Inverse of a function

A inverse of a function is a function that "reverse" the effect of the original function.

If you have a function f that maps an element x from set X to element y in a set Y, the inverse function f^{-1} map y back to x.

Definition

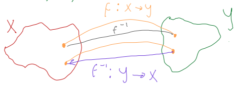
Given a function f : X -> Y , then inverse function $f^{-1} : Y \to X$ for every $y \in Y$, this is a unique $x \in X$ such that

$$f(x) = y$$

The inverse function f^{-1} satisfies the following condition:

1) For all $x \in X : f(f^{-1}(y)) = y$

2) For all $y \in Y : f^{-1}(f(x)) = x$



These conditions imply that applying the function and then it inverse will return the original value.

Identity function

$$I_X : X \to X \Rightarrow I_x(a) = a$$
$$a \in X$$

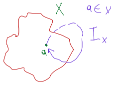
$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$I_v = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

For a set X, the identity function I_X is defined as:

$$I_X(a) = a \quad \text{for all } a \in X$$

Ix is the identity function on the set X and it maps every element x in X to itself.



Properties of Identity function

1) Preservation: Does not alter any element. If x is the domain , then the image of x under the identity function is x.

2) Linearity: Identity function is a linear transformation

$$I(u+v) = I(u) + I(v)$$
$$I(cv) = c\,I(u) = cu$$

3) Identity Matrix: n x n => all the diagonal elements will be 1 and 0's else where.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2 \times 2}$$

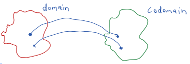
4) Inverserse: The Identity function is its own inverse.

Existence and uniqueness

A function f has an inverse if and only if it is bijective.

Bijective fulfills two propierties:

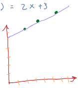
1) Injective (One to one) : Different elements in the domain map to different elements in the codomain.



2) Surjective (onto) : Every elemento in the codomain is the image of atleast one element in the domain.

Example:

Linear function $f(x) = 2x + 3$

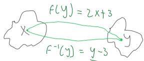


Find the inverse

$y = 2x + 3$

$y - 3 = 2x$

$\boxed{x = \frac{y-3}{2}}$



The inverse function $f^{-1}(y) = \frac{y-3}{2}$

Verification

1) $f(f^{-1}(y)) = f\left(\frac{y-3}{2}\right) = 2\left(\frac{y-3}{2}\right) + 3$
$= y - 3 + 3$
$= y$

2) $f^{-1}(f(x)) = f^{-1}(2x+3)$
$= \frac{(2x+3)-3}{2}$
$= x$