Homework 1

September 17, 2023

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1

1) The maximum number of edges that a simple graph can have is $\frac{n(n-1)}{2}$
2) The maximum number of edges that a simple graph can have is $(n-1)$

2

This graph is not a stronly connected graph. This is because Vertex B and E does not have a path that directly leads to them.

3

This simple graph is not a bipartite graph. This is beause it is not possible to generate two subset graphs that within the same set are adjacent

4

from the graph we can see that node 1, 2, 6 and 7 are symmetric to each other. Node 3 and 5 are also symetric.

$$C_{1,2,6,7} = \frac{7}{0+1+1+2+3+4+4} \quad C_{3,5} = \frac{7}{1+1+0+1+2+3+3} \quad C_4 = \frac{7}{2+2+1+0+1+2+2}$$

$$C_{1,2,6,7} = \frac{7}{15} \quad C_{3,5} = \frac{7}{11} \quad C_4 = \frac{7}{10}$$

so we can say that the Closeness of nodes 1,2,6,7 are $\frac{7}{15}$, nodes 3 and 5 are $\frac{7}{11}$ and node 4 is $\frac{7}{10}$

5

Just by looking at the graph, we can tell that the betweeness for Node 1,3,4,5 are zero.

$$B_2 = \frac{1}{\binom{5-1}{2}} * (\frac{0}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1})$$
$$B_2 = \frac{1}{\binom{4}{2}} * (0 + 1 + 1 + 1 + 1 + 1)$$
$$B_2 = \frac{1}{6} * (5)$$
$$B_2 = \frac{5}{6}$$

So the Betweeness of nodes 1,3,4,5 are zero and the betweeness of node 2 is $\frac{5}{6}$

6

1)
From thie graph, we know that vertices 6 and 7 are symmetric. So both will have the same closeness factor

$$C_1 = \frac{7}{0+1+1+2+2+3+3} = \frac{7}{12}$$
$$C_2 = \frac{7}{1+0+1+2+1+2+2} = \frac{7}{9}$$
$$C_3 = \frac{7}{1+1+0+1+2+3+3} = \frac{7}{11}$$
$$C_4 = \frac{7}{2+2+1+0+3+4+4} = \frac{7}{16}$$
$$C_5 = \frac{7}{2+1+2+3+0+1+1} = \frac{7}{10}$$
$$C_{6,7} = \frac{7}{3+2+3+4+1+2+0} = \frac{7}{15}$$

Vertex 2 is the one with the higest Closeness 2)
All combination of paths have a total of 1 shoretst path. So fro $P(u, v)$ in which u and v do not equal each other is 1. From the Graph we can see that for vertices 2,3 and 5, thier betweeness is non zero. While the rest are 0.

$$B_2 = \frac{1}{\binom{7-1}{2}} * 9 = \frac{9}{15}$$
$$B_3 = \frac{1}{\binom{7-1}{2}} * 4 = \frac{4}{15}$$
$$B_5 = \frac{1}{\binom{7-1}{2}} * 8 = \frac{8}{15}$$

So the vertex with the largest betweenness is vertex 2