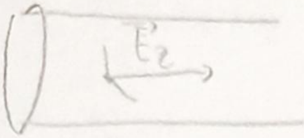
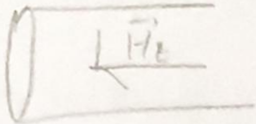


TM



TE

Ecuaciones de Maxwell con cilíndricas

$$\frac{\partial H_z}{\rho\,\partial\phi} + \gamma H_\phi = j\omega\epsilon E_\rho \qquad \frac{\partial E_z}{\rho\,\partial\phi} + \gamma E_\phi = -j\omega\mu H_\rho$$

$$-\gamma H_\rho - \frac{\partial H_z}{\partial\rho} = j\omega\epsilon E_\phi \qquad -\gamma E_\rho - \frac{\partial E_z}{\partial\rho} = -j\omega\mu H_\phi$$

$$\frac{1}{\rho}\left(\frac{\partial(\rho H_\phi)}{\partial\rho} - \frac{\partial H_\rho}{\partial\phi}\right) = j\omega\epsilon E_z \qquad \frac{1}{\rho}\left(\frac{\partial(\rho E_\phi)}{\partial\rho} - \frac{\partial E_\rho}{\partial\phi}\right) = -j\omega\mu H_z$$

se obtiene:

$$h^2 H_\rho = j\frac{\omega\epsilon}{\rho}\frac{\partial E_z}{\partial\phi} - \gamma\frac{\partial H_z}{\partial\rho}$$

$$h^2 H_\phi = -j\omega\epsilon\frac{\partial E_z}{\partial\rho} - \frac{\gamma}{\rho}\frac{\partial H_z}{\partial\phi} \qquad ;\ h^2 = \gamma^2 + \omega^2\mu\epsilon$$

$$h^2 E_\rho = -\gamma\frac{\partial E_z}{\partial\rho} - j\frac{\omega\mu}{\rho}\frac{\partial H_z}{\partial\phi}$$

$$h^2 E_\phi = -\frac{\gamma}{\rho}\frac{\partial E_z}{\partial\phi} + j\omega\mu\frac{\partial H_z}{\partial\rho}$$

se obtiene:

$$\frac{\partial^2 E_z}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial\phi^2} + \frac{\partial^2 E_z}{\partial z^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial\rho} = -\omega^2\mu\epsilon E_z$$

Se resuelve usando el polinomio de Bessel

$$N_m(K_c r) = \frac{\cos(m\pi)\, J_m(K_c r) - J_{-m}(K_c r)}{\operatorname{sen}(m\pi)} \qquad ;\ K_c = \gamma^2 + \omega^2\mu\epsilon$$

↳ familia de soluciones de Neumann

$$J_m(K_c r) = -\frac{i^{-m}}{\pi}\int_0^{\pi} e^{jK_c r\cos(\alpha)}\cos(m\alpha)\,d\alpha$$

↳ familia de soluciones de Bessel.

Neumann se descarta

$$E_z(r,\phi) = A J_m(k_c r)\left(B\cos(m\phi) + C\sin(m\phi)\right)$$

Ceros de Función de Bessel $(J_m(k_c a))$

m\n	1	2	3	4
0	2.405	5.520	8.654	11.792
1	3.832	7.016	10.173	13.324
2	5.136	8.417	11.620	14.796
3	6.380	9.761	13.015	16.223

Modos TM ($\mathcal{E}_z \neq 0$)

En TM

$$\mathcal{E}_\rho = \frac{1}{k_c^2}\left[-j\beta \frac{\partial \mathcal{E}_z}{\partial \rho}\right]$$

$$\mathcal{E}_\phi = \frac{1}{k_c^2}\left[-j\frac{\beta}{\rho} \frac{\partial \mathcal{E}_z}{\partial \phi}\right]$$

$$\mathcal{H}_\rho = \frac{1}{k_c^2}\left[j\frac{\omega\varepsilon}{\rho} \frac{\partial \mathcal{E}_z}{\partial \phi}\right]$$

$$\mathcal{H}_\phi = \frac{1}{k_c^2}\left[-j\omega\varepsilon \frac{\partial \mathcal{E}_z}{\partial \rho}\right]$$

$$\mathcal{E}_z(\rho,\phi) = E_0 J_m(k_c\rho)\begin{Bmatrix}\cos(m\phi)\\ \mathrm{sen}(m\phi)\end{Bmatrix}$$

Modos TE ($\mathcal{H}_z \neq 0$)

En TE

$$\mathcal{E}_\rho = \frac{1}{k_c^2}\left[-j\frac{\omega\mu}{\rho} \frac{\partial \mathcal{H}_z}{\partial \phi}\right]$$

$$\mathcal{E}_\phi = \frac{1}{k_c^2}\left[j\omega\mu \frac{\partial \mathcal{H}_z}{\partial \rho}\right]$$

$$\mathcal{H}_\rho = \frac{1}{k_c^2}\left[-j\beta \frac{\partial \mathcal{H}_z}{\partial \rho}\right]$$

$$\mathcal{H}_\phi = \frac{1}{k_c^2}\left[-j\frac{\beta}{\rho} \frac{\partial \mathcal{H}_z}{\partial \phi}\right]$$

Frecuencia escogida para f_c: 15 GHz

Frecuencia menor a f_c: $F = 7$ GHz

Frecuencia mayor a f_c: $F = 18$ GHz

Modo ~~TE~~ TM11

$m = 1$ $n = 1$ Se asume vacía $\epsilon = \epsilon_0$ y $\mu = \mu_0$

$$h_{c_{mn}}^2 = \omega^2 \mu \epsilon + \gamma^2 \rightarrow \gamma^2 = h_{c_{mn}}^2 - \omega^2 \mu \epsilon$$

a)

$$\omega_{c_{mn}} = \frac{1}{\sqrt{\mu \epsilon}} h_{c_{mn}} \qquad h_{c_{11}} a = 1.841$$

$$\omega_{c_{mn}} = \frac{1}{\sqrt{\mu \epsilon}} \rightarrow \omega_{c_{mn}} = \frac{1}{\sqrt{\mu \epsilon}} \left(\frac{1.841}{a} \right)$$

$$a = \frac{1.8412}{\sqrt{(12.57 \times 10^{-7})(8.84 \times 10^{-12})}\,(15 \times 10^{9} Hz) 2\pi} = 0.036 m = 3.68 cm$$

b)

$$\beta = \omega \sqrt{\mu_0 \epsilon_0} \sqrt{1 - \left(\frac{F_{c_{mn}}}{F} \right)^2}$$

Para $F < f_c \rightarrow F = 7$ GHz

$$\beta = 2\pi (7 \times 10^9) \sqrt{\mu_0 \epsilon_0} \sqrt{1 - \left(\frac{15}{7} \right)^2} = 277.83j \ rad/m$$

Para $F > f_c \rightarrow F = 18$ GHz

$$\beta = 2\pi (18 \times 10^9) \sqrt{\mu_0 \epsilon_0} \sqrt{1 - \left(\frac{15}{18} \right)^2} = 208.37 \ rad/m$$

c)

$$\eta_{TM} = \frac{E_\rho}{H_\phi} = -\frac{E_\phi}{H_\rho} = \sqrt{\frac{\mu}{\epsilon}} \sqrt{1 - \left(\frac{f_c}{f} \right)^2}$$

Para $F < f_c \rightarrow F = 7$ GHz

$$\eta_{TM} = \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{1 - \left(\frac{15}{7} \right)^2} = 714.66j \ \Omega$$

Para $F > F_c \rightarrow F = 18\,GHz$

$$\eta_{TM} = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \sqrt{1-\left(\frac{15}{18}\right)^2} = 208.44\,\Omega$$

d)

$$\lambda = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} \qquad V_F = \lambda f$$

Para $F < F_c \rightarrow F = 7\,GHz$

$$\lambda = \frac{2\pi}{2\pi(7\times10^9)\sqrt{\mu_0\epsilon_0}} = 0.043\,m$$

$$V_F = (0.043\,m)(7\times10^9) = 3\times10^8\,m/s = c$$

Para $F > F_c \rightarrow F = 18\,GHz$

$$V_F = \frac{2\pi}{2\pi(18\times10^9)\sqrt{\mu_0\epsilon_0}} \cdot f = 3\times10^8\,m/s = c$$

e)

Para $F < F_c \rightarrow F = 7\,GHz$

$$\lambda = \frac{2\pi}{2\pi(7\times10^9)\sqrt{\mu_0\epsilon_0}} = 0.043\,m = 4.30\,cm$$

Para $F > F_c \rightarrow F = 18\,GHz$

$$\lambda = \frac{2\pi}{2\pi(18\times10^9)\sqrt{\mu_0\epsilon_0}} = 0.0167\,m = 1.67\,cm$$

f)

Para $F > F_c \rightarrow F = 7\,GHz$

$$h_c = \frac{P_{mn}}{a} = \frac{1.841}{0.037} = 50$$

$$\mathcal{E}_z = \mathcal{E}_z\ V/m \qquad \mathcal{H}_z = 0$$

g)

$k_c^2 = 1.02 \times 10^5$

Para $f = 7\,GHz$

$$\mathcal{E}_\rho = \frac{1}{2500}\left[277.83 \frac{\partial \mathcal{E}_z}{\partial \rho}\right]$$

$$\mathcal{E}_\phi = \frac{1}{2500}\left[\frac{277.83}{\rho} \frac{\partial \mathcal{E}_z}{\partial \phi}\right]$$

$$\mathcal{H}_\rho = \frac{1}{2500}\left[\frac{0.389j}{\rho} \frac{\partial \mathcal{E}_z}{\partial \phi}\right]$$

$$\mathcal{H}_\phi = \frac{1}{2500}\left[-0.389j \frac{\partial \mathcal{E}_z}{\partial \rho}\right]$$

Para $f = 18\,GHz$

$$\mathcal{E}_\rho = \frac{1}{2500}\left[-208.37j \frac{\partial \mathcal{E}_z}{\partial \rho}\right]$$

$$\mathcal{E}_\phi = \frac{1}{2500}\left[\frac{-208.37j}{\rho} \frac{\partial \mathcal{E}_z}{\partial \phi}\right]$$

$$\mathcal{H}_\rho = \frac{1}{2500}\left[\frac{j}{\rho} \frac{\partial \mathcal{E}_z}{\partial \phi}\right]$$

$$\mathcal{H}_\phi = \frac{1}{2500}\left[-j \frac{\partial \mathcal{E}_z}{\partial \rho}\right]$$

h)

Modo ~~TE~~ TE01

$m = 0 \quad n = 1 \quad P_{mn} = 3.832 \qquad f_c = 15\,GHz$

Se asume el vacío $\epsilon = \epsilon_0 \quad \mu = \mu_0$

$K_c^2 = \omega^2 \mu \epsilon + \gamma^2 \rightarrow \gamma = K_{c\,mn}^2 - \omega^2 \mu \epsilon$

a)

$\omega_{c\,mn} = \frac{1}{\sqrt{\mu \epsilon}} K_{c\,mn} \qquad K_{c_{01}}\, a = 3.832$

$\omega_{c\,mn} = \frac{1}{\sqrt{\mu \epsilon}} \left(\frac{3.832}{a} \right) \rightarrow a = \frac{3.832}{2\pi f_c \sqrt{\mu_0 \epsilon_0}} = 0.0122\,m$

$a = 1.22\,cm$

b)

$\beta = \omega \sqrt{\mu_0 \epsilon_0} \sqrt{1 - \left(\frac{f_{c\,mn}}{f} \right)^2}$

Para $f < f_c \rightarrow f = 7\,GHz$

$\beta = 2\pi (7 \times 10^9) \sqrt{\mu_0 \epsilon_0} \sqrt{1 - \left(\frac{15}{7} \right)^2} = 277.83 j\ rad/m$

Para $f > f_c \rightarrow f = 18\,GHz$

$\beta = 2\pi (18\,GHz) \sqrt{\mu_0 \epsilon_0} \sqrt{1 - \left(\frac{15}{18} \right)^2} = 208.37\ rad/m$

c)

$\eta_{TE} = \frac{\sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{1 - \left(\frac{f_{c\,mn}}{f} \right)^2}}$

Para $f < f_c \rightarrow f = 7\,GHz$

$\eta_{TE} = \frac{\sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{1 - \left(\frac{15}{7} \right)^2}} = -148.97 j\ \Omega$

Para $f > f_c \rightarrow f = 18\,GHz$

$\eta_{TE} = \frac{\sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{1 - \left(\frac{15}{18} \right)^2}} = 682.18\ \Omega$

d)

$$V_F = \lambda F \; ; \; \lambda = \frac{2\pi}{2\pi F\sqrt{\mu\epsilon}} \rightarrow V_F = \frac{f}{f\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}}$$

Para $F < F_c \rightarrow f = 7\,GHz$

$$V_F = \frac{1}{\sqrt{\mu_0\epsilon_0}} = c \approx 3\times10^8\, m/s$$

Para $f > f_c \rightarrow f = 18\,GHz$

$$V_F = \frac{1}{\sqrt{\mu_0\epsilon_0}} = c \approx 3\times10^8\, m/s$$

e)

Para $f < F_c \rightarrow F = 7\,GHz$

$$\lambda = \frac{2\pi}{2\pi(7\times10^9)\sqrt{\mu_0\epsilon_0}} = 0.043\,m = 4.30\,cm$$

Para $F > F_c \rightarrow f = 18\,GHz$

$$\lambda = \frac{2\pi}{2\pi(18\times10^9)\sqrt{\mu_0\epsilon_0}} = 0.0167\,m = 1.67\,cm$$

f)

$$\mathcal{E}_z = 0 \qquad \mathcal{H}_z = \mathcal{H}_z$$

g)

$$K_c = \frac{P_{mn}}{a} = \frac{3.832}{0.0122} = 314.10$$

Para $f < f_c \rightarrow f = 7GHz$

$$\mathcal{E}_\rho = \frac{1}{(314.10)^2}\left[-\frac{17598\pi j}{\rho}\frac{\partial \mathcal{H}_z}{\partial \phi}\right]$$

$$\mathcal{E}_\phi = \frac{1}{(314.10)^2}\left[17598\pi j\frac{\partial \mathcal{H}_z}{\partial \rho}\right]$$

$$\mathcal{H}_\rho = \frac{1}{(314.10)^2}\left[277.87\frac{\partial \mathcal{H}_z}{\partial \rho}\right]$$

$$\mathcal{H}_\phi = \frac{1}{(314.10)^2}\left[\frac{277.87}{\rho}\frac{\partial \mathcal{H}_z}{\partial \phi}\right]$$

Para $f > f_c \rightarrow f = 18GHz$

$$\mathcal{E}_\rho = \frac{1}{(314.10)^2}\left[-\frac{45252\pi j}{\rho}\frac{\partial \mathcal{H}_z}{\partial \phi}\right]$$

$$\mathcal{E}_\phi = \frac{1}{(314.10)^2}\left[45252\pi j\frac{\partial \mathcal{H}_z}{\partial \phi}\right]$$

$$\mathcal{H}_\rho = \frac{1}{(314.10)^2}\left[-208.37j\frac{\partial \mathcal{H}_z}{\partial \rho}\right]$$

$$\mathcal{H}_\phi = \frac{1}{(314.10)^2}\left[-\frac{208.37j}{\rho}\frac{\partial \mathcal{H}_z}{\partial \phi}\right]$$

h)