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Compact telephoto objectives with zero Petzval sum using varifocal lenses

Jorge Ojeda-Castaneda, Cristian M Gómez-Sarabia* and Sergio Ledesma
*Electronics Department and *Digital Arts, Universidad de Guanajuato*
Campus Salamanca, 36885, Guanajuato, México

Dedicated to Dr A K Gupta

We present two non-conventional telephoto objectives that have zero Petzval sum. These devices employ two or three varifocal lenses, which may be implemented by using a pair of free-form optical elements. Every pair is here denoted as a vortex pair. To our end, we revisit the Gaussian optics of a zoom system having a single varifocal lens. Next, we unveil a telephoto system that uses two varifocal lenses with zero Petzval sum. This system does not require of any axial movement. The optical powers of the varifocal lenses are related by the condition $K_1 = - K_2 = q\, K_0$. These lenses are separated by a distance $d = 1/QK_0$; where q and Q are positive real numbers such that $1 \leq q < Q$. We show that this optical system has extremely low telephoto ratios, $M = 1 - q/Q$. Finally, we describe a 3-lens solution with zero Petzval sum, which uses three varifocal lenses and it requires an axial displacement. This latter solution may be useful for reducing other type of aberration rather than field curvature. We discuss what we believe to be the first formulas representing these imaging devices.

1 Introduction

There is a research trend addressed to the use of free-form elements in optical instruments [1-6]. Some researchers have considered the use of liquid lenses for implementing optical systems with varifocal lenses[7-10]. Some other authors have considered the use of Alvarez–Lohmann lenses [11-15],which are apparently useful for designing nonconventional imaging devices [16-17]. Recently we have discussed the possibility of implementing varifocal lenses, and other type of focalizers, by using a pair of vortex lenses, which are denoted as a vortex pair [18-20].

Here our aim is to present two telephoto objectives, which use two or more varifocal lenses. These novel imaging devices have zero Petzval sum. To our end, in section 2, we review the concept of a vortex pair for setting nonconventional varifocal lenses. In section 3, we revisit and report new formulas that describe a zoom system that uses a single varifocal lens. In section 4, we discuss the Gaussian optics formulas for describing a telephoto objective with zero Petzval sum. Then, in section 5, we present a 3-lens solution for a telephoto objective that has zero Petzval sum. In section 6, we summarize our presentation.

2 Vortex pairs

We start by considering a free-form refractive element whose complex amplitude transmittance is

$$P_1\,(r, \varphi) = \exp\ \{- i\ [N - 1] \left(\frac{t}{\lambda R^2}\right) r^2\ \varphi\}\ \mathrm{circ}\left(\frac{r}{R}\right) \tag{1}$$

In Eq.(1) the letters r and φ are the polar coordinates over the pupil aperture that limits the optical system. We represent the finite size of the pupil aperture by using the circ function, which is equal unity if $r \leq R$. Otherwise, the circ function is equal to zero. The letter N is the value of the refractive index of the material used for building the free-form refractive element. We employ the letter "t" for denoting the maximum thickness of the refractive element; and the Greek letter lambda represents the wavelength of the optical radiation. Next, we consider a second free-form element whose complex amplitude transmittance is the complex conjugate of Eq.(1). That is,

$$P_2\,(r, \varphi) = \exp\ \{- i\ [N - 1] \left(\frac{t}{\lambda R^2}\right) r^2\ \varphi\}\ \mathrm{circ}\left(\frac{r}{R}\right) \tag{2}$$

Corresponding author :
e-mail: jorge_ojedacastaneda@yahoo.com (Jorge Ojeda-Castaneda)

We recognize that the complex amplitude transmittances in Eqs. (1) and (2) have helical variations. Or equivalently, they have vortex variations. Now, it is convenient to combine the two above refractive elements for setting the vortex pair. By introducing an in-plane rotation (say by an angle β) between the elements of the pair, we obtain the following remarkable result

$$\mathrm{P}\,(\mathrm{r},\,\varphi;\,\beta) = \mathrm{P}_1\left(\mathrm{r},\,\varphi + \frac{\beta}{2}\right)\mathrm{P}_2\left((\mathrm{r},\,\varphi - \frac{\beta}{2}\right) = \exp\,\{-\,i\,[N-1]\left(\frac{t\beta}{\lambda R^2}\right)\mathrm{r}^2\}\,\mathrm{circ}\left(\frac{r}{R}\right) \tag{3}$$

It is apparent from Eq.(3) that the angle β controls the maximum value of the optical path difference. And hence the angle β tunes usefully the optical power, or if you will the focal length, of the lens represented by the expression

$$\mathrm{P}\,(\mathrm{r},\,\varphi;\,\beta) = \exp\,\{-\,i\,\frac{r^2}{\lambda}\,K\,(\beta)\}\;\mathrm{circ}\left(\frac{r}{R}\right) = \exp\left\{-\,i\,\frac{r^2}{\lambda f(\beta)}\right\}\mathrm{circ}\left(\frac{r}{R}\right) \tag{4}$$

In Eq.(4) we use the following relationship

$$K\,(\beta) = (N-1)\,\frac{t}{R^2}\,\beta = \frac{1}{f(\beta)} \tag{5}$$

Along this paper, the first lens of our proposed devices has a variable optical power of the form

$$K_1 = (N-1)\,\frac{t}{R^2}\,q = K_0\,q;\; 1 \le q < Q. \tag{6}$$

This simple result is applied next for designing two nonconventional telephoto objectives that have zero Petzval sum. However, before discussing the proposed devices, we emphasize our approach by revisiting a zoom system that uses a single varifocal lens.

3 Zoom systems using a single varifocal lens

In Figs 1 and 2 we depict schematically the image formation process between two conjugated planes, which are separated by a distance T that is here denoted as the throw [21].

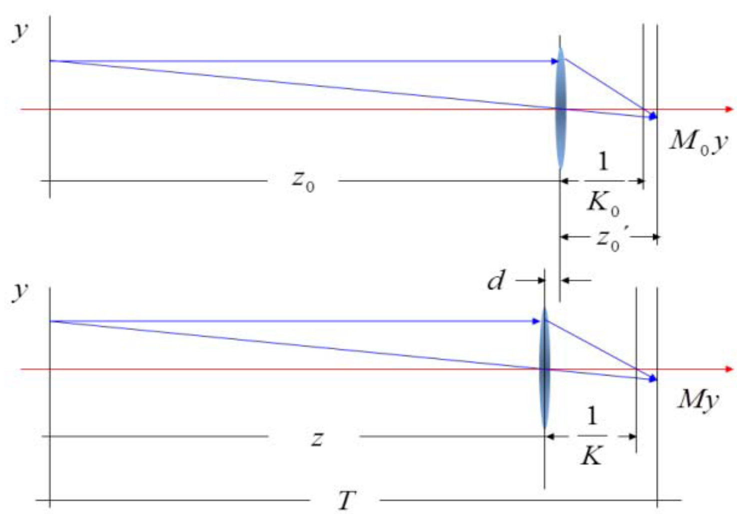

Fig 1. Optical setup for imaging with magnification M, two planes separated by a distance T.

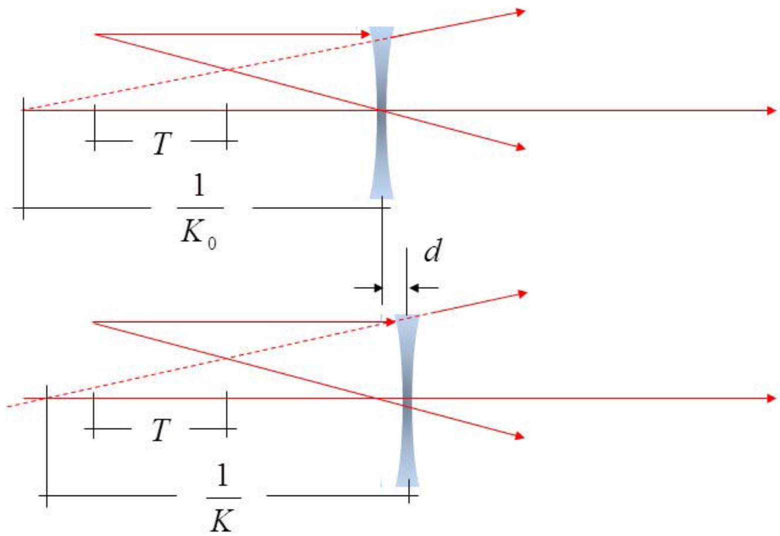

Fig 2. Same as Fig 1, but for a negative lens.

As is indicated in reference [21], the values of the magnification M and the throw T are sufficient for specifying the optical power of the lens, namely

$$K = -\frac{(1-M)^2}{MT} \tag{7}$$

Furthermore, the distance from the input plane to the lens is

$$z = \frac{T}{M-1} \tag{8}$$

For a fixed value of T, we need to place the lens at two different positions (say z_0 and z) for obtaining two different magnifications (say M_0 and M). And accordingly, the optical power of the lens must change from K_0 to K. From Eq. (7) it is straightforward to show that the ratio between two optical powers is

$$\frac{K}{K_0} = -\frac{M_0}{M}\frac{(1-M)^2}{(1-M_0)^2} \tag{9}$$

Hence, if one changes the magnification from M_0 to M then the focal length changes as follows

$$f = -\frac{M}{M_0}\left[\frac{(1-M_0}{(1-M}\right]^2 f_0 \tag{10}$$

In a similar manner, from Eq.(8), we have that the ratio between two positions is

$$\frac{z}{z_0} = \frac{M_0-1}{M-1} \tag{11}$$

Therefore, for changing the magnification from M_0 to M, one needs to move the lens, along the optical axis, by the distance

$$d \equiv -(z-z_0) - \frac{(M_0-M)}{(M-1)} z_0 \tag{12}$$

In other words, for a single lens zoom system, if one wishes to modify the magnification then one needs to modify both the optical power K, and the position of the lens with respect to the input plane z. If the reference magnification is set to $M_0 = -1$, then $z_0 = T/2$ and consequently Eq. (12) becomes

$$d = \frac{(M+1)}{2(M-1)} T \tag{13}$$

In Fig 3 we display the variations of focal length, as well as the variations of the distance d, as a function of the magnification for the optical setup in Fig 1. We note that as the absolute value of the magnification decreases, the focal length decreases monotonically , and the distance from the input plane increase monotonically.

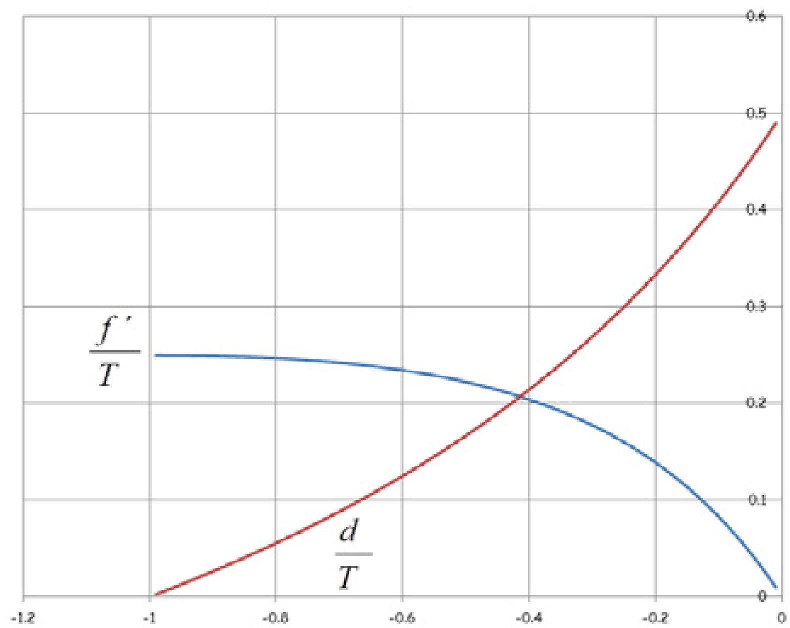

Fig 3. Variations in focal length and distance from the input plane as a function of the magnification.

In what follows we describe first a telephoto objective with zero Petzval sum, which does not require a longitudinal displacement. And then, we describe a 3-lens solution with zero Petzval sum. For this latter solution, two lenses remain at fixed positions, while the third lens requires an axial displacement for preserving the same detection plane.

4 Telephoto without longitudinal displacement and zero Petzval sum

One can obtain a zero Petzval sum if one uses a positive lens, say with optical power K_1, and a negative lens with optical power $K_2 = - K_1$; as is depicted in Fig 4. Here, we use two varifocal lenses, which are separated by a distance

$$d = \frac{1}{QK_0} \tag{14}$$

In Eq. (14) the letter Q is a real positive number greater than unity; and the letter K_0 is the initial value of the positive varifocal lens. In other words, the optical power of the varifocal lens varies linearly as follows

$$K_1 = qK_0 = -K_2 \tag{15}$$

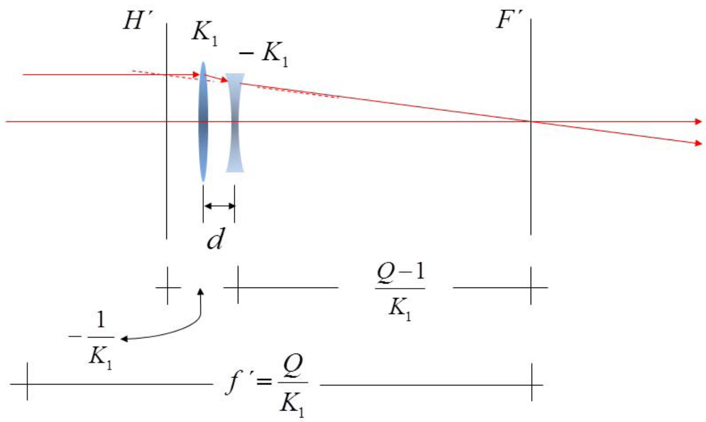

Fig 4. Telephoto system with zero Petzval sum.

In Eq. (15) the letter q denotes a real positive number such that $1 \le q < Q$. The total optical power is

$$K = \frac{(qK_0)^2}{QK_0} = \left(\frac{q^2}{Q}\right) K_0 \tag{16}$$

By Gaussian ray tracing it is straightforward to obtain that the back focal length of this optical system is

$$f_{back} = \frac{Q - q}{q^2\, K_0} \tag{17}$$

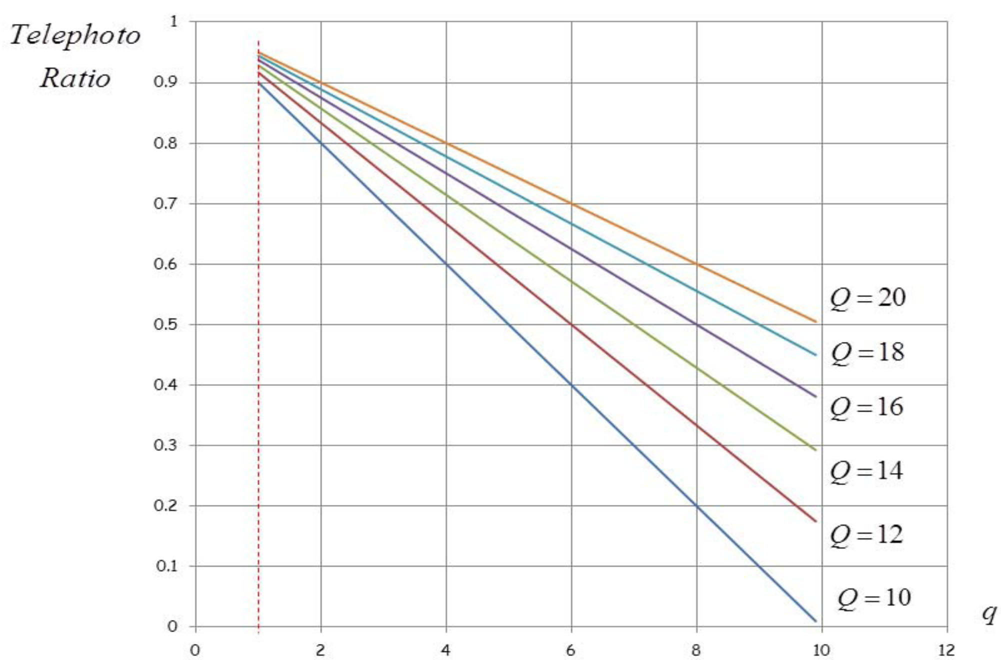

Fig 5. Telephoto ratio as function of q, for several values of Q.

Hence, according to Cox [22], the telephoto ratio is

$$M = \frac{f_{back}}{f'} = 1 - \frac{q}{Q} \tag{18}$$

Here it is relevant to note that that in a conventional telephoto system one changes the separation between the lenses. In our present formulation, this situation is equivalent to q = 1 and varying the parameter Q. Thus, in a conventional telephoto system the telephoto ratio is

$$M = \frac{f_{back}}{f'} = 1 - \frac{1}{Q} \tag{19}$$

In Fig 5 we plot the telephoto ratio; in Eq. (18) as a function of the parameter q, for Q = 10, 12, 14, 16, 18, 20. The classical telephoto ratio, in Eq. (19), is the vertical line located at q = 1 with variable Q.

It is apparent from Eqs.(18) and (19), as well as from Fig 5, that by using varifocal lenses one can obtain a compact device (that is Q > 10) and yet with extremely low values of telephoto ratio.

5 A 3-lens solution for telephoto objective with zero Petzval sum

Now, we consider the use of three varifocal lenses that have zero Petzval sum. As is indicated in Fig 6, this optical objective uses two varifocal lenses for implementing a telescopic device. And then, it requires a third lens that one needs to move along the optical axis by a distance d. The two lenses for the telescopic device are separated by the distance equal to 1/ (Q K_0), where Q is a real and positive number greater than unity, and as before K_0 is the initial optical power of the first lens, whose optical power changes as follows

$$K_1 = q K_0 \tag{20}$$

In Eq.(20) we use the letter q for denoting a real positive number such that $1 \leq q < Q$. Since we set a telescopic system using lens one and lens two, then

$$K_2 = -\frac{qQ}{Q-q} K_0 \tag{21}$$

Next, we recognize that for satisfying Petzval condition the third lens should have an optical power equal to

$$K_3 = -(K_1 + K_2) = \frac{q^2}{Q-q} K_0 \tag{22}$$

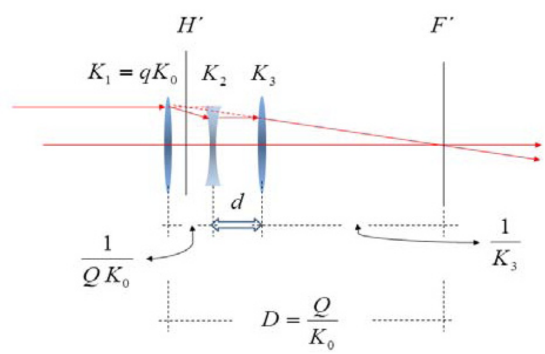

Fig 6. Telephoto objective that uses three varifocal lenses.

The back focal length of this device is $1/K_3$, while the distance from the last lens to the back principal plane is

$$s = -\frac{1}{q\,K_0} \tag{23}$$

Consequently, the equivalent optical power is

$$K = \frac{1}{f} = \frac{q^2\,K_0}{Q} \tag{24}$$

The equivalent optical power is identical to that in Eq. (16). And therefore, the telephoto ratio is the same as that in Eq. (18). However, we believe that this latter solution may be useful for compensating another type of aberration than field curvature. However, this task is beyond our present scope.

6 Conclusions

We have presented two telephoto objectives that are able to generate extremely low telephoto ratios, with zero Petzval sum. For achieving these goals, the proposed telephoto objectives employ two or three varifocal lenses. For clarifying our proposal we have revisited the concept of a vortex pair for setting nonconventional varifocal lenses. Then, we have discussed and reported new formulas describing a zoom system that uses a single varifocal lens.

Our first telephoto objective uses two varifocal lenses with zero Petzval sum. This device does not require of any axial movement. The second telephoto objective is a 3-lens solution that has zero Petzval sum. In this latter solution two varifocal lenses are employed for setting a telecentric subsystem, and a third lens is used for focalizing an incoming plane wave. This proposal may be useful for reducing other type of aberration than field curvature. We have reported first formulas representing the propsed imaging devices.

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