p	e	$\nu(N_E)$	Type	Base change of	Curve	Relevant extensions
2	8	5	$\tau_{ex}(5,8)_1$		5E1	$[K_4 \mid M_3, M_{31}, M_{35}]$
			$\tau_{ex}(5,8)_2$		5E2	$[K_4 \mid M_1, M_{15}, M_{17}]$
			$\tau_{ex}(5,8)_1 \otimes \varepsilon_5$		5E3	$[K_4 \mid M_3, M_{31}, M_{35}]$
			$\tau_{ex}(5,8)_1 \otimes \varepsilon_3$		5E4	$[K_4 \mid M_3, M_{31}, M_{35}]$
			$\tau_{ex}(5,8)_2 \otimes \varepsilon_3$		5E5	$[K_4 \mid M_1, M_{15}, M_{17}]$
			$\tau_{ex}(5,8)_2 \otimes \varepsilon_5$		5E6	$[K_4 \mid M_1, M_{15}, M_{17}]$
			$\tau_{ex}(5,8)_2 \otimes \varepsilon_1$		5E7	$[K_4 \mid M_1, M_{15}, M_{17}]$
			$\tau_{ex}(5,8)_1 \otimes \varepsilon_1$		5E8	$[K_4 \mid M_3, M_{31}, M_{35}]$
		6	$\tau_{ex}(6,8)_1$		6E1	$[K_4 \mid M_{69}, M_{113}, M_{115}]$
			$\tau_{ex}(6,8)_1 \otimes \varepsilon_1$		6E2	$[K_4 \mid M_{69}, M_{113}, M_{115}]$
			$\tau_{ex}(6,8)_1 \otimes \varepsilon_3$		6E3	$[K_4 \mid M_{69}, M_{113}, M_{115}]$
			$\tau_{ex}(6,8)_1 \otimes \varepsilon_5$		6E4	$[K_4 \mid M_{69}, M_{113}, M_{115}]$
			$\tau_{ex}(6,8)_1 \otimes \varepsilon_7$		6E5	$[K_4 \mid M_{69}, M_{113}, M_{115}]$
			$\tau_{ex}(6,8)_1 \otimes \varepsilon_9$		6E6	$[K_4 \mid M_{69}, M_{113}, M_{115}]$
			$\tau_{ex}(6,8)_1 \otimes \varepsilon_{11}$		6E7	$[K_4 \mid M_{69}, M_{113}, M_{115}]$
			$\tau_{ex}(6,8)_1 \otimes \varepsilon_{13}$		6E8	$[K_4 \mid M_{69}, M_{113}, M_{115}]$
			$\tau_{ex}(5,8)_1 \otimes \varepsilon_7$		6E30	$[K_4 \mid M_3, M_{31}, M_{35}]$
			$\tau_{ex}(5,8)_1 \otimes \varepsilon_9$		6E31	$[K_4 \mid M_3, M_{31}, M_{35}]$
			$\tau_{ex}(5,8)_1 \otimes \varepsilon_{11}$		6E32	$[K_4 \mid M_3, M_{31}, M_{35}]$
			$\tau_{ex}(5,8)_1 \otimes \varepsilon_{13}$		6E33	$[K_4 \mid M_3, M_{31}, M_{35}]$
			$\tau_{ex}(5,8)_2 \otimes \varepsilon_7$		6E34	$[K_4 \mid M_1, M_{15}, M_{17}]$
			$\tau_{ex}(5,8)_2 \otimes \varepsilon_9$		6E35	$[K_4 \mid M_1, M_{15}, M_{17}]$
			$\tau_{ex}(5,8)_2 \otimes \varepsilon_{11}$		6E36	$[K_4 \mid M_1, M_{15}, M_{17}]$
			$\tau_{ex}(5,8)_2 \otimes \varepsilon_{13}$		6E37	$[K_4 \mid M_1, M_{15}, M_{17}]$
	24	3	$\tau_{ex}(3,24)_1$		3E1	$[K_2 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_2$	$\tau_{ex,1,1}$	3E2	$[K_1 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_3$		3E3	$[K_3 \mid M_1, M_3, M_5]$
		4	$\tau_{ex}(3,24)_2 \otimes \varepsilon_3$	$\tau_{ex,1,1} \otimes \eta_{-1}$	4E1	$[K_1 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_2 \otimes \varepsilon_5$		4E2	$[K_1 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_1 \otimes \varepsilon_5$		4E3	$[K_2 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_1 \otimes \varepsilon_3$		4E4	$[K_2 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_3 \otimes \varepsilon_1$		4E5	$[K_3 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_1 \otimes \varepsilon_1$		4E6	$[K_2 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_3 \otimes \varepsilon_3$		4E7	$[K_3 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_3 \otimes \varepsilon_5$		4E8	$[K_3 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_2 \otimes \varepsilon_1$		4E9	$[K_1 \mid M_1, M_3, M_5]$
		6	$\tau_{ex}(3,24)_1 \otimes \varepsilon_7$		6E18	$[K_2 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_1 \otimes \varepsilon_9$		6E19	$[K_2 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_1 \otimes \varepsilon_{11}$		6E20	$[K_2 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_1 \otimes \varepsilon_{13}$		6E21	$[K_2 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_2 \otimes \varepsilon_7$	$\tau_{ex,1,1} \otimes \eta_{-2}$	6E22	$[K_1 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_2 \otimes \varepsilon_9$		6E23	$[K_1 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_2 \otimes \varepsilon_{11}$	$\tau_{ex,1,1} \otimes \eta_{-2}$	6E24	$[K_1 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_2 \otimes \varepsilon_{13}$		6E25	$[K_1 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_3 \otimes \varepsilon_7$		6E26	$[K_3 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_3 \otimes \varepsilon_9$		6E27	$[K_3 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_3 \otimes \varepsilon_{11}$		6E28	$[K_3 \mid M_1, M_3, M_5]$
			$\tau_{ex}(3,24)_3 \otimes \varepsilon_{13}$		6E29	$[K_3 \mid M_1, M_3, M_5]$

TABLE 1. Exceptional inertial types of elliptic curves defined over $\mathbb{Q}_4$ with additive potentially good reduction and $\nu(N_E) \leq 6$; the explicit definition of the curves is given Tables 3 and the inertia field of each type in Table 5

p	e	$\nu(N_E)$	Type	Base change of	Curve	Relevant extensions
			$\tau_{ex}(7,24)_1$		7E1	$[K_1 \vert M_{57}, M_{83}, M_{110}]$
			$\tau_{ex}(7,24)_2$		7E2	$[K_1 \vert M_{55}, M_{79}, M_{104}]$
			$\tau_{ex}(7,24)_1 \otimes \varepsilon_{13}$		7E3	$[K_1 \vert M_{57}, M_{83}, M_{110}]$
			$\tau_{ex}(7,24)_3$		7E4	$[K_2 \vert M_{39}, M_{88}, M_{112}]$
			$\tau_{ex}(7,24)_2 \otimes \varepsilon_{13}$	$\tau_{ex,2,2} \otimes \eta_2$	7E5	$[K_1 \vert M_{55}, M_{79}, M_{104}]$
			$\tau_{ex}(7,24)_4$		7E6	$[K_2 \vert M_{43}, M_{94}, M_{114}]$
			$\tau_{ex}(7,24)_3 \otimes \varepsilon_{13}$		7E7	$[K_2 \vert M_{39}, M_{88}, M_{112}]$
			$\tau_{ex}(7,24)_1 \otimes \varepsilon_9$		7E8	$[K_1 \vert M_{57}, M_{83}, M_{110}]$
			$\tau_{ex}(7,24)_4 \otimes \varepsilon_1$		7E9	$[K_2 \vert M_{43}, M_{94}, M_{114}]$
			$\tau_{ex}(7,24)_2 \otimes \varepsilon_9$	$\tau_{ex,2,2} \otimes \eta_{-2}$	7E10	$[K_1 \vert M_{55}, M_{79}, M_{104}]$
			$\tau_{ex}(7,24)_1 \otimes \varepsilon_{11}$		7E11	$[K_1 \vert M_{57}, M_{83}, M_{110}]$
			$\tau_{ex}(7,24)_1 \otimes \varepsilon_5$		7E12	$[K_1 \vert M_{57}, M_{83}, M_{110}]$
			$\tau_{ex}(7,24)_2 \otimes \varepsilon_1$	$\tau_{ex,2,2}$	7E13	$[K_1 \vert M_{55}, M_{79}, M_{104}]$
			$\tau_{ex}(7,24)_5$		7E14	$[K_3 \vert M_{48}, M_{72}, M_{120}]$
			$\tau_{ex}(7,24)_6$		7E15	$[K_3 \vert M_{54}, M_{74}, M_{124}]$
			$\tau_{ex}(7,24)_5 \otimes \varepsilon_9$		7E16	$[K_3 \vert M_{48}, M_{72}, M_{120}]$
			$\tau_{ex}(7,24)_6 \otimes \varepsilon_{11}$		7E17	$[K_3 \vert M_{54}, M_{74}, M_{124}]$
			$\tau_{ex}(7,24)_6 \otimes \varepsilon_9$		7E18	$[K_3 \vert M_{54}, M_{74}, M_{124}]$
			$\tau_{ex}(7,24)_6 \otimes \varepsilon_3$		7E19	$[K_3 \vert M_{54}, M_{74}, M_{124}]$
			$\tau_{ex}(7,24)_1 \otimes \varepsilon_1$		7E20	$[K_1 \vert M_{57}, M_{83}, M_{110}]$
			$\tau_{ex}(7,24)_2 \otimes \varepsilon_7$		7E21	$[K_1 \vert M_{55}, M_{79}, M_{104}]$
			$\tau_{ex}(7,24)_1 \otimes \varepsilon_3$		7E22	$[K_1 \vert M_{57}, M_{83}, M_{110}]$
			$\tau_{ex}(7,24)_3 \otimes \varepsilon_7$		7E23	$[K_2 \vert M_{39}, M_{88}, M_{112}]$
2	24	7	$\tau_{ex}(7,24)_4 \otimes \varepsilon_3$		7E24	$[K_2 \vert M_{43}, M_{94}, M_{114}]$
			$\tau_{ex}(7,24)_3 \otimes \varepsilon_{11}$		7E25	$[K_2 \vert M_{39}, M_{88}, M_{112}]$
			$\tau_{ex}(7,24)_4 \otimes \varepsilon_5$		7E26	$[K_2 \vert M_{43}, M_{94}, M_{114}]$
			$\tau_{ex}(7,24)_6 \otimes \varepsilon_1$		7E27	$[K_3 \vert M_{54}, M_{74}, M_{124}]$
			$\tau_{ex}(7,24)_5 \otimes \varepsilon_5$		7E28	$[K_3 \vert M_{48}, M_{72}, M_{120}]$
			$\tau_{ex}(7,24)_6 \otimes \varepsilon_7$		7E29	$[K_3 \vert M_{54}, M_{74}, M_{124}]$
			$\tau_{ex}(7,24)_5 \otimes \varepsilon_1$		7E30	$[K_3 \vert M_{48}, M_{72}, M_{120}]$
			$\tau_{ex}(7,24)_5 \otimes \varepsilon_{13}$		7E31	$[K_3 \vert M_{48}, M_{72}, M_{120}]$
			$\tau_{ex}(7,24)_3 \otimes \varepsilon_1$		7E32	$[K_2 \vert M_{39}, M_{88}, M_{112}]$
			$\tau_{ex}(7,24)_5 \otimes \varepsilon_3$		7E33	$[K_3 \vert M_{48}, M_{72}, M_{120}]$
			$\tau_{ex}(7,24)_2 \otimes \varepsilon_{11}$		7E34	$[K_1 \vert M_{55}, M_{79}, M_{104}]$
			$\tau_{ex}(7,24)_4 \otimes \varepsilon_{11}$		7E35	$[K_2 \vert M_{43}, M_{94}, M_{114}]$
			$\tau_{ex}(7,24)_5 \otimes \varepsilon_7$		7E36	$[K_3 \vert M_{48}, M_{72}, M_{120}]$
			$\tau_{ex}(7,24)_3 \otimes \varepsilon_5$		7E37	$[K_2 \vert M_{39}, M_{88}, M_{112}]$
			$\tau_{ex}(7,24)_3 \otimes \varepsilon_3$		7E38	$[K_2 \vert M_{39}, M_{88}, M_{112}]$
			$\tau_{ex}(7,24)_2 \otimes \varepsilon_5$	$\tau_{ex,2,2} \otimes \eta_{-1}$	7E39	$[K_1 \vert M_{55}, M_{79}, M_{104}]$
			$\tau_{ex}(7,24)_4 \otimes \varepsilon_{13}$		7E40	$[K_2 \vert M_{43}, M_{94}, M_{114}]$
			$\tau_{ex}(7,24)_5 \otimes \varepsilon_{11}$		7E41	$[K_3 \vert M_{48}, M_{72}, M_{120}]$
			$\tau_{ex}(7,24)_4 \otimes \varepsilon_7$		7E42	$[K_2 \vert M_{43}, M_{94}, M_{114}]$
			$\tau_{ex}(7,24)_2 \otimes \varepsilon_3$		7E43	$[K_1 \vert M_{55}, M_{79}, M_{104}]$
			$\tau_{ex}(7,24)_3 \otimes \varepsilon_9$		7E44	$[K_2 \vert M_{39}, M_{88}, M_{112}]$
			$\tau_{ex}(7,24)_6 \otimes \varepsilon_5$		7E45	$[K_3 \vert M_{54}, M_{74}, M_{124}]$
			$\tau_{ex}(7,24)_6 \otimes \varepsilon_{13}$		7E46	$[K_3 \vert M_{54}, M_{74}, M_{124}]$
			$\tau_{ex}(7,24)_1 \otimes \varepsilon_7$		7E47	$[K_1 \vert M_{57}, M_{83}, M_{110}]$
			$\tau_{ex}(7,24)_4 \otimes \varepsilon_9$		7E48	$[K_2 \vert M_{43}, M_{94}, M_{114}]$

TABLE 2. Exceptional inertial types of elliptic curves defined over $\mathbb{Q}_4$ with additive potentially good reduction and $\nu(N_E) = 7$; the explicit definition of the curves is given Tables 4 and the inertia field of each type in Table 6

Label	Curve (ϕ root of x^2-x-1)	Type
3E1	$y^2=x^3+x^2+2x+(\phi+1)$	$\tau_{ex}(3,24)_1$
3E2	$y^2=x^3+x^2+x+2$	$\tau_{ex}(3,24)_2$
3E3	$y^2=x^3+2^2x^2+2^2x+(\phi+1)\cdot 2^2$	$\tau_{ex}(3,24)_3$
4E1	$y^2=x^3-x^2+x-2$	$\tau_{ex}(3,24)_2\otimes\varepsilon_3$
4E2	$y^2=x^3+(-2\phi+11)x^2+(-40\phi+125)x+(-610\phi+1455)\cdot 2$	$\tau_{ex}(3,24)_2\otimes\varepsilon_5$
4E3	$y^2=x^3+(-2\phi+11)x^2+(-40\phi+125)\cdot 2x+235\phi+845$	$\tau_{ex}(3,24)_1\otimes\varepsilon_5$
4E4	$y^2=x^3-x^2+2x+(-\phi-1)$	$\tau_{ex}(3,24)_1\otimes\varepsilon_3$
4E5	$y^2=x^3+(2\phi-11)\cdot 2^2x^2+(-40\phi+125)\cdot 2^2x+(-235\phi-845)\cdot 2^2$	$\tau_{ex}(3,24)_3\otimes\varepsilon_1$
4E6	$y^2=x^3+(2\phi-11)x^2+(-40\phi+125)\cdot 2x+(-235\phi-845)$	$\tau_{ex}(3,24)_1\otimes\varepsilon_1$
4E7	$y^2=x^3-2^2x^2+2^2x+(-\phi-1)\cdot 2^2$	$\tau_{ex}(3,24)_3\otimes\varepsilon_3$
4E8	$y^2=x^3+(-2\phi+11)\cdot 2^2x^2+(-40\phi+125)\cdot 2^2x+(235\phi+845)\cdot 2^2$	$\tau_{ex}(3,24)_3\otimes\varepsilon_5$
4E9	$y^2=x^3+(2\phi-11)x^2+(-40\phi+125)x+(610\phi-1455)\cdot 2$	$\tau_{ex}(3,24)_2\otimes\varepsilon_1$
5E1	$y^2=x^3+x^2+2\phi x+(\phi+1)\cdot 2^2$	$\tau_{ex}(5,8)_1$
5E2	$y^2=x^3+x^2+2^2x+\phi\cdot 2^2$	$\tau_{ex}(5,8)_2$
5E3	$y^2=x^3+\phi x^2+2x+\phi\cdot 2^2$	$\tau_{ex}(5,8)_1\otimes\varepsilon_5$
5E4	$y^2=x^3+\phi x^2+2x+(\phi+1)\cdot 2^2$	$\tau_{ex}(5,8)_1\otimes\varepsilon_3$
5E5	$y^2=x^3+\phi\cdot 2^2x^2+x+2$	$\tau_{ex}(5,8)_2\otimes\varepsilon_3$
5E6	$y^2=x^3+(\phi+1)x^2+2x+\phi\cdot 2^2$	$\tau_{ex}(5,8)_2\otimes\varepsilon_5$
5E7	$y^2=x^3+\phi x^2+2\phi x+\phi\cdot 2^2$	$\tau_{ex}(5,8)_2\otimes\varepsilon_1$
5E8	$y^2=x^3+\phi x^2+2x+2^2$	$\tau_{ex}(5,8)_1\otimes\varepsilon_1$
6E1	$y^2=x^3+x^2+2x+2\phi$	$\tau_{ex}(6,8)_1$
6E2	$y^2=x^3+(2\phi-11)x^2+(-40\phi+125)\cdot 2x+(-845\phi+610)\cdot 2$	$\tau_{ex}(6,8)_1\otimes\varepsilon_1$
6E3	$y^2=x^3-x^2+2x-2\phi$	$\tau_{ex}(6,8)_1\otimes\varepsilon_3$
6E4	$y^2=x^3+(-2\phi+11)x^2+(-40\phi+125)\cdot 2x+(845\phi-610)\cdot 2$	$\tau_{ex}(6,8)_1\otimes\varepsilon_5$
6E5	$y^2=x^3-2x^2+2^3x-\phi\cdot 2^4$	$\tau_{ex}(6,8)_1\otimes\varepsilon_7$
6E6	$y^2=x^3+(-2\phi+11)\cdot 2x^2+(-40\phi+125)\cdot 2^3x+(845\phi-610)\cdot 2^4$	$\tau_{ex}(6,8)_1\otimes\varepsilon_9$
6E7	$y^2=x^3+2x^2+2^3x+\phi\cdot 2^4$	$\tau_{ex}(6,8)_1\otimes\varepsilon_{11}$
6E8	$y^2=x^3+(-2\phi+1)\cdot 2x^2+5\cdot 2^3x+(-5\phi-10)\cdot 2^4$	$\tau_{ex}(6,8)_1\otimes\varepsilon_{13}$
6E18	$y^2=x^3-2x^2+2^3x+(-\phi-1)\cdot 2^3$	$\tau_{ex}(3,24)_1\otimes\varepsilon_7$
6E19	$y^2=x^3+(-2\phi+11)\cdot 2x^2+(-40\phi+125)\cdot 2^3x+(235\phi+845)\cdot 2^3$	$\tau_{ex}(3,24)_1\otimes\varepsilon_9$
6E20	$y^2=x^3+2x^2+2^3x+(\phi+1)\cdot 2^3$	$\tau_{ex}(3,24)_1\otimes\varepsilon_{11}$
6E21	$y^2=x^3+(-2\phi+1)\cdot 2x^2+5\cdot 2^3x+(-15\phi-5)\cdot 2^3$	$\tau_{ex}(3,24)_1\otimes\varepsilon_{13}$
6E22	$y^2=x^3-2x^2+2^2x-2^4$	$\tau_{ex}(3,24)_2\otimes\varepsilon_7$
6E23	$y^2=x^3+(-2\phi+11)\cdot 2x^2+(-40\phi+125)\cdot 2^2x+(-610\phi+1455)\cdot 2^4$	$\tau_{ex}(3,24)_2\otimes\varepsilon_9$
6E24	$y^2=x^3+2x^2+2^2x+2^4$	$\tau_{ex}(3,24)_2\otimes\varepsilon_{11}$
6E25	$y^2=x^3+(-2\phi+1)\cdot 2x^2+5\cdot 2^2x+(-10\phi+5)\cdot 2^4$	$\tau_{ex}(3,24)_2\otimes\varepsilon_{13}$
6E26	$y^2=x^3-2^3x^2+2^4x+(-\phi-1)\cdot 2^5$	$\tau_{ex}(3,24)_3\otimes\varepsilon_7$
6E27	$y^2=x^3+(-2\phi+11)\cdot 2^3x^2+(-40\phi+125)\cdot 2^4x+(235\phi+845)\cdot 2^5$	$\tau_{ex}(3,24)_3\otimes\varepsilon_9$
6E28	$y^2=x^3+2^3x^2+2^4x+(\phi+1)\cdot 2^5$	$\tau_{ex}(3,24)_3\otimes\varepsilon_{11}$
6E29	$y^2=x^3+(-2\phi+1)\cdot 2^3x^2+5\cdot 2^4x+(-15\phi-5)\cdot 2^5$	$\tau_{ex}(3,24)_3\otimes\varepsilon_{13}$
6E30	$y^2=x^3-2x^2+\phi\cdot 2^3x+(-\phi-1)\cdot 2^5$	$\tau_{ex}(5,8)_1\otimes\varepsilon_7$
6E31	$y^2=x^3+(-2\phi+11)\cdot 2x^2+(85\phi-40)\cdot 2^3x+(235\phi+845)\cdot 2^5$	$\tau_{ex}(5,8)_1\otimes\varepsilon_9$
6E32	$y^2=x^3+2x^2+\phi\cdot 2^3x+(\phi+1)\cdot 2^5$	$\tau_{ex}(5,8)_1\otimes\varepsilon_{11}$
6E33	$y^2=x^3+(-2\phi+1)\cdot 2x^2+5\phi\cdot 2^3x+(-15\phi-5)\cdot 2^5$	$\tau_{ex}(5,8)_1\otimes\varepsilon_{13}$
6E34	$y^2=x^3-2x^2+2^4x-\phi\cdot 2^5$	$\tau_{ex}(5,8)_2\otimes\varepsilon_7$
6E35	$y^2=x^3+(-2\phi+11)\cdot 2x^2+(-40\phi+125)\cdot 2^4x+(845\phi-610)\cdot 2^5$	$\tau_{ex}(5,8)_2\otimes\varepsilon_9$
6E36	$y^2=x^3+2x^2+2^4x+\phi\cdot 2^5$	$\tau_{ex}(5,8)_2\otimes\varepsilon_{11}$
6E37	$y^2=x^3+(-2\phi+1)\cdot 2x^2+5\cdot 2^4x+(-5\phi-10)\cdot 2^5$	$\tau_{ex}(5,8)_2\otimes\varepsilon_{13}$

TABLE 3. Curves realizing exceptional types with $\nu(N_E)\leq 7$

Label	Curve (ϕ root of $x^2 - x - 1$)	Type
7E1	$y^2 = x^3 + ((\phi+1)\cdot 2^2)x^2 + (\phi\cdot 2)x + 2^2$	$\tau_{ex}(7,24)_1$
7E2	$y^2 = x^3 + ((-120\phi+375)\cdot 2^2)x + ((610\phi-1455)\cdot 2^4)$	$\tau_{ex}(7,24)_2$
7E3	$y^2 = x^3 + ((-3\phi-1)\cdot 2^3)x^2 + 5\phi\cdot 2^3x + ((-10\phi+5)\cdot 2^5)$	$\tau_{ex}(7,24)_1\otimes\varepsilon_{13}$
7E4	$y^2 = x^3 + ((\phi+1)\cdot 2)x^2 + \phi x + ((\phi+1)\cdot 2^2)$	$\tau_{ex}(7,24)_3$
7E5	$y^2 = x^3 + 2x^2 + 2^2x + 2^3$	$\tau_{ex}(7,24)_2\otimes\varepsilon_{13}$
7E6	$y^2 = x^3 + (\phi\cdot 2^2)x^2 + (\phi\cdot 2^2)x + (\phi\cdot 2^4)$	$\tau_{ex}(7,24)_4$
7E7	$y^2 = x^3 + 2^2x^2 + ((\phi+1)\cdot 2)x + \phi\cdot 2^2$	$\tau_{ex}(7,24)_3\otimes\varepsilon_{13}$
7E8	$y^2 = x^3 + ((7\phi+9)\cdot 2^3)x^2 + ((85\phi-40)\cdot 2^3)x + ((-610\phi+1455)\cdot 2^5)$	$\tau_{ex}(7,24)_1\otimes\varepsilon_9$
7E9	$y^2 = x^3 + ((-9\phi+2)\cdot 2^2)x^2 + ((85\phi-40)\cdot 2^2)x + ((-845\phi+610)\cdot 2^4)$	$\tau_{ex}(7,24)_4\otimes\varepsilon_1$
7E10	$y^2 = x^3 + 2^2x^2 + 2x + 2^2$	$\tau_{ex}(7,24)_2\otimes\varepsilon_9$
7E11	$y^2 = x^3 + (\phi\cdot 2)x^2 + 2x + ((\phi+1)\cdot 2^2)$	$\tau_{ex}(7,24)_1\otimes\varepsilon_{11}$
7E12	$y^2 = x^3 + (\phi\cdot 2^2)x^2 + (\phi\cdot 2)x + 2^2$	$\tau_{ex}(7,24)_1\otimes\varepsilon_5$
7E13	$y^2 = x^3 + x^2 + x + 1$	$\tau_{ex}(7,24)_2\otimes\varepsilon_1$
7E14	$y^2 = x^3 + (\phi\cdot 2)x^2 + (\phi\cdot 2)x + (\phi\cdot 2^3)$	$\tau_{ex}(7,24)_5$
7E15	$y^2 = x^3 - 2^2x^2 + 2^3x - (\phi\cdot 2^5)$	$\tau_{ex}(7,24)_6$
7E16	$y^2 = x^3 + ((9\phi-2)\cdot 2^2)x^2 + ((85\phi-40)\cdot 2^3)x + ((845\phi-610)\cdot 2^6)$	$\tau_{ex}(7,24)_5\otimes\varepsilon_9$
7E17	$y^2 = x^3 + 2x^2 + (\phi\cdot 2)x + 2^2$	$\tau_{ex}(7,24)_6\otimes\varepsilon_{11}$
7E18	$y^2 = x^3 + ((2\phi-11)\cdot 2)x^2 + ((-40\phi+125)\cdot 2)x + ((-845\phi+610)\cdot 2^2)$	$\tau_{ex}(7,24)_6\otimes\varepsilon_9$
7E19	$y^2 = x^3 + \phi x^2 + (\phi\cdot 2)x + ((\phi+1)\cdot 2)$	$\tau_{ex}(7,24)_6\otimes\varepsilon_3$
7E20	$y^2 = x^3 + 2^2x^2 + (\phi\cdot 2)x + 2^2$	$\tau_{ex}(7,24)_1\otimes\varepsilon_1$
7E21	$y^2 = x^3 + (\phi\cdot 2)x^2 + (\phi\cdot 2)x + (\phi\cdot 2^2)$	$\tau_{ex}(7,24)_2\otimes\varepsilon_7$
7E22	$y^2 = x^3 + ((\phi+1)\cdot 2)x^2 + 2x + (\phi\cdot 2^2)$	$\tau_{ex}(7,24)_1\otimes\varepsilon_3$
7E23	$y^2 = x^3 + 2^2x^2 + 2x + (\phi\cdot 2^2)$	$\tau_{ex}(7,24)_3\otimes\varepsilon_7$
7E24	$y^2 = x^3 + \phi x^2 + 2x + (\phi\cdot 2)$	$\tau_{ex}(7,24)_4\otimes\varepsilon_3$
7E25	$y^2 = x^3 + ((\phi+1)\cdot 2^2)x^2 + 2x + (\phi\cdot 2^2)$	$\tau_{ex}(7,24)_3\otimes\varepsilon_{11}$
7E26	$y^2 = x^3 + ((9\phi-2)\cdot 2^2)x^2 + ((85\phi-40)\cdot 2^2)x + ((845\phi-610)\cdot 2^4)$	$\tau_{ex}(7,24)_4\otimes\varepsilon_5$
7E27	$y^2 = x^3 + (\phi\cdot 2)x^2 + 2x + 2^2$	$\tau_{ex}(7,24)_6\otimes\varepsilon_1$
7E28	$y^2 = x^3 + ((9\phi-2)\cdot 2)x^2 + ((85\phi-40)\cdot 2)x + ((845\phi-610)\cdot 2^3)$	$\tau_{ex}(7,24)_5\otimes\varepsilon_5$
7E29	$y^2 = x^3 + 2x^2 + 2x + (\phi\cdot 2^2)$	$\tau_{ex}(7,24)_6\otimes\varepsilon_7$
7E30	$y^2 = x^3 + 2^2x^2 + ((\phi+1)\cdot 2)x + ((\phi+1)\cdot 2^2)$	$\tau_{ex}(7,24)_5\otimes\varepsilon_1$
7E31	$y^2 = x^3 + ((-\phi-2)\cdot 2^2)x^2 + 5\phi\cdot 2^3x + ((-5\phi-10)\cdot 2^6)$	$\tau_{ex}(7,24)_5\otimes\varepsilon_{13}$
7E32	$y^2 = x^3 + ((\phi+1)\cdot 2)x^2 + \phi x + (\phi\cdot 2^2)$	$\tau_{ex}(7,24)_3\otimes\varepsilon_1$
7E33	$y^2 = x^3 + ((\phi+1)\cdot 2^2)x^2 + 2x + ((\phi+1)\cdot 2^2)$	$\tau_{ex}(7,24)_5\otimes\varepsilon_3$
7E34	$y^2 = x^3 + ((\phi+1)\cdot 2^2)x^2 + 2x + 2^2$	$\tau_{ex}(7,24)_2\otimes\varepsilon_{11}$
7E35	$y^2 = x^3 + (\phi\cdot 2)x^2 + \phi x + (\phi\cdot 2)$	$\tau_{ex}(7,24)_4\otimes\varepsilon_{11}$
7E36	$y^2 = x^3 + 2^2x^2 + (\phi\cdot 2)x + ((\phi+1)\cdot 2^2)$	$\tau_{ex}(7,24)_5\otimes\varepsilon_7$
7E37	$y^2 = x^3 + x^2 + (\phi\cdot 2)x + (\phi\cdot 2)$	$\tau_{ex}(7,24)_3\otimes\varepsilon_5$
7E38	$y^2 = x^3 + 2^2x^2 + \phi x + 2$	$\tau_{ex}(7,24)_3\otimes\varepsilon_3$
7E39	$y^2 = x^3 + x^2 + 2x + 2$	$\tau_{ex}(7,24)_2\otimes\varepsilon_5$
7E40	$y^2 = x^3 + (\phi\cdot 2^2)x^2 + (\phi\cdot 2)x + (\phi\cdot 2^2)$	$\tau_{ex}(7,24)_4\otimes\varepsilon_{13}$
7E41	$y^2 = x^3 + (\phi\cdot 2^2)x^2 + (\phi\cdot 2)x + ((\phi+1)\cdot 2^2)$	$\tau_{ex}(7,24)_5\otimes\varepsilon_{11}$
7E42	$y^2 = x^3 - (\phi\cdot 2^3)x^2 + (\phi\cdot 2^4)x - (\phi\cdot 2^7)$	$\tau_{ex}(7,24)_4\otimes\varepsilon_7$
7E43	$y^2 = x^3 + ((-120\phi+375)\cdot 2^2)x + ((-610\phi+1455)\cdot 2^4)$	$\tau_{ex}(7,24)_2\otimes\varepsilon_3$
7E44	$y^2 = x^3 + ((\phi+1)\cdot 2^2)x^2 + \phi x + 2$	$\tau_{ex}(7,24)_3\otimes\varepsilon_9$
7E45	$y^2 = x^3 + ((2\phi-11)\cdot 2^2)x^2 + ((-40\phi+125)\cdot 2^3)x + ((-845\phi+610)\cdot 2^5)$	$\tau_{ex}(7,24)_6\otimes\varepsilon_5$
7E46	$y^2 = x^3 + x^2 + ((\phi+1)\cdot 2)x + ((\phi+1)\cdot 2)$	$\tau_{ex}(7,24)_6\otimes\varepsilon_{13}$
7E47	$y^2 = x^3 + ((-\phi-1)\cdot 2^3)x^2 + (\phi\cdot 2^3)x - 2^5$	$\tau_{ex}(7,24)_1\otimes\varepsilon_7$
7E48	$y^2 = x^3 + 2^2x^2 + (\phi\cdot 2)x + (\phi\cdot 2^2)$	$\tau_{ex}(7,24)_4\otimes\varepsilon_9$

TABLE 4. Curves realizing exceptional types with $\nu(N_E) = 7$

Curve	Defining Polynomial of Inertia Field over $\mathbb{Q}_4$ (ϕ root of $x^2 - x - 1$)	Inertial Type
3E1	$x^{24} + 2\phi x^9 + 2\phi x^6 + 2\phi$	$\tau_{ex}(3,24)_1$
3E2	$x^{24} + 2x^9 + 2x^6 + 2$	$\tau_{ex}(3,24)_2$
3E3	$x^{24} + (2\phi+2)x^9 + (2\phi+2)x^6 + 2\phi + 2$	$\tau_{ex}(3,24)_3$
4E1	$x^{24} + 2x^{15} + 2x^6 + 2$	$\tau_{ex}(3,24)_2 \otimes \varepsilon_3$
4E2	$x^{24} + 2x^{21} + (2\phi+2)x^{15} + 2x^6 + 2$	$\tau_{ex}(3,24)_2 \otimes \varepsilon_5$
4E3	$x^{24} + 2\phi x^{15} + 2\phi x^6 + 2\phi$	$\tau_{ex}(3,24)_1 \otimes \varepsilon_5$
4E4	$x^{24} + 2\phi x^{21} + (2\phi+2)x^{15} + 2\phi x^6 + 2\phi + 4$	$\tau_{ex}(3,24)_1 \otimes \varepsilon_3$
4E5	$x^{24} + (2\phi+2)x^{15} + (2\phi+2)x^6 + 2\phi + 2$	$\tau_{ex}(3,24)_3 \otimes \varepsilon_1$
4E6	$x^{24} + 2\phi x^{21} + 2x^{15} + 2\phi x^6 + 6\phi$	$\tau_{ex}(3,24)_1 \otimes \varepsilon_1$
4E7	$x^{24} + (2\phi+2)x^{21} + 2\phi x^{15} + (2\phi+2)x^6 + 2\phi + 2$	$\tau_{ex}(3,24)_3 \otimes \varepsilon_3$
4E8	$x^{24} + (2\phi+2)x^{21} + 2x^{15} + (2\phi+2)x^6 + 2\phi + 2$	$\tau_{ex}(3,24)_3 \otimes \varepsilon_5$
4E9	$x^{24} + 2x^{21} + 2\phi x^{15} + 2x^6 + 2$	$\tau_{ex}(3,24)_2 \otimes \varepsilon_1$
5E1	$x^8 + (2\phi+2)x^6 + 4x^4 + (4\phi+4)x^3 + 4x + 2$	$\tau_{ex}(5,8)_1$
5E2	$x^8 + 2\phi x^6 + (4\phi+4)x^3 + 4x + 2$	$\tau_{ex}(5,8)_2$
5E3	$x^8 + (2\phi+2)x^6 + 4x^4 + 4x^3 + 4x + 2$	$\tau_{ex}(5,8)_1 \otimes \varepsilon_5$
5E4	$x^8 + (2\phi+2)x^6 + 4x^4 + 4\phi x^3 + 4x + 2$	$\tau_{ex}(5,8)_1 \otimes \varepsilon_3$
5E5	$x^8 + 2\phi x^6 + 4x^4 + 4\phi x^3 + 4x + 2$	$\tau_{ex}(5,8)_2 \otimes \varepsilon_3$
5E6	$x^8 + 2\phi x^6 + 4x^3 + 4x + 2$	$\tau_{ex}(5,8)_2 \otimes \varepsilon_5$
5E7	$x^8 + 2\phi x^6 + 4x^4 + 4x + 2$	$\tau_{ex}(5,8)_2 \otimes \varepsilon_1$
5E8	$x^8 + (2\phi+2)x^6 + 4x^4 + 4x + 2$	$\tau_{ex}(5,8)_1 \otimes \varepsilon_1$
6E1	$x^8 + 2x^6 + 2x^4 + (4\phi+4)x^3 + 2$	$\tau_{ex}(6,8)_1$
6E2	$x^8 + (4\phi+4)x^7 + 2x^6 + 2x^4 + (4\phi+4)x^3 + 2$	$\tau_{ex}(6,8)_1 \otimes \varepsilon_1$
6E3	$x^8 + 4\phi x^7 + 2x^6 + 2x^4 + (4\phi+4)x^3 + 2$	$\tau_{ex}(6,8)_1 \otimes \varepsilon_3$
6E4	$x^8 + 4x^7 + 2x^6 + 2x^4 + (4\phi+4)x^3 + 2$	$\tau_{ex}(6,8)_1 \otimes \varepsilon_5$
6E5	$x^8 + (4\phi+4)x^7 + 2x^6 + 2x^4 + 4\phi x^3 + 2$	$\tau_{ex}(6,8)_1 \otimes \varepsilon_7$
6E6	$x^8 + 4\phi x^7 + 2x^6 + 2x^4 + 4\phi x^3 + 2$	$\tau_{ex}(6,8)_1 \otimes \varepsilon_9$
6E7	$x^8 + 2x^6 + 2x^4 + 4\phi x^3 + 2$	$\tau_{ex}(6,8)_1 \otimes \varepsilon_{11}$
6E8	$x^8 + 4x^7 + 2x^6 + 2x^4 + 4\phi x^3 + 10$	$\tau_{ex}(6,8)_1 \otimes \varepsilon_{13}$
6E9	$x^{24} + (4\phi+4)x^{21} + 4\phi x^{15} + 2\phi x^6 + 4\phi x^3 + 2\phi + 8$	$\tau_{ex}(3,24)_1 \otimes \varepsilon_7$
6E10	$x^{24} + 4x^{21} + 2\phi x^6 + 4\phi x^3 + 2\phi$	$\tau_{ex}(3,24)_1 \otimes \varepsilon_9$
6E11	$x^{24} + 4x^{15} + 2\phi x^6 + 4\phi x^3 + 2\phi$	$\tau_{ex}(3,24)_1 \otimes \varepsilon_{11}$
6E12	$x^{24} + (4\phi)x^{21} + (4\phi+4)x^{15} + 2\phi x^6 + 4\phi x^3 + 2\phi$	$\tau_{ex}(3,24)_1 \otimes \varepsilon_{13}$
6E13	$x^{24} + 4x^{21} + 2x^6 + 4x^3 + 2$	$\tau_{ex}(3,24)_2 \otimes \varepsilon_7$
6E14	$x^{24} + (4\phi)x^{21} + (4\phi+4)x^{15} + 2x^6 + 4x^3 + 2$	$\tau_{ex}(3,24)_2 \otimes \varepsilon_9$
6E15	$x^{24} + 4x^{15} + 2x^6 + 4x^3 + 2$	$\tau_{ex}(3,24)_2 \otimes \varepsilon_{11}$
6E16	$x^{24} + (4\phi+4)x^{21} + 4\phi x^{15} + 2x^6 + 4x^3 + 8\phi + 2$	$\tau_{ex}(3,24)_2 \otimes \varepsilon_{13}$
6E17	$x^{24} + 4x^{21} + (2\phi+2)x^6 + (4\phi+4)x^3 + 2\phi + 2$	$\tau_{ex}(3,24)_3 \otimes \varepsilon_7$
6E18	$x^{24} + 4x^{15} + (2\phi+2)x^6 + (4\phi+4)x^3 + 2\phi + 2$	$\tau_{ex}(3,24)_3 \otimes \varepsilon_9$
6E19	$x^{24} + (4\phi+4)x^{21} + 4\phi x^{15} + (2\phi+2)x^6 + (4\phi+4)x^3 + 2\phi + 2$	$\tau_{ex}(3,24)_3 \otimes \varepsilon_{11}$
6E20	$x^{24} + 4\phi x^{21} + (4\phi+4)x^{15} + (2\phi+2)x^6 + (4\phi+4)x^3 + 2\phi + 10$	$\tau_{ex}(3,24)_3 \otimes \varepsilon_{13}$
6E21	$x^8 + (4\phi+4)x^7 + (2\phi+2)x^6 + (4\phi+4)x^5 + 4\phi x^3 + 8\phi + 2$	$\tau_{ex}(5,8)_1 \otimes \varepsilon_7$
6E22	$x^8 + 4x^7 + (2\phi+2)x^6 + (4\phi+4)x^5 + 4\phi x^3 + 8\phi + 2$	$\tau_{ex}(5,8)_1 \otimes \varepsilon_9$
6E23	$x^8 + (2\phi+2)x^6 + (4\phi+4)x^5 + 4\phi x^3 + 8\phi + 2$	$\tau_{ex}(5,8)_1 \otimes \varepsilon_{11}$
6E24	$x^8 + 4\phi x^7 + (2\phi+2)x^6 + (4\phi+4)x^5 + 4\phi x^3 + 2$	$\tau_{ex}(5,8)_1 \otimes \varepsilon_{13}$
6E25	$x^8 + 4\phi x^7 + 2\phi x^6 + 4\phi x^5 + (4\phi+4)x^3 + 2$	$\tau_{ex}(5,8)_2 \otimes \varepsilon_7$
6E26	$x^8 + (4\phi+4)x^7 + 2\phi x^6 + 4\phi x^5 + (4\phi+4)x^3 + 2$	$\tau_{ex}(5,8)_2 \otimes \varepsilon_9$
6E27	$x^8 + 2\phi x^6 + 4\phi x^5 + (4\phi+4)x^3 + 2$	$\tau_{ex}(5,8)_2 \otimes \varepsilon_{11}$
6E28	$x^8 + 4x^7 + 2\phi x^6 + 4\phi x^5 + (4\phi+4)x^3 + 8\phi + 2$	$\tau_{ex}(5,8)_2 \otimes \varepsilon_{13}$

TABLE 5. Polynomial defining the inertia field for each type with $\nu(N_E) \le 6$

Curve	Defining Polynomial of Inertia Field over $\mathbb{Q}_4$ (ϕ root of $x^2 - x - 1$)	Inertial Type
7E1	$x^{24} + 4x^{21} + 4x^{12} + 8\phi x^9 + 4x^6 + (8\phi + 8)x^3 + 2$	$\tau_{ex}(7, 24)_1$
7E2	$x^{24} + 4x^{21} + (8\phi + 8)x^9 + 4x^6 + 2$	$\tau_{ex}(7, 24)_2$
7E3	$x^{24} + 4x^{21} + (8\phi + 4)x^{12} + 4x^6 + 8\phi x^3 + 2$	$\tau_{ex}(7, 24)_1 \otimes \varepsilon_{13}$
7E4	$x^{24} + (4\phi + 4)x^{21} + (8\phi + 4)x^{12} + (8\phi + 8)x^9 + (4\phi + 4)x^6 + 8\phi x^3 + 2\phi$	$\tau_{ex}(7, 24)_3$
7E5	$x^{24} + 4x^{21} + 4x^6 + 8x^3 + 2$	$\tau_{ex}(7, 24)_2 \otimes \varepsilon_{13}$
7E6	$x^{24} + (4\phi + 4)x^{21} + (8\phi + 8)x^9 + (4\phi + 4)x^6 + (8\phi + 8)x^3 + 2\phi$	$\tau_{ex}(7, 24)_4$
7E7	$x^{24} + (4\phi + 4)x^{21} + 4x^{12} + 8\phi x^9 + (4\phi + 4)x^6 + 2\phi$	$\tau_{ex}(7, 24)_3 \otimes \varepsilon_{13}$
7E8	$x^{24} + 4x^{21} + 4x^{12} + 8x^9 + 4x^6 + 8\phi x^3 + 2$	$\tau_{ex}(7, 24)_1 \otimes \varepsilon_9$
7E9	$x^{24} + (4\phi + 4)x^{21} + 8\phi x^{12} + 8x^9 + (4\phi + 4)x^6 + (8\phi + 8)x^3 + 2\phi$	$\tau_{ex}(7, 24)_4 \otimes \varepsilon_1$
7E10	$x^{24} + 4x^{21} + 8x^9 + 4x^6 + 8x^3 + 2$	$\tau_{ex}(7, 24)_2 \otimes \varepsilon_9$
7E11	$x^{24} + 4x^{21} + (8\phi + 4)x^{12} + (8\phi + 8)x^9 + 4x^6 + 8\phi x^3 + 2$	$\tau_{ex}(7, 24)_1 \otimes \varepsilon_{11}$
7E12	$x^{24} + 4x^{21} + (8\phi + 4)x^{12} + 4x^6 + (8\phi + 8)x^3 + 2$	$\tau_{ex}(7, 24)_1 \otimes \varepsilon_5$
7E13	$x^{24} + 4x^{21} + 4x^6 + 2$	$\tau_{ex}(7, 24)_2 \otimes \varepsilon_1$
7E14	$x^{24} + 4\phi x^{21} + (8\phi + 8)x^9 + 4\phi x^6 + 8x^3 + 2\phi + 2$	$\tau_{ex}(7, 24)_5$
7E15	$x^{24} + 4\phi x^{21} + (8\phi + 4)x^{12} + 4\phi x^6 + 2\phi + 2$	$\tau_{ex}(7, 24)_6$
7E16	$x^{24} + 4\phi x^{21} + 4\phi x^6 + (8\phi)x^3 + 2\phi + 2$	$\tau_{ex}(7, 24)_5 \otimes \varepsilon_9$
7E17	$x^{24} + 4\phi x^{21} + 4x^{12} + 8x^9 + 4\phi x^6 + (8\phi + 8)x^3 + 2\phi + 2$	$\tau_{ex}(7, 24)_6 \otimes \varepsilon_{11}$
7E18	$x^{24} + 4\phi x^{21} + (8\phi + 4)x^{12} + 8\phi x^9 + 4\phi x^6 + (8\phi + 8)x^3 + 2\phi + 2$	$\tau_{ex}(7, 24)_6 \otimes \varepsilon_9$
7E19	$x^{24} + 4\phi x^{21} + (8\phi + 4)x^{12} + 8\phi x^9 + 4\phi x^6 + 2\phi + 2$	$\tau_{ex}(7, 24)_6 \otimes \varepsilon_3$
7E20	$x^{24} + 4x^{21} + 4x^{12} + 8x^9 + 4x^6 + (8\phi + 8)x^3 + 2$	$\tau_{ex}(7, 24)_1 \otimes \varepsilon_1$
7E21	$x^{24} + 4x^{21} + 8\phi x^{12} + 8\phi x^9 + 4x^6 + 8x^3 + 2$	$\tau_{ex}(7, 24)_2 \otimes \varepsilon_7$
7E22	$x^{24} + 4x^{21} + (8\phi + 4)x^{12} + (8\phi + 8)x^9 + 4x^6 + (8\phi + 8)x^3 + 2$	$\tau_{ex}(7, 24)_1 \otimes \varepsilon_3$
7E23	$x^{24} + (4\phi + 4)x^{21} + (8\phi + 4)x^{12} + (8\phi + 8)x^9 + (4\phi + 4)x^6 + 2\phi$	$\tau_{ex}(7, 24)_3 \otimes \varepsilon_7$
7E24	$x^{24} + (4\phi + 4)x^{21} + 8\phi x^{12} + (4\phi + 4)x^6 + (8\phi + 8)x^3 + 2\phi$	$\tau_{ex}(7, 24)_4 \otimes \varepsilon_3$
7E25	$x^{24} + (4\phi + 4)x^{21} + (8\phi + 4)x^{12} + (4\phi + 4)x^6 + 2\phi$	$\tau_{ex}(7, 24)_3 \otimes \varepsilon_{11}$
7E26	$x^{24} + (4\phi + 4)x^{21} + 8\phi x^9 + (4\phi + 4)x^6 + (8\phi + 8)x^3 + 2\phi$	$\tau_{ex}(7, 24)_4 \otimes \varepsilon_5$
7E27	$x^{24} + 4\phi x^{21} + (8\phi + 4)x^{12} + 8x^9 + 4\phi x^6 + 2\phi + 2$	$\tau_{ex}(7, 24)_6 \otimes \varepsilon_1$
7E28	$x^{24} + 4\phi x^{21} + 4\phi x^6 + 8x^3 + 2\phi + 2$	$\tau_{ex}(7, 24)_5 \otimes \varepsilon_5$
7E29	$x^{24} + 4\phi x^{21} + (8\phi + 4)x^{12} + (8\phi + 8)x^9 + 4\phi x^6 + (8\phi + 8)x^3 + 2\phi + 2$	$\tau_{ex}(7, 24)_6 \otimes \varepsilon_7$
7E30	$x^{24} + 4\phi x^{21} + 8\phi x^9 + 4\phi x^6 + 8x^3 + 2\phi + 2$	$\tau_{ex}(7, 24)_5 \otimes \varepsilon_1$
7E31	$x^{24} + 4\phi x^{21} + 8\phi x^{12} + 8\phi x^9 + 4\phi x^6 + 8\phi x^3 + 2\phi + 2$	$\tau_{ex}(7, 24)_5 \otimes \varepsilon_{13}$
7E32	$x^{24} + (4\phi + 4)x^{21} + 4x^{12} + 8x^9 + (4\phi + 4)x^6 + 8\phi x^3 + 2\phi$	$\tau_{ex}(7, 24)_3 \otimes \varepsilon_1$
7E33	$x^{24} + 4\phi x^{21} + 8x^9 + 4\phi x^6 + 8x^3 + 2\phi + 2$	$\tau_{ex}(7, 24)_5 \otimes \varepsilon_3$
7E34	$x^{24} + 4x^{21} + 8\phi x^{12} + (8\phi + 8)x^9 + 4x^6 + 8x^3 + 2$	$\tau_{ex}(7, 24)_2 \otimes \varepsilon_{11}$
7E35	$x^{24} + (4\phi + 4)x^{21} + 8x^9 + (4\phi + 4)x^6 + 8x^3 + 2\phi$	$\tau_{ex}(7, 24)_4 \otimes \varepsilon_{11}$
7E36	$x^{24} + 4\phi x^{21} + 8x^9 + 4\phi x^6 + 8\phi x^3 + 2\phi + 2$	$\tau_{ex}(7, 24)_5 \otimes \varepsilon_7$
7E37	$x^{24} + (4\phi + 4)x^{21} + 4x^{12} + 8\phi x^9 + (4\phi + 4)x^6 + 8\phi x^3 + 2\phi$	$\tau_{ex}(7, 24)_3 \otimes \varepsilon_5$
7E38	$x^{24} + (4\phi + 4)x^{21} + (8\phi + 4)x^{12} + (4\phi + 4)x^6 + 8\phi x^3 + 2\phi$	$\tau_{ex}(7, 24)_3 \otimes \varepsilon_3$
7E39	$x^{24} + 4x^{21} + 8x^9 + 4x^6 + 2$	$\tau_{ex}(7, 24)_2 \otimes \varepsilon_5$
7E40	$x^{24} + (4\phi + 4)x^{21} + (8\phi + 8)x^9 + (4\phi + 4)x^6 + 8x^3 + 2\phi$	$\tau_{ex}(7, 24)_4 \otimes \varepsilon_{13}$
7E41	$x^{24} + 4\phi x^{21} + 8\phi x^{12} + (8\phi + 8)x^9 + 4\phi x^6 + 8\phi x^3 + 2\phi + 2$	$\tau_{ex}(7, 24)_5 \otimes \varepsilon_{11}$
7E42	$x^{24} + (4\phi + 4)x^{21} + 8\phi x^{12} + 8\phi x^9 + (4\phi + 4)x^6 + 8x^3 + 2\phi$	$\tau_{ex}(7, 24)_4 \otimes \varepsilon_7$
7E43	$x^{24} + 4x^{21} + 8\phi x^9 + 4x^6 + 2$	$\tau_{ex}(7, 24)_2 \otimes \varepsilon_3$
7E44	$x^{24} + (4\phi + 4)x^{21} + (8\phi + 4)x^{12} + 8x^9 + (4\phi + 4)x^6 + 2\phi$	$\tau_{ex}(7, 24)_3 \otimes \varepsilon_9$
7E45	$x^{24} + 4\phi x^{21} + (8\phi + 4)x^{12} + (8\phi + 8)x^9 + 4\phi x^6 + 2\phi + 2$	$\tau_{ex}(7, 24)_6 \otimes \varepsilon_5$
7E46	$x^{24} + 4\phi x^{21} + 4x^{12} + 4\phi x^6 + (8\phi + 8)x^3 + 2\phi + 2$	$\tau_{ex}(7, 24)_6 \otimes \varepsilon_{13}$
7E47	$x^{24} + 4x^{21} + 4x^{12} + 8\phi x^9 + 4x^6 + 8\phi x^3 + 2$	$\tau_{ex}(7, 24)_1 \otimes \varepsilon_7$
7E48	$x^{24} + (4\phi + 4)x^{21} + (4\phi + 4)x^6 + 8x^3 + 2\phi$	$\tau_{ex}(7, 24)_4 \otimes \varepsilon_9$

TABLE 6. Polynomial defining the inertia field for each type with $\nu(N_E) = 7$