

Homework 4 UG

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4.6)a

$$\begin{aligned}P(S|UG) &= .15 \\P(S|G) &= .23 \\P(G) &= .2 \\P(UG) &= .8\end{aligned}\tag{1}$$

These are the known probabilities.

From this we can find $P(G|S)$.

Because of Bayes Theorem $P(G|S)$ is the same as the following

$$P(G|S) = \frac{P(S|G) * P(G)}{P(S)}\tag{1}$$

$P(S)$ can be found as

$$\begin{aligned}P(S) &= P(S|G) * P(G) + P(S|UG) * P(UG) \\P(S) &= .23 * .2 + .15 * .8 \\P(S) &= .166\end{aligned}\tag{2}$$

Therefore

$$\begin{aligned}P(G|S) &= \frac{.23 * .2}{.166} \\P(G|S) &= .277\end{aligned}\tag{3}$$

So the probability that a smoker is a graduate student is .277

4.6)c

The probability that a smoker is a graduated student can be written as $P(UG|S)$.

$$\begin{aligned}P(UG|S) &= \frac{P(S|UG) * P(UG)}{P(S)} \\P(UG|S) &= \frac{.15 * .8}{.166} \\P(UG|S) &= .857\end{aligned}\tag{4}$$

So the probability that a smoker is an undergrad is .857.

Since $P(UG|S) > P(G|S)$ we can conclude we have a higher chance of finding an undergrad that is a smoker

4.6)d

$$\begin{aligned}
P(D|UG) &= .1 \\
P(D|G) &= .3 \\
P(D) &= P(D|UG) * P(UG) + P(D|G) * P(G) \\
P(D) &= 0.1 * .8 + .2 * .3 \\
P(D) &= .14 \\
P(D,S|G) &= P(D|G) * P(S|G) \\
P(D,S|G) &= .3 * .23 \\
P(D,S|G) &= .069 \\
P(D,S|UG) &= P(D|UG) * P(S|UG) \\
P(D,S|UG) &= .1 * .15 \\
P(D,S|UG) &= 0.015 \\
P(D,S) &= Q
\end{aligned} \tag{5}$$

These are the known probabilities. Since we don't know what $P(D,S)$ is, we set it as a constant Q

Now we can find the values for $P(G|D,S)$ and $P(UG|D,S)$

$$\begin{aligned}
P(UG|D,S) &= \frac{P(D,S|UG) * P(UG)}{P(D,S)} \\
P(UG|D,S) &= \frac{.015 * .8}{Q} \\
P(UG|D,S) &= \frac{.012}{Q} \\
P(G|D,S) &= \frac{P(D,S|UG) * P(UG)}{P(D,S)} \\
P(G|D,S) &= \frac{.069 * .2}{Q} \\
P(G|D,S) &= \frac{.0139}{Q}
\end{aligned} \tag{6}$$

From these results we can conclude that the chance that we find a graduate that lives in a dorm and is a smoker is higher than the chance that we find an undergraduate that lives in a dorm and is a smoker.

4.7)a

$$\begin{aligned}
P(A=0|+) &= \frac{2}{5} = .4 \\
P(A=0|-) &= \frac{3}{5} = .6 \\
P(A=1|+) &= \frac{3}{5} = .6 \\
P(A=1|-) &= \frac{2}{5} = .4 \\
P(B=0|+) &= \frac{4}{5} = .8 \\
P(B=0|-) &= \frac{3}{5} = .6 \\
P(B=1|+) &= \frac{1}{5} = .2 \\
P(B=1|-) &= \frac{2}{5} = .4 \\
P(C=0|+) &= \frac{3}{5} = .6 \\
P(C=0|-) &= \frac{0}{5} = 0 \\
P(C=1|+) &= \frac{2}{5} = .4 \\
P(C=1|-) &= \frac{5}{5} = .1
\end{aligned}
\tag{7}$$

4.7)b

4.7)c

4.7)d

4.7)e