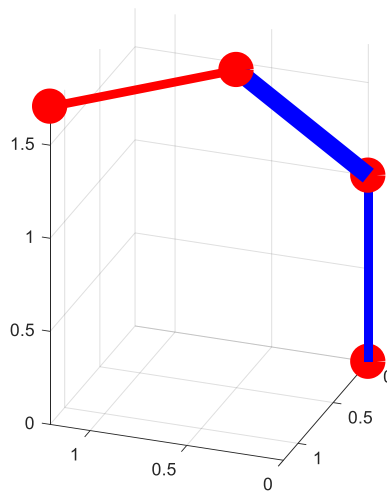


Home Task 4

To run the file with name "Program.m" in MATLAB.

1. Calculate Jacobian (numeric method)



$$FK = R_z(q(1)) * T_z(L(1)) * R_y(q(2)) * T_x(L(2)) * R_y(q(3)) * T_x(L(3)) ;$$

```
[ cos(q2 + q3)*cos(q1), -sin(q1), sin(q2 + q3)*cos(q1), cos(q1)*(d3*cos(q2 + q3) + d2*cos(q2))]
[ cos(q2 + q3)*sin(q1), cos(q1), sin(q2 + q3)*sin(q1), sin(q1)*(d3*cos(q2 + q3) + d2*cos(q2))]
[      -sin(q2 + q3),      0,      cos(q2 + q3),      d1 - d3*sin(q2 + q3) - d2*sin(q2)]
[      0,      0,      0,      1]
```

I used the numeric method to get the jacobian.

Jq1 =

```
[ -sin(q1)*(d3*cos(q2 + q3) + d2*cos(q2)), -cos(q1)*(d3*sin(q2 + q3) + d2*sin(q2)), -d3*sin(q2 + q3)*cos(q1)]
[ cos(q1)*(d3*cos(q2 + q3) + d2*cos(q2)), -sin(q1)*(d3*sin(q2 + q3) + d2*sin(q2)), -d3*sin(q2 + q3)*sin(q1)]
[      0,      -d3*cos(q2 + q3) - d2*cos(q2),      -d3*cos(q2 + q3)]
[      0,      -sin(q1),      -sin(q1)]
[      0,      cos(q1),      cos(q1)]
[      1,      0,      0]
```

2. Joint trajectory $q(t)$ from $q(0) = (0, 0, 0)$ to $q(2) = (2, 3, 4)$ with null initial and final velocities and accelerations. (polynomial)

A polynomial solution for each joint is solved as like in presentation, since it has 6 constrains, is needed a polynomial of fifth solution.

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

$$\begin{bmatrix} \theta_0 \\ \omega_0 \\ \alpha_0 \\ \theta_f \\ \omega_f \\ \alpha_f \end{bmatrix} = \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 & t_0^4 & t_0^5 \\ 0 & 1 & 2t_0 & 3t_0^2 & 4t_0^3 & 5t_0^4 \\ 0 & 0 & 2 & 6t_0 & 12t_0^2 & 20t_0^3 \\ 1 & t_f & t_f^2 & t_f^3 & t_f^4 & t_f^5 \\ 0 & 1 & 2t_f & 3t_f^2 & 4t_f^3 & 5t_f^4 \\ 0 & 0 & 2 & 6t_f & 12t_f^2 & 20t_f^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$$

For the joint 1:

So the coefficients for the polynomial are:

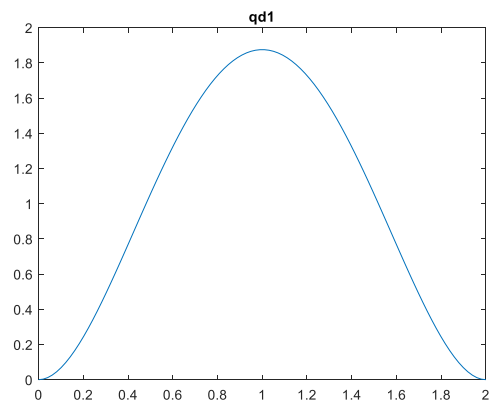
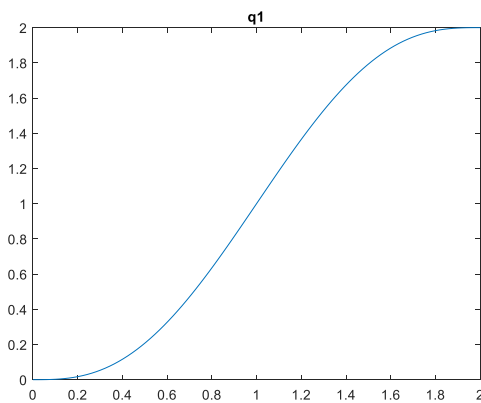
b =

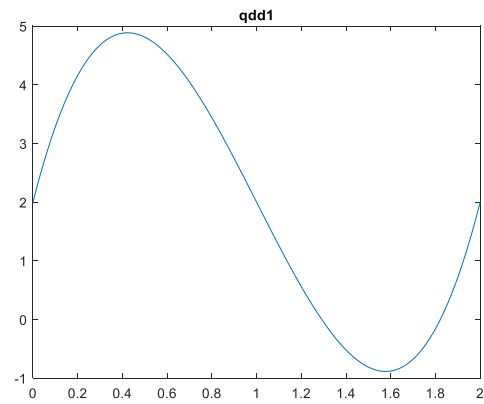
```

0
0.0000
0
2.5000
-1.8750
0.3750

```

$$q1 = (3*t^5)/8 - (15*t^4)/8 + (5*t^3)/2 + t/562949953421312$$

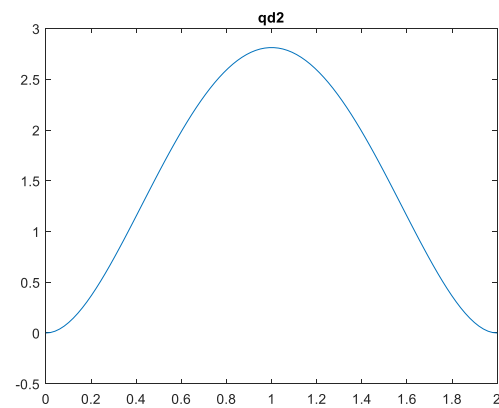
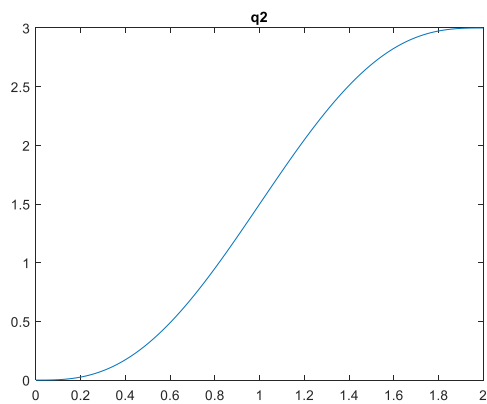


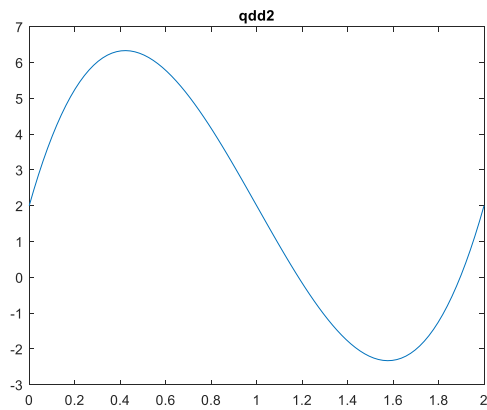


For the joint 2:

```
b =
    0
-0.0000
    0
    3.7500
-2.8125
    0.5625
```

$$q2 = (9 \cdot t^5)/16 - (45 \cdot t^4)/16 + (15 \cdot t^3)/4 - t/562949953421312$$



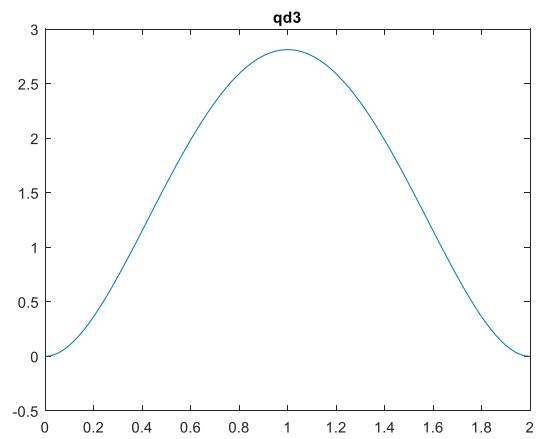
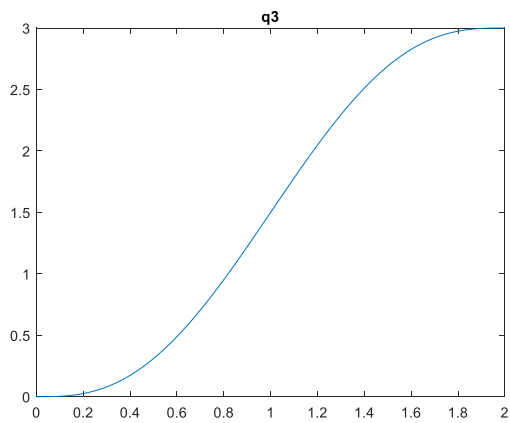


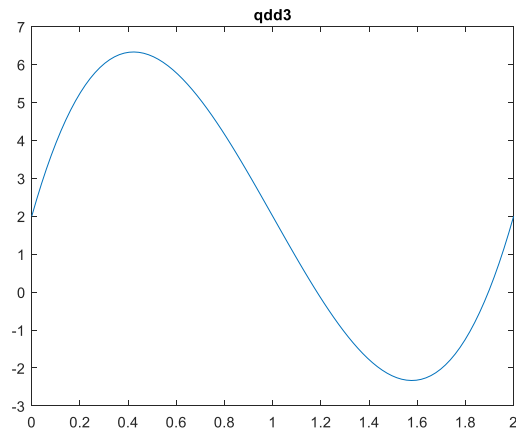
For the joint 3:

$b =$

0
0.0000
0
5.0000
-3.7500
0.7500

$$q3 = \frac{3t^5}{4} - \frac{15t^4}{4} + 5t^3 + \frac{t}{281474976710656}$$





3. Joint trajectory for the following commands: PTP – $q_1 = (0, 0, 0)$ to $q_2 = (2, 3, 4)$ (trapezoidal)

Fist we have to calculate the times “ts” and “tf” for all trapezoidal velocity for each joints, taking into account the controller command interpretation frequency of 10hz.

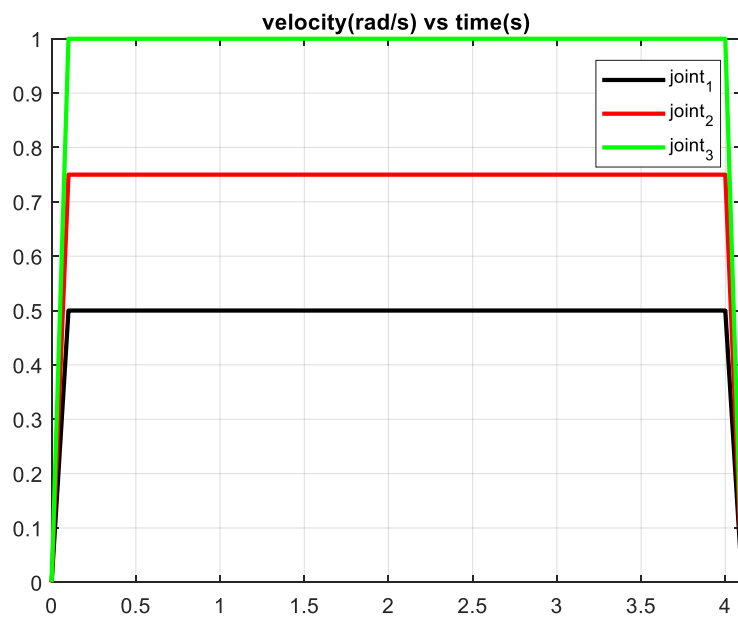
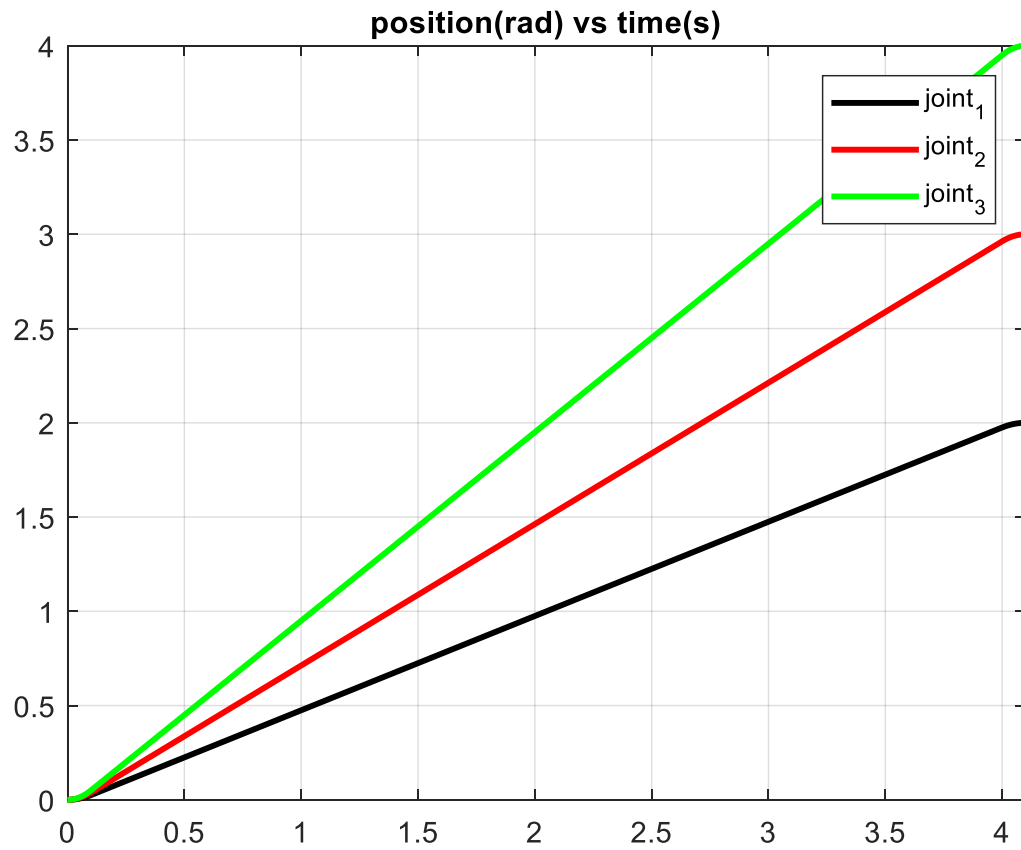
So, the values of “ts” and “tf” for all the joins is :

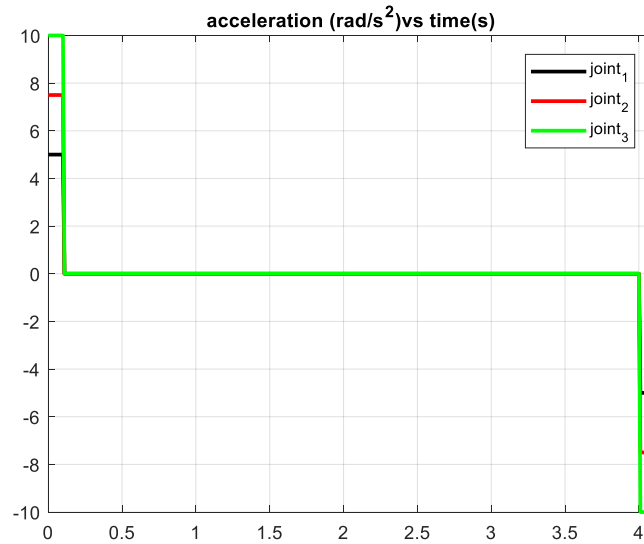
ts_new =	tf_new =
0.1000	4.1000

And the new values of max velocity and acceleration for each joint are:

vmax1_new =	vmax2_new =	vmax3_new =
0.5000	0.7500	1.0000
amax1_new =	amax2_new =	amax3_new =
5.0000	7.5000	10.0000

And the plot of the position, velocity and aceleration are:





4. Joint trajectory for the following commands: LIN – $p_1 = (1, 0, 1)$ to $p_2 = (\sqrt{2}/2, \sqrt{2}/2, 1.2)$ (trapezoidal)

First we have to calculate the joints position for p_1 and p_2 , so we use the inverse kinematics. I develop a function for IK (file named "IK.m").

```
e_p = p_d - p; %is the error in position, destination position - position of this iteration

J = J(1:3,:); %first 3 rows, Jacobian section for joints velocity
Ji = J'/(J*J'+0.1*eye(3)); %add small increment

e = e_p;

q = q + Ji*e;
```

So for point p_1 and p_2 I get the joints position and checked they were correct, with my forward kinematics:

p1 =	p2 =
1	0.7071
0	0.7071
1	1.2000
q_p1 =	q_p2 =
0	0.7854
-1.0472	-1.2331
2.0944	2.0715
p1_ik =	p2_ik =
1.0000	0.7071
0	0.7071
1.0000	1.2000

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% inverse jacobian to convert task space velocities to joint space velocities
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
J = J(1:3,:); %first 3 rows, Jacobian section for joints velocity
J_inv=inv(J);
task_space_max_velocities= [1,1,1]'
q_max_velocities =J_inv* task_space_max_velocities

task_space_max_aceleration= [10,10,10]'
q_max_aceleration =J_inv* task_space_max_aceleration

```