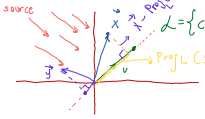
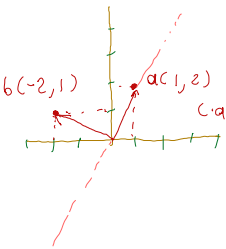
Introduction to Projections



$L = \{c\vec{v} \mid c \in \mathbb{R}\}$

$\text{Proj}_L(\vec{x}) \Rightarrow$ Project the $\vec{x}$ on the line L

$\text{Proj}_L(\vec{x}) \Rightarrow$ Some vector in Line where $\vec{x} - \text{Proj}_L(\vec{x})$ is perpendicular to L



$$a = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad b = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$a \cdot b = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$a \cdot b = a_1 b_1 + a_2 b_2$$

$$a \cdot b = (1)(-2) + (2)(1)$$

$$a \cdot b = 0$$

When two vectors are perpendicular the dot product betwen this vectors is 0

$\text{Proj}_L(\vec{x}) \Rightarrow$ Some vector in Line where $\vec{x} - \text{Proj}_L(\vec{x})$ is perpendicular to L

$$= (\vec{x} - c\vec{v}) \cdot \vec{v} = 0$$

$$\vec{x} \cdot \vec{v} - c\vec{v} \cdot \vec{v} = 0 \rightarrow \vec{x} \cdot \vec{v} = c\vec{v}\vec{v}$$

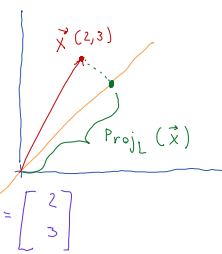
$$\rightarrow c = \frac{\vec{x}\,\vec{v}}{\vec{v}\,\vec{v}}$$

$$\text{Proj}_L(\vec{x}) = c\vec{v}$$

$$= \left(\frac{\vec{x} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\right) \cdot \vec{v}$$

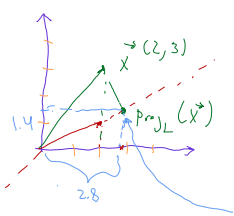
$$\text{Proj}_b(a) = \left(\frac{a \cdot b}{b \cdot b}\right) \cdot b$$

$$\text{Proj}_L(\vec{x}) = \left(\frac{\vec{x} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\right) \cdot \vec{v}$$



$$L = \left\{ c\underbrace{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}_{c\vec{v}_2} \mid c \in \mathbb{R} \right\}$$

$$\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$



$$\text{Proj}_L(\vec{x}) = \left(\frac{\begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}}{\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}}\right) \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \left(\frac{7}{5}\right) \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\text{Proj}_L(\vec{x}) = \begin{bmatrix} \frac{14}{5} \\ \frac{7}{5} \end{bmatrix} = \begin{bmatrix} 2.8 \\ 1.4 \end{bmatrix}$$