W	Wheel inertia	2.00	kgm^2
k_C	Clutch stiffness	500	Nm/rad
k_S	Driveshaft stiffness	10000	Nm/rad
R_e	Damping of crankshaft	0.01	Nms/rad
R_{Cv}	Internal damping of clutch	10	Nms/rad
R_g	Damping in transmission	0.50	Nms/rad
R_p	Damping in differential input	0.50	Nms/rad
R_d	Damping in differential output	0.10	Nms/rad
R_S	Damping of driveshaft output	0.1	Nms/rad
R_{Sv}	Internal damping of driveshaft	200	Nms/rad
R_v	Damping in vehicle motion	20	Ns/m
m	Vehicle mass	1200	kg
n_g	Transmission ratios (gear 1)	3	-
n_f	Final drive ratio	4	-
r_W	Tyre effective radius	30	cm
m	Vehicle mass	1200	kg

(a) Find steady values for state variables at speed of 5 m/s.
(b) At t = 1s the throttle is suddenly fully depressed. Find the variations of engine speed, clutch spring depression angle, vehicle speed and acceleration.

Solution

(a) Steady state values of system states can simply be obtained from Equations 6.35–6.37 by equating them to zero. The results for the fully engaged clutch ($R_C = \infty$) are:

$$\omega_{e0} = \frac{n}{r_W} v0$$

$$\theta_{C0} = \frac{r_W R_{CC}}{n k_C} v0$$

The value of engine torque to maintain steady motion is:

$$T_{e0} = R_e \omega_{e0} + k_C \theta_{C0}$$

(b) For this part two MATLAB programs are needed, one main program and one function including the system differential equations. Typical programs for this purpose are provided in Figures 6.24 and 6.25.

```
% Example 6.5.1
% A sample program to investigate the driveline vibrations with only Clutch
compliance

clc,  close all,   clear all

global thrtl Te0 RCv kC Ie rW n Rcc mcc Re p

% Vehicle information (see Table 6.1):
m=1200; Ie=0.3; Ic=0.04; Ig=0.05; Icw=0.1; Ip=0.01;  Id=0.1; Is=0.5; Iw=2;
kC=500;
Re=0.01; RCv=10; Rg=0.5; Rd=0.1; Rp=0.02; Rs=0.1; Rv=20; ng=3.0;
nf=4.0; rW=0.3;
n=ng*nf;

% Engine Full throttle information:
te=[80 98 100 105 110 112 109 111 104 96.6];
ome=[1000 1500 2000 2500 3000 3500 4000 4500 5000 5300];
% Fit a curve to WOT data
[p,s]=polyfit(ome,te,2);

% Define equivalent mcc and Rcc:
mcc=m+(Icw+Id+Is+Iw+(Ig+Ip)*nf^2+Ic*n^2)/rW^2;
Rcc=Rv+(Rd+Rs+(Rg+Rp)*nf^2)/rW^2;
```

Figure 6.24 MATLAB program listing of Example 6.5.1

```
% Initial conditions:
v0=5;
omegae0=v0*n/rW;
thetac0=Rcc*v0*rW/n/kC;
x0=[omegae0 thetac0 v0];
Te0=Re*omegae0+Rcc*v0*rW/n;                % Steady torque

% Specify throttle value:
thrtl=100;

% Specify differentiation interval t0-tf:
t0=0; tf=5;

% Invoke ode15s (stiff equations):
[t,x]=ode15s(@driveline_cc, [t0 tf], x0);    % Calls 'driveline_cc' function

% Plot the variation of parameters versus time
plot(t,x(:,1)*30/pi),
xlabel('Time (s)')
ylabel('Engine speed (rpm)')
grid
figure
plot(t,x(:,2)*180/pi)
xlabel('Time (s)')
ylabel('Clutch spring angle (degree)')
grid
figure
plot(t,x(:,3))
xlabel('Time (s)')
ylabel('Vehicle speed (m/s)')
grid
% Example 6.5.1 (continued)
% Generate the acceleration
j=length(t);
for i=1: j
   a(i)=(-Rcc*x(i,3)+kC*x(i,2)*n/rW)/mcc;
end
figure
plot(t,a)
xlabel('Time (sec)')
ylabel('Vehicle acceleration (m/s2)')
grid
```

Figure 6.24 (*Continued*)

It is worth noting that the equations of motion describe an oscillatory system and instead of '*ode45*' function, '*ode15s*' for stiff differential equations is used for integration.

The outputs of the programs are illustrated in Figures 6.26–6.29.

```
% Function called in Example 6.5.1
function f=driveline_cc(t,x)

global thrtl Te0 RCv kC Ie rW n Rcc mcc Re p

omegae=x(1);
thetac=x(2);
v=x(3);
   if t<1
      Te=Te0;
   else
   % Engine MT formula
   pow=(1.003*omegae*30/pi)^1.824;
   den=(1+exp(-11.12-0.0888*thrtl))^pow;
   Te=polyval(p,omegae*30/pi)/den;
   end

   f2=omegae-v*n/rW-omc;
   T1=kC*thetac+RCv*f2;
   f1=(Te-Re*omegae-T1)/Ie;
   f3=(-Rcc*v+T1*n/rW)/mcc;

   f=[f1
      f2
      f3];
```

Figure 6.25 MATLAB function of Example 6.5.1

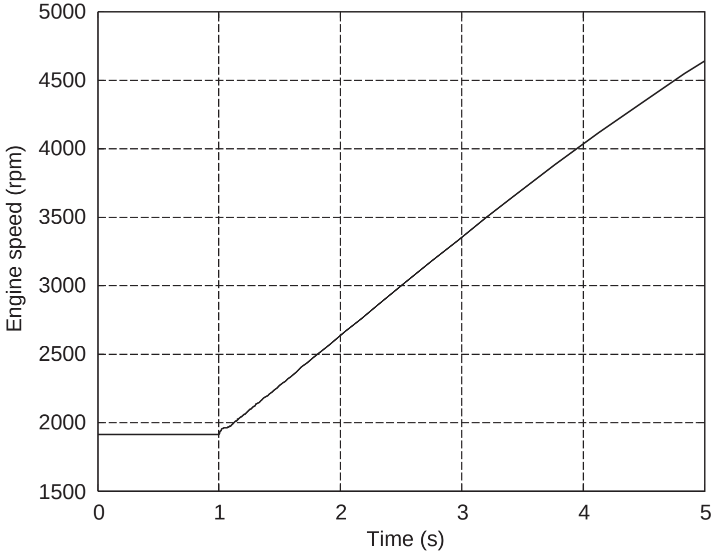

Figure 6.26 Variations of engine speed for sudden throttle input of Example 6.5.1

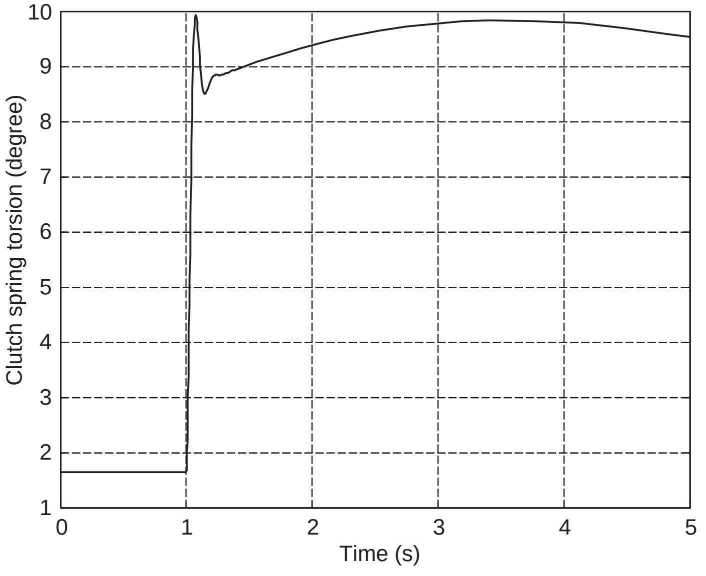

Figure 6.27 Variations of clutch spring torsion for sudden throttle input of Example 6.5.1

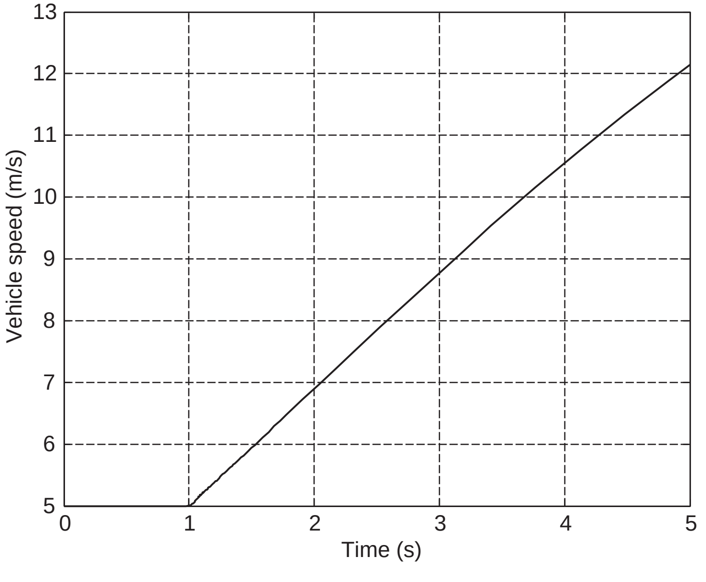

Figure 6.28 Variations of vehicle speed for sudden throttle input of Example 6.5.1

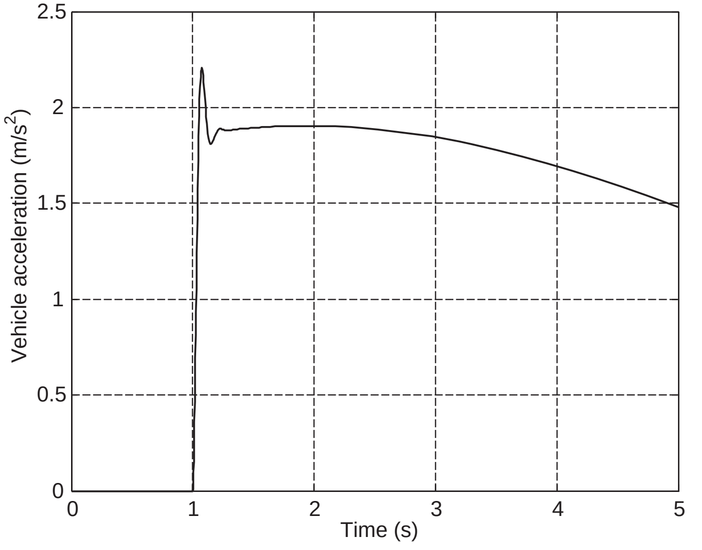

Figure 6.29 Variations of vehicle acceleration for sudden throttle input of Example 6.5.1

6.5.2 Effect of Driveshaft Compliance

The equations of motion of the vehicle driveline in simplified form for this case are:

$$I_{Eq}\frac{d\omega_e}{dt} = T_e - R_{Eq}\omega_e - T_1 \tag{6.39}$$

$$\frac{d\theta_s}{dt} = \frac{1}{n}\omega_e - \frac{1}{r_W}v \tag{6.40}$$

$$m_{DC}\frac{dv}{dt} = \frac{n}{r_W}T_1 - R_{DC}v \tag{6.41}$$

with:

$$T_1 = \frac{1}{n}(k_S\theta_S + R_{Sv}\dot{\theta}_S) \tag{6.42}$$

which are very similar to equations when only the clutch compliance was present. Thus the solution method is exactly similar.

Example 6.5.2

For the vehicle of Example 6.5.1 use a driveline model with only driveshaft compliance:

(a) Find steady values for state variables at speed of 5 m/s.
(b) At $t = 1$s the throttle is suddenly fully depressed. Find the variations of engine speed, driveshaft torsion angle, vehicle speed and acceleration.

Solution

The MATLAB program of Figures 6.24 and 6.25 can simply be modified for this case. The main changes are for the initial conditions and derivative equations inside the function file.

(a) Initial conditions or steady values of state variables read:

$$\omega_{e0} = \frac{n}{r_W} v0$$

$$\theta_{S0} = \frac{r_W R_{DC}}{k_S} v0$$

The initial steady value of engine torque is:

$$T_{e0} = R_{Eq}\omega_{e0} + \frac{1}{n} k_S \theta_{S0}$$

(b) The results for this case are plotted in Figures 6.30–6.33. It is clear that the initial and final conditions for this case are exactly equal to those of previous section and only the oscillations are different.

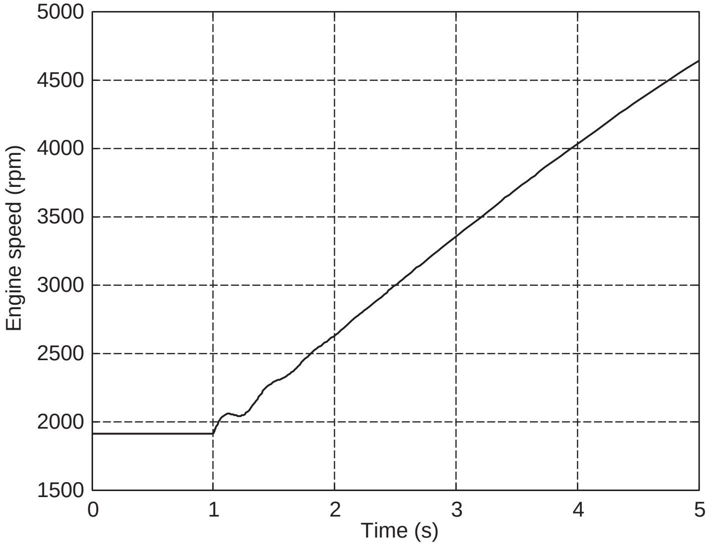

Figure 6.30 Variations of engine speed for sudden throttle input of Example 6.5.2

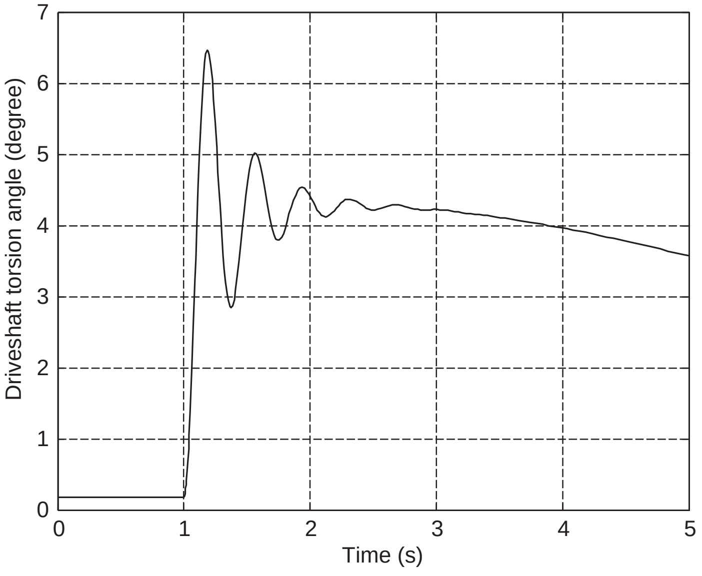

Figure 6.31 Variations of driveshaft torsion for sudden throttle input of Example 6.5.2

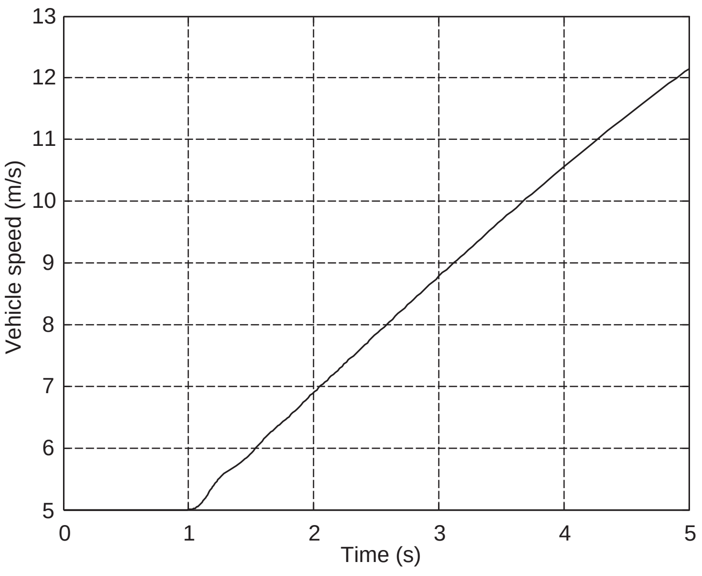

Figure 6.32 Variations of vehicle speed for sudden throttle input of Example 6.5.2

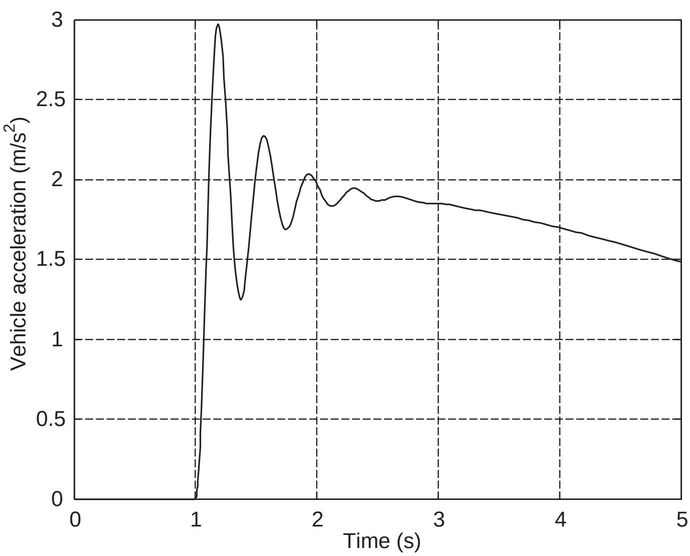

Figure 6.33 Variations of vehicle acceleration for sudden throttle input of Example 6.5.2

6.5.3 Effect of Clutch and Driveshaft Compliances

The equations of motion of driveline for this case are (see Equations 6.28–6.32):

$$I_e \frac{d\omega_e}{dt} = T_e - R_e\omega_e - T_1 \tag{6.43}$$

$$\frac{d\theta_C}{dt} = \omega_e - \frac{T_1}{R_{Cd}} - n\omega_d \tag{6.44}$$

$$I_{deq} \frac{d\omega_d}{dt} = nT_1 - R_{deq}\omega_d - T_2 \tag{6.45}$$

$$\frac{d\theta_S}{dt} = \omega_d - \frac{v}{r_W} \tag{6.46}$$

$$I_{CDC} \frac{dv}{dt} = \frac{1}{r_W} T_2 - vR_{CDC} \tag{6.47}$$

in which:

$$T_1 = k_C\theta_C + R_{Cv}\dot{\theta}_C \tag{6.48}$$

$$T_2 = k_S\theta_S + R_{Sv}\dot{\theta}_S \tag{6.49}$$

Example 6.5.3

For the vehicle of Example 6.5.1 use a driveline model with both clutch and driveshaft compliances:

(a) Find the steady values for state variables at speed of 5 m/s.
(b) At t = 1s the throttle is suddenly fully depressed. Find the variations of engine speed, clutch torsion, driveshaft torsion, vehicle speed and acceleration.

Solution

(a) The steady-state values are obtained by equating the time derivatives of Equations 6.43–6.49 to zero. The results are:

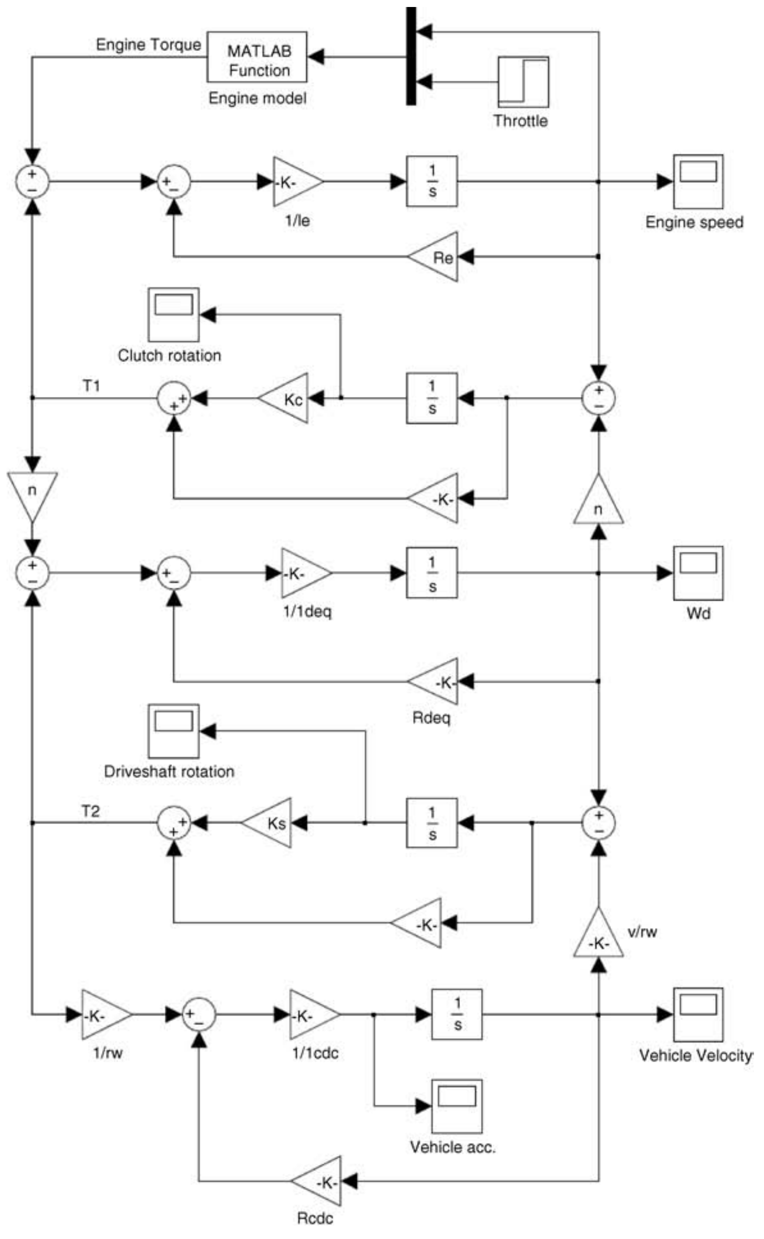

Figure 6.34 Simulink model for driveline including clutch and driveshaft compliances

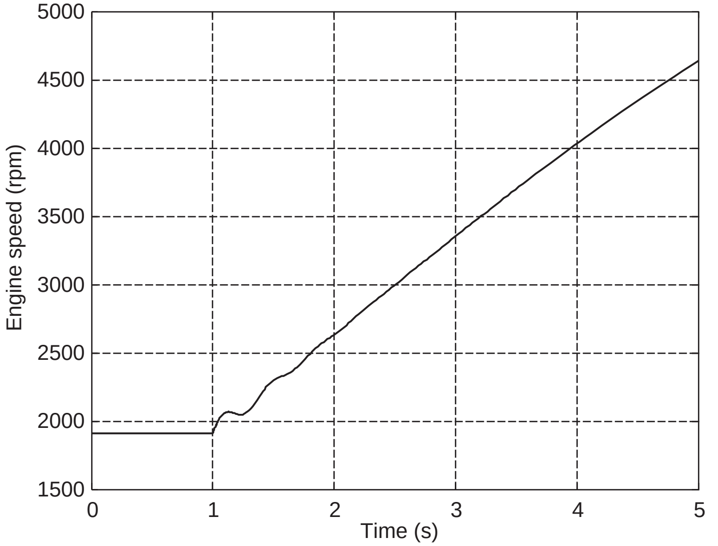

Figure 6.35 Variations of engine speed for sudden throttle input of Example 6.5.3

$$\omega_{e0} = \frac{n}{r_W} v0$$

$$\theta_{S0} = \frac{r_W R_{CDC}}{k_S} v0$$

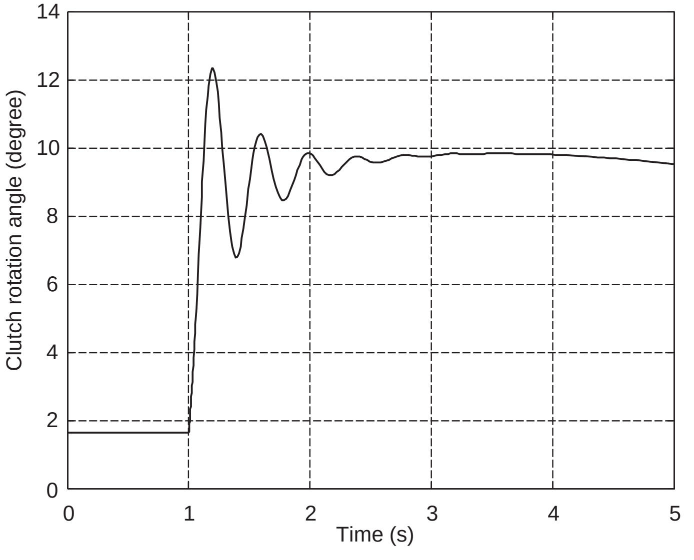

Figure 6.36 Variations of clutch spring torsion for sudden throttle input of Example 6.5.3

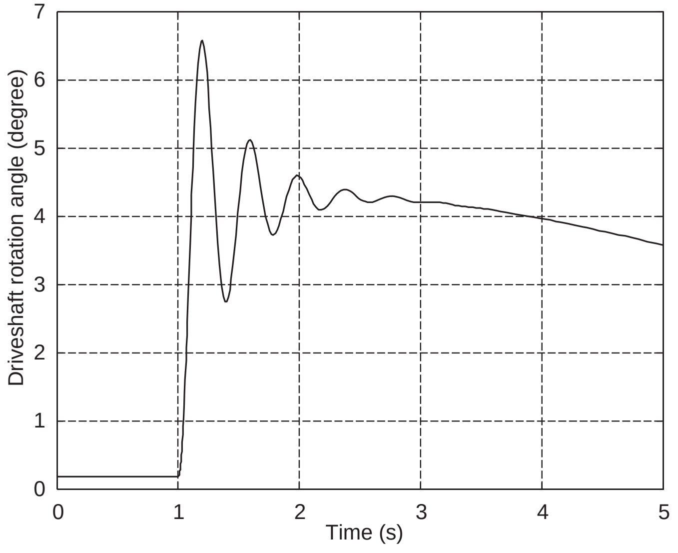

Figure 6.37 Variations of driveshaft torsion for sudden throttle input of Example 6.5.3

$$\theta_{C0} = \frac{r_W k_S \theta_{S0} + R_{deq}}{n r_W k_C} v0$$

The initial steady value of engine torque is:

$$T_{e0} = R_e \omega_{e0} + k_C \theta_{C0}$$

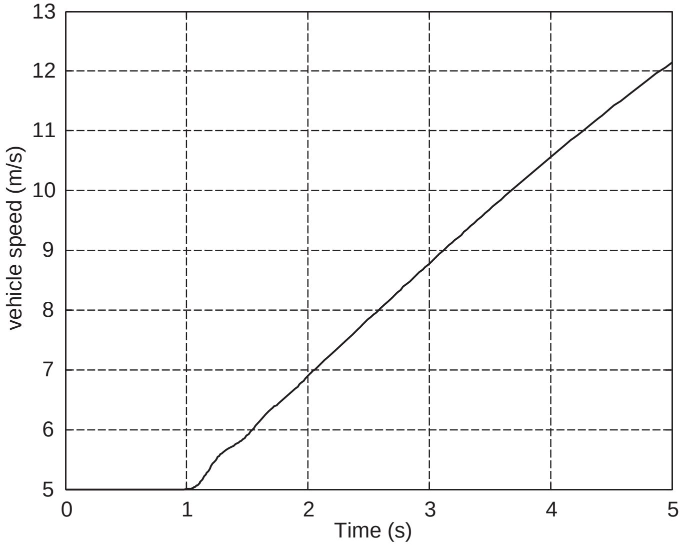

Figure 6.38 Variations of vehicle speed for sudden throttle input of Example 6.5.3

(b) As an alternative method to our solutions in two previous examples, let us this time use Simulink. A model for this purpose is shown in Figure 6.34. The output results are given in Figures 6.35–6.39.

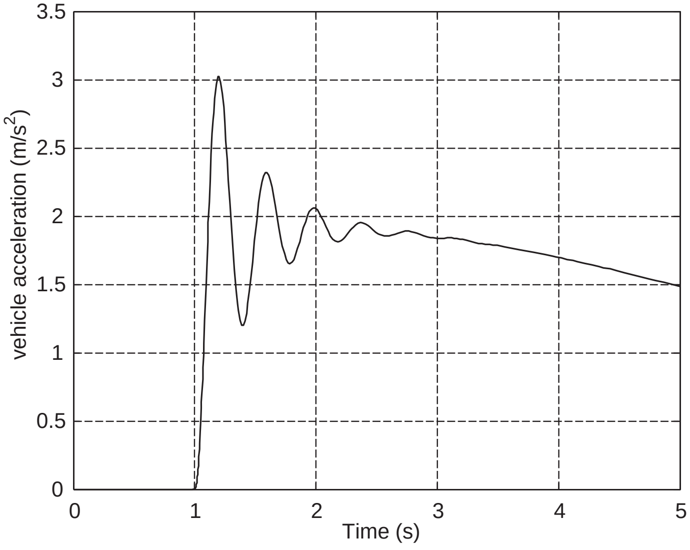

Figure 6.39 Variations of vehicle acceleration for sudden throttle input of Example 6.5.3

6.5.4 *Frequency Responses*

The equations of motion for the driveline models are all in the form of a set of first order ordinary differential equations that can easily be written in state space form. With the use of specialized software such as MATLAB, obtaining the frequency response analysis is rather simple. In addition to useful information regarding the frequencies and damping ratios of the driveline systems, transfer functions between inputs and outputs can also be produced. This can help in performing additional linear analyses studies and controller designs for active elements.

The state space representation of a system is in the basic form of:

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{AX} + \boldsymbol{BU} \tag{6.50}$$

in which $\boldsymbol{X}$ is an $n \times 1$ column matrix containing the system state variables, $\boldsymbol{U}$ is an $m \times 1$ column matrix containing the system inputs, $\boldsymbol{A}$ is an $n \times n$ square matrix and $\boldsymbol{B}$ is an $n \times m$ matrix. Elements of $\boldsymbol{A}$ and $\boldsymbol{B}$ are obtained from the governing equations of the system.

If specific system outputs given in an $r \times 1$ column matrix, $\boldsymbol{Y}$ is chosen, it can be represented by following expression:

$$\boldsymbol{Y} = \boldsymbol{CX} + \boldsymbol{DU} \tag{6.51}$$

with $\boldsymbol{C}$ and $\boldsymbol{D}$ as two $r \times n$ and $r \times m$ matrices. The following examples in this section are designed to familiarize the reader with developing models for frequency response analysis.

Example 6.5.4

For the three driveline models developed in Sections 6.5.1, 6.5.2 and 6.5.3 derive the state space matrices. Engine torque T_e is input whereas vehicle speed and acceleration are outputs.

Solution

(a) Model with clutch compliance: for this case Equations 6.35–6.37 can be written in the form of Equation 6.50. Matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are:

$$\boldsymbol{A} = \begin{bmatrix} -\dfrac{R_e + R_{Cv}}{I_e} & -\dfrac{k_C}{I_e} & \dfrac{nR_{Cv}}{r_W I_e} \\ 1 & 0 & -\dfrac{n}{r_W} \\ \dfrac{nR_{Cv}}{r_W I_{CC}} & \dfrac{nk_C}{r_W I_{CC}} & -\dfrac{R_{CC}}{I_{CC}} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \dfrac{1}{I_e} \\ 0 \\ 0 \end{bmatrix}$$

Matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ must be generated for the two outputs v and a and the input T_e:

$$\boldsymbol{C} = \begin{bmatrix} 0 & 0 & 1 \\ \dfrac{nR_{Cv}}{r_W I_{CC}} & \dfrac{nk_C}{r_W I_{CC}} & -\dfrac{R_{CC}}{I_{CC}} \end{bmatrix}, \boldsymbol{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(b) Model with driveshaft compliance: for this case Equations 6.39–6.41 are used. Matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ this time are:

$$\boldsymbol{A} = \begin{bmatrix} -\dfrac{R_{Eq} + \dfrac{1}{n^2} R_{Sv}}{I_{Eq}} & -\dfrac{k_S}{nI_{Eq}} & \dfrac{R_{Sv}}{nr_W I_{Eq}} \\ \dfrac{1}{n} & 0 & -\dfrac{1}{r_W} \\ \dfrac{R_{Sv}}{nr_W I_{DC}} & \dfrac{k_S}{r_W I_{DC}} & -\dfrac{R_{Sv} + r_W^2 R_{DC}}{r_W^2 I_{DC}} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \dfrac{1}{I_{Eq}} \\ 0 \\ 0 \end{bmatrix}$$

Matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ for the same input and outputs are:

$$\boldsymbol{C} = \begin{bmatrix} 0 & 0 & 1 \\ \dfrac{R_{Sv}}{nr_W I_{DC}} & \dfrac{k_S}{r_W I_{DC}} & -\dfrac{R_{Sv} + r_W^2 R_{DC}}{r_W^2 I_{DC}} \end{bmatrix}, \boldsymbol{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(c) Model with both clutch and driveshaft compliances: for this case Equations 6.43–6.47 are used and thus the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ include five rows:

$$\boldsymbol{A} = \begin{bmatrix} -\dfrac{R_e + R_{Cv}}{I_e} & -\dfrac{k_C}{I_e} & \dfrac{nR_{Cv}}{I_e} & 0 & 0 \\ 1 & 0 & -n & 0 & 0 \\ \dfrac{nR_{Cv}}{r_W I_{deq}} & \dfrac{nk_C}{I_{deq}} & -\dfrac{R_{Sv} + n^2 R_{Cv} + R_{deq}}{I_{deq}} & -\dfrac{k_S}{I_{deq}} & \dfrac{R_{Sv}}{r_W I_{deq}} \\ 0 & 0 & 1 & 0 & -\dfrac{1}{r_W} \\ 0 & 0 & \dfrac{R_{Sv}}{r_W I_{CDC}} & \dfrac{k_S}{r_W I_{CDC}} & -\dfrac{R_{Sv} + r_W^2 R_{CDC}}{r_W^2 I_{CDC}} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \dfrac{1}{I_e} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ for the same input and outputs are:

$$\boldsymbol{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{R_{Sv}}{r_W I_{CDC}} & \dfrac{k_S}{r_W I_{CDC}} & -\dfrac{R_{Sv} + r_W^2 R_{CDC}}{r_W^2 I_{CDC}} \end{bmatrix}, \ \boldsymbol{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Example 6.5.5

For the driveline models with the state space matrices obtained in previous example, use the driveline information of Example 6.5.1:

(a) Determine the frequencies and damping ratios for each driveline model.
(b) Plot the Bode diagrams for the outputs for the third driveline model.

Solution

(a) The frequencies and damping ratios depend only on the matrix A of each system. In the MATLAB environment a simple command '[wn, z]=damp(A)' produces the results by displaying natural frequencies (wn) and damping ratios (z).

For the three driveline models, the values of natural frequencies and damping ratios are summarized in Table 6.2.

(b) The state space from can be converted into a MATLAB system by using the following command:

$$sys = ss(\mathrm{A},\ \mathrm{B},\ \mathrm{C},\ \mathrm{D});$$

Then the Bode plot is simply obtained by issuing command: '*bode(sys)*'. For the third driveline model the result is illustrated in Figure 6.40.

Table 6.2 Frequencies and damping ratios of Example 6.5.5

Item	Modes	Clutch compliance	Driveshaft compliance	Clutch and driveshaft compliances
Natural Frequency (rad/s) [Hz]	First	0.073 [0.012]	0.073 [0.012]	0.073 [0.012]
	Second	47.75 [7.6]	17.03 [2.7]	16.97 [2.6]
	Third	47.75 [7.6]	17.03 [2.7]	16.97 [2.6]
	Fourth	–	–	66.11 [10.5]
	Fifth	–	–	201.87 [32.1]
Damping ratio	First	1	1	1
	Second	0.478	0.175	0.165
	Third	0.478	0.175	0.165
	Fourth	–	–	1
	Fifth	–	–	1

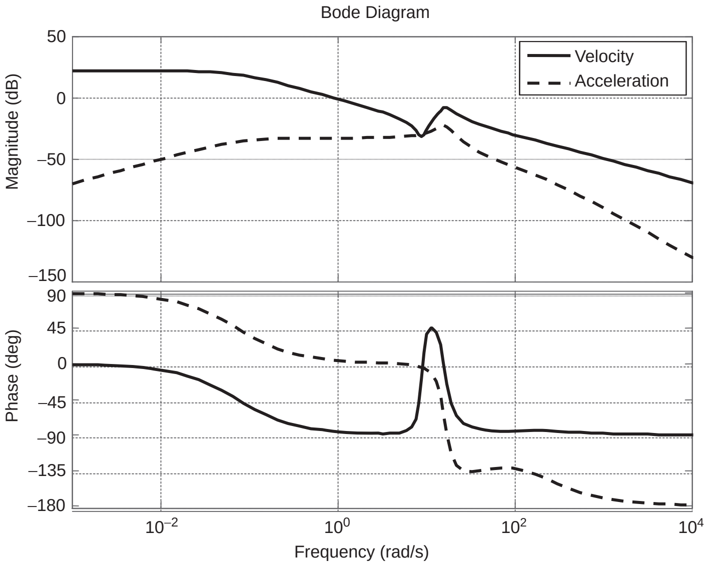

Figure 6.40 Bode plots of system outputs for Example 6.5.5

6.5.5 Improvements

In the design process with the help of software simulations, the driveline parameters can be altered to get better responses in terms of driveline oscillations at different working conditions. The potential approaches that are possible for reducing driveline vibrations in general are:

1. Tuning the system natural frequencies by changing the stiffnesses or inertias of the components.
2. Adjusting the damping of the driveline system. Viscous damping in the system decreases the amplitude of the oscillations and must be adjusted to the desired values to obtain faster responses and at the same time low amplitude oscillations.
3. Regulating the input power to the system by controlling the engine torque.

Engine torque fluctuations can be a source of excitation of driveline vibration. Engine torque fluctuations can initiate or magnify the oscillations. Passive crankshaft torsional dampers are useful in reducing the amplitude of vibrations. These include very simple rubber dampers or sophisticated dual-mass systems. The appropriate control of engine torque also can lead to smooth power transfers at different driving conditions. Engine management systems can achieve this by controlling the throttle and ignition for smoothing the torque outputs. The controller calculates a spark advance or fuel injection timing as well as the throttle position that is needed in order to counteract the torque changes and reduce the overall driveline vibrations.

6.6 Conclusion

In this chapter, the modelling of the vehicle driveline was discussed and the development of several models using bond graph method was presented. This method allows the inclusion of the detailed component properties of the driveline system. The derivation of the governing equations is straightforward and the resulting equations are in the form of a set of first order ordinary differential equations that are suitable for direct numerical integration. MATLAB/Simulink programs were used to demonstrate the method of solving system equations for three different cases including stiffness of clutch plate, stiffness of driveshaft and both stiffnesses together.

Other analysis techniques are also available, once the equations of motion are derived. These include state space and frequency response analyses. The study of driveline behaviour can be extended by observing the effect of changing parameters on the oscillation of the system and modification of parameter values in order to reduce the vibrations.

6.7 Review Questions

6.1 Describe why vehicle drivelines are prone to vibration.
6.2 Explain the difference in concept of the bond graph method with other methods such as block diagrams and signal flow graphs.
6.3 Explain why there are two friction elements in the bond graph of a clutch plate.
6.4 What is a modulated transformer?
6.5 Explain how the inertia is lumped on one side of an element. Does it make any difference about which side to include?
6.6 Is the vehicle differential a true transformer? Why?
6.7 Explain why the final steady values for Examples 6.5.1, 6.5.2 and 6.5.3 are similar.
6.8 Describe how the system natural frequencies can be obtained once the equations of motion are derived.
6.9 What are the practical ways of reducing the driveline vibrations at the design stage?
6.10 What are the practical ways of reducing the vehicle vibration during operation?

6.8 Problems

Problem 6.1

For a single cylinder internal combustion engine with a schematic diagram shown in Figure P6.1:

(a) Explain clearly how many *transformers*, *gyrators*, *inertia elements*, *capacitor elements* and *resistor elements* you find in the bond graph.

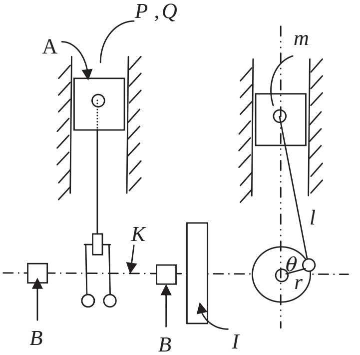

Figure P6.1 Schematic diagram of engine for Problem 6.1

(b) Construct the complete bond graph, number it and insert the causal strokes.
(c) Specify the state variables of the system.

Problem 6.2

For a two-cylinder internal combustion engine with a schematic diagram shown in Figure P6.2:

(a) Construct the bond graph.
(b) Assign the causal strokes and specify the state variables.

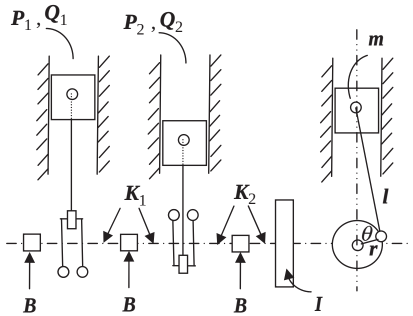

Figure P6.2 Schematic diagram of engine for Problem 6.2

Problem 6.3

For a four-cylinder internal combustion engine with a schematic diagram shown in Figure P6.3:

(a) Construct the bond graph.
(b) Assign causal strokes and specify the state variables.

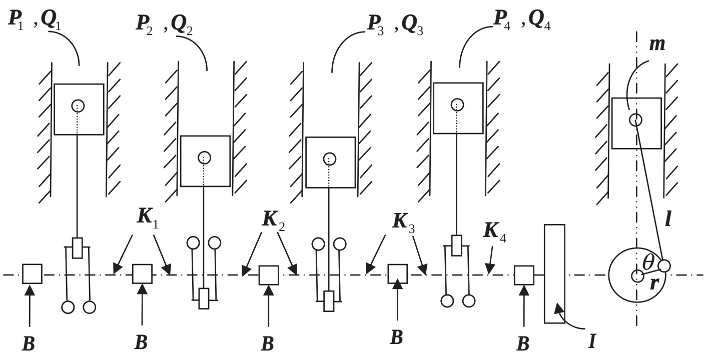

Figure P6.3 Schematic diagram of engine for Problem 6.3

Problem 6.4

Consider a five-speed constant mesh gearbox as shown in Figure P6.4:

(a) Construct a bond graph for the system when gear 1 is engaged.
(b) Insert causal strokes and then specify the state variables.
(c) What changes will occur to the bond graph when another gear is engaged? Explain why.

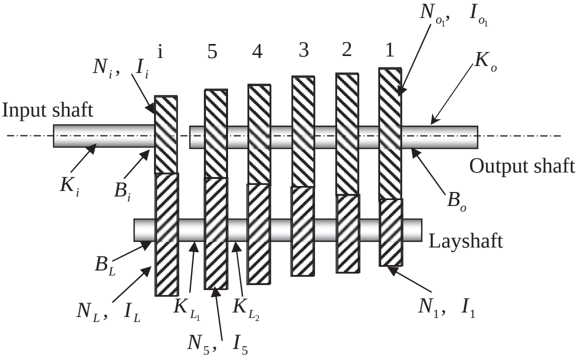

Figure P6.4 Schematic diagram of gearbox for Problem 6.4

Problem 6.5

For a vehicle differential with details shown in Figure P6.5:

(a) Draw a complete bond graph.
(b) Assign proper causal strokes.
(c) Specify the appropriate state variables.

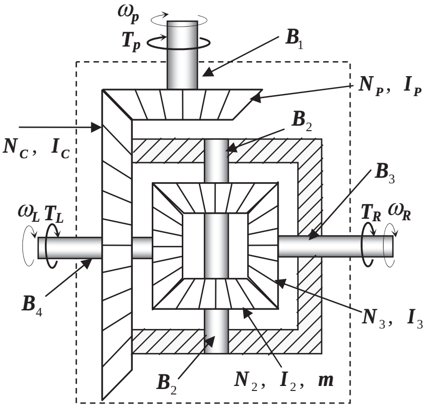

Figure P6.5 Schematic diagram of differential for Problem 6.5

Problem 6.6

For the planetary gear set shown in Figure P6.6, construct the bond graph for following cases:

(a) When the Carrier C is fixed (T_R input and $T_S$output).
(b) When the Sun S is fixed (T_R input and $T_C$output).
(c) When the Ring R is fixed (T_S input and $T_C$output).
(d) Write the equations of motion of system in case (a).

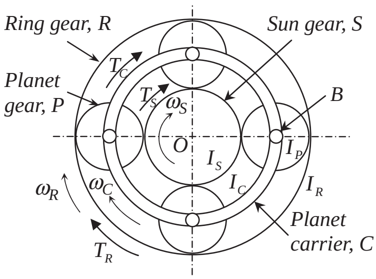

Figure P6.6 Schematic diagram of epicyclic gear set for Problem 6.6

Problem 6.7

A rigid body model of the driveline is represented as a simplified model in Figure P6.7 by ignoring damping:

(a) Derive the equations of motion of the system (note the differential causalities).
(b) Find the equation for the angular acceleration α_W of the wheel
(c) Find an expression for the overall gear ratio n that maximizes α_W
(d) Is the result useful?

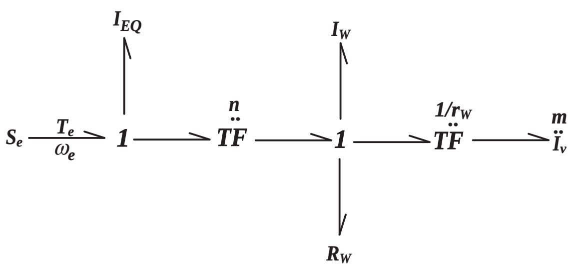

Figure P6.7 Rigid body bond graph model of Problem 6.7

Problem 6.8

Repeat Examples 6.5.1, 6.5.2 and 6.5.3 for an initial speed of 15 m/s and gear ratio of 2.

Problem 6.9

Repeat Examples 6.5.1, 6.5.2 and 6.5.3 for an initial speed of 10 m/s and a sudden release of the accelerator pedal. In this case the engine will generate a braking torque that can be modelled by the relation $T_{be} = -0.1\omega_e$ (ω_e in rad/s).

Problem 6.10

Repeat Problem 6.9 for an initial speed of 15 m/s and gear ratio of 2.

Problem 6.11

Repeat Examples 6.5.1, 6.5.2 and 6.5.3 for a 3 second pulse of throttle shown in Figure P6.11. Use the braking torque of Problem 6.9.

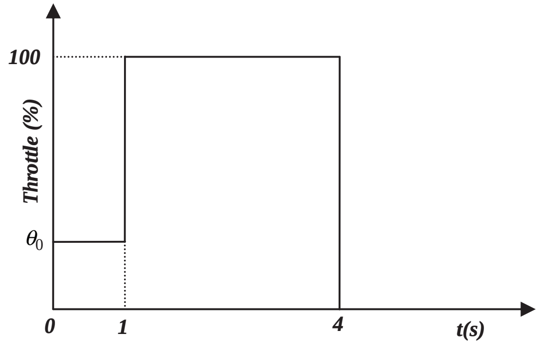

Figure P6.11 Throttle pulse of Problem 6.11

Problem 6.12

Consider the bond graph of a whole vehicle for straight ahead motion and ignore propeller shaft elasticity.

(a) Simplify it by combining the elements around the middle transformers.
(b) Insert causal stokes and specify the state variables of system.
(c) Derive the equations of motion of the resulting bond graph.

Problem 6.13

Consider the bond graph of whole vehicle in straight-ahead motion.

(a) Simplify it by combining the elements around the transformers.
(b) For (a), insert causal strokes and specify the state variables of system.
(c) Derive the equations of motion of system.

Problem 6.14

Consider the rigid-body model of driveline.

(a) Compare the system of Problem 6.7 with the rigid-body model and determine R_W and I_{EQ}.
(b) Specify which components have been ignored.
(c) Describe what the system of Problem 6.7 is telling you and if it is comparable to what was discussed in Chapter 3.

Further Reading

The torsional behaviour of the interconnected components in a driveline is important in controlling the overall refinement of the powertrain system and its effects on noise, vibration and harshness (NVH).

There are several methods for tackling this dynamic problem, and the method adopted in this book is the bond graph approach. Although there are several texts available, the best and most comprehensive introduction to the bond graph method is that by Borutzky [4]. Although there have been plenty of published studies of automotive driveline dynamics, little of this work has found its way into reference books. However, there is a good section in Kienke [12] (Chapter 5 in that book) which provides a simple driveline analysis using a Newtonian approach, and then goes on to discuss control issues relating to functionality and driveability.

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