p	e	$\nu(N_E)$	τ_E	Base Change of	Curve	Description	Character
3	2	2	ε_1	ε_1	2E1	Principal series	$\chi(2)$
	3	4	$\tau_{ps}(1,2,3)_1$	$\tau_{ps,3}(1,2,3)$	4E5	Principal series	$\eta(0,1)$
			$\tau_{ps}(1,2,3)_2$	$\tau_{sc,3}(-1,2,3)$	4E2		$\eta(2,0)$
			$\tau_{ps}(1,2,3)_3$		4E3		$\eta(2,1)$
			$\tau_{ps}(1,2,3)_4$		4E4		$\eta(2,2)$
			$\tau_{sc}(1+\sqrt{2},2,3)_1$		4E8	Supercuspidal $\mathbb{Q}_9(\sqrt{1+\sqrt{2}})$	$\chi_3(0,1)$
			$\tau_{sc}(1+\sqrt{2},2,3)_2$		4E7		$\chi_3(2,0)$
			$\tau_{sc}(1+\sqrt{2},2,3)_3$		4E1		$\chi_3(2,1)$
			$\tau_{sc}(1+\sqrt{2},2,3)_4$		4E6		$\chi_3(2,2)$
	4	2	$\tau_{ps}(1,1,4)$	$\tau_{sc,3}(-1,1,4)$	2E2	Principal series	$\chi(1)$
	6	4	$\tau_{ps}(1,2,3)_1 \otimes \varepsilon_1$	$\tau_{ps,3}(1,2,3) \otimes \varepsilon_1$	4E13	Principal series	$\eta(3,1)$
			$\tau_{ps}(1,2,3)_2 \otimes \varepsilon_1$	$\tau_{sc,3}(-1,2,3) \otimes \varepsilon_1$	4E10		$\eta(5,0)$
			$\tau_{ps}(1,2,3)_3 \otimes \varepsilon_1$		4E11		$\eta(5,1)$
			$\tau_{ps}(1,2,3)_4 \otimes \varepsilon_1$		4E12		$\eta(5,2)$
			$\tau_{sc}(1+\sqrt{2},2,3)_1 \otimes \varepsilon_1$		4E16	Supercuspidal $\mathbb{Q}_9(\sqrt{1+\sqrt{2}})$	$\chi_3(3,1)$
			$\tau_{sc}(1+\sqrt{2},2,3)_2 \otimes \varepsilon_1$		4E15		$\chi_3(5,0)$
			$\tau_{sc}(1+\sqrt{2},2,3)_3 \otimes \varepsilon_1$		4E9		$\chi_3(5,1)$
			$\tau_{sc}(1+\sqrt{2},2,3)_4 \otimes \varepsilon_1$		4E14		$\chi_3(5,2)$
	12	3	$\tau_{sc}(3,2,6)_1$	$\tau_{sc,3}(-3,2,6)$	3E5	Supercuspidal $\mathbb{Q}_9(\sqrt{3})$	$\chi_1(1,0,0)$
			$\tau_{sc}(3,2,6)_2$		3E3		$\chi_1(1,1,0)$
			$\tau_{sc}(3,2,6)_3$		3E8		$\chi_1(1,2,0)$
			$\tau_{sc}(3,2,6)_4$	$\tau_{sc,3}(3,2,6)$	3E6		$\chi_1(3,1,0)$
			$\tau_{sc}(3+3\sqrt{2},2,6)_1$		3E4	Supercuspidal $\mathbb{Q}_9(\sqrt{3+3\sqrt{2}})$	$\chi_2(1,0)$
			$\tau_{sc}(3+3\sqrt{2},2,6)_2$		3E2		$\chi_2(1,1)$
			$\tau_{sc}(3+3\sqrt{2},2,6)_3$		3E1		$\chi_2(1,2)$
			$\tau_{sc}(3+3\sqrt{2},2,6)_4$		3E7		$\chi_2(3,1)$
		5	$\tau_{sc}(3,4,6)_1$		5E2	Supercuspidal $\mathbb{Q}_9(\sqrt{3})$	$\chi_1(0,0,1)$
			$\tau_{sc}(3,4,6)_2$	$\tau_{sc,3}(-3,4,6)_3$	5E8		$\chi_1(1,0,1)$
			$\tau_{sc}(3,4,6)_3$		5E1		$\chi_1(1,0,2)$
			$\tau_{sc}(3,4,6)_4$	$\tau_{sc,3}(-3,4,6)_1$	5E5		$\chi_1(1,1,1)$
			$\tau_{sc}(3,4,6)_5$		5E3		$\chi_1(1,1,2)$
			$\tau_{sc}(3,4,6)_6$	$\tau_{sc,3}(-3,4,6)_2$	5E4		$\chi_1(1,2,1)$
			$\tau_{sc}(3,4,6)_7$		5E9		$\chi_1(1,2,2)$
			$\tau_{sc}(3,4,6)_8$		5E7		$\chi_1(3,1,1)$
			$\tau_{sc}(3,4,6)_9$		5E6		$\chi_1(3,1,2)$

TABLE 1. Inertial types of elliptic curves over $\mathbb{Q}_9$ with potentially good reduction. Curves are defined in Table 2, characters in Table 4 and the inertia field of each curve in Table 3

Label	Curve ($a = \sqrt{2}$)	Inertial Type
2E1	$y^2 = x^3 + 3^2x + 2\cdot 3^3$	$\varepsilon_1 \oplus \varepsilon_1$
2E2	$y^2 = x^3 + 3ax^2 + 3ax + 3^2a$	$\tau_{ps}(1,1,4)$
3E1	$y^2 + 3axy + 3ay = x^3 + 3ax^2 + 3^2ax + 3^2a$	$\tau_{sc}(3+3\sqrt{2},2,6)_3$
3E2	$y^2 = x^3 + 3^3ax^2 + 3^3ax + 3^3a$	$\tau_{sc}(3+3\sqrt{2},2,6)_2$
3E3	$y^2 = x^3 + 3ax^2 + 2\cdot 3ax + 3a$	$\tau_{sc}(3,2,6)_2$
3E4	$y^2 = x^3 + 3^3ax^2 + 3^3ax + 2\cdot 3^3$	$\tau_{sc}(3+3\sqrt{2},2,6)_1$
3E5	$y^2 = x^3 + 3ax^2 + 2\cdot 3x + 2\cdot 3$	$\tau_{sc}(3,2,6)_1$
3E6	$y^2 = x^3 + 3ax^2 + 2\cdot 3x + 3a$	$\tau_{sc}(3,2,6)_4$
3E7	$y^2 = x^3 + 3^2ax^2 + 3^3ax + 2\cdot 3^3a$	$\tau_{sc}(3+3\sqrt{2},2,6)_4$
3E8	$y^2 = x^3 + 3ax^2 + 3ax + 3a$	$\tau_{sc}(3,2,6)_3$
4E1	$y^2 = x^3 + (-200a - 379)3^8x + (6520a + 10398)3^{11}$	$\tau_{sc}(1+\sqrt{2},2,3)_3$
4E2	$y^2 = x^3 + 3^3ax^2 + 2\cdot 3^4x + 3^5a$	$\tau_{ps}(1,2,3)_2$
4E3	$y^2 = x^3 + 3^3ax^2 + 3^4ax + 2\cdot 3^5$	$\tau_{ps}(1,2,3)_3$
4E4	$y^2 = x^3 + 3^2ax^2 + 3^3ax + 3a$	$\tau_{ps}(1,2,3)_4$
4E5	$y^2 = x^3 + 3^3ax^2 + 2\cdot 3^4x + 2\cdot 3^5$	$\tau_{ps}(1,2,3)_1$
4E6	$y^2 = x^3 + (176a - 611)3^8x + (-7568a + 18598)3^{11}$	$\tau_{sc}(1+\sqrt{2},2,3)_4$
4E7	$y^2 = x^3 + (-104a - 347)3^8x + (3160a + 8126)3^{11}$	$\tau_{sc}(1+\sqrt{2},2,3)_2$
4E8	$y^2 = x^3 + (-176a - 379)3^8x + (5776a + 9982)3^{11}$	$\tau_{sc}(1+\sqrt{2},2,3)_1$
4E9	$y^2 + (2a+9)xy + 3\cdot 2ay = x^3 + x^2 + 3^2x + 1$	$\tau_{sc}(1+\sqrt{2},2,3)_3 \otimes \varepsilon_1$
4E10	$y^2 = x^3 + 3^2ax^2 + 2\cdot 3^2x + 3^2a$	$\tau_{ps}(1,2,3)_2 \otimes \varepsilon_1$
4E11	$y^2 = x^3 + 3^2ax^2 + 3^2ax + 2\cdot 3^2$	$\tau_{ps}(1,2,3)_3 \otimes \varepsilon_1$
4E12	$y^2 = x^3 + 3^2ax^2 + 3^2ax + 3^2a$	$\tau_{ps}(1,2,3)_4 \otimes \varepsilon_1$
4E13	$y^2 = x^3 + 3^2ax^2 + 2\cdot 3^2x + 2\cdot 3^2$	$\tau_{ps}(1,2,3)_1 \otimes \varepsilon_1$
4E14	$y^2 + 11xy + 2\cdot 3^2ay = x^3 + 2x^2 + 3x + 1$	$\tau_{sc}(1+\sqrt{2},2,3)_4 \otimes \varepsilon_1$
4E15	$y^2 + (2a+9)xy + 2\cdot 3^2ay = x^3 + x^2 + 3x + 1$	$\tau_{sc}(1+\sqrt{2},2,3)_2 \otimes \varepsilon_1$
4E16	$y^2 + (2a+9)xy + 3^2ay = x^3 + x^2 + 3x + 1$	$\tau_{sc}(1+\sqrt{2},2,3)_1 \otimes \varepsilon_1$
5E1	$y^2 = x^3 + 3^2ax^2 + 3^2ax + 2\cdot 3$	$\tau_{sc}(3,4,6)_3$
5E2	$y^2 = x^3 + 3^2ax^2 + 2\cdot 3^2ax + 2\cdot 3$	$\tau_{sc}(3,4,6)_1$
5E3	$y^2 = x^3 + 3^2ax^2 + 2\cdot 3^2ax + 3a$	$\tau_{sc}(3,4,6)_5$
5E4	$y^2 = x^3 + 3^2ax^2 + 3^3ax + 3a$	$\tau_{sc}(3,4,6)_6$
5E5	$y^2 = x^3 + 3^2ax^2 + 3^3ax + 2\cdot 3a$	$\tau_{sc}(3,4,6)_4$
5E6	$y^2 = x^3 + 3^2ax^2 + 3^2ax + 3a$	$\tau_{sc}(3,4,6)_9$
5E7	$y^2 = x^3 + 3^2ax^2 + 3^2ax + 2\cdot 3a$	$\tau_{sc}(3,4,6)_8$
5E8	$y^2 = x^3 + 3^2ax^2 + 3^3ax + 2\cdot 3$	$\tau_{sc}(3,4,6)_2$
5E9	$y^2 = x^3 + 3^2ax^2 + 2\cdot 3^2ax + 2\cdot 3a$	$\tau_{sc}(3,4,6)_7$

TABLE 2. Curves realizing inertial types of curves over $\mathbb{Q}_9$ with potentially good reduction

Label	Defining Polynomial of Inertia Field over $\mathbb{Q}_9$ $\quad (a=\sqrt{2})$	Inertial Type
2E1	x^2-3	$\varepsilon_1 \oplus \varepsilon_1$
2E2	x^4+6a	$\tau_{ps}(1,1,4)$
3E1	$x^{12}+(3a+6)x^4+3a+3$	$\tau_{sc}(3+3\sqrt{2},2,6)_3$
3E2	$x^{12}+(6a+3)x^4+3a+3$	$\tau_{sc}(3+3\sqrt{2},2,6)_2$
3E3	$x^{12}+6a\,x^4+6a$	$\tau_{sc}(3,2,6)_2$
3E4	$x^{12}+(6a+6)x^4+6a+3$	$\tau_{sc}(3+3\sqrt{2},2,6)_1$
3E5	$x^{12}+6x^4+3$	$\tau_{sc}(3,2,6)_1$
3E6	$x^{12}+3x^4+3$	$\tau_{sc}(3,2,6)_4$
3E7	$x^{12}+(3a+3)x^4+6a+3$	$\tau_{sc}(3+3\sqrt{2},2,6)_4$
3E8	$x^{12}+3a\,x^4+6a$	$\tau_{sc}(3,2,6)_3$
4E1	$x^3+(6a+3)x^2+3$	$\tau_{sc}(1+\sqrt{2},2,3)_3$
4E2	x^3+3x^2+3	$\tau_{ps}(1,2,3)_2$
4E3	$x^3+3a\,x^2+3$	$\tau_{ps}(1,2,3)_3$
4E4	$x^3+6a\,x^2+3$	$\tau_{ps}(1,2,3)_4$
4E5	x^3+6x^2+3	$\tau_{ps}(1,2,3)_1$
4E6	$x^3+(6a+6)x^2+3$	$\tau_{sc}(1+\sqrt{2},2,3)_4$
4E7	$x^3+(3a+6)x^2+3$	$\tau_{sc}(1+\sqrt{2},2,3)_2$
4E8	$x^3+(3a+3)x^2+3$	$\tau_{sc}(1+\sqrt{2},2,3)_1$
4E9	$x^6+(6a+3)x^4+3$	$\tau_{sc}(1+\sqrt{2},2,3)_3 \otimes \varepsilon_1$
4E10	x^6+3x^4+3	$\tau_{ps}(1,2,3)_2 \otimes \varepsilon_1$
4E11	$x^6+3a\,x^4+3$	$\tau_{ps}(1,2,3)_3 \otimes \varepsilon_1$
4E12	$x^6+6a\,x^4+3$	$\tau_{ps}(1,2,3)_4 \otimes \varepsilon_1$
4E13	x^6+6x^4+3	$\tau_{ps}(1,2,3)_1 \otimes \varepsilon_1$
4E14	$x^6+(6a+6)x^4+3$	$\tau_{sc}(1+\sqrt{2},2,3)_4 \otimes \varepsilon_1$
4E15	$x^6+(3a+6)x^4+3$	$\tau_{sc}(1+\sqrt{2},2,3)_2 \otimes \varepsilon_1$
4E16	$x^6+(3a+3)x^4+3$	$\tau_{sc}(1+\sqrt{2},2,3)_1 \otimes \varepsilon_1$
5E1	$x^{12}+(9a+18)x^4+3$	$\tau_{sc}(3,4,6)_3$
5E2	$x^{12}+(18a+18)x^4+3$	$\tau_{sc}(3,4,6)_1$
5E3	$x^{12}+(9a+9)x^4+3$	$\tau_{sc}(3,4,6)_5$
5E4	$x^{12}+9x^4+3$	$\tau_{sc}(3,4,6)_6$
5E5	$x^{12}+3$	$\tau_{sc}(3,4,6)_4$
5E6	$x^{12}+(18a+9)x^4+3$	$\tau_{sc}(3,4,6)_9$
5E7	$x^{12}+18a\,x^4+3$	$\tau_{sc}(3,4,6)_8$
5E8	$x^{12}+18x^4+3$	$\tau_{sc}(3,4,6)_2$
5E9	$x^{12}+9a\,x^4+3$	$\tau_{sc}(3,4,6)_7$

TABLE 3. Inertial field of types of curves over $\mathbb{Q}_9$ with potentially good reduction

p	Label of character χ	$\chi(g_1)$	$\chi(g_2)$	$\chi(g_3)$	$\chi(g_4)$
3	$\chi(r)$	i^r	1	-	-
	$\eta(r,s)$	ζ_6^r	ζ_3^s	-	-
	$\chi_1(r,s,t)$	ζ_6^r	ζ_3^s	ζ_3^t	-
	$\chi_2(r,s)$	ζ_6^r	ζ_3^s	-	-
	$\chi_3(r,s)$	ζ_6^r	ζ_3^s	-	-

TABLE 4. Explicit definition of characters appearing in Table 1