

# FUNCTIONS AND LINEAR TRANSFORMATIONS

## Functions

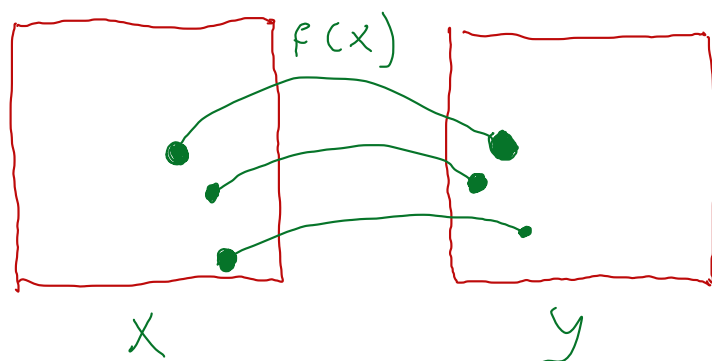
A function is a mathematical relationship that uniquely associates elements of one set (called the domain) with elements of another set (called the codomain). In simpler terms, a function maps inputs to outputs in a specific way.

Notation: A function  $f$  mapping elements from set  $X$  (domain) to set  $Y$  (codomain) is denoted by  $f: X \rightarrow Y$

If  $x$  is an element of  $X$ , then  $f(x)$  is corresponding element in  $Y$ .

Example:  $f(n) = 2n + 3 \rightarrow$  maps each real number  $n$  to real number  $2n + 3$

$f:$   
 $x \mapsto y$   
 $f(2) = 2(2) + 3 = 7$   $f(n) \mapsto$  mapping  $z \in \mathbb{R}$  to  $7 \in \mathbb{R}$



$g(n)$   $f: X \rightarrow Y$

$$f: \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \Rightarrow \begin{bmatrix} x+y \\ 6z \end{bmatrix} \in \mathbb{R}^2 \rightarrow \begin{matrix} 3 \text{ dimension} \\ \text{vector} \end{matrix} \rightarrow \begin{matrix} 2 \text{ dimension} \\ \text{vector} \end{matrix}$$

$$f(n) = \begin{bmatrix} x+y \\ 6z \end{bmatrix}$$

$$\begin{matrix} f: \mathbb{R}^3 & \rightarrow & \mathbb{R}^2 \\ \downarrow & & \downarrow \\ \text{domain} & & \text{codomain} \end{matrix}$$

Example: Dimensionality Reduction  $\leftarrow f(n) \Rightarrow$  Transformation