camera model becomes

$$\begin{bmatrix} u_\ell \\ v_\ell \\ u_r \\ v_r \end{bmatrix} = \begin{bmatrix} f_u & 0 & c_u & 0 \\ 0 & f_v & c_v & 0 \\ f_u & 0 & c_u & -f_u b \\ 0 & f_v & c_v & 0 \end{bmatrix} \frac{1}{z} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}. \tag{6.140}$$

Typically, in this form, the v_r equation is dropped and the u_r equation is replaced with one for the *disparity*[8], d, given by

$$d = u_\ell - u_r = \frac{1}{z} f_u b, \tag{6.141}$$

so that we can write

$$\begin{bmatrix} u_\ell \\ v_\ell \\ d \end{bmatrix} = \mathbf{s}(\boldsymbol{\rho}) = \begin{bmatrix} f_u & 0 & c_u & 0 \\ 0 & f_v & c_v & 0 \\ 0 & 0 & 0 & f_u b \end{bmatrix} \frac{1}{z} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}, \tag{6.142}$$

for the stereo model. This form has the appealing property that we are going from three point parameters, (x, y, z), to three observations, (u_ℓ, v_ℓ, d). A similar model can be developed for the right camera.

6.4.3 Range-Azimuth-Elevation

Some sensors, such as lidar (light detection and ranging), can be modelled as a *range-azimuth-elevation (RAE)*, which essentially observes a point, P, in spherical coordinates. For lidar, which can measure distance by reflecting laser pulses off a scene, the azimuth and elevation are the angles of the mirrors that are used to steer the laser beam and the range is the reported distance determined by time of flight. The geometry of this sensor type is depicted in Figure 6.13.

The coordinates of point P in the sensor frame, $\underrightarrow{\mathcal{F}}_s$, are

$$\boldsymbol{\rho} = \mathbf{r}_s^{ps} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}. \tag{6.143}$$

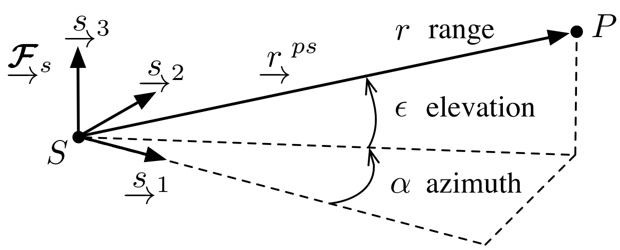

Figure 6.13 A range-azimuth-elevation sensor model observes a point P in spherical coordinates.

[8] The disparity equation can be used as a one-dimensional stereo camera model, as we have already seen in the earlier chapter on nonlinear estimation.

These can also be written as

$$\boldsymbol{\rho} = \mathbf{C}_3^T(\alpha)\,\mathbf{C}_2^T(-\epsilon)\begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix}, \tag{6.144}$$

where α is the azimuth, ϵ is the elevation, r is the range, and $\mathbf{C}_i$ is the principal rotation about axis i. The elevation rotation indicated in Figure 6.13 is negative according to the right-hand rule. Inserting the principal-axis rotation formulas and multiplying out, we find that

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r\cos\alpha\cos\epsilon \\ r\sin\alpha\cos\epsilon \\ r\sin\epsilon \end{bmatrix}, \tag{6.145}$$

which are the common spherical-coordinate expressions. Unfortunately, this is the inverse of the sensor model we desire. We can invert this expression to show that the RAE sensor model is

$$\begin{bmatrix} r \\ \alpha \\ \epsilon \end{bmatrix} = \mathbf{s}(\boldsymbol{\rho}) = \begin{bmatrix} \sqrt{x^2+y^2+z^2} \\ \tan^{-1}(y/x) \\ \sin^{-1}\left(z/\sqrt{x^2+y^2+z^2}\right) \end{bmatrix}. \tag{6.146}$$

In the case that the point P lies in the xy-plane, we have $z = 0$ and hence $\epsilon = 0$, so that the RAE model simplifies to the range-bearing model:

$$\begin{bmatrix} r \\ \alpha \end{bmatrix} = \mathbf{s}(\boldsymbol{\rho}) = \begin{bmatrix} \sqrt{x^2+y^2} \\ \tan^{-1}(y/x) \end{bmatrix}, \tag{6.147}$$

which is commonly used in mobile robotics.

6.4.4 Inertial Measurement Unit

Another common sensor that functions in three-dimensional space is the *inertial measurement unit (IMU)*. An ideal IMU comprises three orthogonal linear accelerometers and three orthogonal rate gyros[9]. All quantities are measured in a sensor frame, $\underrightarrow{\mathcal{F}}_s$, which is typically not located at the vehicle frame, $\underrightarrow{\mathcal{F}}_v$, as shown in Figure 6.14.

To model an IMU, we assume that the state of the vehicle can be captured by the quantities

$$\underbrace{\mathbf{r}_i^{vi},\quad \mathbf{C}_{vi},}_{\text{pose}} \qquad \underbrace{\boldsymbol{\omega}_v^{vi},}_{\text{angular velocity}} \qquad \underbrace{\dot{\boldsymbol{\omega}}_v^{vi},}_{\text{angular acceleration}} \tag{6.148}$$

and that we know the fixed pose change between the vehicle and sensor frames given by $\mathbf{r}_v^{sv}$ and $\mathbf{C}_{sv}$, which is typically determined by calibration.

[9] Typically, calibration is required, as the axes are never perfectly orthogonal due to manufacturing tolerances.