How to find Inverse of a matrix

1. Derminant
2. How to inverse

Example 2x2 Matrix

A => find its inverse and also verify inverse using a transformation.

$$A = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix} \qquad T: \mathbb{R} \to \mathbb{R}$$

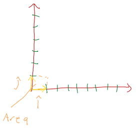
$$A \cdot X = y$$

$$A^{-1}$$

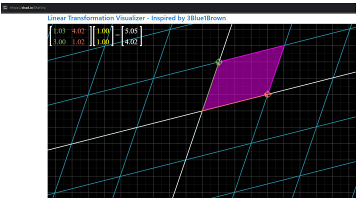
Find the inverse of A

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & d \end{bmatrix}$$

Determinant



The determinant is a scalar value tha can be computed from a square matrix. It provides important information about the matrix, such as whether the matriz is invertible (i.e., has an inverse), and it also has geometric interpretations, such as describing the scaling factor of linear transformations represented by the matrix.



Determinant

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow ad - cb \Rightarrow \text{scalar} \Rightarrow \text{Determinant}$$

step 1

$$A = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix} \qquad \det(A) = 24 - 14 = 10$$

$\det(A)$ = Non zero => Inverse of the matrix

Since the determinant is non zero the matrix A is invertible

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{10} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} \frac{6}{10} & -\frac{7}{10} \\ -\frac{2}{10} & \frac{4}{1} \end{bmatrix}$$

3 Verify using a vector

$$X = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow$$ X using A then use A^{-1} to recover the original vector

$$y = Ax = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4+7 \\ 2+6 \end{bmatrix} = \begin{bmatrix} 11 \\ 8 \end{bmatrix}$$

Recovery x using A^{-1}

$$x = A^{-1} y = \begin{bmatrix} \frac{6}{10} & -\frac{1}{7} \\ -\frac{2}{10} & \frac{4}{10} \end{bmatrix} \begin{bmatrix} 11 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} \frac{11 \cdot 6}{10} - \frac{8}{7} \\ -\frac{11 \cdot 2}{10} + \frac{8 \cdot 4}{10} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Thus A^{-1} succesfuly recover the original vector X