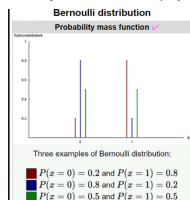Definition: The bernoulli distribution is the simplest discrete probability distribution. It represent the probability distribution of a random variable that has exactly two possible outcomes: success (with probability p) and failure (with probability 1-p). Its is used to model binary outcomes, such as a coin flip or yes/no question.

Bernoulli distribution
Probability mass function ✓

Three examples of Bernoulli distribution:
- $P(x = 0) = 0.2$ and $P(x = 1) = 0.8$
- $P(x = 0) = 0.8$ and $P(x = 1) = 0.2$
- $P(x = 0) = 0.5$ and $P(x = 1) = 0.5$

1) Discrete Random Variable (PMF)
2) Outcomes are Binary
Example:
1) Tossing a coin $\{H, T\}$

$Pr(X = H) = 0.5 = P$ P, q
$Pr(x = T) = 1 - 0.5 = q$ $q = (1-p)$

2) Whether the person will Pass/Fail
$Pr(X = pass) = 0.4$
$Pr(x = Fail) = 1 - 0.4 = 0.6$

Parameters

$0 \le p \le 1$ $q = 1 - p$ $K = \{0, 1\} \Rightarrow 2$ outcomes
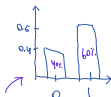$Pr(success) \Rightarrow K = 1$
$Pr(fail) \Rightarrow K = 0$

PMF → Company has launched a new Smartphone "A".
k
(1) Use = 60% → P
(0) not use = 40% → q = 1-p

$PMF = P^K \cdot (1-P)^{1-K}$ if $K = 1$
$Pr(K = 1) = P^1 (1-P)^{1-1} \rightarrow P$
$Pr(K = 0) = P^0 (1-P)^{1-0} \rightarrow 1 - P = q$

Simplified

$PMF \begin{cases} q = 1-p & \text{if } K = 0 \\ p & \text{if } K = 1 \end{cases}$
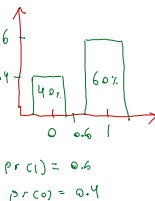
* Mean of Bernoulli Distribution

$E(x) = \sum_{K=0}^{1} K \cdot P(K) \quad \{0, 1\}$

$= 0 \cdot (0.4) + 1 \cdot (0.6)$
$E(x) = 0.6 \rightarrow P$ $P = 0.6$
$= \quad q = 0.4$ $Pr(1) = 0.6$
                        $Pr(0) = 0.4$

2) Median of Bernoulli Distribution

$Median \begin{cases} 0 & \text{if } p < 1/2 \\ [0,1] & \text{if } p = 1/2 \\ 1 & \text{if } p > 1/2 \end{cases}$

median = 0 if $q > p$ median = 0.5 if $q = p$ median = 1 if $q < p$

3) Mode $p > q \Rightarrow p$ will be the mode
                            else will be the mode

4) Variance

σ^2 $K = 0$ and 1 $Pr(K = 0) = 0.4 \rightarrow q$
                                $Pr(K = 1) = 0.6 \rightarrow P$

$\sigma^2 = 0.4 * (0 - 0.6)^2 + 0.6 (1 - 0.6)^2$
$= 0.4 + 0.36 + 0.6 (0.16)$
$\sigma^2 = 0.24 \rightarrow Pr(K = 0) \cdot Pr(K = 1)$
$\sigma^2 = P \cdot q$
$\sigma = \sqrt{P \cdot q}$