

Homework 1

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1

- 1) The maximum number of edges that a simple graph can have is $\frac{n(n-1)}{2}$
- 2) The maximum number of edges that a simple graph can have is $(n-1)$

2

This graph is not a strongly connected graph. This is because Vertex B and E does not have a path that directly leads to them.

3

This simple graph is not a bipartite graph. This is because it is not possible to generate two subset graphs that within the same set are adjacent

4

from the graph we can see that node 1, 2, 6 and 7 are symmetric to each other. Node 3 and 5 are also symmetric.

$$\begin{array}{lll} C_{1,2,6,7} = \frac{7}{0+1+1+2+3+4+4} & C_{3,5} = \frac{7}{1+1+0+1+2+3+3} & C_4 = \frac{7}{2+2+1+0+1+2+2} \\ C_{1,2,6,7} = \frac{7}{15} & C_{3,5} = \frac{7}{11} & C_4 = \frac{7}{10} \end{array}$$

so we can say that the Closeness of nodes 1,2,6,7 are $\frac{7}{15}$, nodes 3 and 5 are $\frac{7}{11}$ and node 4 is $\frac{7}{10}$

5

Just by looking at the graph, we can tell that the betweenness for Node 1,3,4,5 are zero.

$$B_2 = \frac{1}{\binom{5-1}{2}} * \left(\frac{0}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} \right)$$

$$B_2 = \frac{1}{\binom{4}{2}} * (0 + 1 + 1 + 1 + 1 + 1)$$

$$B_2 = \frac{1}{6} * (5)$$

$$B_2 = \frac{5}{6}$$

So the Betweenness of nodes 1,3,4,5 are zero and the betweenness of node 2 is $\frac{5}{6}$

6

1)

From this graph, we know that vertices 6 and 7 are symmetric. So both will have the same closeness factor

$$C_1 = \frac{7}{0 + 1 + 1 + 2 + 2 + 3 + 3} = \frac{7}{12}$$

$$C_2 = \frac{7}{1 + 0 + 1 + 2 + 1 + 2 + 2} = \frac{7}{9}$$

$$C_3 = \frac{7}{1 + 1 + 0 + 1 + 2 + 3 + 3} = \frac{7}{11}$$

$$C_4 = \frac{7}{2 + 2 + 1 + 0 + 3 + 4 + 4} = \frac{7}{16}$$

$$C_5 = \frac{7}{2 + 1 + 2 + 3 + 0 + 1 + 1} = \frac{7}{10}$$

$$C_{6,7} = \frac{7}{3 + 2 + 3 + 4 + 1 + 2 + 0} = \frac{7}{15}$$

Vertex 2 is the one with the highest Closeness 2)

All combination of paths have a total of 1 shortest path. So from $P(u, v)$ in which u and v do not equal each other is 1. From the Graph we can see that for vertices 2,3 and 5, their betweenness is non zero. While the rest are 0.

$$B_2 = \frac{1}{\binom{7-1}{2}} * 9 = \frac{9}{15}$$

$$B_3 = \frac{1}{\binom{7-1}{2}} * 4 = \frac{4}{15}$$

$$B_5 = \frac{1}{\binom{7-1}{2}} * 8 = \frac{8}{15}$$

So the vertex with the largest betweenness is vertex 2