

# 14.4 Monte Carlo simulation of 2-d U(1) lattice gauge theory

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A U(1) variable at point  $x$  in the  $\mu$  direction is denoted by  $U_\mu(x)$ . For U(1), the lattice gauge action reads

$$S[U] = \beta \sum_x \sum_{\mu < \nu} \text{Re} [1 - U_{\mu\nu}(x)], \quad (1)$$

where  $\beta = \frac{2}{e^2}$  and  $e$  being the gauge coupling. The plaquettes are defined by

$$U_{\mu\nu}(x) = U_\mu(x)U_\nu(x + \hat{\mu})U_\mu^*(x + \hat{\mu} + \hat{\nu})U_\nu^*(x) \quad (2)$$

The sum of plaquette variables is computed using

$$S_P = \sum_x \sum_{\mu < \nu} \text{Re} U_{\mu\nu}(x), \quad (3)$$

so the expectation value of an elementary Wilson loop or plaquette reads

$$E_P = \frac{\langle S_P \rangle}{DV}, \quad (4)$$

where  $V = L^d$  is the volume of the lattice and  $D = \frac{d(d-1)}{2}$  is the number of planes of rotation in  $d$  dimensions.

We define the sum of staples as

$$\Sigma_\mu(x) = \sum_{\nu \neq \mu} [U_\nu(x)U_\mu(x + \hat{\nu})U_\nu^*(x + \hat{\mu}) + U_\nu^*(x - \hat{\nu})U_\mu(x - \hat{\nu})U_\nu(x + \hat{\mu} - \hat{\nu})]. \quad (5)$$

The change in the action by a local update when changing  $U_\mu(x) \rightarrow U'_\mu(x)$  is

$$\Delta S = -\beta \text{Re} [(U'_\mu(x) - U_\mu(x)) \Sigma_\mu^*]. \quad (6)$$

We use the Metropolis algorithm for the simulation as decribed in the following steps:

1. Given some gauge field configuration, we go through all the lattice points in a lexicographic way. At each point  $x$  in the  $\mu$  direction we generate a random U(1) variable  $U'_\mu(x)$ .
2. We compute the sum of the staples and compute the change of the action according to eq. (6). We generate a uniform random number  $r \in [0, 1)$  and accept the change if  $r \leq p$ , where  $p$  is

$$p = \min(1, \exp(-\Delta S)). \quad (7)$$

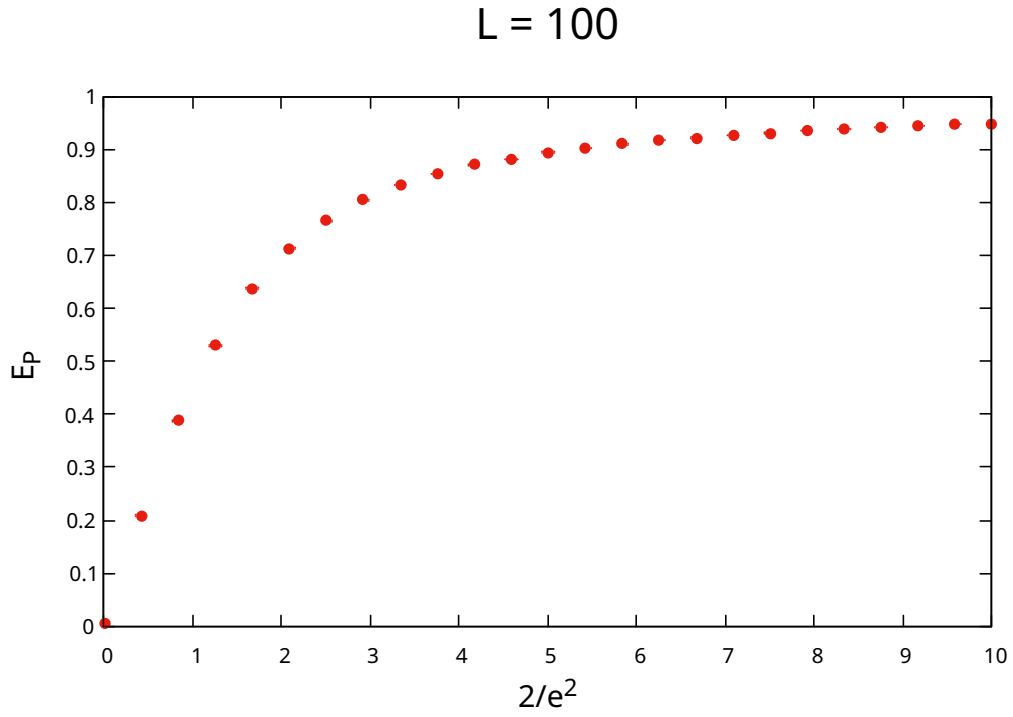


Figure 1: Expectation value of the plaquette variable vs.  $\beta = 2/e^2$  with a volume  $V = 20^2$ . The errors were computed using the Jackknife error. The parameters of the simulation were: at each value of  $\beta$ , 5000 measurements were made separated by 20 sweeps and a thermalization of 1000 sweeps.