Multichannel states with open and closed channels.

Let us consider a coupled channels system, with N open channels $|i\rangle, |j\rangle$. $i,j = \{1 \dots N\}$, and M closed channels $|k\rangle, |\ell\rangle$; $k, \ell = 1 \dots M$.

We can obtain $M+N$ independent solutions, $|\Psi_j(r)\rangle$, $|\tilde{\Psi}_\ell(r)\rangle$, which for $r \to 0$ behave as: $|\Psi_j(r)\rangle = r^{\ell_j} |j\rangle$ $(j = 1 \dots N)$; $|\tilde{\Psi}_\ell(r)\rangle = r^{\ell_\ell} |\ell\rangle$ $(\ell = 1 \dots M)$.

Asymptotically, for large r, these solutions will behave as:

$$|\Psi_j(r)\rangle = \sum_{i=1}^{N} \frac{A_{ij}^* e^{ik_i r} - A_{ij} e^{-ik_i r}}{2i\sqrt{k_i}\sqrt{k_0}\, r} |i\rangle + \sum_{k=1}^{M} \frac{C_{kj} e^{+q_k r} + D_{kj} e^{-q_k r}}{q_k r} |k\rangle$$

$$|\tilde{\Psi}_\ell(r)\rangle = \sum_{i=1}^{N} \frac{\tilde{A}_{i\ell}^* e^{ik_i r} - \tilde{A}_{i\ell} e^{-ik_i r}}{2i\sqrt{k_i}\sqrt{k_0}\, r} |i\rangle + \sum_{k=1}^{M} \frac{\tilde{C}_{k\ell} e^{+q_k r} + \tilde{D}_{k\ell} e^{-q_k r}}{q_k r} |k\rangle$$

To cure the exponentially growing of the solutions, we take, for each open solution, the following combination:

$$|\Psi_j^C(r)\rangle = |\Psi_j(r)\rangle + \sum_\ell a_{\ell j} |\tilde{\Psi}_\ell(r)\rangle ,$$

where

$$C_{kj} + \sum_\ell a_{\ell j} \tilde{C}_{k\ell} = 0$$

Note that this expresion requires that the $M\times M$ real matrix $\tilde{C}_{kl}$ should have non-zero determinant. This should be the case as the solutions $|\tilde{\psi}_l(r)\rangle$ are independent.

Thus, $$a_{lj} = -\sum_k (\tilde{C})^{-1}_{lk} \cdot C_{kj}$$

With this choice of a_{lj}, the behavior of

$$|\psi_j^c(r)\rangle = r^{l_j}|j\rangle + \sum_l a_{lj} \cdot r^{l_l}|l\rangle \qquad r \ll a$$

so, it is a real combination of one open channel and all closed channels.

For large distances,

$$|\psi_j^c(r)\rangle = \sum_{i=1}^{N} \frac{B^*_{ij} e^{ik_i r} - B_{ij} e^{-ik_i r}}{2i\sqrt{k_i}\sqrt{k_0}\, r}|i\rangle + \sum_{k=1}^{M} \frac{E_{kj} e^{-q_k r}}{q_k r}|k\rangle$$

where

$$B_{ij} = A_{ij} + \sum_{k=1}^{M} a_{kj} \tilde{A}_{ik} \ ; \ E_{kj} = D_{kj} + \sum_{l=1}^{M} a_{lj} \tilde{D}_{kl}$$

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These N "cured" solutions have a norm given by

$$N(i,i') = \sum_j B^*_{ij} B_{i'j}$$

, which is real and allows the Gram-Schmidt orthogonalisation.