Robótica industrial: Algoritmo computacional Lagrange-Euler

BRANKS

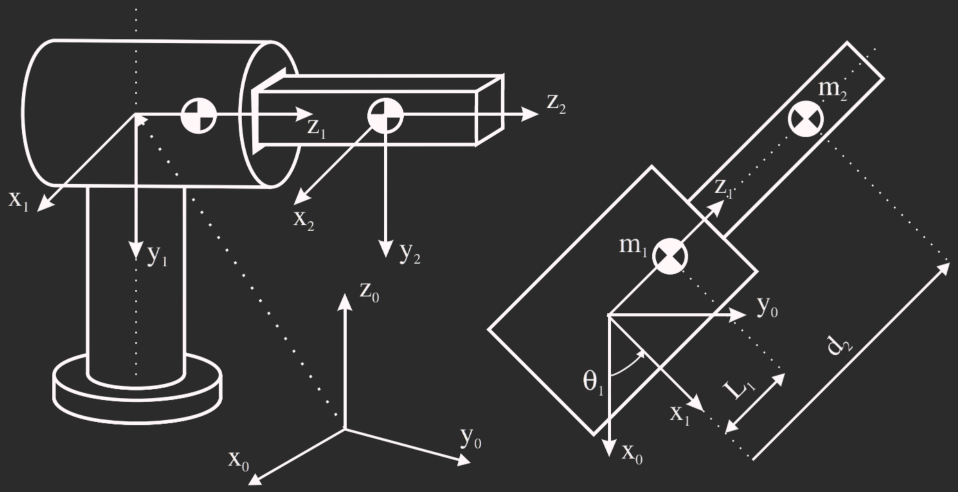
ROBÓTICA COLABORATIVA

Algoritmo computacional:

- Basado en la representación de D-H
- Poca eficiencia computacional: O(n4) (n=nº GDL)
- Ecuaciones finales bien estructuradas (D,H,C por separado)

Se compone de una serie de pasos que veremos a continuación:

1. *Asignar a cada barra un sistema de referencia de acuerdo D-H.*



Articulación	θi	di	ai	αi
1	θ1	0	0	-90
2	0	d2	0	0

2. *Obtener las matrices de transformación 0Ai para cada barra i.*

$$
{}^{0}\mathbf{A}_1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{1}\mathbf{A}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}\mathbf{A}_2 = {}^{0}\mathbf{A}_1\, {}^{1}\mathbf{A}_2 = \begin{bmatrix} C_1 & 0 & -S_1 & -d_2 S_1 \\ S_1 & 0 & C_1 & d_2 C_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

3. *Obtener las matrices Uij definidas por:*

$$
\mathbf{U}_{ij} = \frac{\partial^0 \mathbf{A}_i}{\partial q_j}
$$

$$
\mathbf{U}_{11} = \frac{\partial^0 \mathbf{A}_1}{\partial \theta_1} = \frac{\partial \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}{\partial \theta_1} = \begin{bmatrix} -S_1 & 0 & -C_1 & 0 \\ C_1 & 0 & -S_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$\mathbf{U}_{12} = \frac{\partial^0 \mathbf{A}_1}{\partial d_2} = \frac{\partial \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}{\partial d_2} = [0]$$

$$\mathbf{U}_{21} = \frac{\partial^0 \mathbf{A}_2}{\partial \theta_1} = \frac{\partial \begin{bmatrix} C_1 & 0 & -S_1 & -d_2 S_1 \\ S_1 & 0 & C_1 & d_2 C_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}{\partial \theta_1} = \begin{bmatrix} -S_1 & 0 & -C_1 & -d_2 C_1 \\ C_1 & 0 & -S_1 & -d_2 S_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{U}_{22} = \frac{\partial^0 \mathbf{A}_2}{\partial d_2} = \frac{\partial \begin{bmatrix} C_1 & 0 & -S_1 & -d_2 S_1 \\ S_1 & 0 & C_1 & d_2 C_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}{\partial d_2} = \begin{bmatrix} 0 & 0 & 0 & -S_1 \\ 0 & 0 & 0 & C_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4. Obtener las matrices Uijk definidas por:

$$\mathbf{U}_{ijk} = \frac{\partial \mathbf{U}_{ij}}{\partial q_k}$$

Se va iterando 1,1,1; 1,1,2; 1,2,1; 1,2,2... de manera sucesiva hasta terminar las iteraciones:

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$$\mathbf{U}_{111} = \frac{\partial \mathbf{U}_{11}}{\partial \theta_1} = \frac{\partial \begin{bmatrix} -S_1 & 0 & -C_1 & 0 \\ C_1 & 0 & -S_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}{\partial \theta_1} = \begin{bmatrix} -C_1 & 0 & S_1 & 0 \\ -S_1 & 0 & -C_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{U}_{112} = \frac{\partial \mathbf{U}_{11}}{\partial d_2} = \frac{\partial \begin{bmatrix} -S_1 & 0 & -C_1 & 0 \\ C_1 & 0 & -S_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}{\partial d_2} = [0]$$

$$\mathbf{U}_{121} = \frac{\partial \mathbf{U}_{12}}{\partial \theta_1} = \frac{\partial [0]}{\partial \theta_1} = [0] \qquad \mathbf{U}_{122} = \frac{\partial \mathbf{U}_{12}}{\partial d_2} = \frac{\partial [0]}{\partial d_2} = [0]$$

$$\mathbf{U}_{211} = \frac{\partial \mathbf{U}_{21}}{\partial \theta_1} = \frac{\partial \begin{bmatrix} -S_1 & 0 & -C_1 & -d_2 C_1 \\ C_1 & 0 & -S_1 & -d_2 S_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}{\partial \theta_1} = \begin{bmatrix} -C_1 & 0 & S_1 & d_2 S_1 \\ -S_1 & 0 & -C_1 & -d_2 C_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{U}_{212} = \frac{\partial \mathbf{U}_{21}}{\partial d_2} = \frac{\partial \begin{bmatrix} -S_1 & 0 & -C_1 & -d_2C_1 \\ C_1 & 0 & -S_1 & -d_2S_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}{\partial d_2} = \begin{bmatrix} 0 & 0 & 0 & -C_1 \\ 0 & 0 & 0 & -S_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{U}_{221} = \frac{\partial \mathbf{U}_{22}}{\partial \theta_1} = \frac{\partial \begin{bmatrix} 0 & 0 & 0 & -S_1 \\ 0 & 0 & 0 & C_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}{\partial \theta_1} = \begin{bmatrix} 0 & 0 & 0 & -C_1 \\ 0 & 0 & 0 & -S_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{U}_{222} = \frac{\partial \mathbf{U}_{22}}{\partial d_2} = \frac{\partial \begin{bmatrix} 0 & 0 & 0 & -S_1 \\ 0 & 0 & 0 & C_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}{\partial d_2} = [0]$$

Al terminar las operaciones, el siguiente paso es:

5. Obtener las matrices de PseudoInercias Ji para cada barra i.

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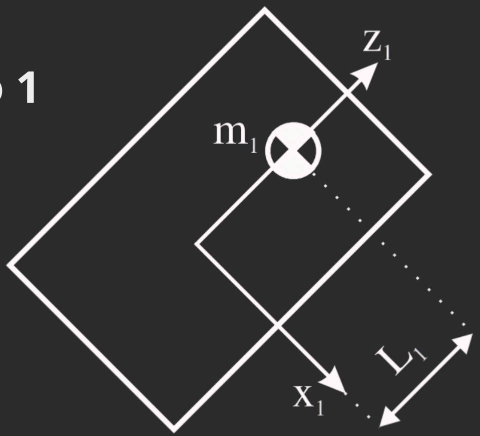
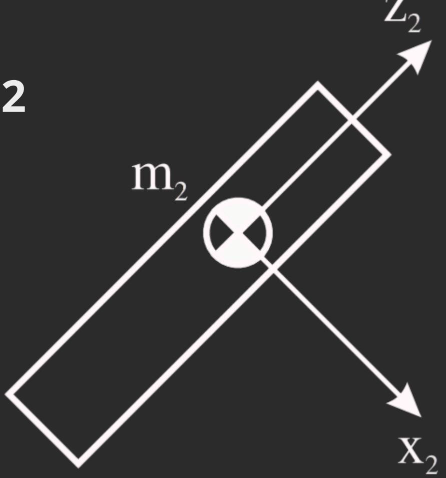
$$\mathbf{J}_i = \begin{bmatrix} \int_i x_i^2 dm & \int_i x_i y_i dm & \int_i x_i z_i dm & \int_i x_i dm \\ \int_i y_i x_i dm & \int_i y_i^2 dm & \int_i y_i z_i dm & \int_i y_i dm \\ \int_i z_i x_i dm & \int_i z_i y_i dm & \int_i z_i^2 dm & \int_i z_i dm \\ \int_i x_i dm & \int_i y_i dm & \int_i z_i dm & \int_i dm \end{bmatrix}$$

Elemento 1



$$\mathbf{J}_1 = \begin{bmatrix} \int_1 x_1^2 dm & \int_1 x_1 y_1 dm & \int_1 x_1 z_1 dm & \int_1 x_1 dm \\ \int_1 y_1 x_1 dm & \int_1 y_1^2 dm & \int_1 y_1 z_1 dm & \int_1 y_1 dm \\ \int_1 z_1 x_1 dm & \int_1 z_1 y_1 dm & \int_1 z_1^2 dm & \int_1 z_1 dm \\ \int_1 x_1 dm & \int_1 y_1 dm & \int_1 z_1 dm & \int_1 dm \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m_1 L_1^2 & m_1 L_1 \\ 0 & 0 & m_1 L_1 & m_1 \end{bmatrix}$$

Elemento 2



Debido a que la masa se considera concentrada en el centro de masas y el origen del sistema de coordenadas del elemento 2 se toma en el mismo centro:

$$\mathbf{J}_2 = \begin{bmatrix} \int_2 x_2^2 dm & \int_2 x_2 y_2 dm & \int_2 x_2 z_2 dm & \int_2 x_2 dm \\ \int_2 y_2 x_2 dm & \int_2 y_2^2 dm & \int_2 y_2 z_2 dm & \int_2 y_2 dm \\ \int_2 z_2 x_2 dm & \int_2 z_2 y_2 dm & \int_2 z_2^2 dm & \int_2 z_2 dm \\ \int_2 x_2 dm & \int_2 y_2 dm & \int_2 z_2 dm & \int_2 dm \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix}$$

6. Obtener la matriz de Inercia D cuyos elementos vienen definidos por:

$$d_{ij} = \sum_{k=(\max\ i,j)}^{n} \mathrm{Traza}\left(\mathbf{U}_{kj}\mathbf{J}_k\mathbf{U}_{ki}^T\right)$$

$$con \quad i,j = 1,2,\ldots,n$$

$$n = \text{número de grados de libertad}$$

$$d_{ij} = \sum_{k=(\max\ i,j)}^{n} \mathrm{Traza}\left(\mathbf{U}_{kj}\mathbf{J}_k\mathbf{U}_{ki}^T\right); \qquad d_{11} = \sum_{k=(\max\ 1,1)}^{2} \mathrm{Traza}\left(\mathbf{U}_{k1}\mathbf{J}_k\mathbf{U}_{k1}^T\right)$$

$$d_{11} = Tr\left(\mathbf{U}_{11}\mathbf{J}_1\mathbf{U}_{11}^T\right) + Tr\left(\mathbf{U}_{21}\mathbf{J}_2\mathbf{U}_{21}^T\right)$$

$$d_{11} = Tr\left(\begin{bmatrix} C_1^2 L_1^2 m_1 & S_1 C_1 L_1^2 m_1 & 0 & 0 \\ C_1 S_1 L_1^2 m_1 & S_1^2 L_1^2 m_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right) + Tr\left(\begin{bmatrix} C_1^2 d_2^2 m_2 & S_1 C_1 d_2^2 m_2 & 0 & 0 \\ C_1 S_1 d_2^2 m_2 & S_1^2 d_2^2 m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 \end{bmatrix}\right)$$

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$$d_{11} = \left(C_1^2 + S_1^2\right) m_1 L_1^2 + \left(C_1^2 + S_1^2\right) d_2^2 m_2 = m_1 L_1^2 + m_2 d_2^2$$

$$d_{ij} = \sum_{k=(\max\ i,j)}^{n} \text{Traza}\left(\mathbf{U}_{kj} \mathbf{J}_k \mathbf{U}_{ki}^T\right); \qquad d_{12} = \sum_{k=(\max\ 1,2)}^{2} \text{Traza}\left(\mathbf{U}_{k2} \mathbf{J}_k \mathbf{U}_{k1}^T\right)$$

$$d_{12} = Tr\left(\mathbf{U}_{22} \mathbf{J}_2 \mathbf{U}_{21}^T\right)$$

$$d_{12} = Tr\left(\begin{bmatrix} S_1 C_1 d_2 m_2 & S_1^2 d_2 m_2 & 0 & 0 \\ -C_1^2 d_2 m_2 & -S_1 C_1 d_2 m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right) = S_1 C_1 d_2 m_2 - S_1 C_1 d_2 m_2 = 0$$

$$d_{ij} = \sum_{k=(\max\ i,j)}^{n} \text{Traza}\left(\mathbf{U}_{kj} \mathbf{J}_k \mathbf{U}_{ki}^T\right); \qquad d_{21} = \sum_{k=(\max\ 2,1)}^{2} \text{Traza}\left(\mathbf{U}_{k1} \mathbf{J}_k \mathbf{U}_{k2}^T\right)$$

$$d_{21} = Tr\left(\mathbf{U}_{21} \mathbf{J}_2 \mathbf{U}_{22}^T\right)$$

$$d_{21} = Tr\left(\begin{bmatrix} S_1 C_1 d_2 m_2 & -C_1^2 d_2 m_2 & 0 & 0 \\ S_1^2 d_2 m_2 & -S_1 C_1 d_2 m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right) = S_1 C_1 d_2 m_2 - S_1 C_1 d_2 m_2 = 0$$

$$d_{ij} = \sum_{k=(\max\ i,j)}^{n} \mathrm{Traza}\left(\mathbf{U}_{kj}\mathbf{J}_k\mathbf{U}_{ki}^T\right); \qquad d_{22} = \sum_{k=(\max\ 2,2)}^{2} \mathrm{Traza}\left(\mathbf{U}_{k2}\mathbf{J}_k\mathbf{U}_{k2}^T\right)$$

$$d_{22} = Tr\left(\mathbf{U}_{22}\mathbf{J}_2\mathbf{U}_{22}^T\right)$$

$$d_{22} = Tr\left(\begin{bmatrix} S_1^2 m_2 & -S_1C_1m_2 & 0 & 0 \\ -S_1C_1m_2 & C_1^2 m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right) = S_1^2 m_2 - C_1^2 m_2 = m_2$$

$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

$$D = \begin{bmatrix} m_1L_1^2 + m_2d_2^2 & 0 \\ 0 & m_2 \end{bmatrix}$$

7. Obtener los término h(ikm) definidos por:

$$h_{ikm} = \sum_{j=(\max\ i,k,m)}^{n} \mathrm{Traza}\left(\mathbf{U}_{jkm}\mathbf{J}_j\mathbf{U}_{ji}^T\right)$$

$$con \quad i,k,m = 1,2,\ldots,n$$

$$n = \text{número de grados de libertad}$$

$$h_{ikm} = \sum_{j=(\max\ i,k,m)}^{n} \mathrm{Traza}\left(\mathbf{U}_{jkm}\mathbf{J}_j\mathbf{U}_{ji}^T\right); \quad h_{111} = \sum_{j=(\max\ 1,1,1)}^{2} \mathrm{Traza}\left(\mathbf{U}_{j11}\mathbf{J}_j\mathbf{U}_{j1}^T\right)$$

$$h_{111} = Tr\left(\mathbf{U}_{111}\mathbf{J}_1\mathbf{U}_{11}^T\right) + Tr\left(\mathbf{U}_{211}\mathbf{J}_2\mathbf{U}_{21}^T\right)$$

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$$h_{111} = Tr\left(\begin{bmatrix} -C_1S_1m_1L_1^2 & -S_1^2m_1L_1^2 & 0 & 0 \\ C_1^2m_1L_1^2 & C_1S_1m_1L_1^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right) + Tr\left(\begin{bmatrix} -S_1C_1d_2^2m_2 & -S_1^2d_2^2m_2 & 0 & 0 \\ C_1^2d_2^2m_2 & S_1C_1d_2^2m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right)$$

$$h_{111} = -C_1S_1m_1L_1^2 + C_1S_1m_1L_1^2 - S_1C_1d_2^2m_2 + S_1C_1d_2^2m_2 = 0$$

$$h_{ikm} = \sum_{j=(\max\ i,k,m)}^{n} \mathrm{Traza}\left(\mathbf{U}_{jkm}\mathbf{J}_j\mathbf{U}_{ji}^T\right); \quad h_{112} = \sum_{j=(\max\ 1,1,2)}^{2} \mathrm{Traza}\left(\mathbf{U}_{j12}\mathbf{J}_j\mathbf{U}_{j1}^T\right)$$

$$h_{112} = Tr\left(\mathbf{U}_{212}\mathbf{J}_2\mathbf{U}_{21}^T\right)$$

$$h_{112} = Tr\left(\begin{bmatrix} C_1^2d_2m_2 & S_1C_1d_2m_2 & 0 & 0 \\ S_1C_1d_2m_2 & S_1^2d_2m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right)$$

$$h_{112} = C_1^2d_2m_2 + S_1^2d_2m_2 = d_2m_2$$

$$h_{ikm} = \sum_{j=(\max\ i,k,m)}^{n} \mathrm{Traza}\left(\mathbf{U}_{jkm}\mathbf{J}_j\mathbf{U}_{ji}^T\right); \quad h_{121} = \sum_{j=(\max\ 1,2,1)}^{2} \mathrm{Traza}\left(\mathbf{U}_{j21}\mathbf{J}_j\mathbf{U}_{j1}^T\right)$$

$$h_{121} = Tr\left(\mathbf{U}_{221}\mathbf{J}_2\mathbf{U}_{21}^T\right)$$

Algoritmo computacional

$$h_{121} = Tr\left(\begin{bmatrix} C_1^2 d_2 m_2 & S_1 C_1 d_2 m_2 & 0 & 0 \\ S_1 C_1 d_2 m_2 & S_1^2 d_2 m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right)$$

$$h_{121} = C_1^2 d_2 m_2 + S_1^2 d_2 m_2 = d_2 m_2$$

$$h_{ikm} = \sum_{j=(\max\ i,k,m)}^{n} \mathrm{Traza}\left(\mathbf{U}_{jkm} \mathbf{J}_j \mathbf{U}_{ji}^T\right); \quad h_{122} = \sum_{j=(\max\ 1,2,2)}^{2} \mathrm{Traza}\left(\mathbf{U}_{j22} \mathbf{J}_j \mathbf{U}_{j1}^T\right)$$

$$h_{122} = Tr\left(\mathbf{U}_{222} \mathbf{J}_2 \mathbf{U}_{21}^T\right)$$

$$h_{122} = Tr\left(\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{J}_2 \mathbf{U}_{21}^T\right)$$

$$h_{122} = 0$$

$$h_{ikm} = \sum_{j=(\max\ i,k,m)}^{n} \mathrm{Traza}\left(\mathbf{U}_{jkm} \mathbf{J}_j \mathbf{U}_{ji}^T\right); \quad h_{211} = \sum_{j=(\max\ 2,1,1)}^{2} \mathrm{Traza}\left(\mathbf{U}_{j11} \mathbf{J}_j \mathbf{U}_{j2}^T\right)$$

$$h_{211} = Tr\left(\mathbf{U}_{211} \mathbf{J}_2 \mathbf{U}_{22}^T\right)$$

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$$h_{211} = Tr\left(\begin{bmatrix} -S_1^2 d_2 m_2 & S_1 C_1 d_2 m_2 & 0 & 0 \\ S_1 C_1 d_2 m_2 & -C_1^2 d_2 m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right)$$

$$h_{211} = -S_1^2 d_2 m_2 - C_1^2 d_2 m_2 = -d_2 m_2$$

$$h_{ikm} = \sum_{j=(\max\ i,k,m)}^{n} \mathrm{Traza}\left(\mathbf{U}_{jkm} \mathbf{J}_j \mathbf{U}_{ji}^T\right); \quad h_{212} = \sum_{j=(\max\ 2,1,2)}^{2} \mathrm{Traza}\left(\mathbf{U}_{j12} \mathbf{J}_j \mathbf{U}_{j2}^T\right)$$

$$h_{212} = Tr\left(\mathbf{U}_{212} \mathbf{J}_2 \mathbf{U}_{22}^T\right)$$

$$h_{212} = Tr\left(\begin{bmatrix} S_1 C_1 m_2 & -C_1^2 m_2 & 0 & 0 \\ S_1^2 m_2 & -S_1 C_1 m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right)$$

$$h_{212} = S_1 C_1 m_2 - S_1 C_1 m_2 = 0$$

$$h_{ikm} = \sum_{j=(\max\ i,k,m)}^{n} \mathrm{Traza}\left(\mathbf{U}_{jkm} \mathbf{J}_j \mathbf{U}_{ji}^T\right); \quad h_{221} = \sum_{j=(\max\ 2,2,1)}^{2} \mathrm{Traza}\left(\mathbf{U}_{j21} \mathbf{J}_j \mathbf{U}_{j2}^T\right)$$

$$h_{221} = Tr\left(\mathbf{U}_{221} \mathbf{J}_2 \mathbf{U}_{22}^T\right)$$

$$h_{221} = Tr\left(\begin{bmatrix} S_1C_1m_2 & -C_1^2m_2 & 0 & 0 \\ S_1^2m_2 & -S_1C_1m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right)$$

$$h_{221} = S_1C_1m_2 - S_1C_1m_2 = 0$$

$$h_{ikm} = \sum_{j=(\max\ i,k,m)}^{n} \mathrm{Traza}\left(\mathbf{U}_{jkm}\mathbf{J}_j\mathbf{U}_{ji}^T\right); \quad h_{222} = \sum_{j=(\max\ 2,2,2)}^{2} \mathrm{Traza}\left(\mathbf{U}_{j22}\mathbf{J}_j\mathbf{U}_{j2}^T\right)$$

$$h_{222} = Tr\left(\mathbf{U}_{222}\mathbf{J}_2\mathbf{U}_{22}^T\right)$$

$$h_{222} = Tr\left(\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\mathbf{J}_2\mathbf{U}_{22}^T\right)$$

$$h_{222} = 0$$

8. Obtener el vector columna H de fuerzas de Coriolis y Centrifugas, cuyos elementos son:

$$h_i = \sum_{k=1}^{n}\sum_{m=1}^{n} h_{ikm}\dot{q}_k\dot{q}_m$$

$$con \quad i = 1,2,\ldots,n$$

$$n = \text{número de grados de libertad}$$

$$h_1 = \sum_{k=1}^{2}\sum_{m=1}^{2} h_{1km}\dot{q}_k\dot{q}_m = h_{111}\dot{\theta}_1\dot{\theta}_1 + h_{112}\dot{\theta}_1\dot{d}_2 + h_{121}\dot{d}_2\dot{\theta}_1 + h_{122}\dot{d}_2\dot{d}_2$$

$$h_1 = 0\dot{\theta}_1^2 + d_2 m_2\dot{\theta}_1\dot{d}_2 + d_2 m_2\dot{d}_2\dot{\theta}_1 + 0\dot{d}_2^2 = 2d_2 m_2\dot{\theta}_1\dot{d}_2$$

$$h_2 = \sum_{k=1}^{2}\sum_{m=1}^{2} h_{2km}\dot{q}_k\dot{q}_m = h_{211}\dot{\theta}_1\dot{\theta}_1 + h_{212}\dot{\theta}_1\dot{d}_2 + h_{221}\dot{d}_2\dot{\theta}_1 + h_{222}\dot{d}_2\dot{d}_2$$

$$h_2 = -d_2 m_2\dot{\theta}_1^2 + 0\dot{\theta}_1\dot{d}_2 + 0\dot{d}_2\dot{\theta}_1 + 0\dot{d}_2^2 = -d_2 m_2\dot{\theta}_1^2$$

$$\mathbf{H} = \begin{bmatrix} 2d_2 m_2\dot{\theta}_1\dot{d}_2 \\ -d_2 m_2\dot{\theta}_1^2 \end{bmatrix}$$

9. Obtener el vector columna C de Fuerzas de Gravedad, cuyos elementos son:

$$c_i = \sum_{j=1}^{n}\left(-m_j \mathbf{g}\mathbf{U}_{ji}\,{}^j\mathbf{r}_j\right)$$

$$con \quad i = 1,2,\ldots,n$$

$$n = \text{número de grados de libertad}$$

g: es el vector de gravedad expresado en el sistema de la base {S0} y viene expresado por (gx0 , gy0 , gz0 , 0)

irj: es el vector de coordenadas homogéneas del centro de masas del elemento j expresado en el sistema de referencia del elemento i.

El vector de gravedad expresado en el sistema de la base del robot {S0}

$$\mathbf{g} = [g, 0, 0, 0]$$

Los vectores de coordenadas homogéneas de posición del centro de masas del eslabón j expresado en el sistema {Sj}

$${}^{1}\mathbf{r}_1 = \begin{bmatrix} 0 & 0 & L & 1 \end{bmatrix}^T$$

$${}^{2}\mathbf{r}_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$c_1 = \sum_{j=1}^{2} \left(-m_j \mathbf{g} \mathbf{U}_{j1}\, {}^{j}\mathbf{r}_j\right) = -m_1 \mathbf{g} \mathbf{U}_{11}\, {}^{1}\mathbf{r}_1 - m_2 \mathbf{g} \mathbf{U}_{21}\, {}^{2}\mathbf{r}_2$$

$$c_1 = -m_1 \begin{bmatrix} 0 & 0 & -g & 0 \end{bmatrix} \begin{bmatrix} -S_1 & 0 & -C_1 & 0 \\ C_1 & 0 & -S_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ L_1 \\ 1 \end{bmatrix} -$$

$$m_2 \begin{bmatrix} 0 & 0 & -g & 0 \end{bmatrix} \begin{bmatrix} -S_1 & 0 & -C_1 & -d_2 C_1 \\ C_1 & 0 & -S_1 & -d_2 S_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$c_2 = \sum_{j=1}^{2} \left(-m_j \mathbf{g} \mathbf{U}_{j2}\, {}^{j}\mathbf{r}_j\right) = -m_1 \mathbf{g} \mathbf{U}_{12}\, {}^{1}\mathbf{r}_1 - m_2 \mathbf{g} \mathbf{U}_{22}\, {}^{2}\mathbf{r}_2$$

$$c_2 = -m_1\begin{bmatrix}0 & 0 & -g & 0\end{bmatrix}\begin{bmatrix}0\end{bmatrix}\begin{bmatrix}0\\0\\L_1\\1\end{bmatrix} - m_2\begin{bmatrix}0 & 0 & -g & 0\end{bmatrix}\begin{bmatrix}0 & 0 & 0 & -S_1\\0 & 0 & 0 & C_1\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\0\\0\\1\end{bmatrix} = 0$$

Por tanto

$$C = \begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

10.La ecuación del modelo Dinámico es:

$$\tau = D\ddot{q} + H + C$$

$$\begin{bmatrix}T_1\\F_2\end{bmatrix} = \begin{bmatrix}m_1L_1^2 + m_2d_2^2 & 0\\0 & m_2\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\\ddot{d}_2\end{bmatrix} + \begin{bmatrix}2d_2m_2\dot{\theta}_1\dot{d}_2\\-d_2m_2\dot{\theta}_1^2\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix}$$