The profile of non-standard cosmic strings

José Antonio García Hernández

Instituto de Ciencias Nucleares, UNAM

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Topological defects

Topological defect: Singularity of the field that cannot be removed without being altered at large distances.

Stable.

Topological defects in a system can be described by analyzing the topology of the vacuum manifold $\mathcal{M} = G/H$.

We study the homotopy group of the vacuum manifold $\pi_n(\mathcal{M})$.

$\pi_n(\mathcal{M})$: contains information on how an n-dimensional *loop* or closed surface cannot be contracted to a point.

If $\pi_n(\mathcal{M}) \neq I \Rightarrow$ topological defects.

If $\pi_n(\mathcal{M}) = \mathbb{Z} \Rightarrow$ topological charge or *winding number*.

There exist several examples of topological defects in condensed matter systems.

In the physics of the early universe they are still hypothetical.

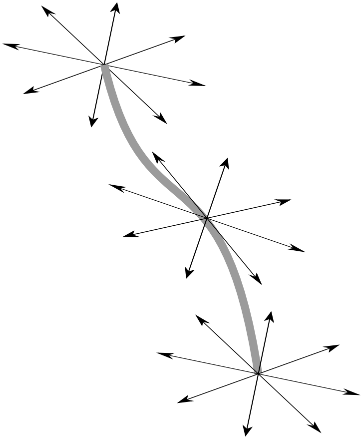
Kibble Mechanism

It is generally assumed that phase transitions occurred in the early universe at the late stages of inflation. These transitions could have formed topological defects.

Examples:

- $\pi_0(\mathcal{M}) \neq I \Rightarrow$ Domain wall
- $\pi_1(\mathcal{M}) \neq I \Rightarrow$ Vortex (2d), cosmic strings (3d)
- $\pi_2(\mathcal{M}) \neq I \Rightarrow$ Monopole

Cosmic Strings



2-dimensional vortices stacked on top of each other, forming a cosmic string in three dimensions

$U(1)_{B-L}$ exact local symmetry

In the Standard Model $U(1)_{B-L}$ is an **exact** global symmetry. B baryon number, L lepton number.

This is strange, an exact symmetry is only natural when it is local.

Gauge symmetry

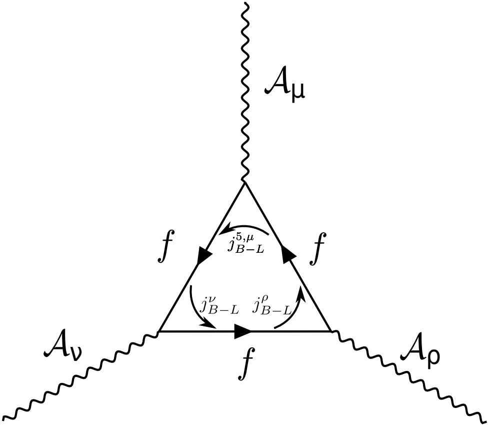
We promote $\mathrm{U}(1)_{B-L}$ to a local symmetry and combine it with $\mathrm{U}(1)_Y$.

We introduce a new gauge coupling h' and define a new charge as

$$Y' \equiv 2hY + \frac{h'}{2}(B - L).$$

We take the gauge group to be $\mathrm{U}(1)_{Y'}$ and we call the gauge field $\mathcal{A}_\mu$.

Gauge Anomaly



In each vertex the quarks of one generation contribute with $B = 4$, and leptons with $L = 3$.

$B - L \neq 0$.

Gauge anomaly. It is cured by adding a ν_R $(L = 1)$ to each generation.

Since the neutrinos have mass, it is natural to introduce a mechanism to give mass to the neutrinos.

We can give mass to the neutrino with a Dirac mass term

$$f_\nu \left[\bar{\nu}_R \begin{pmatrix} -\Phi_0 & \Phi_+ \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + \begin{pmatrix} \bar{\nu}_L & \bar{e}_L \end{pmatrix} \begin{pmatrix} -\Phi_0^* \\ \Phi_+^* \end{pmatrix} \nu_R \right],$$

where f_ν is a Yukawa coupling.

Usually we can also give mass to the right-handed neutrinos with a Majorana mass term

$$M(\bar{\nu}_R + \nu_R^T C)(\nu_R + C\bar{\nu}_R^T).$$

However, in our scenario, this term is forbidden since it breaks the $\mathrm{U}(1)_{Y'}$ gauge symmetry.

To add a mass term solely for ν_R, independently of ν_L, we add a new Higgs field $\chi \in \mathbb{C}$

$$f_{\nu_R} \nu_R^T \chi \nu_R + \text{c.c.},$$

where f_{ν_R} is a Yukawa coupling.

To preserve gauge invariance, the field χ must have a charge $B - L = 2$.

We generate the Majorana mass with the Higgs mechanism using the new Higgs field $\chi \in \mathbb{C}$.

We denote the vacuum expectation value of χ as v'.

χ gives a Majorana mass to the right-handed neutrino $M = f_{\nu_R} v'$.

χ is added to the Lagrangian with the potential

$$V' = \frac{m'^2}{2}\chi^*\chi + \frac{\lambda'}{4}(\chi^*\chi)^2.$$

It is natural to include

$$\frac{\kappa}{2}\Phi^\dagger\Phi\chi^*\chi.$$

We assume that $v' \gg v$ and $f_{\nu_R} \simeq O(1)$ in order to give a large mass to the right-handed neutrino.

Lagrangian

$$
\begin{aligned}
\mathcal{L} \;=\;& \frac{1}{2}(D^{\mu}\Phi)^{\dagger}D_{\mu}\Phi - \frac{m^2}{2}\Phi^{\dagger}\Phi - \frac{\lambda}{4}(\Phi^{\dagger}\Phi)^2 - \frac{\lambda}{4}v^4 \\
&+\frac{1}{2}(D^{\mu}\chi)^{*}D_{\mu}\chi - \frac{m'^2}{2}\chi^{*}\chi - \frac{\lambda'}{4}(\chi^{*}\chi)^2 - \frac{\lambda'}{4}v'^4 \\
&-\frac{\kappa}{2}\Phi^{\dagger}\Phi\chi^{*}\chi - \frac{\kappa}{2}v^2v'^2 - \frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu},
\end{aligned}
$$

- $\Phi = (\phi_{+}, \phi_0)^{\mathsf{T}} \in \mathbb{C}^2$
- $D_{\mu}\Phi = (\partial_{\mu} + ih\mathcal{A}_{\mu})\Phi$
- $D_{\mu}\chi = (\partial_{\mu} + ih'\mathcal{A}_{\mu})\chi$
- $\mathcal{F}_{\mu\nu} = \partial_{\mu}\mathcal{A}_{\nu} - \partial_{\nu}\mathcal{A}_{\mu}$

For the potential to be bounded from below, we need

$$\lambda > 0, \quad \lambda' > 0, \quad \kappa^2 < \lambda\lambda',$$

and for spontaneous symmetry breaking to occur

$$m^2 = -\kappa v'^2 - \lambda v^2 < 0,$$
$$m'^2 = -\kappa v^2 - \lambda' v'^2 < 0.$$

Equations of motion

$$
\begin{aligned}
D^\mu D_\mu \Phi &= -m^2\Phi - \lambda(\Phi^\dagger\Phi)\Phi - \kappa\Phi\chi^*\chi \\
D^\mu D_\mu \chi &= -m'^2\chi - \lambda'(\chi^*\chi)\chi - \kappa\chi\Phi^\dagger\Phi \\
\partial^\lambda \mathcal{F}_{\lambda\nu} &= -\frac{ih}{2}\left[(D_\nu\Phi)^\dagger\Phi - \Phi^\dagger(D_\nu\Phi)\right] \\
&\quad -\frac{ih'}{2}\left[(D_\nu\chi)^*\chi - \chi^*(D_\nu\chi)\right]
\end{aligned}
$$

Ansatz

The Lagrangian has a $\mathrm{U}(1)_{Y'}$ symmetry which can "spontaneously break" down to I.

$\mathcal{M} = \mathrm{U}(1)_{Y'}/I = \mathrm{U}(1) \Rightarrow \pi_1(\mathrm{U}(1)) = \mathbb{Z} \Rightarrow$ cosmic strings.

We only consider the component ϕ_0 of the Higgs field Φ. Cylindrically symmetric ansatz

$$\begin{aligned} \phi_0(r,\varphi) &= \phi(r)e^{in\varphi} \\ \chi(r,\varphi) &= \xi(r)e^{in'\varphi} \\ \mathcal{A}(r) &= \frac{a(r)}{r}\hat{\varphi}. \end{aligned}$$

Equations of motion

$$\partial_r^2\phi + \frac{1}{r}\partial_r\phi - \frac{(n+ha)^2}{r^2}\phi - m^2\phi - \lambda\phi^3 - \kappa\phi\xi^2 = 0$$

$$\partial_r^2\xi + \frac{1}{r}\partial_r\xi - \frac{(n'+h'a)^2}{r^2}\xi - m'^2\xi - \lambda'\xi^3 - \kappa\xi\phi^2 = 0$$

$$\partial_r^2 a - \frac{1}{r}\partial_r a - h(n+ha)\phi^2 - h'(n'+h'a)\xi^2 = 0.$$

Boundary conditions

$$\phi(0) = 0, \qquad \lim_{r\to\infty}\phi(r) = v$$

$$\xi(0) = 0, \qquad \lim_{r\to\infty}\xi(r) = v'$$

$$a(0) = 0, \quad \lim_{r\to\infty} a(r) = -\frac{n}{h} = -\frac{n'}{h'}.$$

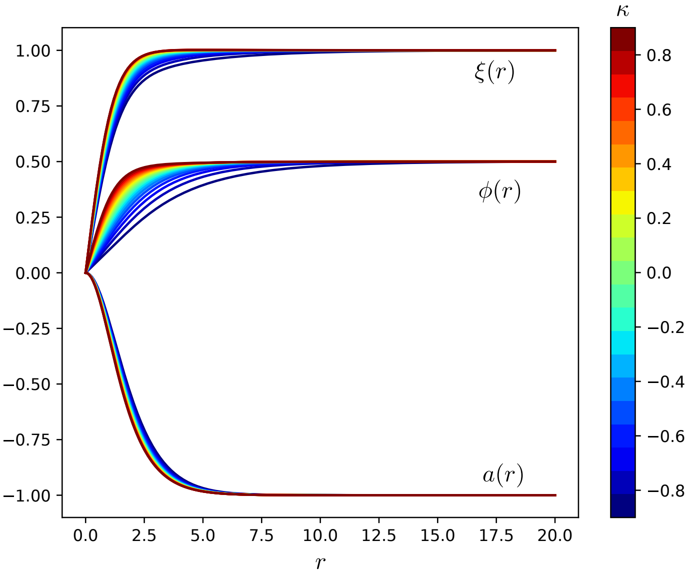
Boundary value problem, numerical solutions with the damped Newton method.

Solutions uniquely defined by inserting v, v', λ, λ', h, h', n and n'.

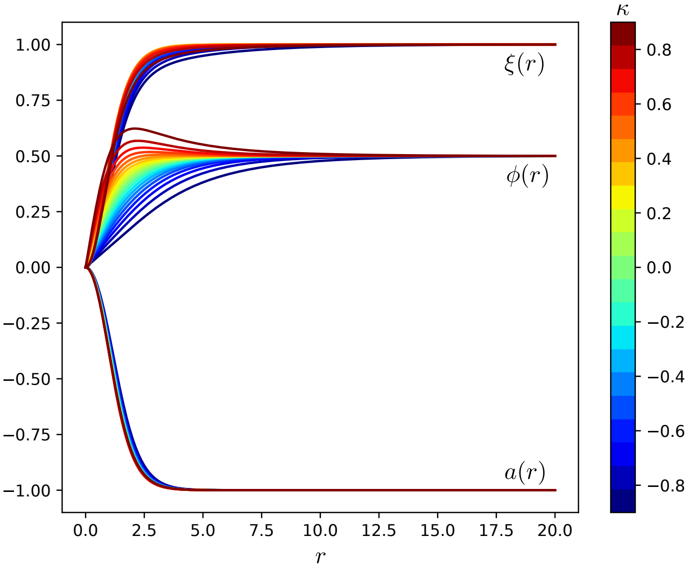
We choose $v' \gg v$.

$v = 246$ GeV is used to convert all dim'less variables to physical units. We display the profile radius r in units of

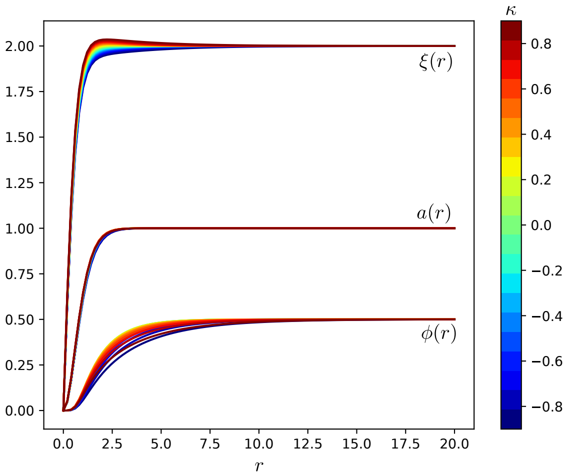
$$v_{\text{dim'less}} \cdot 0.0008 \text{ fm}.$$



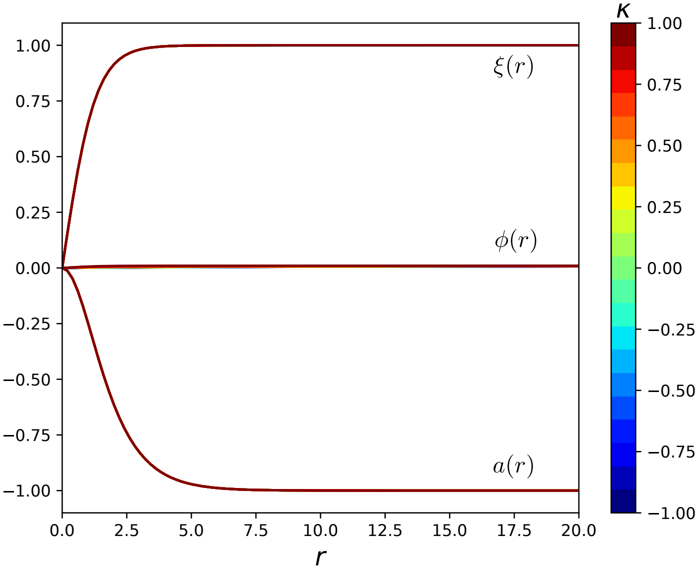
$$v = 0.5,\ v' = 1,\ n = n' = h = h' = \lambda = \lambda' = 1.$$



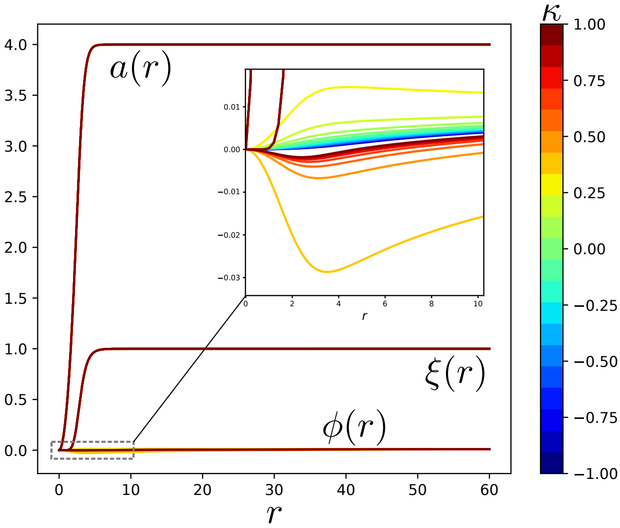
$$v = 0.5,\ v' = 1,\ n = 1,\ n' = 2,\ h = 1,\ h' = 2,$$
$$\lambda = \lambda' = 1.$$



$v = 0.5,\ v' = 2,\ n = -5,\ n' = -1,\ h = 5,\ h' = 1,$
$\lambda = \lambda' = 1.$ This is an example from the SO(10) GUT [Buchmüller/Greub/Minkowski, '91].



$$n = h = n' = h' = \lambda = \lambda' = 1,\ \ v = 0.01,\ \ v' = 1$$



Coaxial string solution, cf. [Bogomol'nyi, 1975] with $n = -2,\ h = 0.5,\ n' = 10,\ h' = -2.5,\ \lambda = 1,\ \lambda' = 1,\ v = 0.01,\ v' = 1.$

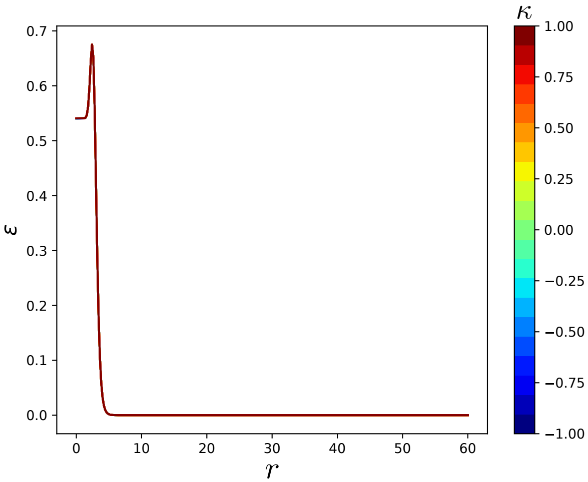


Figure: Energy density, $v = 0.01$, $v' = 1$, $n = -2$, $n' = 10$, $h = 0.5$, $h' = -2.5$, $\lambda = \lambda' = 1$, in units of 4.79×10^{19} GeV/fm^3.

By integrating the energy density we find that the string tension is of the order of

$$\mu \sim 10^{10}\ \text{GeV}^2 = 10^{25}\ \frac{\text{kg}}{\text{pc}}.$$

Therefore

$$G\mu \sim 10^{-28},$$

where $G = \frac{1}{(1.2\times 10^{19}\ \text{GeV})^2}$.

The LIGO/Virgo collaboration set constraints to the string tension

$$G\mu \lesssim 4 \times 10^{-15}.$$

Summary

In this BSM model, we added

- A new gauge coupling h'
- A right-handed neutrino ν_R
- A new Higgs field $\chi \in \mathbb{C}$

A non-standard type of cosmic strings is possible.

Overshoot and coaxial string solutions.

At large distances, they do not affect known physics.

Not observed but detectable, in principle. Like gravitational wave detection, gravitational lensing, CMB anisotropies etc.

No contradictions with SM physics, motivated from the exactness of $\mathrm{U}(1)_{B-L}$.

The end

Can I have my master's now?

Measurements of the CMB power spectrum

A short time after the Big Bang the universe was filled with a plasm of baryons, leptons and photons.

When the universe was $\sim 300,000$ years old, the first atoms were formed.

Since atoms are electrically neutral, photons decoupled from matter. A process known as *recombination*.

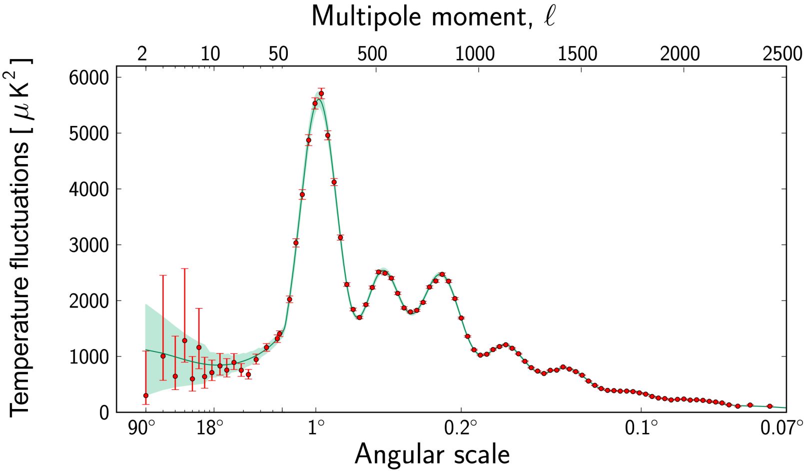
This radiation is called the CMB.

If cosmic strings exist they would have a distinctive footprint in the CMB *power spectrum*.

Photons passing near a cosmic string produce discontinuities in the CMB of the order of

$$\frac{\Delta T}{T} = 8\pi G\mu\beta,$$

where β is the transverse velocity of the string and T is the average temperature and ΔT is the magnitude of temperature fluctuations.


Multipole moment, ℓ
2
10
50
500
1000
1500
2000
2500
Temperature fluctuations [μK^2]
6000
5000
4000
3000
2000
1000
0
90°
18°
1°
0.2°
0.1°
0.07°
Angular scale

According to the measurments obtained by PLANCK, the constriction to the string tension is

$$G\mu \lesssim 1.49 \times 10^{-7}.$$

Gravitational lensing

Energy momentum tensor of a static cosmic string

$$T^{\mu\nu} = \mu\delta(x)\delta(y)\,\mathrm{diag}(1, 0, 0, -1).$$

Far from the core the line element reads

$$ds^2 = dt^2 - dr'^2 - r'^2 d\varphi'^2 - dz^2,$$

where

$$(1-8G\mu\log(r/r_0))r^2 = (1-8G\mu)r'^2,\ \varphi' = (1-4G\mu)\varphi.$$

Here $r \in [0, \infty]$, $\varphi \in [0, 2\pi)$ and r_0 is a constant.

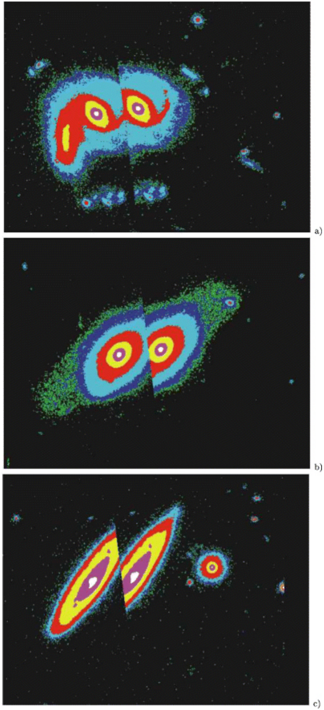
This implies an angular deficit

$$\Delta\varphi = \varphi_{\mathrm{max}} - \varphi'_{\mathrm{max}} = 2\pi - 2\pi(1 - 4G\mu) = 8\pi G\mu.$$

A cosmic string produces a double image of an object behind it separated by the angle

$$\alpha = \frac{l_1}{l_2}\Delta\varphi \sin\theta,$$

assuming $G\mu \ll 1$, where l_1 is the distance between the string and the observer, l_2 is the distance between the object and the string and θ is the angle that the string makes with the plane perpendicular between the observer and the object.


a)
b)
c)

Gravitational waves

One of the most realistic ways of possible detection of cosmic string is by their emission of gravitational radiation.

According to General Relativity, an oscillating *loop* of a cosmic string looses energy with power

$$P = \Gamma G \mu^2,$$

where Γ is a constant.

This *loops* have a characteristic emission of gravitational waves produced by *kinks* and *cusps*.

Kinks are discontinuities in $\dot{x}^\mu$.

Cusps are pointy regions on the string.

Cusps produce signals beamed in the direction of the cusp.

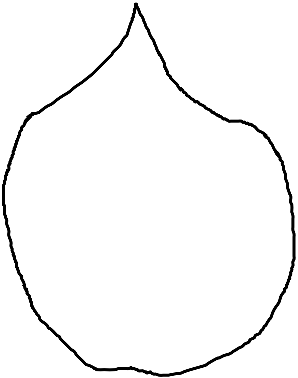


Figure: A loop with a cusp. Near the cusp the speeds of the right and left modes are the speed of light.