DDDM - Project3 mathematical models

June 12, 2020

1 Model 1: Base Model

1.1 Problem data:

1.1.1 In python

M		Number of customer cities
N		Number of distribution cities
P $\leq M$		Number of airports
lat$_k$	k $\in [1, N]$	cities' latitude
lng$_k$	k $\in [1, N]$	cities' longitude
pop$_k$	k $\in [1, N]$	cities' population
D$_k = \lceil \frac{50pop_k}{1000} \rceil$	k $\in [1, N]$	demand at city k
R = 6371.009		earth radius (km)
c = 0.001		unit cost per km, from airport to DC
C = 0.01		unit cost per km, from DC to final dest
f = 100000		cost for opening a DC
Cap = 50000		DC's capacity
w$_{ij} = c\frac{2\pi R(\lvert lat_i - lat_j\rvert + \lvert lng_i - lng_j\rvert)}{360}$	i $\in [1, P], j \in [1, M]$	air transport cost from i to j
W$_{jk} = C\frac{2\pi R(\lvert lat_j - lat_k\rvert + \lvert lng_j - lng_k\rvert)}{360}$	j $\in [1, M], k \in [1, N]$	land transport cost from j to k

1.1.2 In AMPL

M		Number of customer cities
N		Number of distribution cities
P $\leq M$		Number of airports
D$_k$	k $\in [1, N]$	demand at city k
f = 100000		cost for opening a DC (independently of city)
Cap = 50000		DC's capacity
w$_{ij}$	i $\in [1, P], j \in [1, M]$	air transport cost from i to j
W$_{jk}$	j $\in [1, M], k \in [1, N]$	land transport cost from j to k

1.2 Variables:

y_j	$j \in [1, M]$	1 if a DC is placed in city j, 0 otherwise
x_{jk}	$j \in [1, M], k \in [1, N]$	quantity served by DC j to customer k
t_{ij}	$i \in [1, P], j \in [1, M]$	quantity served by airport i to DC j

1.3 formulation:

$$\text{minimize cost} = \sum_{j=1}^{N} y_j f + \sum_{i=1}^{P}\sum_{j=1}^{N} t_{ij} w_{ij} + \sum_{j=1}^{M}\sum_{k=1}^{N} x_{jk} W_{jk} \tag{1}$$

$$\text{subject to:} \sum_{j=1}^{M} x_{jk} = D_k \qquad \forall k \in [1, N] \tag{2}$$

$$\sum_{k=1}^{N} x_{jk} \leq y_j Cap \qquad \forall j \in [1, M] \tag{3}$$

$$\sum_{i=1}^{P} t_{ij} = \sum_{k=1}^{M} x_{jk} \qquad \forall j \in [1, M] \tag{4}$$

$$y_j \in \{0, 1\} \qquad \forall j \in [1, M] \tag{5}$$

$$x_{jk}, tij \geq 0 \qquad \forall i \in [1, P], \forall j \in [1, M], k \in [1, N] \tag{6}$$

2 Model 2: Optimized Model 1

2.1 Problem data:

2.1.1 In python

M		Number of customer cities
N		Number of distribution cities
$P \leq M$		Number of airports
lat_k	$k \in [1, N]$	cities' latitude
lng_k	$k \in [1, N]$	cities' longitude
pop_k	$k \in [1, N]$	cities' population
$D_k = \lceil \frac{50 pop_k}{1000} \rceil$	$k \in [1, N]$	demand at city k
R = 6371.009		earth radius (km)
c = 0.001		unit cost per km, from airport to
C = 0.01		unit cost per km, from DC to fina
f = 100000		cost for opening a DC
Cap = 50000		DC's capacity
$w_j = Min_{i=0}^{p} c \frac{2\pi R(\lvert lat_i - lat_j\rvert + \lvert lng_i - lng_j\rvert)}{360}$	$j \in [1, M]$	min air transport cost to each DC
$W_{jk} = C \frac{2\pi R(\lvert lat_j - lat_k\rvert + \lvert lng_j - lng_k\rvert)}{360} + w_j$	$j \in [1, M], k \in [1, N]$	transport cost from i to j

2.1.2 In AMPL

M		Number of customer cities
N		Number of distribution cities
D_k	k $\in [1, N]$	demand at city k
f = 100000		cost for opening a DC (independently of city)
Cap = 50000		DC's capacity
W_{jk}	j $\in [1, M], k \in [1, N]$	land transport cost from j to k

2.2 Variables:

y_j	j $\in [1, M]$	1 if a DC is placed in city j, 0 otherwise
x_{jk}	j $\in [1, M], k \in [1, N]$	quantity served by DC j to customer k

2.3 formulation:

$$\text{minimize cost} = \sum_{j=1}^{N} y_j f + \sum_{j=1}^{M} \sum_{k=1}^{N} x_{jk} W_{jk} \tag{7}$$

subject to:

$$\sum_{j=1}^{M} x_{jk} = D_k \qquad \forall k \in [1, N] \tag{8}$$

$$\sum_{k=1}^{N} x_{jk} \leq y_j Cap \qquad \forall j \in [1, M] \tag{9}$$

$$\sum_{j=1}^{M} y_j \geq \left\lceil \sum_{k=1}^{M} \frac{D_k}{Cap} \right\rceil \tag{10}$$

$$\sum_{j=1}^{M} y_j \leq Max(\sum_{k=1}^{N} Max_{j=1}^{M} W_{jk} D_k / f, \left\lceil \sum_{k=1}^{N} D_k Cap \right\rceil) \tag{11}$$

$$y_j \in \{0, 1\} \qquad \forall j \in [1, M] \tag{12}$$

$$x_{jk} \geq 0 \qquad \forall j \in [1, M], \forall k \in [1, N] \tag{13}$$

3 Model 3: Greedy Model 1

The problem is slit into several optimization problems.
Each is the same and described below.

3.1 Problem data:

3.1.1 In python

M		Number of customer cities
N		Number of distribution cities
P $\leq M$		Number of airports
lat$_k$	k $\in [1, N]$	cities' latitude
lng$_k$	k $\in [1, N]$	cities' longitude
pop$_k$	k $\in [1, N]$	cities' population
D$_k = \lceil \frac{50 pop_k}{1000} \rceil$	k $\in [1, N]$	demand at city k
R = 6371.009		earth radius (km)
c = 0.001		unit cost per km, from airport to
C = 0.01		unit cost per km, from DC to fina
f = 100000		cost for opening a DC
Cap = 50000		DC's capacity
w$_j = Min_{i=0}^{p} c \frac{2\pi R(\lvert lat_i - lat_j \rvert + \lvert lng_i - lng_j \rvert)}{360}$	j $\in [1, M]$	min air transport cost to each DC
W$_{jk} = C \frac{2\pi R(\lvert lat_j - lat_k \rvert + \lvert lng_j - lng_k \rvert)}{360} + w_j$	j $\in [1, M], k \in [1, N]$	transport cost from i to j

3.1.2 In AMPL

M		Number of customer cities
N		Number of distribution cities
D$_k$	k $\in [1, N]$	demand at city k
f = 100000		cost for opening a DC (independently of city)
Cap = 50000		DC's capacity
W$_{jk}$	j $\in [1, M], k \in [1, N]$	land transport cost from j to k

3.2 Variables:

y$_j$	j $\in [1, M]$	1 if a DC is placed in city j, 0 otherwise
x$_{jk}$	j $\in [1, M], k \in [1, N]$	quantity served by DC j to customer k

3.3 formulation:

$$\text{minimize cost} = f + \sum_{j=1}^{M} \sum_{k=1}^{N} x_{jk} W_{jk} \tag{14}$$

$$\begin{aligned}
\text{subject to:} \quad & \sum_{j=1}^{M} x_{jk} \leq D_k && \forall k \in [1, N] && (15)\\
& \sum_{k=1}^{N} x_{jk} = y_j Cap && \forall j \in [1, M] && (16)\\
& \sum_{j=1}^{M} y_j = 1 && && (17)\\
& y_j \in \{0, 1\} && \forall j \in [1, M] && (18)\\
& x_{jk} \geq 0 && \forall j \in [1, M], \forall k \in [1, N] && (19)
\end{aligned}$$

4 Model 3: Greedy Model 2

The problem is slit into several optimization problems.
Each is the same and described below.

4.1 Problem data:

4.1.1 In python

M		Number of customer cities
N		Number of distribution cities
P $\leq M$		Number of airports
lat$_k$	k $\in [1, N]$	cities' latitude
lng$_k$	k $\in [1, N]$	cities' longitude
pop$_k$	k $\in [1, N]$	cities' population
D$_k = \lceil \frac{50pop_k}{1000} \rceil$	k $\in [1, N]$	demand at city k
R = 6371.009		earth radius (km)
c = 0.001		unit cost per km, from airport to
C = 0.01		unit cost per km, from DC to fina
f = 100000		cost for opening a DC
Cap = 50000		DC's capacity
w$_j = Min_{i=0}^{p} c \frac{2\pi R(\|lat_i - lat_j\| + \|lng_i - lng_j\|)}{360}$	j $\in [1, M]$	min air transport cost to each DC
W$_{jk} = C \frac{2\pi R(\|lat_j - lat_k\| + \|lng_j - lng_k\|)}{360} + w_j$	j $\in [1, M], k \in [1, N]$	transport cost from i to j

4.1.2 In AMPL

M		Number of customer cities
N		Number of distribution cities
D$_k$	k $\in [1, N]$	demand at city k
f = 100000		cost for opening a DC (independently of city)
Cap = 50000		DC's capacity
W$_{jk}$	j $\in [1, M], k \in [1, N]$	land transport cost from j to k
m		Number of facilities to select

4.2 Variables:

y_j	j $\in [1, M]$	1 if a DC is placed in city j, 0 otherwise
x_{jk}	j $\in [1, M], k \in [1, N]$	quantity served by DC j to customer k

4.3 formulation:

$$\text{minimize cost} = f * m + \sum_{j=1}^{M} \sum_{k=1}^{N} x_{jk} W_{jk} \tag{20}$$

$$\text{subject to:} \quad \sum_{j=1}^{M} x_{jk} \leq D_k \qquad \forall k \in [1, N] \tag{21}$$

$$\sum_{k=1}^{N} x_{jk} = y_j Cap \qquad \forall j \in [1, M] \tag{22}$$

$$\sum_{j=1}^{M} y_j = m \tag{23}$$

$$y_j \in \{0, 1\} \qquad \forall j \in [1, M] \tag{24}$$

$$x_{jk} \geq 0 \qquad \forall j \in [1, M], \forall k \in [1, N] \tag{25}$$

5 Model 4: Optimized Model 2

The problem is slit into several optimization problems.
Each is the same and described below.

5.1 Problem data:

5.1.1 In python

M		Number of customer cities
N		Number of distribution cities
P $\leq M$		Number of airports
lat_k	$k \in [1, N]$	cities' latitude
lng_k	$k \in [1, N]$	cities' longitude
pop_k	$k \in [1, N]$	cities' population
$D_k = \lceil \frac{50 pop_k}{1000} \rceil$	$k \in [1, N]$	demand at city k
R = 6371.009		earth radius (km)
c = 0.001		unit cost per km, from airport to
C = 0.01		unit cost per km, from DC to fina
f = 100000		cost for opening a DC
Cap = 50000		DC's capacity
$w_j = Min_{i=0}^{p} c \frac{2\pi R(\lvert lat_i - lat_j\rvert + \lvert lng_i - lng_j\rvert)}{360}$	$j \in [1, M]$	min air transport cost to each DC
$W_{jk} = C \frac{2\pi R(\lvert lat_j - lat_k\rvert + \lvert lng_j - lng_k\rvert)}{360} + w_j$	$j \in [1, M], k \in [1, N]$	transport cost from i to j
LB		total cost from transport model
UB		total cost from greedy 2 model

5.1.2 In AMPL

M		Number of customer cities
N		Number of distribution cities
D_k	$k \in [1, N]$	demand at city k
f = 100000		cost for opening a DC (independently of city)
Cap = 50000		DC's capacity
W_{jk}	$j \in [1, M], k \in [1, N]$	land transport cost from j to k
LB		total cost from transport model plus min facility cost
UB		total cost from greedy 2 model

5.2 Variables:

y_j	$j \in [1, M]$	1 if a DC is placed in city j, 0 otherwise
x_{jk}	$j \in [1, M], k \in [1, N]$	quantity served by DC j to customer k

5.3 formulation:

$$\text{minimize cost} = \sum_{j=1}^{N} y_j f + \sum_{j=1}^{M} \sum_{k=1}^{N} x_{jk} W_{jk} \tag{26}$$

subject to: $$\sum_{j=1}^{M} x_{jk} = D_k \qquad \forall k \in [1, N] \tag{27}$$

$$\sum_{k=1}^{N} x_{jk} \leq y_j Cap \qquad \forall j \in [1, M] \tag{28}$$

$$\left\lceil \sum_{k=1}^{N} \frac{D_k}{Cap} \right\rceil \leq \sum_{j=1}^{M} y_j \leq \left\lceil \frac{UB}{f} \right\rceil \tag{29}$$

$$LB \leq \sum_{j=1}^{M} y_j f + \sum_{j=1}^{M} \sum_{k=1}^{N} x_{jk} W_{jk} \leq UB \tag{30}$$

$$LB \leq \sum_{j=1}^{M} \sum_{k=1}^{N} x_{jk} W_{jk} + \left\lceil \sum_{k=1}^{N} \frac{D_k}{Cap} \right\rceil * f \leq UB \tag{31}$$

$$x_{jk} \leq Cap * y_j \qquad \forall j \in [1, M], \forall k \in [1, N] \tag{32}$$

$$y_j \in \{0, 1\} \qquad \forall j \in [1, M] \tag{33}$$

$$x_{jk} \geq 0 \qquad \forall j \in [1, M], \forall k \in [1, N] \tag{34}$$