The profile of non-standard cosmic strings

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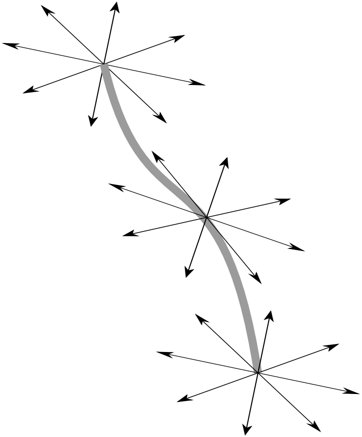
Kibble Mechanism

It is generally assumed that phase transitions occurred in the early universe at the late stages of inflation. These transitions could have formed topological defects.

Examples:

- $\pi_0(\mathcal{M}) \neq I \Rightarrow$ Domain wall
- $\pi_1(\mathcal{M}) \neq I \Rightarrow$ Vortex (2d), cosmic strings (3d)
- $\pi_2(\mathcal{M}) \neq I \Rightarrow$ Monopole

Cosmic Strings



2-dimensional vortices stacked on top of each other, forming a cosmic string in three dimensions

$\mathrm{U}(1)_{B-L}$ exact local symmetry

In the Standard Model $\mathrm{U}(1)_{B-L}$ is an **exact** global symmetry. B baryon number, L lepton number.

This is strange, an exact symmetry is only natural when it is local.

Gauge symmetry

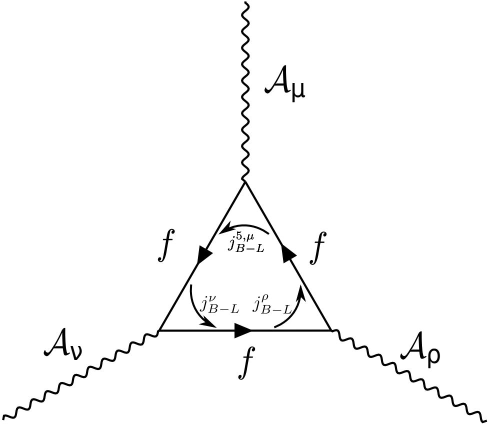
We promote $\mathrm{U}(1)_{B-L}$ to a local symmetry and combine it with $\mathrm{U}(1)_Y$.

We introduce a new gauge coupling h' and define a new charge as

$$Y' \equiv 2hY + \frac{h'}{2}(B-L).$$

We take the gauge group to be $\mathrm{U}(1)_{Y'}$ and we call the gauge field $\mathcal{A}_\mu$.

Gauge Anomaly



In each vertex the quarks of one generation contribute with $B = 4$, and leptons with $L = 3$. $B - L \neq 0$.

Gauge anomaly. It is cured by adding a ν_R $(L = 1)$ to each generation.

Since the neutrinos have mass, it is natural to introduce a mechanism to give mass to the neutrinos.

To add a mass term solely for ν_R, independently of ν_L, we add a new Higgs field $\chi \in \mathbb{C}$

$$f_{\nu_R} \nu_R^T \chi \nu_R + \text{c.c.},$$

where f_{ν_R} is a Yukawa coupling.

To preserve gauge invariance, the field χ must have a charge $B - L = 2$.

We generate the Majorana mass with the Higgs mechanism using the new Higgs field $\chi \in \mathbb{C}$.

We denote the vacuum expectation value of χ as v'.

χ gives a Majorana mass to the right-handed neutrino $M = f_{\nu_R} v'$.

χ is added to the Lagrangian with the potential

$$V' = \frac{m'^2}{2}\chi^*\chi + \frac{\lambda'}{4}(\chi^*\chi)^2.$$

It is natural to include

$$\frac{\kappa}{2}\Phi^\dagger\Phi\chi^*\chi.$$

We assume that $v' \gg v$ and $f_{\nu_R} \simeq O(1)$ in order to give a large mass to the right-handed neutrino.

Lagrangian

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{2}(D^\mu\Phi)^\dagger D_\mu\Phi - \frac{m^2}{2}\Phi^\dagger\Phi - \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 - \frac{\lambda}{4}v^4 \\
& + \frac{1}{2}(D^\mu\chi)^* D_\mu\chi - \frac{m'^2}{2}\chi^*\chi - \frac{\lambda'}{4}(\chi^*\chi)^2 - \frac{\lambda'}{4}v'^4 \\
& - \frac{\kappa}{2}\Phi^\dagger\Phi\chi^*\chi - \frac{\kappa}{2}v^2v'^2 - \frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu},
\end{aligned}
$$

- $\Phi = (\phi_+, \phi_0)^\mathsf{T} \in \mathbb{C}^2$
- $D_\mu\Phi = (\partial_\mu + ih\mathcal{A}_\mu)\Phi$
- $D_\mu\chi = (\partial_\mu + ih'\mathcal{A}_\mu)\chi$
- $\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu$

For the potential to be bounded from below, we need

$$\lambda > 0, \quad \lambda' > 0, \quad \kappa^2 < \lambda\lambda',$$

and for spontaneous symmetry breaking to occur

$$m^2 = -\kappa v'^2 - \lambda v^2 < 0,$$
$$m'^2 = -\kappa v^2 - \lambda' v'^2 < 0.$$

Equations of motion

$$
\begin{aligned}
D^\mu D_\mu \Phi &= -m^2\Phi - \lambda(\Phi^\dagger\Phi)\Phi - \kappa\Phi\chi^*\chi \\
D^\mu D_\mu \chi &= -m'^2\chi - \lambda'(\chi^*\chi)\chi - \kappa\chi\Phi^\dagger\Phi \\
\partial^\lambda \mathcal{F}_{\lambda\nu} &= -\frac{ih}{2}\left[(D_\nu\Phi)^\dagger\Phi - \Phi^\dagger(D_\nu\Phi)\right] \\
&\quad -\frac{ih'}{2}\left[(D_\nu\chi)^*\chi - \chi^*(D_\nu\chi)\right]
\end{aligned}
$$

Ansatz

The Lagrangian has a $\mathrm{U}(1)_{Y'}$ symmetry which can "spontaneously break" down to I.

$\mathcal{M} = \mathrm{U}(1)_{Y'}/I = \mathrm{U}(1) \Rightarrow \pi_1(\mathrm{U}(1)) = \mathbb{Z} \Rightarrow$ cosmic strings.

We only consider the component ϕ_0 of the Higgs field Φ. Cylindrically symmetric ansatz

$$\begin{aligned}
\phi_0(r,\varphi) &= \phi(r)e^{in\varphi} \\
\chi(r,\varphi) &= \xi(r)e^{in'\varphi} \\
\mathcal{A}(r) &= \frac{a(r)}{r}\hat{\varphi}.
\end{aligned}$$

Equations of motion

$$\partial_r^2\phi + \frac{1}{r}\partial_r\phi - \frac{(n+ha)^2}{r^2}\phi - m^2\phi - \lambda\phi^3 - \kappa\phi\xi^2 = 0$$

$$\partial_r^2\xi + \frac{1}{r}\partial_r\xi - \frac{(n'+h'a)^2}{r^2}\xi - m'^2\xi - \lambda'\xi^3 - \kappa\xi\phi^2 = 0$$

$$\partial_r^2 a - \frac{1}{r}\partial_r a - h(n+ha)\phi^2 - h'(n'+h'a)\xi^2 = 0.$$

Boundary conditions

$$\phi(0) = 0, \qquad \lim_{r\to\infty}\phi(r) = v$$

$$\xi(0) = 0, \qquad \lim_{r\to\infty}\xi(r) = v'$$

$$a(0) = 0, \quad \lim_{r\to\infty} a(r) = -\frac{n}{h} = -\frac{n'}{h'}.$$

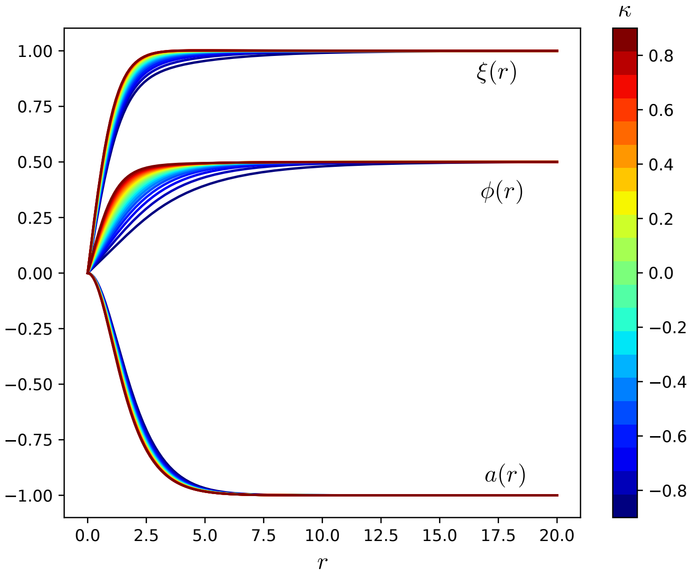
Boundary value problem, numerical solutions with the damped Newton method.

Solutions uniquely defined by inserting v, v', λ, λ', h, h', n and n'.

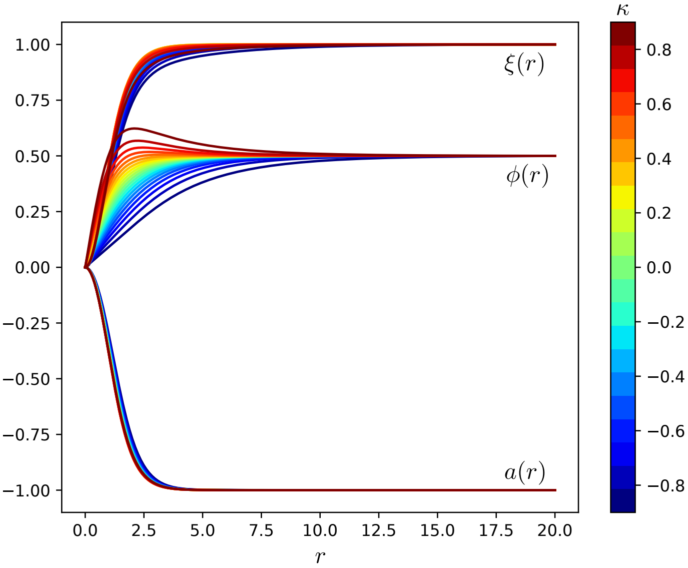
We choose $v' \gg v$.

$v = 246$ GeV is used to convert all dim'less variables to physical units. We display the profile radius r in units of

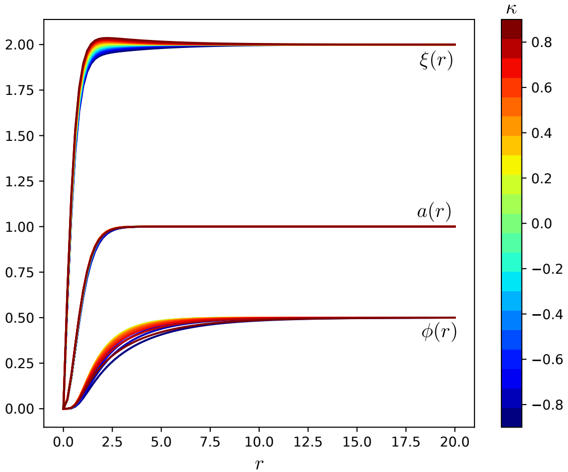
$$v_{\text{dim'less}} \cdot 0.0008 \text{ fm}.$$



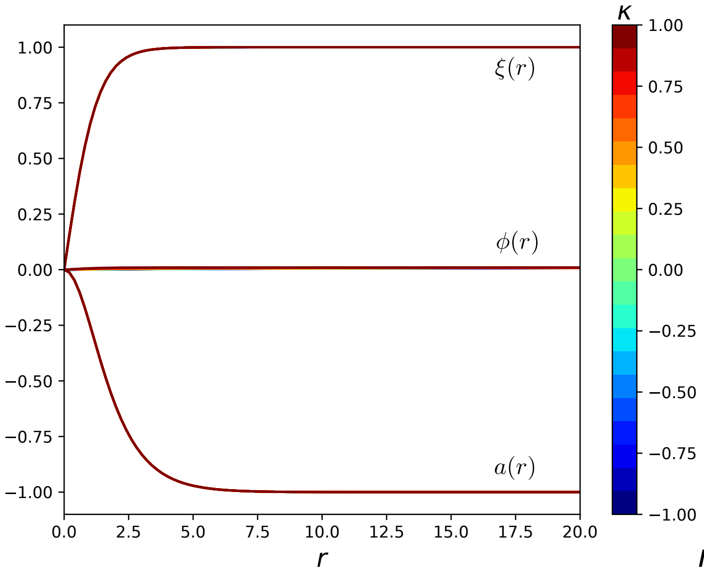
$$v = 0.5,\ v' = 1,\ n = n' = h = h' = \lambda = \lambda' = 1.$$



$v = 0.5,\ v' = 1,\ n = 1,\ n' = 2,\ h = 1,\ h' = 2,$
$\lambda = \lambda' = 1.$

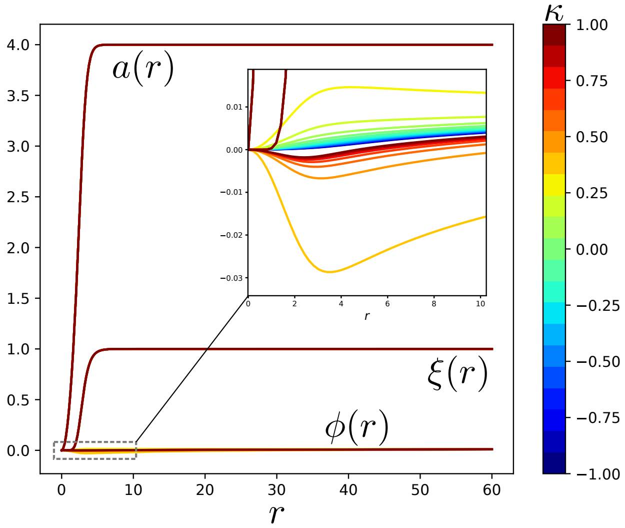


$v = 0.5$, $v' = 2$, $n = -5$, $n' = -1$, $h = 5$, $h' = 1$, $\lambda = \lambda' = 1$. This is an example from the SO(10) GUT [Buchmüller/Greub/Minkowski, '91].



$n =$

$h = n' = h' = \lambda = \lambda' = 1, \;\; v = 0.01, \;\; v' = 1$



Coaxial string solution, cf. [Bogomol'nyi, 1975] with $n = -2$, $h = 0.5$, $n' = 10$, $h' = -2.5$, $\lambda = 1$, $\lambda' = 1$, $v = 0.01$, $v' = 1$.

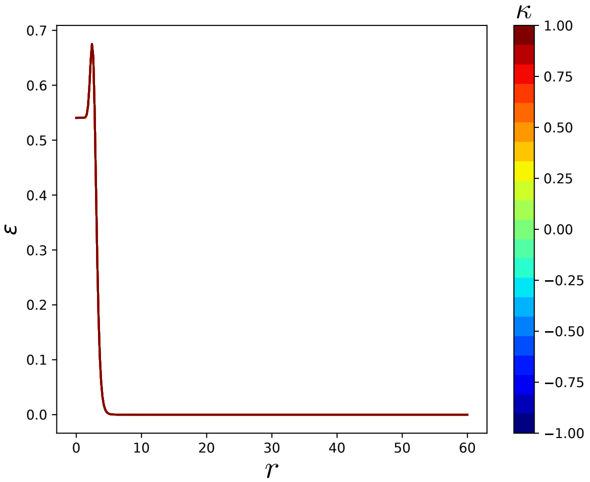


Figure: Energy density, $v = 0.01$, $v' = 1$, $n = -2$, $n' = 10$, $h = 0.5$, $h' = -2.5$, $\lambda = \lambda' = 1$, in units of 4.79×10^{19} GeV/fm^3.

By integrating the energy density we find that the string tension is of the order of

$$\mu \sim 10^{10}\ \mathrm{GeV}^2 = 10^{25}\ \frac{\mathrm{kg}}{\mathrm{pc}}.$$

Therefore

$$G\mu \sim 10^{-28},$$

where $G = \frac{1}{(1.2\times 10^{19}\ \mathrm{GeV})^2}$.

The LIGO/Virgo collaboration set constraints to the string tension

$$G\mu \lesssim 4 \times 10^{-15}.$$

Summary

In this BSM model, we added

- A new gauge coupling h'
- A right-handed neutrino ν_R
- A new Higgs field $\chi \in \mathbb{C}$

A non-standard type of cosmic strings is possible.

Overshoot and coaxial string solutions.

At large distances, they do not affect known physics.

Not observed but detectable, in principle. Like gravitational wave detection, gravitational lensing, CMB anisotropies etc.

No contradictions with SM physics, motivated from the exactness of $U(1)_{B-L}$.