The profile of non-standard cosmic strings

José Antonio García Hernández
Victor Muñoz Vitelly
Wolfgang Bietenholz

Instituto de Ciencias Nucleares, UNAM

XV Latin American Symposium on High Energy Physics
November 4 2024

Kibble Mechanism

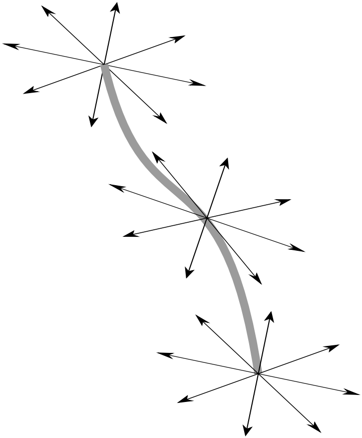
It is generally assumed that phase transitions occurred in the early universe at the late stages of inflation. These transitions could have formed topological defects.

Examples:

- $\pi_0(\mathcal{M}) \neq I \Rightarrow$ Domain wall
- $\pi_1(\mathcal{M}) \neq I \Rightarrow$ Vortex (2d), cosmic strings (3d)
- $\pi_2(\mathcal{M}) \neq I \Rightarrow$ Monopole

$\mathcal{M}$: Vacuum manifold.

Cosmic Strings



2-dimensional vortices stacked on top of each other, forming a cosmic string in three dimensions

$\mathrm{U}(1)_{B-L}$ exact global symmetry

In the Standard Model $\mathrm{U}(1)_{B-L}$ is an **exact** global symmetry. B baryon number, L lepton number.

This is strange, an exact symmetry is only natural when it is local.

We promote $\mathrm{U}(1)_{B-L}$ to a local symmetry and combine it with $\mathrm{U}(1)_Y$.

Gauge symmetry

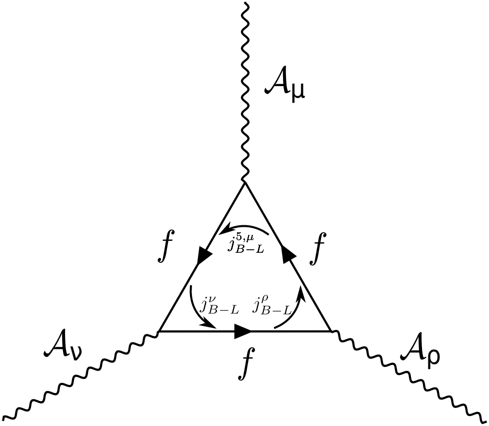
We introduce a new gauge coupling h' and define a new charge as

$$Y' \equiv 2hY + \frac{h'}{2}(B-L),$$

h, h' gauge couplings.

We take the gauge group to be $\mathrm{U}(1)_{Y'}$ and we call the gauge field $\mathcal{A}_\mu$.

Gauge Anomaly



In each vertex the quarks of one generation contribute with $B = 4$, and leptons with $L = 3$.
$B - L \neq 0$.

Gauge anomaly. It is cured by adding a ν_R $(L = 1)$ to each generation.

Since the neutrinos have mass, it is natural to introduce a mechanism to give mass to the neutrinos.

To add a mass term solely for ν_R, independently of ν_L, we add a new Higgs field $\chi \in \mathbb{C}$

$$f_{\nu_R} \nu_R^T \chi C^\dagger \nu_R + \text{c.c.},$$

where f_{ν_R} is a Yukawa coupling.

To preserve gauge invariance, the field χ must have a charge $B - L = 2$.

We generate the Majorana mass with the Higgs mechanism using a non-standard Higgs field $\chi \in \mathbb{C}$.

We denote the vacuum expectation value of χ as v'.

χ gives a Majorana mass to the right-handed neutrino $M = f_{\nu_R} v'$.

χ is added to the Lagrangian with a quartic potential (power counting renormalizable)

$$V' = \frac{m'^2}{2}\chi^*\chi + \frac{\lambda'}{4}(\chi^*\chi)^2.$$

It is natural to include (normalizable, gauge invariant)

$$\frac{\kappa}{2}\Phi^\dagger\Phi\chi^*\chi.$$

We assume that $v' > v$ and f_{ν_R} sufficiently large.

Lagrangian

$$
\begin{aligned}
\mathcal{L} \;=\;& \frac{1}{2}(D^\mu \Phi)^\dagger D_\mu \Phi - \frac{m^2}{2}\Phi^\dagger \Phi - \frac{\lambda}{4}(\Phi^\dagger \Phi)^2 - \frac{\lambda}{4}v^4 \\
&+\frac{1}{2}(d^\mu \chi)^* d_\mu \chi - \frac{m'^2}{2}\chi^*\chi - \frac{\lambda'}{4}(\chi^*\chi)^2 - \frac{\lambda'}{4}v'^4 \\
&-\frac{\kappa}{2}\Phi^\dagger \Phi \chi^* \chi - \frac{\kappa}{2}v^2 v'^2 - \frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu},
\end{aligned}
$$

- $\Phi = (\phi_+, \phi_0)^\mathsf{T} \in \mathbb{C}^2$
- $D_\mu \Phi = (\partial_\mu + ih\mathcal{A}_\mu)\Phi$
- $d_\mu \chi = (\partial_\mu + ih'\mathcal{A}_\mu)\chi$
- $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$

For the potential to be bounded from below, we need

$$\lambda > 0, \quad \lambda' > 0, \quad \kappa^2 < \lambda\lambda',$$

and for spontaneous symmetry breaking to occur

$$m^2 = -\kappa v'^2 - \lambda v^2 < 0,$$
$$m'^2 = -\kappa v^2 - \lambda' v'^2 < 0.$$

Equations of motion

$$
\begin{aligned}
D^\mu D_\mu \Phi &= -m^2\Phi - \lambda(\Phi^\dagger\Phi)\Phi - \kappa\Phi\chi^*\chi \\
d^\mu d_\mu \chi &= -m'^2\chi - \lambda'(\chi^*\chi)\chi - \kappa\chi\Phi^\dagger\Phi \\
\partial^\lambda \mathcal{F}_{\lambda\nu} &= -\frac{ih}{2}\left[(D_\nu\Phi)^\dagger\Phi - \Phi^\dagger(D_\nu\Phi)\right] \\
&\quad -\frac{ih'}{2}\left[(d_\nu\chi)^*\chi - \chi^*(d_\nu\chi)\right]
\end{aligned}
$$

Ansatz

The Lagrangian has a $\mathrm{U}(1)_{Y'}$ symmetry which can "spontaneously break" down to I.

$\mathcal{M} = \mathrm{U}(1)_{Y'}/I = \mathrm{U}(1) \Rightarrow \pi_1(\mathrm{U}(1)) = \mathbb{Z} \Rightarrow$ cosmic strings.

We only consider the component ϕ_0 of the Higgs field Φ. Cylindrically symmetric ansatz

$$
\begin{aligned}
\phi_0(r,\varphi) &= \phi(r)e^{in\varphi} \\
\chi(r,\varphi) &= \xi(r)e^{in'\varphi}, \ \ n, n' \text{ winding numbers} \\
\mathcal{A}(r) &= \frac{a(r)}{r}\hat{\varphi}.
\end{aligned}
$$

Equations of motion

$$\partial_r^2\phi + \frac{1}{r}\partial_r\phi - \frac{(n+ha)^2}{r^2}\phi - m^2\phi - \lambda\phi^3 - \kappa\phi\xi^2 = 0$$

$$\partial_r^2\xi + \frac{1}{r}\partial_r\xi - \frac{(n'+h'a)^2}{r^2}\xi - m'^2\xi - \lambda'\xi^3 - \kappa\xi\phi^2 = 0$$

$$\partial_r^2 a - \frac{1}{r}\partial_r a - h(n+ha)\phi^2 - h'(n'+h'a)\xi^2 = 0.$$

Boundary conditions $(n, n' \neq 0)$

$$\phi(0) = 0, \qquad \lim_{r\to\infty}\phi(r) = v$$

$$\xi(0) = 0, \qquad \lim_{r\to\infty}\xi(r) = v'$$

$$a(0) = 0, \quad \lim_{r\to\infty} a(r) = -\frac{n}{h} = -\frac{n'}{h'}.$$

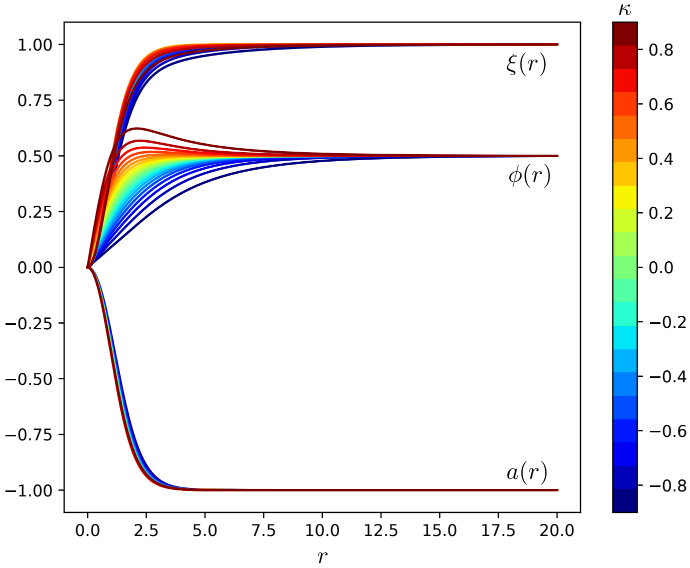
Boundary value problem, numerical solutions with the damped Newton method.

Solutions uniquely defined by inserting v, v', λ, λ', h, h', n and n'.

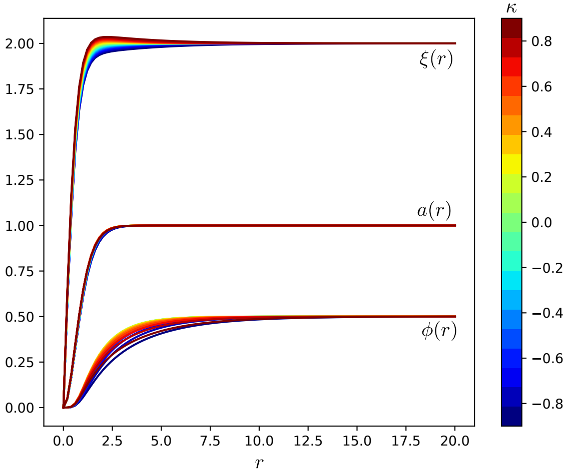
We choose $v' > v$.

$v = 246$ GeV is used to convert all dim'less variables to physical units. We display the profile radius r in units of

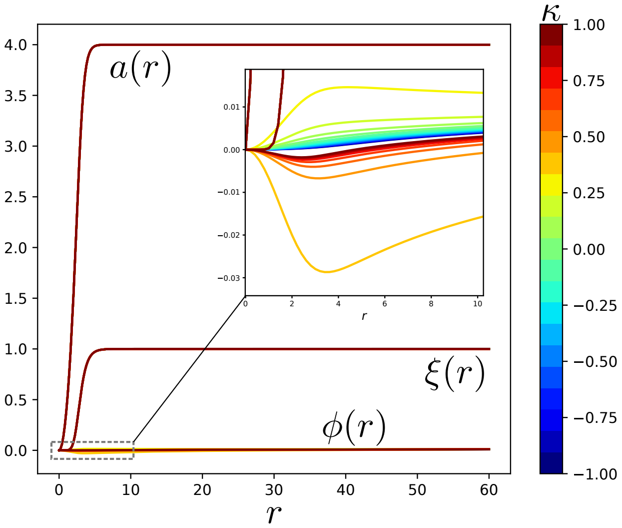
$$v_{\text{dim'less}} \cdot 0.0008 \text{ fm}.$$



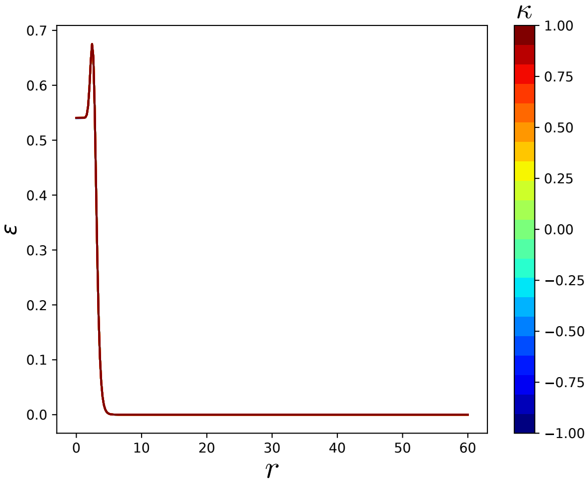
$v = 0.5,\ v' = 1,\ n = 1,\ n' = 2,\ h = 1,\ h' = 2,$
$\lambda = \lambda' = 1.$



$v = 0.5$, $v' = 2$, $n = -5$, $n' = -1$, $h = 5$, $h' = 1$, $\lambda = \lambda' = 1$. This is an example from the SO(10) GUT [Buchmüller/Greub/Minkowski, '91].



Coaxial string solution, cf. [Bogomol'nyi, 1975] with $n = -2,\ h = 0.5,\ n' = 10,\ h' = -2.5,\ \lambda = 1,\ \lambda' = 1,\ v = 0.01,\ v' = 1.$



Energy density, $v = 0.01$, $v' = 1$, $n = -2$, $n' = 10$, $h = 0.5$, $h' = -2.5$, $\lambda = \lambda' = 1$, in units of 4.79×10^{19} GeV/fm^3.

By integrating over the area the energy density, we find that the string tension is of the order of

$$\mu \sim 10^{10}\ \mathrm{GeV}^2 = 10^{25}\ \frac{\mathrm{kg}}{\mathrm{pc}}.$$

Therefore

$$G\mu \sim 10^{-28},$$

where $G = \frac{1}{(1.2\times 10^{19}\ \mathrm{GeV})^2}$.

The LIGO/Virgo collaboration set constraints to the string tension

$$G\mu \lesssim 4 \times 10^{-15}.$$

Summary

In this BSM model, motivated from the exactness of $\mathrm{U}(1)_{B-L}$, we added

- A new Abelian gauge field $\mathcal{A}_\mu$
- A right-handed neutrino ν_R
- A new Higgs field $\chi \in \mathbb{C}$

A non-standard type of cosmic strings is possible.

Overshoot and coaxial string solutions.

At large distances, they do not affect known physics.