Matrix A normal → Matrix Q ortogonal

$$A=\begin{bmatrix}1&0&-1\\0&1&0\\1&1&0\\0&1&1\end{bmatrix} \rightarrow Q=\begin{bmatrix}?&?\\?&?\\?&?\end{bmatrix}$$

$$u_1=\begin{bmatrix}1\\0\\1\\0\end{bmatrix} \quad u_2=\begin{bmatrix}0\\1\\1\\1\end{bmatrix} \quad u_3=\begin{bmatrix}-1\\0\\0\\0\end{bmatrix}$$

$$v_1=\begin{bmatrix}1\\0\\1\\0\end{bmatrix}$$

$$V_2=u_2-\hat{u}_2$$

$$\hat{u}_2=\frac{u_2\cdot v_1}{v_1\,v_1}\cdot v_1=$$

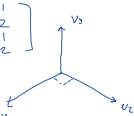
$$\hat{u}_2=\frac{\begin{bmatrix}0\\1\\1\\1\end{bmatrix}\cdot\begin{bmatrix}1\\0\\1\\0\end{bmatrix}}{\begin{bmatrix}1\\0\\1\\0\end{bmatrix}\cdot\begin{bmatrix}1\\0\\1\\0\end{bmatrix}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}$$

$$\hat{u}_2=\begin{bmatrix}1/2\\0\\1/2\\0\end{bmatrix}$$

$$V_2=u_2-\hat{u}_2$$

$$v_2=\begin{bmatrix}0\\1\\1\\1\end{bmatrix}-\begin{bmatrix}1/2\\0\\1/2\\0\end{bmatrix}=\begin{bmatrix}-1/2\\1\\1/2\\1\end{bmatrix} \times 2=\begin{bmatrix}-1\\2\\1\\2\end{bmatrix}$$

$$v_1=\begin{bmatrix}1\\0\\1\\0\end{bmatrix} \qquad v_2=\begin{bmatrix}-1\\2\\1\\2\end{bmatrix}$$



$$\hat{u}_3=\frac{u_3\cdot v_1}{v_1\cdot v_1}+\frac{u_3\cdot v_2}{v_2\,v_2}v_2$$

$$\hat{u}_3=\frac{\begin{bmatrix}-1\\0\\0\\0\end{bmatrix}\cdot\begin{bmatrix}1\\0\\1\\0\end{bmatrix}}{\begin{bmatrix}1\\0\\1\\0\end{bmatrix}\cdot\begin{bmatrix}1\\0\\1\\0\end{bmatrix}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}+\frac{\begin{bmatrix}-1\\2\\1\\2\end{bmatrix}\begin{bmatrix}-1\\2\\1\\2\end{bmatrix}}{\begin{bmatrix}-1\\2\\1\\2\end{bmatrix}\begin{bmatrix}-1\\2\\1\\2\end{bmatrix}}\begin{bmatrix}-1\\2\\1\\2\end{bmatrix}$$

$$\hat{u}_3=\frac{-1}{2}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}+\frac{3}{10}\begin{bmatrix}-1\\2\\1\\2\end{bmatrix}$$

$$\hat{u}_3=\begin{bmatrix}-1/2\\0\\-1/2\\0\end{bmatrix}+\begin{bmatrix}-3/10\\6/10\\3/10\\6/10\end{bmatrix}$$

$$\hat{u}_3=\begin{bmatrix}-4/5\\3/5\\-1/5\\3/5\end{bmatrix} \qquad V_3=u_3-\hat{u}_3$$

$$V_3=\begin{bmatrix}-1/5\\-3/5\\1/5\\2/5\end{bmatrix}\times 5=\begin{bmatrix}-1\\-3\\1\\2\end{bmatrix}$$

$$v_3=\begin{bmatrix}-1\\-3\\1\\2\end{bmatrix} \qquad v_1=\begin{bmatrix}1\\0\\1\\0\end{bmatrix} \qquad v_2=\begin{bmatrix}-1\\2\\1\\2\end{bmatrix}$$

$$w_1=\frac{v_1}{\|v_1\|}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}=\begin{bmatrix}1/\sqrt{2}\\0\\1/\sqrt{2}\\0\end{bmatrix}$$

$$w_2=\frac{v_2}{\|v_2\|}=\frac{1}{\sqrt{10}}\begin{bmatrix}-1\\2\\1\\2\end{bmatrix}=\begin{bmatrix}-1/\sqrt{10}\\2/\sqrt{10}\\1/\sqrt{10}\\2/\sqrt{10}\end{bmatrix}$$

$$w_3=\frac{v_3}{\|v_3\|}=\frac{1}{\sqrt{15}}\begin{bmatrix}-1\\-3\\1\\2\end{bmatrix}=\begin{bmatrix}-1/\sqrt{15}\\-3/\sqrt{15}\\1/\sqrt{15}\\2/\sqrt{15}\end{bmatrix}$$

$$Q=\begin{bmatrix}1/\sqrt{2}&-1/\sqrt{10}&-1/\sqrt{15}\\0&2/\sqrt{10}&-3/\sqrt{15}\\1/\sqrt{2}&1/\sqrt{10}&1/\sqrt{15}\\0&2/\sqrt{10}&2/\sqrt{15}\end{bmatrix}$$

w_1 w_2 w_3