Mathematical elements in the blind separation signals

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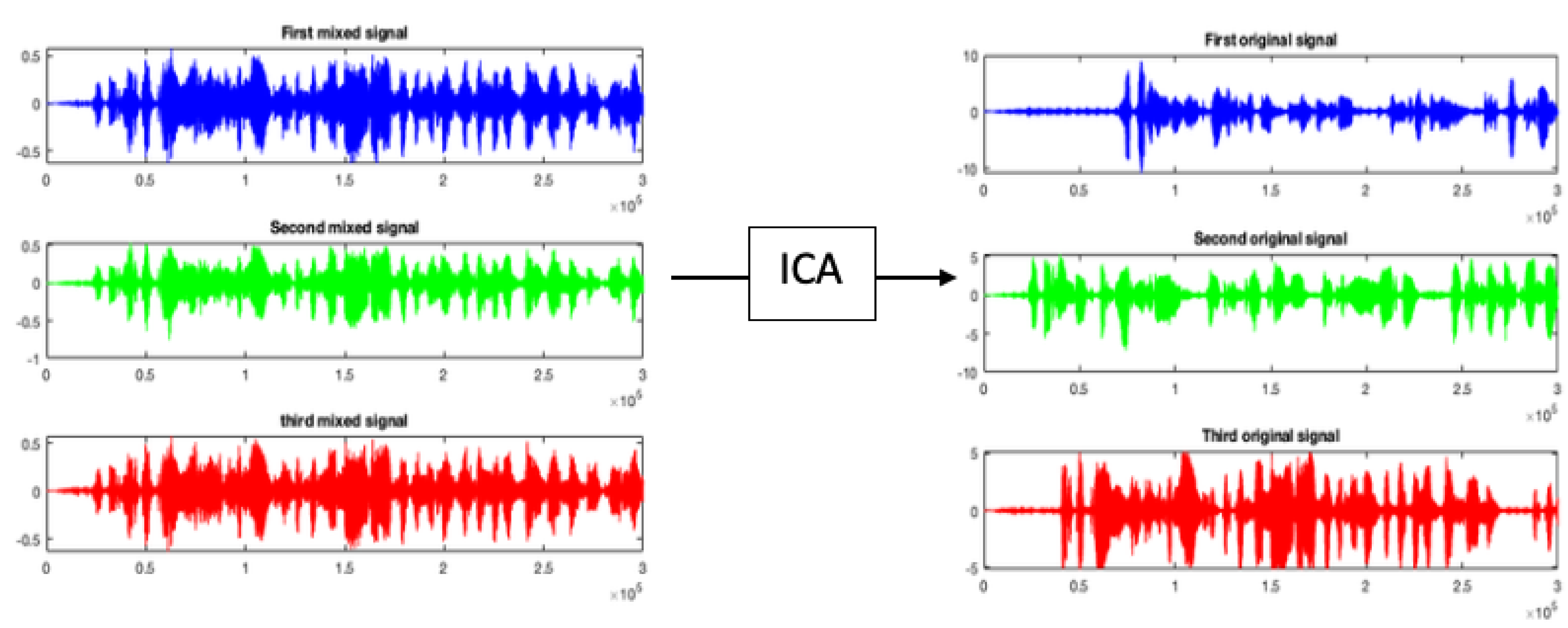
Introduction

When we are having a conversation in a noisy environment, our brain has the capacity to separate sounds and focus its attention on an only conversation. This behavior is known as the cocktail party problem and can be replicated through independent component analysis. This method gets independent and non-Gaussian mixed signals, starting from sensors, and using mathematical and statistical elements, like the negentropy and the kurtosis it can separate those signals. In the following images we can see that on the left there are three mixed voice signals and on the right can be seen the estimated signals of each one of the original ones.

Figure 1: ICA method



For the validation of the method we use some examples on Matlab, with images and sounds. The aim is to find the original signals, starting from the statistical data of the mixed signals, using the ICA method and the fixed-point algorithm (Figure 2).

Mathematical formulation

Let $\mathbf{x} = [x_1, x_2, .., x_m]^T$ a vector of m components that are observed, we assume that this vector is produced as a linear combination of m independent and non-Gaussian signals, denoted by $\mathbf{s} = [s_1, s_2, .., s_m]^T$ and $\mathbf{A}_{m\times m}$ a mixing matrix, as follows:

$$\mathbf{x} = \mathbf{A}\mathbf{s} \tag{1}$$

The aim is to find signals $\mathbf{s}$, but we do not know the $\mathbf{A}$ matrix, for this we estimate the unmixing matrix $\mathbf{W} \approx \mathbf{A}^{-1}$, such that.

$$\mathbf{y} = \mathbf{W}\mathbf{x} \quad \text{with} \quad \mathbf{y} \approx \mathbf{s} \tag{2}$$

For this estimation we need the data to have zero mean and unit variance, for this we use the centralization and whitening process of the data using the convariance matrix. By the Central limit theorem, we know that the sum of independent and identically distributed (i.i.d.) random variables are more Gaussian that each of them. Using this we can conclude that $\mathbf{x}$ has tendency to be more Gaussian. To separate the signals we need to minimize the Gaussianity of $\mathbf{x}$, for this we use the negentropy which is a measure of non-Gaussianity, that is defined for our problem, as:

$$J(\mathbf{w}) = E\left\{G\left(\mathbf{w}^T\mathbf{x}\right)\right\} - E\left\{G(\mathbf{v})\right\} \tag{3}$$

Where V is a Gaussian variable of zero mean and unit variance, and G is a nonquadratic function, called Contrast Function. While data is more approximated to a Gaussian distribution their separation, to independent components, is more difficult. For this we want to maximize the non-Gaussianity of the data, but with the constraint that these are not correlated. Some authors propose the following maximization problem.

$$\max \sum_{i=1}^{n} J(\mathbf{w}_i) \tag{4}$$

$$s.t \quad E\left\{(\mathbf{y}_i)(\mathbf{y}_j))\right\} = \delta_{ij} \tag{5}$$

In the literature there are many algorithms that solve this kind of problem. For this project we use the fixed-point algorithm, that uses the Kuhn–Tucker conditions and solve the problem through Newton's Method, as follows:

$$\mathbf{w}^+ = E\left\{\mathbf{x}g(\mathbf{w}^T\mathbf{x})\right\} - E\left\{g'(\mathbf{w}^T\mathbf{x})\right\}\mathbf{w} \tag{6}$$

Where $g = G'$. To improve the stability of the solution, we can normalize the $\mathbf{w}^+$ vector.

$$\mathbf{w}^* = \mathbf{w}^+ / \left\|\mathbf{w}^+\right\| \tag{7}$$

Results

Using the follow algorithm we can separate the voice signals of the Figure 1, and also use this algorithm to separate the images of the Figure 3 and the results that we obtain are on the Figure 4.

1: Centered the data to make its mean zero
2: Whiten the data to give x
3: Choose an initial (eg.random) vector $\mathbf{w}$ of unit norm
4: Let $\mathbf{w} \leftarrow E\{\mathbf{x}g(\mathbf{w}^T\mathbf{x})\} - E\{g'(\mathbf{w}^T\mathbf{x})\}\mathbf{w}$, where g is Contrast function
5: Let $\mathbf{w} \leftarrow \mathbf{w}/\|\mathbf{w}\|$
6: If not converged, go back to step 4

Figure 2: ICA method

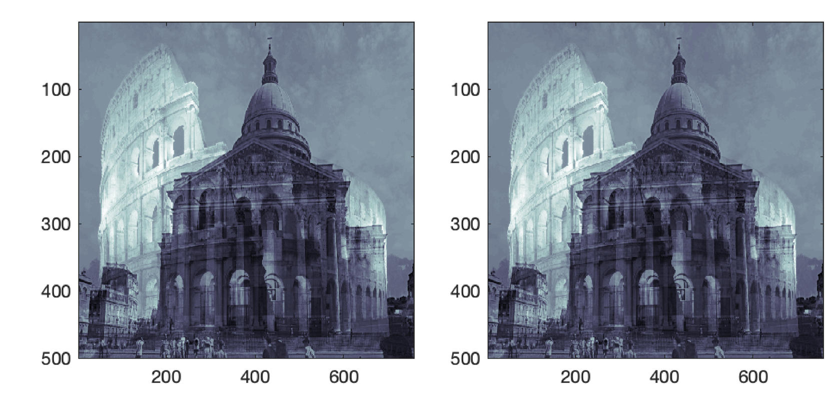


Figure 3: Mixed images

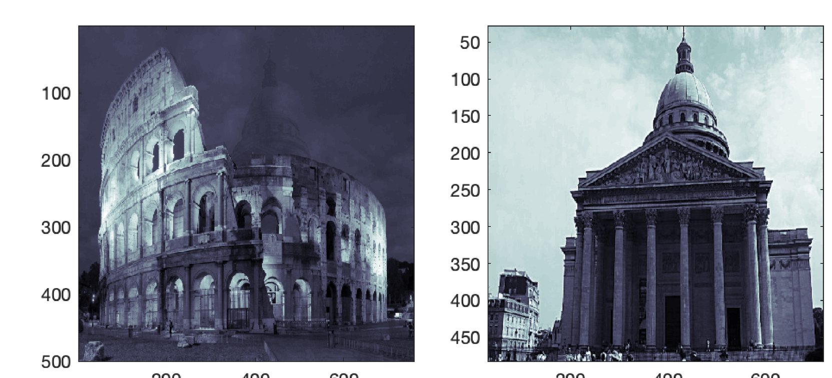


Figure 4: Estimated images

Conclusions and future work

The ICA method has a lot of applications in the signals processing and it is a powerful math and statistic tool in the blind separation signal (BSS), when we showed examples in the section of results, we could see that , in the estimation signals, results were optimum in all the examples independently of the different contrast function that we used in each example. In the literature there are many algorithms that solve the optimization problem of nongaussianity, which use different techniques to solve this problem. In this research we use the fixed-point algorithm and the results that we obtained with this were satisfactory, moreover the development of this algorithm is not complicated.

In the ICA formulation we use some mathematical elements like matrix operations, a numerical method for solving the optimization problem using Khun-Tucker conditions and Newton's method, some basic statistical elements, like the means vector, the covariance matrix, whitening and centralize of the data and finally we did a study about the negentropy definition, because it was a new concept. Is important to said that over the last years ICA has been evolving and improving such that now it is a fundamental method in the BSS.

In a future research we want use the ICA method as a tool that allows to study the brain activity using the electrocardiography and some results of the literature that have been given using wavelets.

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