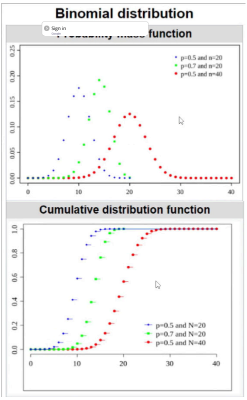
Binomial Distribution

In probability theory and statistics, the binomial distribution with parameters n and p is the discrete probability distribution of the number of sucesses in a sequence of n independet experiments, each asking a yes-no question, and each with its own Boolean-Valued outcome: success (with probability p) or failure (with probability q = 1 - p). A single success/failure experiment is also called a Bernuolli trial or Bernoulli experiment, and a sequence of outcomes is called a Bernoulli process; for a single trial, i.e n=1, the binomial distribution is a Bernoilli distribution. The binomial distribution is the basic for the popular binomial test of statistical significance.



*) Discrete Random Variable

*) Every outcome of the experiment is binary

*) These experiments are performed for n trials

Example: Tossing a coin 10 times → {H, T}

Notation: $B(n, p)$

Parameters: $n \in \{0, 1, 2, 3 ..\}$ → number of trails of experiments

$p \in [0, 1]$ → success probability for each trial

$q = 1 - p$

Support: $k \in \{0, 1, 2, 3 ... n\}$ → Number of success

PMF: $Pr(k, n, p) = {}^{n}C_{k}\, p^{k} (1-p)^{n-k}$

for $k = 0, 1, 2, 3 ..., n$ where

$${}^{n}C_{K} = \frac{n!}{K!(n-K)!}$$ → Binomial Coefficient

Mean $n*p$

Variance npq

σ $\sqrt{npq}$

Example Coin flip

No of trial $(n) = 5$ probability of success $(p) = 0.5$

No of success (K) = varies from 0 to 5

what is the probability of getting exactly 3 head in 5 flips?

$n = 5$ $K = 3$

$Pr(X=3) = {}^{5}C_{3} (0.5)^{3} (1-0.5)^{5-3} = 0.3125$

Example:

Quality Control

Scenario: Inspecting 10 items in a factory where each item has a 10% chance of being defective

1) Number of trials $(n) = 10$

2) Probability of success $(p) = 0.1$ (defective item)

3) Number of success (K) = varies from 0 to 10

Question: what is the probability of finding exactly 2 defective items in a sample of 10?

$Pr(X=2) = {}^{10}C_{2} (0.1)^{2} (1-0.1)^{10-2} = 0.1937$