p	e	$v(N_E)$	τ_E	Base Change of	Curve	Description	Character
			$\varepsilon_1 \oplus \varepsilon_1$		4E1		ε_1
		4	$\varepsilon_3 \oplus \varepsilon_3$	$\eta_{-1} \oplus \eta_{-1}$	4E2	Principal Series	ε_3
			$\varepsilon_5 \oplus \varepsilon_5$		4E3		ε_5
	2		$\varepsilon_7 \oplus \varepsilon_7$	$\eta_{-2} \oplus \eta_{-2}$	6E12		ε_7
		6	$\varepsilon_9 \oplus \varepsilon_9$		6E16	Principal Series	ε_9
			$\varepsilon_{11} \oplus \varepsilon_{11}$	$\eta_2 \oplus \eta_2$	6E7		ε_{11}
			$\varepsilon_{13} \oplus \varepsilon_{13}$		6E9		ε_{13}
	3	2	$\tau_{ps,4}(1,1,3)$	$\tau_{sc,2}(5,1,3)$	2E1	Principal Series	$\eta(2,0,0)$
			$\tau_{ps,4}(1,3,4)_1$		6E4		$\chi(1,3,1)$
			$\tau_{ps,4}(1,3,4)_2$		6E8	Principal Series	$\chi(1,3,0)$
			$\tau_{ps,4}(1,3,4)_3$		6E14		$\chi(1,1,1)$
		6	$\tau_{ps,4}(1,3,4)_4$		6E15		$\chi(1,1,0)$
			$\tau_{sc}(y_{15},3,4)_1$		6E1		$\chi_{15}(0,0,1)$
			$\tau_{sc}(y_{15},3,4)_2$		6E2	Supercuspidal	$\chi_{15}(1,2,1)$
			$\tau_{sc}(y_{15},3,4)_3$		6E3		$\chi_{15}(1,2,0)$
			$\tau_{sc}(y_{15},3,4)_4$		6E6		$\chi_{15}(1,0,1)$
			$\tau_{ps,4}(1,4,4)_1$	$\tau_{sc,2}(5,4,4) \otimes \eta_{-1}$	8E5		$\chi(2,1,0)$
			$\tau_{ps,4}(1,4,4)_2$	$\tau_{ps,2}(1,4,4) \otimes \eta_{-1}$	8E10		$\chi(1,2,0)$
			$\tau_{ps,4}(1,4,4)_3$		8E18		$\chi(1,0,1)$
2	4		$\tau_{ps,4}(1,4,4)_4$		8E21	Principal Series	$\chi(2,1,1)$
			$\tau_{ps,4}(1,4,4)_5$		8E31		$\chi(0,1,1)$
			$\tau_{ps,4}(1,4,4)_6$		8E32		$\chi(1,2,1)$
			$\tau_{ps,4}(1,4,4)_7$	$\tau_{sc,2}(5,4,4)$	8E37		$\chi(0,1,0)$
		8	$\tau_{ps,4}(1,4,4)_8$	$\tau_{ps,2}(1,4,4)$	8E40		$\chi(1,0,0)$
			$\tau_{sc}(y_{15},4,4)_1$		8E3		$\chi_{15}(1,1,1)$
			$\tau_{sc}(y_{15},4,4)_2$		8E15		$\chi_{15}(0,1,0)$
			$\tau_{sc}(y_{15},4,4)_3$		8E17		$\chi_{15}(1,3,1)$
			$\tau_{sc}(y_{15},4,4)_4$		8E20	Supercuspidal	$\chi_{15}(2,1,0)$
			$\tau_{sc}(y_{15},4,4)_5$		8E25		$\chi_{15}(1,3,0)$
			$\tau_{sc}(y_{15},4,4)_6$		8E34		$\chi_{15}(2,1,1)$
			$\tau_{sc}(y_{15},4,4)_7$		8E38		$\chi_{15}(1,1,0)$
			$\tau_{sc}(y_{15},4,4)_8$		8E39		$\chi_{15}(0,1,1)$
			$\tau_{ps,4}(1,1,3) \otimes \varepsilon_1$		4E4		$\eta(5,0,1)$
		4	$\tau_{ps,4}(1,1,3) \otimes \varepsilon_3$	$\tau_{sc,2}(5,1,3) \otimes \eta_{-1}$	4E5		$\eta(5,1,0)$
			$\tau_{ps,4}(1,1,3) \otimes \varepsilon_5$		4E6		$\eta(2,1,1)$
	6		$\tau_{ps,4}(1,1,3) \otimes \varepsilon_7$	$\tau_{sc,2}(5,1,3) \otimes \eta_{-2}$	6E17	Principal Series	$\eta(2,1,0)$
		6	$\tau_{ps,4}(1,1,3) \otimes \varepsilon_9$		6E18		$\eta(5,1,1)$
			$\tau_{ps,4}(1,1,3) \otimes \varepsilon_{11}$	$\tau_{sc,2}(5,1,3) \otimes \eta_2$	6E19		$\eta(5,0,0)$
			$\tau_{ps,4}(1,1,3) \otimes \varepsilon_{13}$		6E20		$\eta(2,0,1)$

TABLE 1. Principal series and supercuspidal unramified types of elliptic curves over $\mathbb{Q}_4$. Curves are defined in tables 6-8 and characters in Table 4

p	e	$v(N_E)$	τ_E	Base Change of	Curve	Description	Characters
	8	5	$\tau_{tri}(1,3,5)_1$	$\tau_{sc,2}(-20,3,4)$	5E3	Triply Imprimitive	Table 3
			$\tau_{tri}(1,4,5)_1$		5E4		
			$\tau_{tri}(2,3,6)_1$	$\tau_{sc,2}(-4,3,4)$	5E2		
			$\tau_{tri}(2,4,5)_1$		5E1		
		6	$\tau_{tri}(1,3,5)_2$	$\tau_{sc,2}(-20,3,4)\otimes\eta_2$	6E10		
			$\tau_{tri}(1,4,5)_2$		6E13		
			$\tau_{tri}(2,3,6)_2$	$\tau_{sc,2}(-4,3,4)\otimes\eta_2$	6E11		
			$\tau_{tri}(2,4,5)_2$		6E5		
		8	$\tau_{tri}(5,8,13)_1$		8E1		
			$\tau_{tri}(1,8,10)_1$		8E2		
			$\tau_{tri}(2,12,14)_1$		8E4		
			$\tau_{tri}(3,8,12)_1$		8E6		
			$\tau_{tri}(1,7,9)_1$		8E7		
			$\tau_{tri}(2,12,14)_2$		8E8		
			$\tau_{tri}(3,7,11)_1$		8E9		
			$\tau_{tri}(5,7,14)_1$		8E11		
			$\tau_{tri}(2,11,13)_1$		8E12		
			$\tau_{tri}(6,10,11)_1$		8E13		
			$\tau_{tri}(3,9,14)_1$		8E14		
			$\tau_{tri}(1,8,10)_2$		8E16		
			$\tau_{tri}(2,11,13)_2$		8E19		
			$\tau_{tri}(5,8,13)_2$		8E22		
			$\tau_{tri}(3,8,12)_2$		8E23		
			$\tau_{tri}(3,9,14)_2$		8E24		
			$\tau_{tri}(5,7,14)_2$		8E26		
			$\tau_{tri}(6,10,11)_2$		8E27		
			$\tau_{tri}(3,10,13)_1$	$\tau_{sc}(-4,6,4)\otimes\eta_2$	8E28		
			$\tau_{tri}(1,7,9)_2$		8E29		
			$\tau_{tri}(3,7,11)_2$		8E30		
			$\tau_{tri}(3,10,13)_2$	$\tau_{sc}(-4,6,4)\otimes\eta_2$	8E33		
			$\tau_{tri}(6,9,12)_1$		8E35		
			$\tau_{tri}(6,9,12)_2$		8E36		

TABLE 2. Triply imprimitive inertial types of elliptic curves defined over $\mathbb{Q}_4$. Curves are defined in tables 6-8, the three characters that define the type are in 3 and the explicit description of each character in Table 4

N_E	τ_E	Character 1	Character 2	Character 3	E
2^5	$\tau_{tri}(1,3,5)_1$	$\chi_1(1,1,2)$	$\chi_3(1,0,1,1)$	$\chi_5(1,1,0)$	$5E3$
	$\tau_{tri}(1,4,6)_1$	$\chi_1(1,3,2)$	$\chi_4(1,0,1,1)$	$\chi_6(1,3,0)$	$5E4$
	$\tau_{tri}(2,3,6)_1$	$\chi_2(1,3,2)$	$\chi_3(1,0,1,0)$	$\chi_6(1,1,0)$	$5E2$
	$\tau_{tri}(2,4,5)_1$	$\chi_2(1,1,2)$	$\chi_4(1,0,1,0)$	$\chi_5(1,3,0)$	$5E1$
2^6	$\tau_{tri}(1,3,5)_2$	$\chi_1(1,1,0)$	$\chi_3(1,2,1,1)$	$\chi_5(1,1,2)$	$6E21$
	$\tau_{tri}(1,4,6)_2$	$\chi_1(1,3,0)$	$\chi_4(1,2,1,0)$	$\chi_6(1,3,2)$	$6E26$
	$\tau_{tri}(2,3,6)_2$	$\chi_2(1,1,0)$	$\chi_3(1,2,1,0)$	$\chi_6(1,1,2)$	$6E22$
	$\tau_{tri}(2,4,5)_2$	$\chi_2(1,3,0)$	$\chi_4(1,2,1,1)$	$\chi_5(1,3,2)$	$6E10$
2^8	$\tau_{tri}(5,8,13)_1$	$\chi_5(1,1,1)$	$\chi_8(1,1,1)$	$\chi_{13}(1,1,1)$	$8E1$
	$\tau_{tri}(1,8,10)_1$	$\chi_1(1,1,3)$	$\chi_8(1,2,1)$	$\chi_{10}(1,3,1)$	$8E2$
	$\tau_{tri}(2,12,14)_1$	$\chi_2(0,1,1)$	$\chi_{12}(1,2,1)$	$\chi_{14}(1,1,1)$	$8E4$
	$\tau_{tri}(3,8,12)_1$	$\chi_3(0,1,0,1)$	$\chi_8(0,1,1)$	$\chi_{12}(0,1,1)$	$8E6$
	$\tau_{tri}(1,7,9)_1$	$\chi_1(1,3,3)$	$\chi_7(1,3,1)$	$\chi_9(1,0,1)$	$8E7$
	$\tau_{tri}(2,12,14)_2$	$\chi_2(2,1,3)$	$\chi_{12}(1,0,1)$	$\chi_{14}(1,3,1)$	$8E8$
	$\tau_{tri}(3,7,11)_1$	$\chi_3(0,1,0,0)$	$\chi_7(0,1,1)$	$\chi_{11}(2,1,1)$	$8E9$
	$\tau_{tri}(5,7,14)_1$	$\chi_5(1,3,3)$	$\chi_7(1,2,1)$	$\chi_{14}(1,0,1)$	$8E11$
	$\tau_{tri}(2,11,13)_1$	$\chi_2(2,1,1)$	$\chi_{11}(1,1,1)$	$\chi_{13}(1,1,1)$	$8E12$
	$\tau_{tri}(6,10,11)_1$	$\chi_6(1,3,3)$	$\chi_{10}(1,2,1)$	$\chi_{11}(1,2,1)$	$8E13$
	$\tau_{tri}(3,9,14)_1$	$\chi_3(1,1,0,0)$	$\chi_9(2,1,1)$	$\chi_{14}(2,1,1)$	$8E14$
	$\tau_{tri}(1,8,10)_2$	$\chi_1(1,1,1)$	$\chi_8(1,0,1)$	$\chi_{10}(1,1,1)$	$8E16$
	$\tau_{tri}(2,11,13)_2$	$\chi_2(0,1,3)$	$\chi_{11}(1,3,1)$	$\chi_{13}(1,3,1)$	$8E19$
	$\tau_{tri}(5,8,13)_2$	$\chi_5(1,1,3)$	$\chi_8(1,3,1)$	$\chi_{13}(1,0,1)$	$8E22$
	$\tau_{tri}(3,8,12)_2$	$\chi_3(2,1,0,1)$	$\chi_8(2,1,1)$	$\chi_{12}(2,1,1)$	$8E23$
	$\tau_{tri}(3,9,14)_2$	$\chi_3(1,3,0,0)$	$\chi_9(0,1,1)$	$\chi_{14}(0,1,1)$	$8E24$
	$\tau_{tri}(5,7,14)_2$	$\chi_5(1,3,1)$	$\chi_7(1,0,1)$	$\chi_{14}(1,2,1)$	$8E26$
	$\tau_{tri}(6,10,11)_2$	$\chi_6(1,3,1)$	$\chi_{10}(1,0,1)$	$\chi_{11}(1,0,1)$	$8E27$
	$\tau_{tri}(3,10,13)_1$	$\chi_3(1,3,0,1)$	$\chi_{10}(2,1,1)$	$\chi_{13}(2,1,1)$	$8E28$
	$\tau_{tri}(1,7,9)_2$	$\chi_1(1,3,1)$	$\chi_7(1,1,1)$	$\chi_9(1,2,1)$	$8E29$
	$\tau_{tri}(3,7,11)_2$	$\chi_3(2,1,0,1)$	$\chi_7(2,1,1)$	$\chi_{11}(2,1,1)$	$8E30$
	$\tau_{tri}(3,10,13)_2$	$\chi_3(1,1,0,1)$	$\chi_{10}(0,1,1)$	$\chi_{13}(0,1,1)$	$8E33$
	$\tau_{tri}(6,9,12)_1$	$\chi_6(1,1,1)$	$\chi_9(1,1,1)$	$\chi_{12}(1,1,1)$	$8E35$
	$\tau_{tri}(6,9,12)_2$	$\chi_6(1,1,3)$	$\chi_9(1,3,1)$	$\chi_{12}(1,3,1)$	$8E36$

TABLE 3. Triply imprimitive inertial types

p	Label of character χ	$\chi(g_1)$	$\chi(g_2)$	$\chi(g_3)$	$\chi(g_4)$
2	$\chi(r,s,t)$	i^r	i^s	$(-1)^t$	-
	$\eta(r,s,t)$	ζ_6^r	$(-1)^s$	$(-1)^t$	-
	$\chi_i(r,s,t)$ with $i \in \{1,2,5,6\}$	i^r	i^s	i^t	-
	$\chi_i(r,s,t,w)$ with $i \in \{3,4\}$	i^r	i^s	$(-1)^t$	$(-1)^w$
	$\chi_i(r,s,t)$ with $7 \leq i \leq 14$	i^r	i^s	$(-1)^t$	-
	χ_{15}	i^r	i^s	$(-1)^t$	-

TABLE 4. Explicit definition of characters appearing tables 1, 2 and 3

Label	Curve (ϕ root of $x^2 - x - 1$)
2E1	$y^2 = x^3 + x^2 + 2^2x + 2^2$
4E1	$y^2 = x^3 + 2^4(-135)x + 2^4(-5130\phi + 2565)$
4E2	$y^2 = x^3 + 2^4(-27)x + 2^4(-513)$
4E3	$y^2 = x^3 + 2^4(-135)x + 2^4(5130\phi - 2565)$
4E4	$y^2 = x^3 + (2\phi - 1)x^2 + 2^2 \cdot 5x + 2^2(10\phi - 5)$
4E5	$y^2 = x^3 - x^2 + 2^2x - 2^2$
4E6	$y^2 = x^3 + (-2\phi + 1)x^2 + 2^2 \cdot 5x + 2^2(-10\phi + 5)$
5E1	$y^2 = x^3 + x^2 + 2x + 2^2\phi$
5E2	$y^2 = x^3 - x$
5E3	$y^2 = x^3 + x^2 + 2x + 2^2$
5E4	$y^2 = x^3 + x^2 + 2x + 2^2(\phi + 1)$
6E1	$y^2 = x^3 + x^2 + 2(3\phi + 2)x + 2(\phi + 1)$
6E2	$y^2 = x^3 + (-2\phi + 1)x^2 + 2(15\phi + 10)x + 2(-15\phi - 5)$
6E3	$y^2 = x^3 + 2(2\phi - 1)x^2 + 2^3(15\phi + 10)x + 2^4(15\phi + 5)$
6E4	$y^2 = x^3 + x^2 + 2\phi x + 2(\phi + 1)$
6E5	$y^2 = x^3 + x^2 + 2^2x + 2(2\phi + 1)$
6E6	$y^2 = x^3 + 2x^2 + 2^3(3\phi + 2)x + 2^4(\phi + 1)$
6E7	$y^2 = x^3 + 2^6(-27)x + 2^7(513)$
6E8	$y^2 = x^3 + 2(2\phi - 1)x^2 + 2^3 \cdot 5\phi x + 2^4(15\phi + 5)$
6E9	$y^2 = x^3 + 2^6(-135)x + 2^7(-5130\phi + 2565)$
6E10	$y^2 = x^3 + x^2 + 2^2x + 2$
6E11	$y^2 = x^3 + x^2 + 2^3x + 2$
6E12	$y^2 = x^3 + 2^6(-27)x + 2^7(-513)$
6E13	$y^2 = x^3 + x^2 + 2^4x + 2(2\phi + 1)$
6E14	$y^2 = x^3 + x^2 + 2(\phi + 1)x + 2\phi$
6E15	$y^2 = x^3 + (2\phi - 1)x^2 + 2 \cdot 5\phi x + 2(15\phi + 5)$
6E16	$y^2 = x^3 + 2^6(-135)x + 2^7(5130\phi - 2565)$
6E17	$y^2 = x^3 + 2(-1)x^2 + 2^4x + 2^5(-1)$
6E18	$y^2 = x^3 + 2(2\phi - 1)x^2 + 2^4 \cdot 5x + 2^5(10\phi - 5)$
6E19	$y^2 = x^3 + 2x^2 + 2^4x + 2^5$
6E20	$y^2 = x^3 + 2(-2\phi + 1)x^2 + 2^4 \cdot 5x + 2^5(-10\phi + 5)$

TABLE 5. Curves realizing nonexceptional inertial types with $\nu(N_E) \leq 6$ over $\mathbb{Q}_4$

Label	Curve (ϕ root of $x^2 - x - 1$)
8E1	$y^2 = x^3 + 2x^2 + 2x + 2^2(34\phi + 21)$
8E2	$y^2 = x^3 + x^2 + x + (8\phi + 5)$
8E3	$y^2 = x^3 + 2(3\phi + 1)x^2 + 2(5\phi + 5)x + 2^2(5\phi - 10)$
8E4	$y^2 = x^3 - 2\phi x^2 + 2(3\phi + 2)x + 2^2(-\phi - 1)$
8E5	$y^2 = x^3 - x^2 - 3x - 1$
8E6	$y^2 = x^3 + 2(\phi - 1)x$
8E7	$y^2 = x^3 + 2\phi x^2 + 2(2\phi + 1)x + 2^2(3\phi + 2)$
8E8	$y^2 = x^3 + 2\phi x^2 + 2(3\phi + 2)x + 2^2(\phi + 1)$
8E9	$y^2 = x^3 + 2x^2 + 2^2x + 2(2\phi + 1)x + 2^3(\phi + 1)$
8E10	$y^2 = x^3 + 2\phi x^2 + 2(34\phi + 21)x + 2^2\phi$
8E11	$y^2 = x^3 + 2^2\phi x^2 + 2\phi x + 2^4\phi$
8E12	$y^2 = x^3 - x^2 + (-8\phi - 5)x + (24\phi - 3)$
8E13	$y^2 = x^3 - x^2 + (2\phi - 3)x + (2\phi - 1)$
8E14	$y^2 = x^3 + 2(1 + \phi)x^2 + (2\phi + 1) + 2^2(1 + \phi)$
8E15	$y^2 = x^3 + 2^2\phi x^2 + 2\phi x + 2^3\phi$
8E16	$y^2 = x^3 - x^2 + x + (-8\phi - 5)$
8E17	$y^2 = x^3 + 2(-3\phi - 1)x^2 + 2(5\phi + 5)x + 2^2(-5\phi + 10)$
8E18	$y^2 = x^3 + 2(1 + \phi)x^2 + 2(170\phi + 105)x + 2^2(5\phi + 10)$
8E19	$y^2 = x^3 + x^2 + (-8\phi - 5)x + (-24\phi + 3)$
8E20	$y^2 = x^3 - 2^2\phi x^2 + 2\phi x - 2^3\phi$
8E21	$y^2 = x^3 + (-2\phi + 1)x^2 - 15x + (-10\phi + 5)$
8E22	$y^2 = x^3 - 2x^2 + 2x + 2^2(-34\phi - 21)$
8E23	$y^2 = x^3 + 2(5\phi - 5)x$
8E24	$y^2 = x^3 + 2^7(-135\phi - 54)x + 2^{13}(297\phi + 270)$
8E25	$y^2 = x^3 + 2^2(-3\phi - 1)x^2 + 2^3(5\phi + 5)x + 2^5(-5\phi + 10)$
8E26	$y^2 = x^3 - 2^2\phi x^2 + 2\phi x - 2^4\phi$
8E27	$y^2 = x^3 + x^2 + (2\phi - 3)x + (-2\phi + 1)$
8E28	$y^2 = x^3 - 2x$
8E29	$y^2 = x^3 - 2\phi x^2 + 2(2\phi + 1)x + 2^2(-3\phi - 2)$
8E30	$y^2 = x^3 + 2^5(810\phi + 135)x + 2^{10}(-6345\phi + 135)$
8E31	$y^2 = x^3 + (2\phi - 1)x^2 - 15x + (10\phi - 5)$
8E32	$y^2 = x^3 + 2(-\phi - 2)x^2 + 2(170\phi + 105)x + 2^2(-5\phi - 10)$
8E33	$y^2 = x^3 - 2^3x$
8E34	$y^2 = x^3 - 2^3\phi x^2 + 2^3\phi x - 2^6\phi$
8E35	$y^2 = x^3 + 2\phi x^2 + 2(5\phi + 3)x + 2^2(2\phi + 1)$
8E36	$y^2 = x^3 - 2\phi x^2 + 2(5\phi + 3)x + 2^2(-2\phi - 1)$
8E37	$y^2 = x^3 + x^2 - 3x + 1$
8E38	$y^2 = x^3 + 2^2(3\phi + 1)x^2 + 2^3(5\phi + 5)x + 2^5(5\phi - 10)$
8E39	$y^2 = x^3 + 2^3\phi x^2 + 2^3\phi x + 2^6\phi$
8E40	$y^2 = x^3 - 2\phi x^2 + 2(34\phi + 21)x - 2^2\phi$

TABLE 6. Curves realizing nonexceptional inertial types with $\nu(N_E) = 8$ over $\mathbb{Q}_4$

Curve	Defining Polynomial of Inertia Field over $\mathbb{Q}_4$ (ϕ root of $x^2 - x - 1$)	Type
2E1	$x^3 + 2$	$\tau_{ps,4}(1,1,3)$
4E1	$x^2 - (1 - 2\phi)$	$\varepsilon_1 \oplus \varepsilon_1$
4E2	$x^2 + 1$	$\varepsilon_3 \oplus \varepsilon_3$
4E3	$x^2 + (1 - 2\phi)$	$\varepsilon_5 \oplus \varepsilon_5$
4E4	$x^6 + (2\phi + 2)x^3 + 6$	$\tau_{ps,4}(1,1,3) \otimes \varepsilon_1$
4E5	$x^6 + 2x^3 + 2$	$\tau_{ps,4}(1,1,3) \otimes \varepsilon_3$
4E6	$x^6 + 2\phi x^3 + 6$	$\tau_{ps,4}(1,1,3) \otimes \varepsilon_5$
5E1	$x^8 + 2x^6 + 4\phi x^4 + 4x^3 + 4x + 4\phi + 2$	$\tau_{tri}(2,4,5)_1$
5E2	$x^8 + 2x^6 + 4x^3 + 4x + 6$	$\tau_{tri}(2,3,6)_1$
5E3	$x^8 + 2x^6 + 4x^3 + 4x + 2$	$\tau_{tri}(1,3,5)_1$
5E4	$x^8 + 2x^6 + 4x^3 + 4x + 4\phi + 6$	$\tau_{tri}(1,4,5)_1$
6E1	$x^4 + 4x^3 + 2x^2 + (4\phi + 4)x + 2$	$\tau_{sc}(y_{15},3,4)_1$
6E2	$x^4 + (4\phi + 4)x^3 + 2x^2 + (4\phi + 4)x + 10$	$\tau_{sc}(y_{15},3,4)_2$
6E3	$x^4 + 4x^3 + 2x^2 + 4\phi x + 10$	$\tau_{sc}(y_{15},3,4)_3$
6E4	$x^4 + 4\phi x^3 + 2x^2 + (4\phi + 4)x + 4\phi + 2$	$\tau_{ps,4}(1,3,4)_1$
6E5	$x^8 + (4\phi + 4)x^7 + 2x^6 + 4x^5 + 4x^3 + 12\phi + 2$	$\tau_{tri}(2,4,5)_2$
6E6	$x^4 + 4\phi x^3 + 2x^2 + 4\phi x + 2$	$\tau_{sc}(y_{15},3,4)_4$
6E7	$x^2 - 2$	$\varepsilon_{11} \oplus \varepsilon_{11}$
6E8	$x^4 + 4x^3 + 2x^2 + 4\phi x + 4\phi + 10$	$\tau_{ps,4}(1,3,4)_2$
6E9	$x^2 + (4\phi - 2)$	$\varepsilon_{13} \oplus \varepsilon_{13}$
6E10	$x^8 + 2x^6 + 4x^5 + 4x^3 + 2$	$\tau_{tri}(1,3,5)_2$
6E11	$x^8 + 4x^7 + 2x^6 + 4x^5 + 4x^3 + 6$	$\tau_{tri}(2,3,6)_2$
6E12	$x^2 + 2$	$\varepsilon_7 \oplus \varepsilon_7$
6E13	$x^8 + 4\phi x^7 + 2x^6 + 4x^5 + 4x^3 + 12\phi + 6$	$\tau_{tri}(1,4,5)_2$
6E14	$x^4 + 4\phi x^3 + 2x^2 + 4\phi x + 4\phi + 2$	$\tau_{ps,4}(1,3,4)_3$
6E15	$x^4 + 2x^2 + (4\phi + 4)x + 4\phi + 10$	$\tau_{ps,4}(1,3,4)_4$
6E16	$x^2 - (4\phi - 2)$	$\varepsilon_9 \oplus \varepsilon_9$
6E17	$x^6 + 2$	$\tau_{ps,4}(1,1,3) \otimes \varepsilon_7$
6E18	$x^6 + (4\phi + 4)x^3 + 8\phi + 2$	$\tau_{ps,4}(1,1,3) \otimes \varepsilon_9$
6E19	$x^6 + 4x^3 + 2$	$\tau_{ps,4}(1,1,3) \otimes \varepsilon_{11}$
6E20	$x^6 + 4\phi x^3 + 8\phi + 2$	$\tau_{ps,4}(1,1,3) \otimes \varepsilon_{13}$

TABLE 7. Inertia fields of types with $\nu(N_E) \leq 6$ over $\mathbb{Q}_4$

Curve	Defining Polynomial of Inertia Field over $\mathbb{Q}_4$ (ϕ root of x^2-x-1)	Type
8E1	$x^8+8x^5+(2\phi+2)x^4+4\phi x^2+(8\phi+8)x+24\phi+2$	$\tau_{tri}(5,8,13)_1$
8E2	$x^8+(8\phi+8)x^7+4\phi x^6+8\phi x^5+2\phi x^4+(8\phi+8)x^3+(4\phi+4)x^2+8\phi x+24\phi+2$	$\tau_{tri}(1,8,10)_1$
8E3	$x^4+8\phi x^3+4\phi x^2+8\phi x+24\phi+2$	$\tau_{sc}(y_{15},4,4)_1$
8E4	$x^8+8\phi x^7+4x^6+2\phi x^4+8x^3+(4\phi+4)x^2+8\phi x+8\phi+6$	$\tau_{tri}(2,12,14)_1$
8E5	x^4+8x^3+8x+2	$\tau_{ps,4}(1,4,4)_1$
8E6	$x^8+(8\phi+8)x^7+2x^4+8x^3+4x^2+8x+8\phi+2$	$\tau_{tri}(3,8,12)_1$
8E7	$x^8+4\phi x^6+8\phi x^5+2\phi x^4+(8\phi+8)x^3+(4\phi+4)x^2+8\phi x+2$	$\tau_{tri}(1,7,9)_1$
8E8	$x^8+8x^7+4x^6+2\phi x^4+8x^3+(4\phi+4)x^2+8\phi x+8\phi+6$	$\tau_{tri}(2,12,14)_2$
8E9	$x^8+8x^7+2x^4+8x^3+4x^2+8x+16\phi+2$	$\tau_{tri}(3,7,11)_1$
8E10	x^4+4x^2+2	$\tau_{ps,4}(1,4,4)_2$
8E11	$x^8+(8\phi+8)x^7+8x^5+(2\phi+2)x^4+4\phi x^2+(8\phi+8)x+16\phi+2$	$\tau_{tri}(5,7,14)_1$
8E12	$x^8+8\phi x^7+4x^6+2\phi x^4+8x^3+(4\phi+4)x^2+8\phi x+6$	$\tau_{tri}(2,11,13)_1$
8E13	$x^8+4\phi x^6+8\phi x^5+(2\phi+2)x^4+(8\phi+8)x^3+4\phi x^2+(8\phi+8)x+6$	$\tau_{tri}(6,10,11)_1$
8E14	$x^8+8\phi x^7+4x^6+8x^5+2x^4+4x^2+8x+16\phi+2$	$\tau_{tri}(3,9,14)_1$
8E15	$x^4+8x^3+(4\phi+4)x^2+(8\phi+8)x+2$	$\tau_{sc}(y_{15},4,4)_2$
8E16	$x^8+4\phi x^6+8\phi x^5+2\phi x^4+(8\phi+8)x^3+(4\phi+4)x^2+8\phi x+24\phi+2$	$\tau_{tri}(1,8,10)_2$
8E17	$x^4+(8\phi+8)x^3+4\phi x^2+8\phi x+24\phi+2$	$\tau_{sc}(y_{15},4,4)_3$
8E18	$x^4+(8\phi+8)x^3+4x^2+16\phi+2$	$\tau_{ps,4}(1,4,4)_3$
8E19	$x^8+8x^7+4x^6+2\phi x^4+8x^3+(4\phi+4)x^2+8\phi x+6$	$\tau_{tri}(2,11,13)_2$
8E20	$x^4+(4\phi+4)x^2+(8\phi+8)x+16\phi+2$	$\tau_{sc}(y_{15},4,4)_4$
8E21	$x^4+8\phi x^3+8x+16\phi+2$	$\tau_{ps,4}(1,4,4)_4$
8E22	$x^8+8\phi x^7+8x^5+(2\phi+2)x^4+4\phi x^2+(8\phi+8)x+24\phi+2$	$\tau_{tri}(5,8,13)_2$
8E23	$x^8+8\phi x^7+2x^4+8x^3+4x^2+8x+24\phi+2$	$\tau_{tri}(3,8,12)_2$
8E24	$x^8+(8\phi+8)x^7+4x^6+8x^5+2x^4+4x^2+8x+16\phi+2$	$\tau_{tri}(3,9,14)_2$
8E25	$x^4+8x^3+4\phi x^2+8\phi x+24\phi+2$	$\tau_{sc}(y_{15},4,4)_5$
8E26	$x^8+8x^7+8x^5+(2\phi+2)x^4+4\phi x^2+(8\phi+8)x+16\phi+2$	$\tau_{tri}(5,7,14)_2$
8E27	$x^8+8\phi x^7+4\phi x^6+8\phi x^5+(2\phi+2)x^4+(8\phi+8)x^3+4\phi x^2+(8\phi+8)x+6$	$\tau_{tri}(6,10,11)_2$
8E28	$x^8+8x^7+4x^6+8x^5+2x^4+4x^2+8x+8\phi+2$	$\tau_{tri}(3,10,13)_1$
8E29	$x^8+(8\phi+8)x^7+4\phi x^6+8\phi x^5+2\phi x^4+(8\phi+8)x^3+(4\phi+4)x^2+8\phi x+2$	$\tau_{tri}(1,7,9)_2$
8E30	$x^8+2x^4+8x^3+4x^2+8x+2$	$\tau_{tri}(3,7,11)_2$
8E31	$x^4+(8\phi+8)x^3+8x+16\phi+2$	$\tau_{ps,4}(1,4,4)_5$
8E32	$x^4+8\phi x^3+4x^2+16\phi+2$	$\tau_{ps,4}(1,4,4)_6$
8E33	$x^8+4x^6+8x^5+2x^4+4x^2+8x+24\phi+2$	$\tau_{tri}(3,10,13)_2$
8E34	$x^4+(8\phi+8)x^3+(4\phi+4)x^2+(8\phi+8)x+2$	$\tau_{sc}(y_{15},4,4)_6$
8E35	$x^8+(8\phi+8)x^7+4\phi x^6+8\phi x^5+(2\phi+2)x^4+(8\phi+8)x^3+4\phi x^2+(8\phi+8)x+8\phi+6$	$\tau_{tri}(6,9,12)_1$
8E36	$x^8+8x^7+4\phi x^6+8\phi x^5+(2\phi+2)x^4+(8\phi+8)x^3+4\phi x^2+(8\phi+8)x+8\phi+6$	$\tau_{tri}(6,9,12)_2$
8E37	x^4+8x+2	$\tau_{ps,4}(1,4,4)_7$
8E38	$x^4+4\phi x^2+8\phi x+8\phi+2$	$\tau_{sc}(y_{15},4,4)_7$
8E39	$x^4+8\phi x^3+(4\phi+4)x^2+(8\phi+8)x+2$	$\tau_{sc}(y_{15},4,4)_8$
8E40	$x^4+8x^3+4x^2+2$	$\tau_{ps,4}(1,4,4)_8$

TABLE 8. Curves realizing nonexceptional inertial types with $\nu(N_E)=8$ over $\mathbb{Q}_4$