Econometrics

Time Series Brief Extension

Jose Angel Garcia Sanchez

Université Paris 1 Panthéon-Sorbonne
jagarsanc@gmail.com

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Outline

1. Beyond Classical Methods
   - ARIMA Models
   - VAR Models
   - Volatility Models
   - State Space Models
   - Machine Learning for Time Series
   - Structural Breaks and Regime Switching
   - Summary and Further Reading

What Comes Next?

- This extension provides a brief overview of modern developments beyond the classical methods covered in this course
- These are generalities to guide further study
- Topics covered: ARIMA, VAR, ARCH/GARCH, State Space, Machine Learning, Structural Breaks
- Each topic would require a full course for rigorous treatment

ARIMA Models

- **ARIMA**: AutoRegressive Integrated Moving Average
- ARIMA(p, d, q) combines three components to model a wide range of time series behaviors
- General form:

$$(1 - L)^d Y_t = c + \phi(L) Y_t + \theta(L)\varepsilon_t$$

- p = AR order (number of autoregressive lags)
- d = differencing order (integration parameter)
- q = MA order (moving average of past errors)
- Model selection via AIC/BIC criteria
- Box-Jenkins methodology for model identification

Stata: `arima`, `sarima` commands

ARIMA Extensions

- **SARIMA**: Seasonal ARIMA
  - ARIMA$(p, d, q)(P, D, Q)_s$ adds seasonal AR and MA terms at lag s
  - Example: quarterly data with $s = 4$
- **ARIMAX**: ARIMA with exogenous regressors
  - Includes external variables X_t to improve forecasts
- **ARFIMA**: Fractional ARIMA
  - Allows $d \in \mathbb{R}$ (not just integers)
  - For long-memory processes with slow autocorrelation decay

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VAR Models

- **VAR**: Vector AutoRegression
- Extends univariate analysis to systems of equations
- Each variable is regressed on lagged values of itself and all other variables
- VAR(p) specification:

$$Y_t = c + A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + \varepsilon_t$$

- $Y_t = k \times 1$ vector of endogenous variables
- $A_i = k \times k$ coefficient matrices
- $\varepsilon_t =$ white noise vector with covariance matrix Σ

Stata: `var`, `svar`, `vec` commands

Classic references: Sims (1980), Johansen (1988)

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VAR Applications and Extensions

Key applications:

- *Impulse Response Functions*: how shocks propagate through the system
- *Variance Decomposition*: contribution of each shock to forecast error variance
- *Granger Causality*: testing predictive relationships between variables
- *Forecasting*: joint prediction of multiple series

Extensions:

- **SVAR**: Structural VAR imposes economic theory via identifying restrictions
- **VECM**: Vector Error Correction Model for cointegrated systems (uses Johansen test)
- **Bayesian VAR**: shrinkage priors help with high-dimensional systems

ARCH Model

- Financial returns exhibit *volatility clustering*: large changes tend to follow large changes
- ARCH/GARCH models capture this time-varying conditional variance
- **ARCH**(q): AutoRegressive Conditional Heteroskedasticity
- Engle (1982) - Nobel Prize 2003

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$$

- Variance depends on past squared residuals
- Captures how recent shocks affect current volatility
- Constraint: $\omega > 0$, $\alpha_i \geq 0$

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GARCH Model

- **GARCH**(p, q): Generalized ARCH
- Bollerslev (1986)

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

- Adds lagged variance terms to the ARCH model
- More parsimonious than high-order ARCH
- GARCH(1,1) is often sufficient in practice
- Persistence: $\alpha_1 + \beta_1$ close to 1 indicates high persistence

Stata: `arch` command

Essential for: risk management, option pricing, Value-at-Risk (VaR) estimation

GARCH Extensions

- **EGARCH**: Exponential GARCH
  - Asymmetric effects: bad news increases volatility more than good news
  - This is called the *leverage effect*
- **GJR-GARCH**: Threshold GARCH
  - Separate coefficients for positive and negative shocks
- **GARCH-M**: GARCH-in-Mean
  - Volatility enters the mean equation
  - Risk premium varies with volatility
- **Multivariate GARCH**: DCC, BEKK models
  - For volatility spillovers and dynamic correlations across assets

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State Space Models

- A general framework that nests ARIMA, regression, and many other models
- Distinguishes between **observed** measurements and **unobserved** states

General formulation:

- Observation equation: $y_t = Z_t \alpha_t + \varepsilon_t$
- State equation: $\alpha_{t+1} = T_t \alpha_t + R_t \eta_t$

Where:

- α_t = unobserved state vector (trend, cycle, seasonal, etc.)
- y_t = observed data
- Z, T, R = system matrices (can be time-varying)

Stata: `sspace` command

Reference: Harvey (1989) *Forecasting, Structural Time Series, and the Kalman Filter*

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The Kalman Filter

- Recursive algorithm that provides optimal estimates of unobserved states
- Steps at each time t:
  - (1) Predict state and its variance
  - (2) Compute forecast error
  - (3) Update estimate with new observation
  - (4) Repeat for $t + 1$

Applications:

- *Unobserved Components*: decompose into trend + cycle + seasonal + irregular
- *Time-Varying Parameters*: regression coefficients that evolve over time
- *Missing Data*: natural handling of irregular observations
- *Mixed Frequencies*: combine monthly indicators with quarterly GDP

Machine Learning for Time Series

- Modern ML methods complement traditional econometrics
- Especially useful for *high-dimensional* problems and *nonlinear* patterns
- Key challenge: respecting temporal structure (no random train/test splits!)

Main approaches:

- **Deep Learning**: LSTM, GRU (recurrent networks with memory), Transformers (attention mechanisms), N-BEATS/N-HiTS (purpose-built for forecasting)
- **Tree-Based**: Random Forests, XGBoost, LightGBM with lagged features
- **Hybrid**: Prophet (trend + seasonality), ensemble methods combining ARIMA + ML

Python: `scikit-learn`, `pytorch-forecasting`, `darts`, `statsforecast`

ML: Important Considerations

- **Validation**: use time-based splits (rolling window, expanding window)
    - Never use random cross-validation for time series!
- **Interpretability**: black-box models are harder to explain
    - Trade-off between accuracy and interpretability
- **Stationarity**: still matters for many ML methods
    - Differencing often helps ML models too
- **Sample size**: deep learning needs large datasets
    - Classical methods often better for small samples
- **Feature engineering**: critical for tree-based methods
    - Lags, rolling statistics, calendar effects

M-competitions show that hybrid methods often perform best.

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Structural Breaks

- Economic relationships are not always stable
- Policy changes, crises, and regime shifts require models that can detect *parameter instability*
- **Structural breaks**: permanent, discrete changes at unknown dates

Tests for structural breaks:

- **Chow Test**: known break date; F-test for parameter equality before/after
- **Bai-Perron**: unknown break dates; test and estimate multiple breaks
- **CUSUM/CUSUMSQ**: recursive residuals detect gradual instability

Stata: `estat sbsingle`

References: Perron (1989), Bai & Perron (1998)

Regime Switching Models

- **Regime switching**: recurrent changes driven by latent (unobserved) states
- Unlike structural breaks, regimes can recur

Main models:

- **Markov-Switching**: Hamilton (1989)
  - Parameters depend on unobserved regime $S_t \in \{1, 2, \ldots, k\}$
  - Transitions follow a Markov chain with probabilities $P(S_t = j | S_{t-1} = i)$
- **Threshold Models**: TAR, STAR
  - Regime depends on an observable threshold variable

Applications: business cycles (expansion/recession), bull/bear markets, policy regimes
Stata: `mswitch` command

Example: The Great Moderation

- US GDP volatility dropped significantly around 1984
- Researchers debate the interpretation:

Two views:

- **Structural break**: one-time permanent shift in volatility
  - Better monetary policy, inventory management, or luck
- **Regime-switching**: economy switches between high and low volatility regimes
  - Could return to high volatility

The 2008 financial crisis suggests that regimes may indeed recur, supporting the regime-switching interpretation.

Summary

Key takeaways:

- **ARIMA**: benchmark for univariate forecasting; handles non-stationarity via differencing
- **VAR**: captures interdependencies between multiple series; impulse responses and Granger causality
- **GARCH**: essential for financial volatility modeling; captures volatility clustering
- **State Space**: unifying framework; Kalman filter for unobserved components
- **Machine Learning**: complements but doesn't replace theory; validation is crucial
- **Structural Breaks**: test for parameter stability; distinguish breaks from regime switches

Software:

- Stata: `arima`, `var`, `arch`, `sspace`, `mswitch`
- R: `forecast`, `vars`, `rugarch`, `dlm`
- Python: `statsmodels`, `arch`, `darts`

Recommended Textbooks

Introductory:

- Hamilton (1994) *Time Series Analysis*
- Enders (2014) *Applied Econometric Time Series*

Advanced:

- Lütkepohl (2005) *New Introduction to Multiple Time Series Analysis*
- Durbin & Koopman (2012) *Time Series Analysis by State Space Methods*

Modern/ML:

- Hyndman & Athanasopoulos *Forecasting: Principles and Practice* (free online)
- Shumway & Stoffer *Time Series Analysis and Its Applications: With R Examples*

These topics build on the foundations covered in this course. Each deserves deeper study for practical application.