Serie geométrica.

$$\boxed{\begin{aligned} &\sum_{k=0}^{n-1} a r^k = \frac{a(1-r^n)}{1-r} \qquad r \neq 1 \\ &\sum_{k=0}^{\infty} a r^k = \frac{a}{1-r} \qquad |r| < 1 \end{aligned}}$$

Transformada z → contraparte Laplace para tiempo discreto

Laplace → ec integro-diff en algebraicas

z → ec. diferenciales a algebraicas

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}};$$

bilateral

$z = re^{j\Omega}$

forma polar variable compleja

$s = \sigma + j\omega$

$s = re^{j\theta}$; $r = \sqrt{\sigma^2 + \omega^2}$

$\theta = \tan^{-1}\left(\frac{\omega}{\sigma}\right)$

$$\boxed{X(z) = \sum_{n=0}^{\infty} x[n] z^{-n}}$$

unilateral

$x[n] = 0 \ ; \ n < 0 \quad ; \quad X(z) = \mathcal{Z}\{x[n]\}$

Region de convergencia:

Ej: $x[n] = a^n u[n] \qquad a \in \mathbb{R}$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{\infty} a^n u[n] z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^n \ ; \quad \sum_{n=0}^{\infty} |az^{-1}|^n < \infty \ ?$$

Por serie geométrica: $\displaystyle\sum_{n=0}^{\infty} ar^n = a \frac{1}{1-r}$

$|r| < 1.$

$$\boxed{\sum_{n=0}^{\infty} (az^{-1})^n = \frac{1}{1-az^{-1}}} \ ; \quad |az^{-1}| < 1. \qquad \left|\frac{a}{z}\right| < 1 \qquad \boxed{|z| > a}$$

$$X(z) = \frac{z}{z - zaz^{-1}} = \frac{z}{z-a} \rightarrow$$ cero en $z=0$
polo en $z=a$

Ej: $x[n] = -a^n u[-n-1]$; $u[-n-1] = \begin{cases} 1 & n \leq -1 \\ 0 & n \geq -1. \end{cases}$

$-n-1 \geq 0$

$n \leq -1.$

$$X(z) = \sum_{n=-\infty}^{\infty} -a^n u[-n-1] z^{-n}$$

$$X(z) = -\sum_{n=-\infty}^{-1} a^n z^{-n} = -\sum_{n=1}^{\infty} a^{-n} z^{n} = -\sum_{n=1}^{\infty} (a^{-1}z)^n$$

$$\boxed{\sum_{n=+a}^{b} c^n = \sum_{n=-b}^{-a} c^{-n}}$$

$n = 0$ $\quad (a^{-1}z)^0 = 1$

$$X(z) = 1 - \sum_{n=0}^{\infty} (a^{-1}z)^n = 1 - \frac{1}{1-a^{-1}z}$$

$|a^{-1}z| < 1$, $\left|\frac{z}{a}\right| < 1.$ $\quad |z| < |a|.$

$$X(z) = \frac{1 - a^{-1}z - 1}{1 - a^{-1}z} = \frac{-a^{-1}z}{1-a^{-1}z}$$

$$X(z) = \frac{-z}{a-z} = \frac{z}{z-a} \quad |z| < |a|$$

Ej: $\delta[n-m_0]$.

$$X(z) = \sum_{n=-\infty}^{\infty} \delta[n-m_0] z^{-n} ; \quad n' = n - m_0$$
$$n = n' + m_0.$$

$$X(z) = \sum_{n'=-\infty}^{\infty} \delta[n'] z^{-(n'+m_0)} = \sum_{n'=-\infty}^{\infty} \delta[n'] z^{-n'} z^{-m_0}.$$

$$X(z) = z^{-m_0} \sum_{n=-\infty}^{\infty} \delta[n'] z^{-n'} = z^{-m_0}$$

Ej: $x[n] = \{4, 5, \underset{\uparrow}{2}, 1, 2\}$.

$$x[n] = 4\delta[n+2] + 5\delta[n+1] + 2\delta[n] + \delta[n-1] + 2\delta[n-2].$$

$$X(z) = 4z^{+2} + 5z + 2 + z^{-1} + 2z^{-2}.$$

$z \to$ operador de adelanto

$z^{-1} \to$ operador de retraso.

Transformada Inversa Z.

$$x[n] = Z^{-1}\{X(z)\} = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} \, dz$$

↗ difícil.

Opción 2:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$; expansión series de potencia.

Opción 3: fracciones parciales

$$X(z) = K \frac{\prod_{c=1}^{M} (z - a_c)}{\prod_{p=1}^{N} (z - b_p)}$$

$$\frac{X(z)}{z} = \frac{C_0}{z} + \frac{C_1}{z - b_1} + \cdots + \frac{C_N}{z - b_N}$$

$$X(z) = C_0 + C_1 \frac{z}{z - b_1} + \cdots + C_N \frac{z}{z - b_N}$$