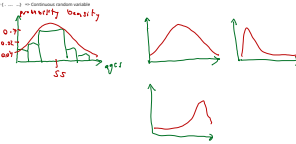
Probability Distribution Functions

Probability distribution function describe how the probabilities are distributed over the values of a random variable.

Ages = {, ... ,} => Continuous random variable



2 Main of probability distribution functions:

1) Probability Mass Functions (PMF) : Used for descrete random variables.

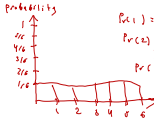
2) Probability density function (PDF) : Used for continuous random variable.

1) Probability Mass Function [Descrete Random Variable]

Example Rolling a dice {1,2,3,4,5,6} => Fair Dice

P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6

PMF



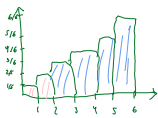
$Pr(1) = 1/6$

$Pr(2) = 1/6$

$\vdots$

$Pr(6) = 1/6$

Cumulative Density Function (CDF)



$Pr(x \leq 6) = 1$

$Pr(x \leq 2) = Pr(x=1) + Pr(x=2)$

$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

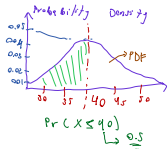
$Pr(x \leq 6) = Pr(x=1) + Pr(x=2) + \dots + Pr(x=6)$

$= 1$

2) Probability Density Function (PDF)

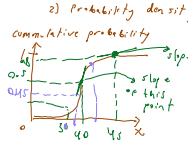
1) Distribution of continuous random variable

$X = Ages = \{- - - - - - -\}$



$Pr(X \leq 40) \rightarrow 0.5 \Rightarrow 50\%$

1) Area under the curve
2) Probability density



$Pr(X \leq 45) = 0.8 = 80\%$

slope = $\left\{ \frac{0.65 - 0.45}{44 - 38} \right\} \Rightarrow$ probability density

(x_1, y_1)

(x_2, y_2)

slope = $\left[\frac{x_2 - x_1}{y_2 - y_1} \right]$ ← gradient => Probability Density

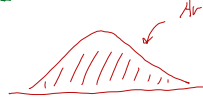
Probability Density = Gradient of cumulative density function

PDF properties

1) Non negative $f(x) \geq 0$ for all x

2) The total area under PDF curve is equal to 1

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$



with respect to different distribution f(x) fution is going to change