~ Descomposición LU

~ Despejar

$$x_1 + x_2 - x_3 = 4$$
$$x_1 - 2x_2 + 3x_3 = -6$$
$$2x_1 + 3x_2 + x_3 = 7$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 3 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix}$$

$A \qquad X = V$

$A = L \cdot U$

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 3 \\ 2 & 3 & 1 \end{bmatrix} \xrightarrow{-1F_1 + F_2}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -3 & 4 \\ 2 & 3 & 1 \end{bmatrix} \xrightarrow{-2F_1 + F_3}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -3 & 4 \\ 0 & 1 & 3 \end{bmatrix} \xrightarrow{\frac{1}{3}F + F_3}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -3 & 4 \\ 0 & 0 & \frac{13}{3} \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -\frac{1}{3} & 1 \end{bmatrix}$$

$Ax = U$

$LUx = V$

$Ux = y \qquad Ly = v$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -1/3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix}$$

$L \qquad y \qquad v$

$y_1 = 4$

$y_1 + y_2 = -6$

$2y_1 - \frac{1}{3}y_2 + y_3 = 7$

$4 + y_2 = -6$

$y_2 = -6 - 4$

$y_2 = -10$

$2(4) - \frac{1}{3}(-10) + y_3 = 7$

$8 + \frac{10}{3} + y_3 = 7$

$y_3 = 7 - 8 - \frac{10}{3}$

$y_3 = -\frac{13}{3}$

$y_1 = 4$

$y_2 = -10$

$y_3 = -\frac{13}{3}$

$U \qquad x \qquad y$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -3 & 4 \\ 0 & 0 & \frac{13}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \\ -\frac{13}{3} \end{bmatrix}$$

$x_1 + x_2 - x_3 = 4$

$-3x_2 + 4x_3 = -10$

$\frac{13}{3}x_3 = -\frac{13}{3}$

$x_3 = -1$

$-3x_2 - 4 = -10$

$-3x_2 = -6$

$x_2 = 2$

$x_1 + 2 + 1 = 4$

$x_1 = 1$

$x_1 = 1$

$x_2 = 2$

$x_3 = -1$

$A = LU$

$LUx = v$

$Ux = y \qquad Ly = v$