Linear Algebra: Intro to Linear Algebra

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Learning outcomes

After this lecture you should be able to:

1. Explain the role of linear algebra in machine learning
2. Explain the concept of "linear combinations" and its connection to equation solving

Linear algebra in machine learning

Linear algebra is used in machine learning as:

- a concise language for expressing and thinking about problems
- an algorithmic tool box of efficient solutions for a certain class of problems

Example: Solving linear regression

Find the parameters of your linear model with:

$$\hat{\beta} = (X^T X)^{-1} X^T y$$

Linear algebra in ML (cont'd.)

An excerpt from the intro to "Neighbourhood Components Analysis" by Goldberger et al:

We estimate such metrics through their inverse square roots, by learning a *linear transformation of the input space such that in the transformed space, KNN performs well*. If we denote the transformation by a matrix A we are effectively learning a metric $Q = A^T A$ such that $d(x, y) = (x - y)^T Q(x - y) = (Ax - Ay)^T (Ax - Ay)$.

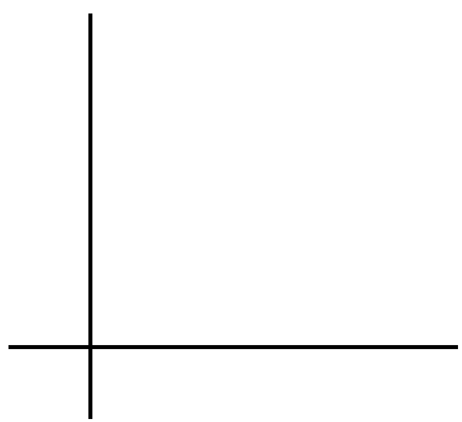
Linear combinations

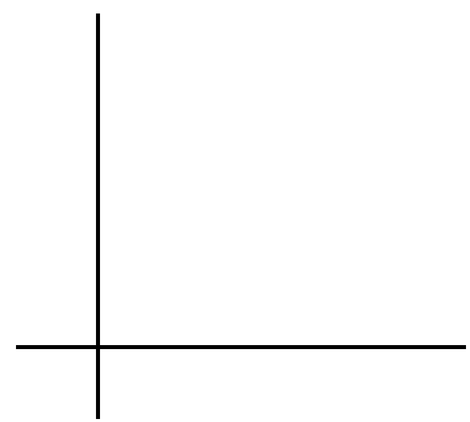
Linear algebra is based on the concept of linear combinations.

Let's look at a couple of vectors:

$\begin{pmatrix}1\\1\end{pmatrix}$ $\begin{pmatrix}2\\3\end{pmatrix}$

Visually:

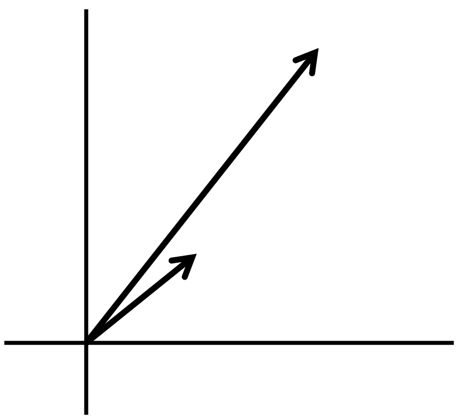




Vector operations

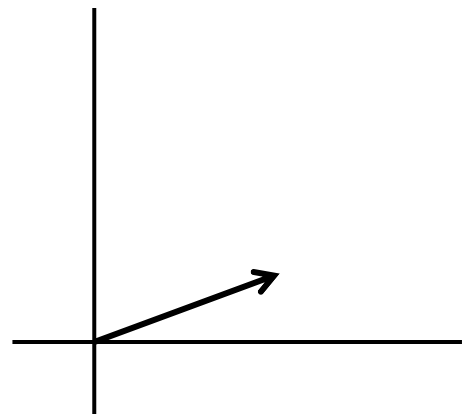
Operations on vectors:

add two vectors



$$\begin{pmatrix}1\\1\end{pmatrix}+\begin{pmatrix}2\\3\end{pmatrix}$$

multiply a vector by a number



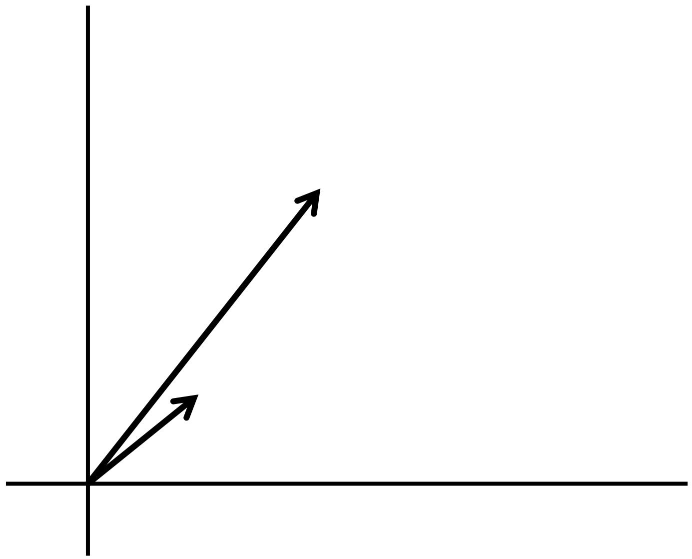
$$c\begin{pmatrix}2\\1\end{pmatrix}$$

Linear combinations of vectors

$$c\begin{pmatrix}1\\1\end{pmatrix} + d\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}c+2d\\c+3d\end{pmatrix}$$

This uses the two operations we have.

This is a linear combination of $\begin{pmatrix}1\\1\end{pmatrix}$ and $\begin{pmatrix}2\\3\end{pmatrix}$.



Exercise

Could you get the blue vector through a linear combination of the black vectors?

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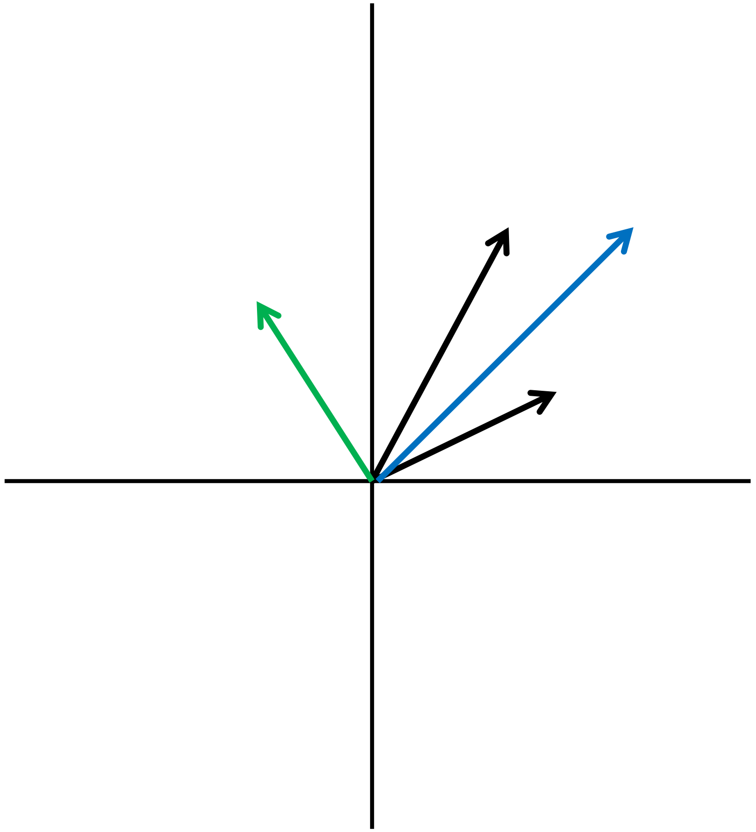
What do you get if you take all combinations of the black vectors?



Equation solving

Linear algebra is used for equation solving

$$\begin{aligned} 2x \ - y &= 0 \\ -x + 2y &= 3 \end{aligned}$$

Can you solve it? How?

The linear algebra view of the equations:

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

matrix of coefficients

vectors

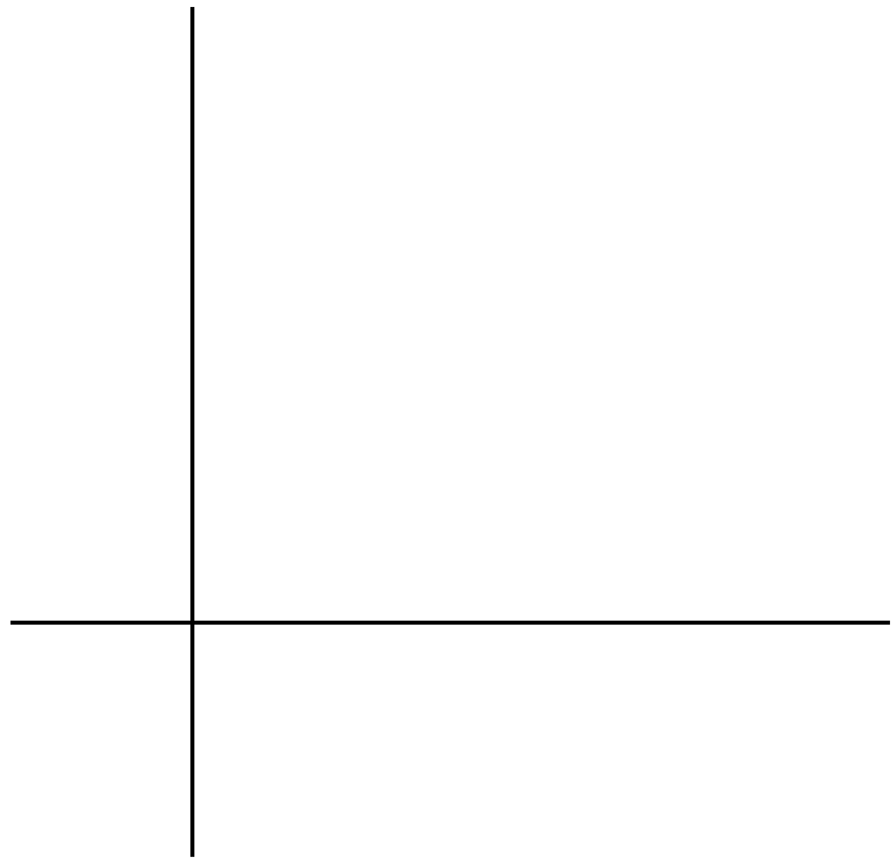
The row view

$$\begin{aligned} 2x\ \ - y &= 0 \\ -x + 2y &= 3 \end{aligned}$$

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

The rows of the matrix correspond
to equations.

What line does the first equation describe?



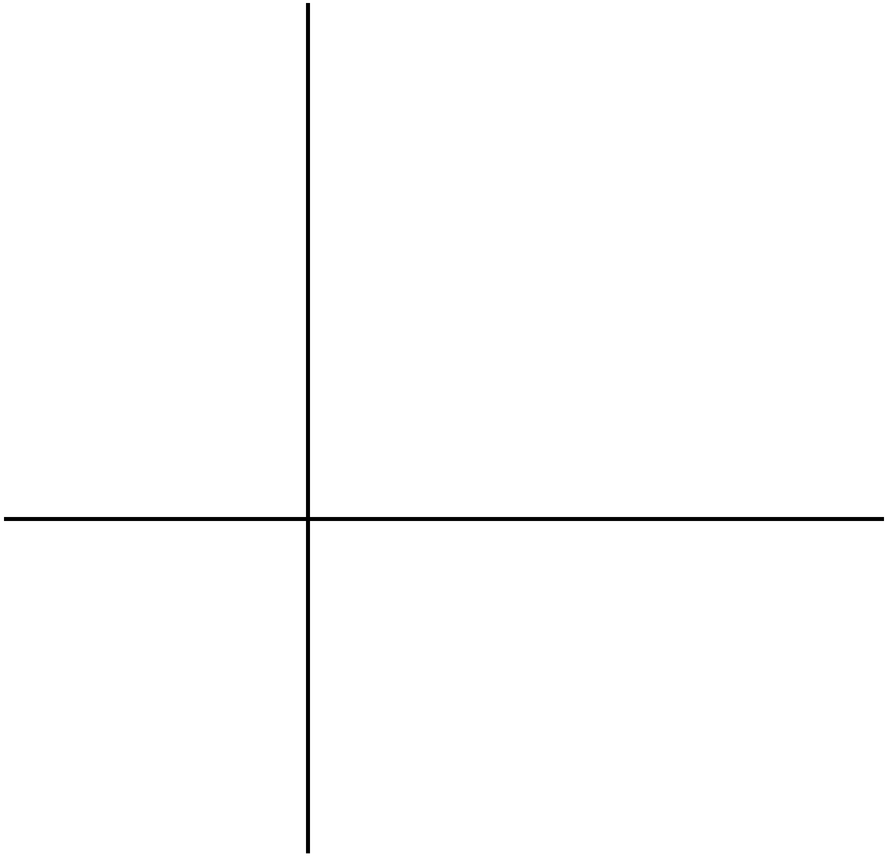
The column view

$$\begin{aligned} 2x \;\; - y &= 0 \\ -x + 2y &= 3 \end{aligned}$$

The equations can be written like this:

$$x\begin{pmatrix} 2 \\ -1 \end{pmatrix} + y\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

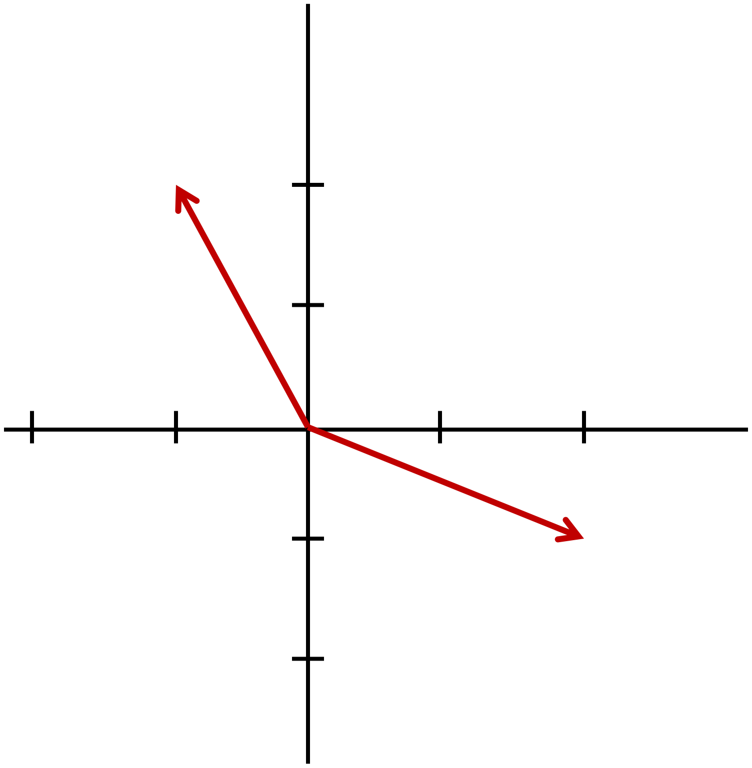
A linear combination of $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$.



Taking all combinations

$$2x \;\; - y = 0$$
$$-x + 2y = 3$$

$$x\begin{pmatrix}2\\-1\end{pmatrix} + y\begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}0\\3\end{pmatrix}$$



What points will you get if you use **all** values of x and y?

Resources

1. Jupyter notebook from author of our text:

   https://github.com/ageron/handson-ml

2. Beezer's Online course on linear algebra

   http://linear.ups.edu/html/fcla.html

3. Hefferon's free text

   http://joshua.smcvt.edu/linearalgebra/book.pdf

4. Khan Academy

Summary

1. Linear algebra is a main math foundation for machine learning.
2. Linear algebra is about taking linear combinations
3. If we write a system of linear equations in matrix form, it's clear the goal is to find the right linear combination of vectors.