Simulating New Keynesian Model with Cost-Push Shock using JuliaPerturbation Toolbox

José Elías Durán Roa
Universidad Carlos III de Madrid

Abstract

This work focuses on a medium-scale New-Keynesian model in a closed economy with complete financial markets and capital accumulation, and its implementation in Julia using the JuliaPerturbation toolbox created by Alvaro Salazar-Perez and Hernan D. Seoane (2023). The model is extended by adding a cost-push shock, which is described in detail in a paper by Peter N. Ireland (2002) . The impulse response functions (IRFs) are simulated for three different types of shocks: a monetary policy shock, a TFP shock, and a cost-push shock. The JuliaPerturbation toolbox is used to perform the perturbation and estimation of the model, and finally the Impulse Response Functions are compared and discussed.

1 Perturbating and estimating DSGE models in Julia

The research paper "Perturbating and estimating DSGE models in Julia" by Salazar-Perez and Seoane (2023), demonstrates the use of the Julia programming language for solution and estimation of Dynamic Stochastic General Equilibrium (DSGE) models. The authors highlight the significant gains achieved through the Julia implementation of the Perturbation solution (first and higher orders) and Bayesian estimation methods. The paper also cover an introduction to DSGE models, to perturbation method and sequential Monte Carlo solution methods and aims to showcase the power of the Julia language in solving and estimating DSGE models over the traditionally used Matlab.

Additionally, by solving and estimating three different models: the standard Real Business Cycle Model (RBC), a Medium scale New-Keynesian Model (NK) and a large model as in the Smets and Wouters (2007), they demonstrate that Julia significantly improves solution and estimation times compared to other programming languages like Matlab. Specifically, they highlight the computation of the log-linear solution, which involves the time-consuming Generalized Schur decomposition. In Julia, this step takes about $14\mu s$, while in Matlab, it takes $128\mu s$, making Julia approximately 10 times faster for this particular task.

1.1 The New - Keynesian model

Following Salazar-Perez and Seoane (2023) we start by presenting and defining the characteristics of the NK model used in this project. We work with a medium-scale New-Keynesian model in a closed economy with complete financial markets and capital accumulation, featuring households, monopolistic intermediate goods producers, competitive final goods producers, and a government responsible for monetary policy and maintaining a balanced budget

Intermediate Goods Producers

In this economic model, there are multiple producers of intermediate goods, each manufacturing a distinct product within the range $[0, 1]$. These producers possess market power and set prices for their unique goods. The demand for each product is determined by the formula:

$$a_{it} = \frac{P_{it}}{P_t} - \eta a_t$$

Where P_{it} is the price of product i, P_t is the average price of the economy, η controls the elasticity of substitutions among products, and a_t represents aggregate demand.

Firms have the option to adjust their prices with a probability α in each period, and only a fraction of firms $(1-\alpha)$ will not be able to change their prices. The firms that do not change their prices maintain the same price as in the previous period ($P_{it} = P_{it-1}$). These firms aim to maximize the expected present discounted value of profits given by:

$$E_0 \sum_{t=0}^{\infty} r_{0,t} P_t \Phi_{it}$$

Here, $r_{0,t}$ is the discount rate, and Φ_{it} represents the period's t profits in real terms, calculated as:

$$\Phi_{it} = \frac{P_{it} a_{it}}{P_t} - u_t k_{it} - w_t h_{it}$$

Each firm acquires labor (h_{it}) and capital (k_{it}) in competitive markets at prices w_t and u_t, respectively, to produce using a Cobb-Douglas technology subject to an aggregate productivity shock:

$$y_{it} = z_t F(h_{it}, k_{it})$$

Firms in the intermediate goods sector must meet the demand for their specific product, so $y_{it} \geq a_{it}$. Furthermore, the productivity shock (z_t) follows an AR(1) process in logarithms:

$$\ln(z_{t+1}) = \rho \ln(z_t) + \sigma_{t+1}$$

with normally independent and identically distributed (i.i.d.) innovations.

Households

In this model, households aim to maximize the present discounted value of utility, which depends on the consumption of different varieties and leisure. They also invest in assets and capital while owning intermediate firms.

The objective function is:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

Here, h_t represents labor supply, and c_t is the consumption bundle at time t.

Households face two intratemporal problems: the leisure-consumption choice and the variety choice. The intratemporal allocation between varieties is determined by solving a cost minimization problem, given by:

$$\min P_t c_t = \int_0^1 P_{it} c_{it} \, di$$

subject to the aggregation technology:

$$c_t = \left(\int_0^1 c_{it}^{1-\frac{1}{\eta}} \, di \right)^{\frac{\eta}{\eta-1}}$$

Where η determines the elasticity of substitution between varieties.

With this intratemporal allocation, households solve the dynamic problem by maximizing the present discounted value of utility, subject to an infinite sequence of period t budget constraints:

$$E_t r_{t,t+1} A_{t+1} + P_t c_t + P_t x_t = P_t w_t h_t + P_t u_t k_t + A_t + \int_0^1 \Phi_{it}\, di$$

Here, A_t represents a complete set of assets, x_t denotes investment, Φ_{it} are lump-sum transfers of intermediate goods producers' profits, and $r_{t,t+1}$ denotes the price of financial assets. Finally, the evolution of capital (Law of motion) is given by:

$$k_{t+1} = x_t + (1-\delta)k_t$$

Government

The government in this model is assumed to maintain a balanced budget, and monetary policy is governed by interest rate feedback rules. Specifically:

$$\ln \frac{R_t}{R^*} = \phi_R \ln \frac{R_{t-1}}{R^*} + \phi_\Pi \ln \frac{\Pi_{t-1}}{\Pi^*} + \phi_y \ln \frac{s_t(c_t + i_t)}{s^*(c^* + i^*)} + u_t$$

Here, u_t represents the monetary policy shock, ϕ captures the persistence of the rule, and ψ captures the responsiveness of monetary policy to inflation.

1.2 Julia Perturbation Toolbox

The Julia Pertubation Toolbox can be founded in Github following this link: [Julia Perturbation].

The repository contains three main files: *run_solution*, *run_estimation*, and *solution_functions*. The *run_solution* file can be modified to adapt the codes to solve and simulate a specific model. The file includes sections on the model, parameters, variables, and shocks, and users can modify these sections to fit their specific model. The *run_estimation* file launches the Bayesian estimation of a model through the Sequential Monte Carlo (SMC) method. The *solution_functions* repository contains all the functions required to solve and simulate the model.

2 Technology Shocks in the New Keynesian Model

In the third of simulation presented in the next section, we implemented the cost-push shock developed by Ireland (2002) using the Julia Pertubation toolbox. However, for greater clarity, we will first begin by conceptually explaining what this modification consists of. The paper of Ireland explores the role of technology shocks in the New Keynesian framework and incorporates both technology and cost-push shocks, which compete as terms that randomly shift the Phillips curve. The author aims to quantitatively examine the importance of technology shocks within this framework.

Moreover, this paper presents a version of the NK model that allows for technology shocks to remain dominant as the primary source of business cycle fluctuations, but also considers other shocks such as preference, cost-push, and monetary shocks. The author estimates the key parameters of this more general NK model using maximum likelihood and quarterly data from the postwar United States. The estimations suggests that other shocks, rather than technology shocks, are more important for explaining the behavior of output, inflation, and interest rates for the analyzed U.S data.

The NK Philipps Curve including both a TFP and a cost push shock looks as follows:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_t + 1 + (\frac{1}{\phi})(\theta - 1)\sigma \hat{y}_t - (\frac{1}{\phi})(\theta - 1)\hat{z}_t - (\frac{1}{\phi})\hat{\theta}_t$$

Here the term $\hat{\theta}_t$ represents the cost push shock. Notice that as another term that stochatically shifts the Philipps curve, the cost-push shock compete directly with the technology shock (represented by $\hat{z}_t$) in accounting for fluctuations in output and inflation. In this case the cost push shock represent changes in the elasticity of demand with respect to price for the intermediate good producers.

The cost push-shock follows an autoregressive process:

$$ln(\theta) = (1 - \rho_\theta)ln(\theta) + \rho_\theta ln(\theta_{t-1}) + \epsilon_{\theta t}$$

The results of the paper weaken the links between the current generation of NK models and the real-business-cycle models from which they were originally derived. The author concludes that while technology shocks can be important in shaping the dynamic behavior of key macroeconomic variables, the presence of nominal price rigidities helps determine how shocks of all kinds impact and propagate through the economy.

3 Simulation and Estimation in Julia

We estimate and simulate three shocks in Julia: TFP, monetary and the previously explained cost push shock. In this section we discuss and interpret the Impulse Response Function differentiating and comparing between these shocks.To solve and estimate the model we followed the same parametrization than Salazar - Pérez and Seoane (2023) for the NK case:

Table 1: Parametrization

Parameters	Interpretation	NK
α	Capital exponent in production	0.30
β	Discount factor	$1.04^{-1/4}$
γ	CRRA coefficient	2.00
δ	Capital depreciation	0.01
ρ_z	TFP persistence coefficient	0.8556
σ_z	TFP standard deviation	0.05
Z	TFP mean	1.00
η	Coeff. of elasticity of subst b/w varieties	5.00
θ	Prob. of Calvo lottery	0.80
ϕ_Π	Interest rate response to inflation	3.00
ϕ_Y	Interest rate response to output	0.01
ϕ_R	Smoothing coefficient of the MP rule	0.80

For simplicity we divided the coding in 4 steps. Step 1 is just the modification of the initial toolbox and can be consulted in Appendix A. In this step he code begins by setting up the model and its parameters. It includes equations, variables, and shock specifications. The code also defines the steady-state values for the model, based on the given parameter values. Additionally, it prepares the necessary functions for model processing and solution.In Step 2, the code defines the Impulse Response Function (IRF) for each shock, in step 3 we plot the relevant results of the IRF analysis. Finally in step 4 we constructed a loop to analyse the determinancy region for a given interval of ρ_y, ρ_π and ρ_p.

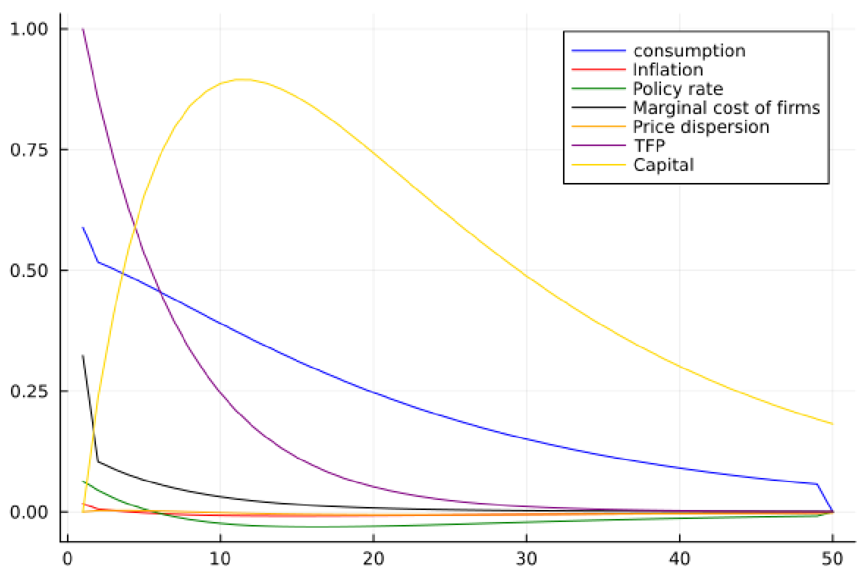
3.1 Total Factor Productivity Shock

To simulate the TFP shock, after solving the model using the Julia Toolbox, we constructed the following function to compute the associated Impulse Response Function (IRF):

```
function TFP(gx, hx, T)
    PVt = zeros(T, 5)
    NPVt = zeros(T, 13)

    PVt[1, :] = [0, 1, 0, 0, 0]

    for i in 2:T
        NPVt[i - 1, :] = gx * PVt[i - 1, :]
        PVt[i, :] = hx * PVt[i - 1, :]
    end

    return NPVt, PVt
end
```

This function calculates and stores the impulse responses of various variables in the model to the TFP shock. The IRF provides insights into how the economy responds to changes in TFP. The function uses the previously computed matrices of model solutions (gx and hx) to propagate the shock over time and generate the IRFs. It stores the responses in matrices PVt and NPVt (predetermined and non predetermined variables).

Figure 1: TFP shock effect - Relevant Variables



Periods

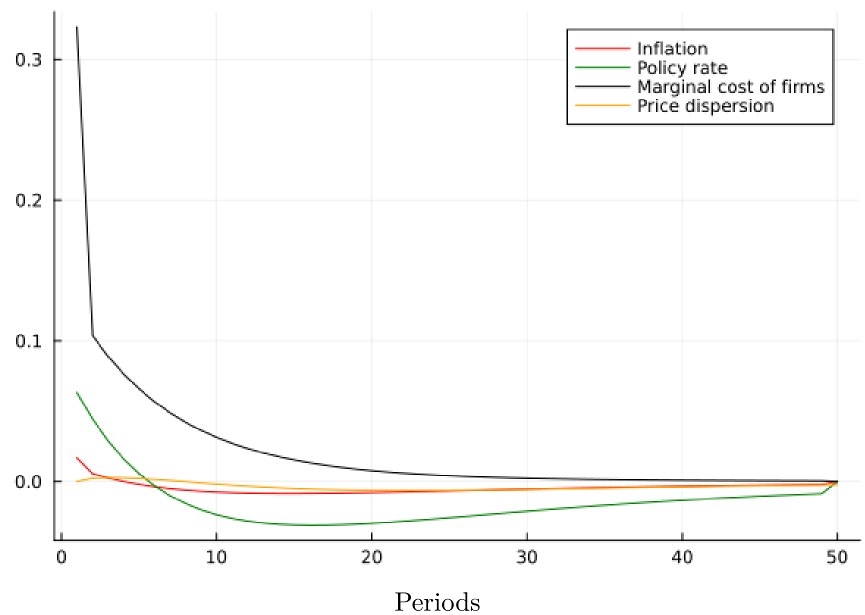
Note: We only included some relevant variables, see the appendix A for complete set of IRF.

For the selected number of periods (T=50) we observe that the variables start to converge to a steady state point. First of all we identify that the parameterization of the TFP's Law of Motion ensures that the stochastic process remains stationary over time. We observe a convex-decreasing behavior of price dispersion in the initial periods. This pattern suggests that in the immediate aftermath of the TFP shock, firms with market power adjust their prices in a manner that leads to increased price dispersion. As some firms opt to adjust their prices while others do not (due to the Calvo pricing mechanism), this convexity arises from the differential speed of price adjustments across firms.

However, as time progresses, price dispersion gradually decreases, converging to a long-term equilibrium. This phenomenon can be attributed to the gradual adjustment of prices across the entire spectrum of intermediate goods producers, as the impact of the TFP shock propagates through the economy. It exemplifies the process of firms gradually aligning their prices with the new productivity level, ultimately contributing to the stabilization of price dispersion. Additionally, the inflation pattern is influenced by the initial convex price dispersion, which contributes to the gradual inflation increase. Second, it underscores the role of price stickiness, a hallmark of New Keynesian models. As prices adjust gradually and asymmetrically in response to the TFP shock, inflation takes time to reach its long-term equilibrium level.

Finally, in period 0 we observe very small rise in consumption which gradually declines from period 1 onward until it stabilizes at its long-term level. Simultaneously, the capital stock remains unchanged in period 0 but begins to grow starting from period 1, eventually approaching its steady state value. These fluctuations in consumption and capital can be succinctly described as a form of consumption smoothing. Additionally, we notice a concave trajectory in the capital stock. This implies that initially, the capital stock experiences a less-than-proportional increase compared to the output growth triggered by the shock.

Figure 2: TFP shock effect - Nominal Variables



Note that the policy rate closely tracks the evolution of inflation. This observation underscores that the TFP shock, being a real shock, has a limited impact on the nominal variables of the model. Real shocks, such as TFP fluctuations, primarily affect the underlying economic fundamentals, like the production technology and efficiency, rather than exerting significant influence on nominal factors, such as inflation or the policy rate.

3.2 Monetary Shock

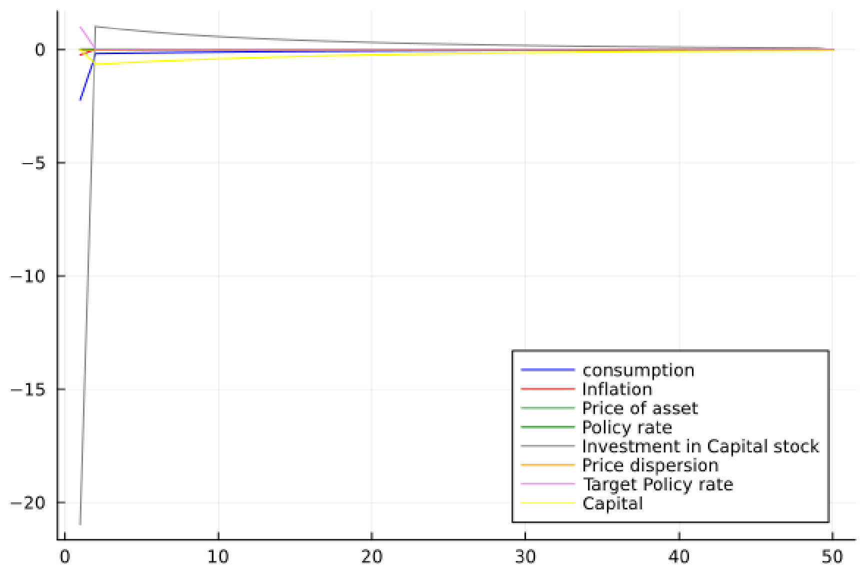
A Monetary Policy Shock, characterized by a 1-percentage-point positive deviation in the Target Policy Rate from the steady-state level, is a deliberate contractionary measure implemented by a central bank to manage economic conditions. This policy action involves raising interest rates, increasing the cost of borrowing for businesses and individuals, and thus curbing spending and investments in the economy. It is a strategic maneuver designed to address issues such as inflation or economic overheating, highlighting the central bank's pivotal role in steering economic stability. This underscores the central bank's direct impact on inflation, output, and overall economic well-being. The function designed to model the IRF for this particular shock is the following:

```
function Monetary(gx, hx, T)

    PVt = zeros(T, 5)
    NPVt = zeros(T, 13)

```

```
    PVt[1, :] = [0, 0, 1, 0, 0]

    for i in 2:T
        NPVt[i - 1, :] = gx * PVt[i - 1, :]
        PVt[i, :] = hx * PVt[i - 1, :]
    end

    return NPVt, PVt
end
```

As we can observe in figure 3 note the monetary shock influences the Policy Target Rate, which, in accordance with the Taylor Rule, leads to an increase in the policy rate. This increase in the policy rate has a ripple effect across the entire economic landscape, with consumption and the capital stock taking a hit in response. As borrowing costs surge due to the elevated interest rates, households and firms reduce their spending and investment activities. This effect underscores the central bank's ability to directly influence financial conditions and economic activity through interest rate adjustments, particularly when it seeks to address inflationary pressures or promote economic stability.

Figure 3: Monetary shock effect - Relevant Variables

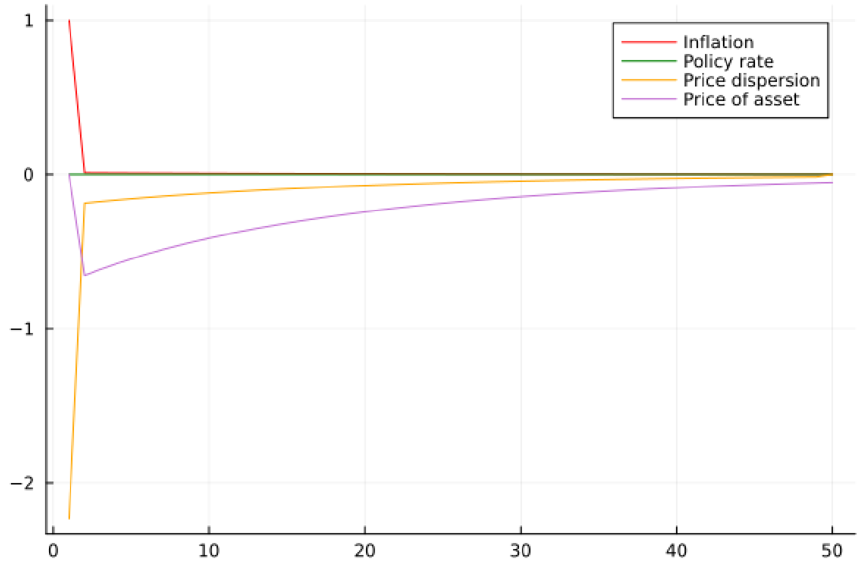


Periods

An upward policy rate adjustment results in a higher real interest rate, driven by the Fisher equation. This tighter monetary policy exerts downward pressure on economic output, reflecting the trade-offs central banks face in balancing inflation control and economic growth support. Additionally, the shock influences the firm's marginal cost by negatively impacting inflation expectations through changes in the Policy Target Rate. Anticipated lower inflation due to a higher policy target influences firms' pricing decisions, illustrating how monetary policy impacts both demand and supply, affecting price dynamics. This interaction between monetary policy and inflation expectations underscores the central bank's pivotal role in shaping real and nominal variables, contributing to macroeconomic stability.

In Figure 4, the New Keynesian Phillips Curve (NKPC) stands out as a central factor governing inflation dynamics. When a shock occurs, we observe a clear decrease in inflation. This decline is primarily due to the simultaneous reduction in the firm's marginal cost. As the policy rate gradually returns to its steady-state level, inflation begins to rise, driven by the underlying dynamics of the model. Importantly, it's crucial to note that this simulation represents a temporary policy change and does not alter the steady state.

Figure 4: Monetary shock effect - Nominal Variables



Periods

The initial drop in inflation also leads to a significant reduction in price dispersion, which is influenced by the dynamics of monetary policy. The asset price, a crucial financial variable, experiences a decline at the start of the shock. This decline can be attributed to its inverse relationship with the policy rate. These findings offer valuable insights into how monetary policy and nominal variables interact in the New Keynesian model. They highlight the essential role of central banks in shaping inflation dynamics and demonstrate how short-term policy adjustments can have substantial effects on inflation, price dispersion, and financial markets.

3.3 Cost Push Shock

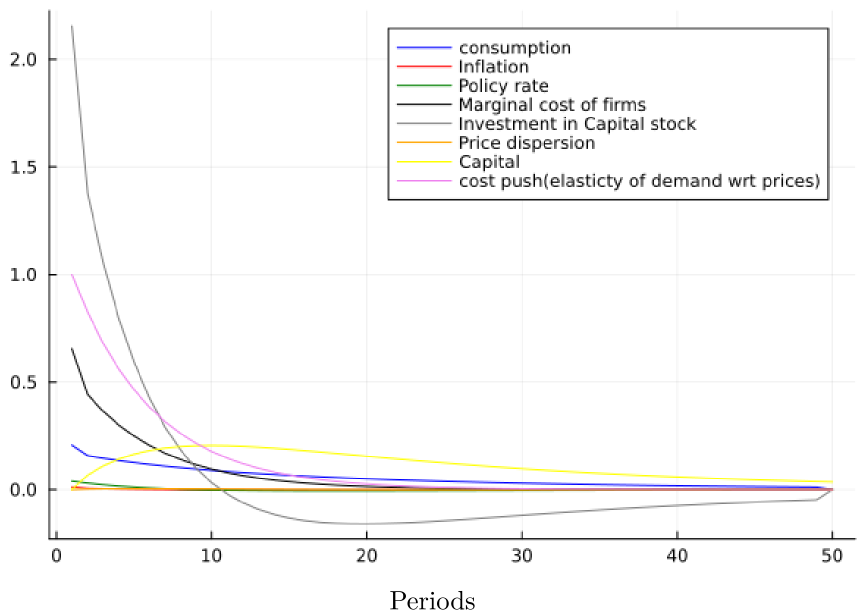
The cost-push shock introduced in the model represents changes in the elasticity of demand with respect to the price for intermediate good producers. Unlike the pure TFP shock, which primarily influences the real side of the economy by affecting productivity and output, the cost-push shock has direct implications for price dynamics. Specifically, it perturbs the market conditions that intermediate goods producers face, impacting the pricing strategies of firms within the economy. The function for the cost push shock IRF is the following:

```
function CostPush(gx, hx, T)

    PVt = zeros(T, 5)
    NPVt = zeros(T, 13)

    PVt[1, :] = [0, 0, 0, 0, 1]

    for i in 2:T
        NPVt[i - 1, :] = gx * PVt[i - 1, :]
        PVt[i, :] = hx * PVt[i - 1, :]
    end

    return NPVt, PVt
end
```

As we can see in Figure 5 the significance of this measure of the cost-push shock lies in its elasticity. A higher elasticity indicates that even a minor uptick in production costs, resulting in a modest price increase, leads to a pronounced decrease in demand. This scenario presents optimizing firms with a notable trade-off between the price effect and the quantity effect. As a result, a higher elasticity diminishes the incentive to raise prices, emphasizing the intricate balance between these factors.

In the dynamics of the model, we observe an initial increase in consumption, followed by a gradual decline towards the steady state. Concurrently, the capital stock remains unchanged at time 0, but subsequently commences a gradual ascent until it reaches a point where it initiates a decline. These trends highlight the evolving patterns in consumption and capital stock over time, underscoring the model's temporal dynamics.

Figure 5: Cost Push shock effect - Relevant Variables



The initial increase in inflation following a cost-push shock, that we observe in Figure 6 can be attributed to specific factors:

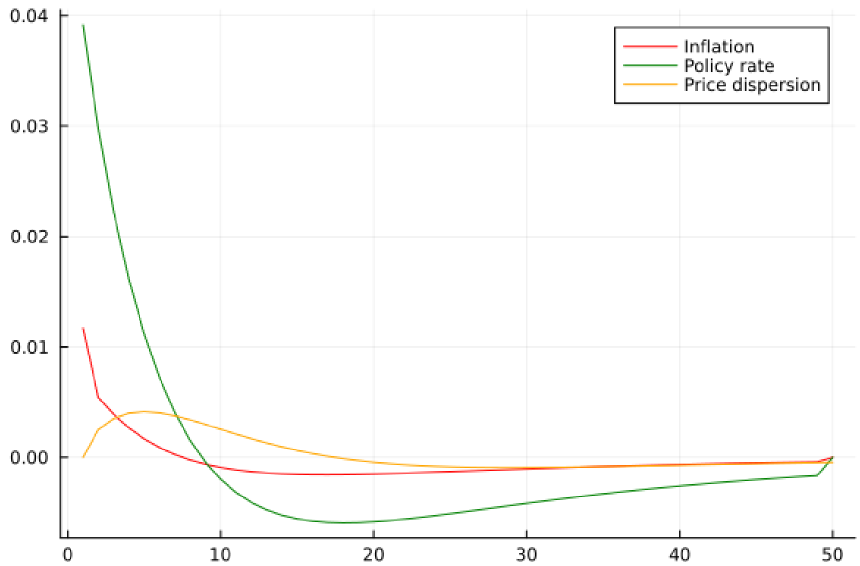
1. **Inflation Expectations**: Economic agents, including consumers and firms, form expectations about future inflation. When a cost-push shock occurs, it creates uncertainty about the future price levels. In some cases, individuals might anticipate that these cost pressures will persist and lead to higher future prices. Consequently, they may adjust their behavior, such as demanding higher wages or increasing prices for their goods and services. These revised expectations can temporarily drive up inflation.

2. **Nominal Rigidities**: The New Keynesian models, incorporate nominal rigidities, such as price stickiness. These rigidities mean that prices and wages do not adjust instantaneously to changes in economic conditions. When a cost-push shock initially impacts the economy, firms may not immediately adjust their prices to reflect the increased production costs. This lag in price adjustment can result in higher inflation rates during the initial shock period.

3. **Policy Response**: Depending on the central bank's response to the cost-push shock, inflation may exhibit a temporary increase. If the central bank decides to accommodate the shock by keeping interest rates low or increasing the money supply, this can contribute to higher inflation. The central bank's actions play a crucial role in shaping the inflation trajectory.

Overall, this initial increase in inflation reflects the complex dynamics of inflationary responses to external shocks in a New Keynesian framework. It underscores the importance of inflation expectations, nominal

Figure 6: Cost Push shock effect - Relevant Variables



rigidities, and monetary policy in shaping the inflation outcomes within the model. This understanding is valuable in the analysis of how economic variables respond to different shocks and policy measures, as demonstrated in our work.

4 Determinancy region analysis

In the final section of our study, we embarked on the task of determining the "determinacy" regions for our macroeconomic model. To accomplish this, we developed a Julia code that allowed us to explore how two key parameters, α_π (PHI_PPI) and α_y (PHI_ Y), influence the stability of the model's solutions.

```
########## Define the matrix  to store 0´s and 1´s dependending on the stability of the solution ###########
SCATTER_MAT=zeros(5,5)

for i in 1:5
    for j in 1:5

## Model
        ...
        ...   NOTE: HERE WE DON´T MODIFY NOTHING ELSE FROM THE TOOOLBOX. SEE APENDIX B FOR COMPLETE CODE.
        ...
        PHI_PPI    =       i
        PHI_Y      =       j
       ...
       ...
       ....
        sol_mat = solve_model(model, deriv, eta)

# Store gx and hx

        gx=sol_mat[1];
        hx=sol_mat[2];

# Define the matrix with 0 and 1 dependending on the stability of the solution #

        if isempty(gx)
            SCATTER_MAT[i,j] = 0
```

```
        else
            if isempty(hx)
                SCATTER_MAT[i,j] = 0
            else
                SCATTER_MAT[i,j] = 1
            end
        end
    end
end
```

The code begins by conducting a stability analysis while varying these parameters within specific intervals. This stability analysis is crucial for understanding the determinacy region of our model, which is where the model's solutions are stable and predictable. In practical terms, the code starts by defining a matrix called "SCATTER_MAT." This matrix stores binary values, 0 or 1, indicating whether the model's solution is stable or not for a specific combination of α_π and α_y.

The stability assessment process is based on calculating the Jacobian matrices (represented as "gx" and "hx") of the model. These matrices provide insights into the stability of the solutions. If both matrices are empty, it is considered that the solution is unstable, and it is marked with a 0 in the "SCATTER_MAT" matrix. Conversely, if at least one of the matrices contains information, the solution is considered stable and marked with a 1. By running this code for various values of α_π and α_y, we obtain a matrix that helps us visualize the determinacy region. This matrix will be used later to create a contour map in which regions with stable solutions (marked with 1) represent the determinacy region, while regions with unstable solutions (marked with 0) fall outside the determinacy region.

This stability analysis is a crucial step in our work because it enables us to identify the areas in which our economic model exhibits stable solutions. This understanding is essential for comprehending the dynamics and behavior of our model in different scenarios.

The resulting matrix is the following:

$$\begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

The matrix highlights how the α_π (PHI_PPI) and α_y (PHI_Y) parameters impact the determinacy of our macroeconomic model. The diagonal shift from 0.0 to 1.0 entries marks a critical threshold, showing that higher values of these parameters lead to increased stability. This analysis reveals the key parameter combinations that promote stability and emphasizes their role in shaping our model's economic dynamics.

5 Appendix A: Additional graphs

Figure 7: TFP shock - Non predetermined variables

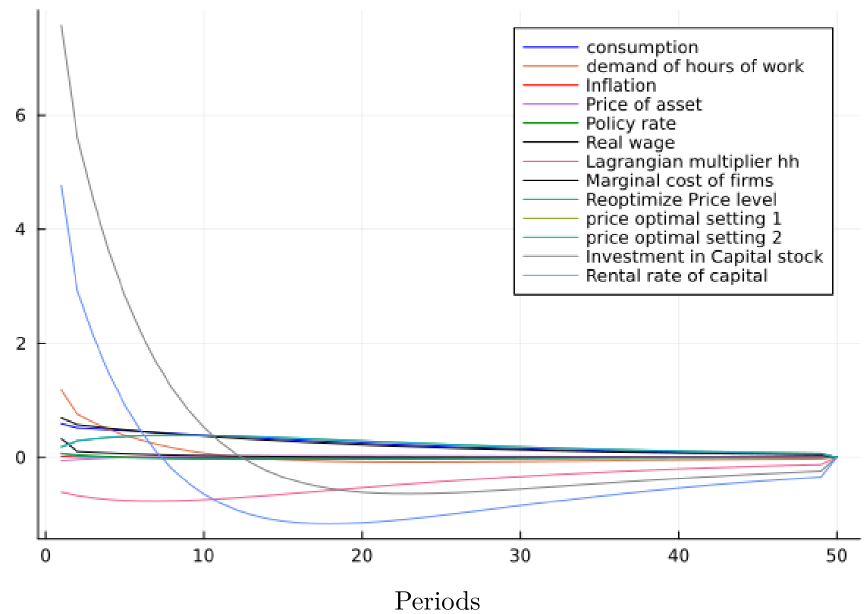


Figure 8: TFP shock - Predetermined variables

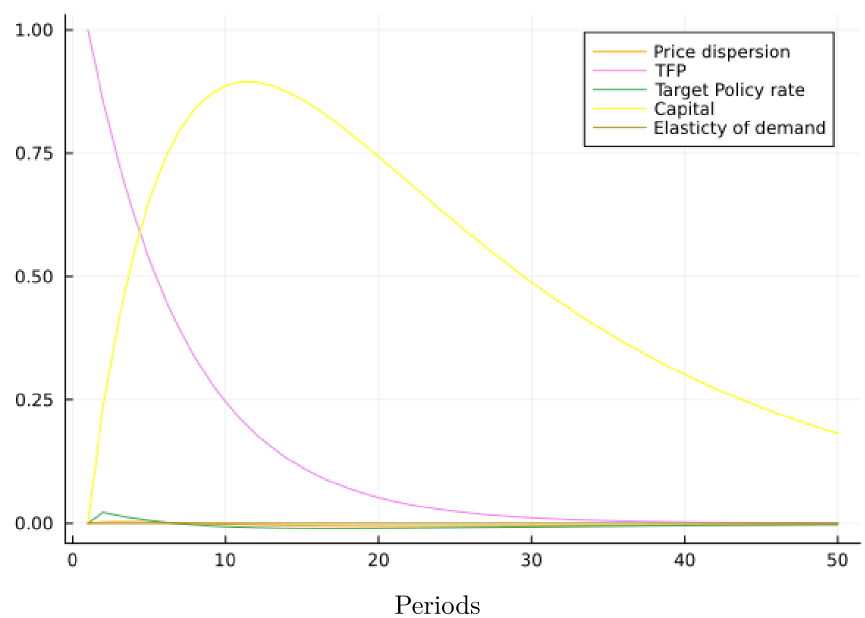
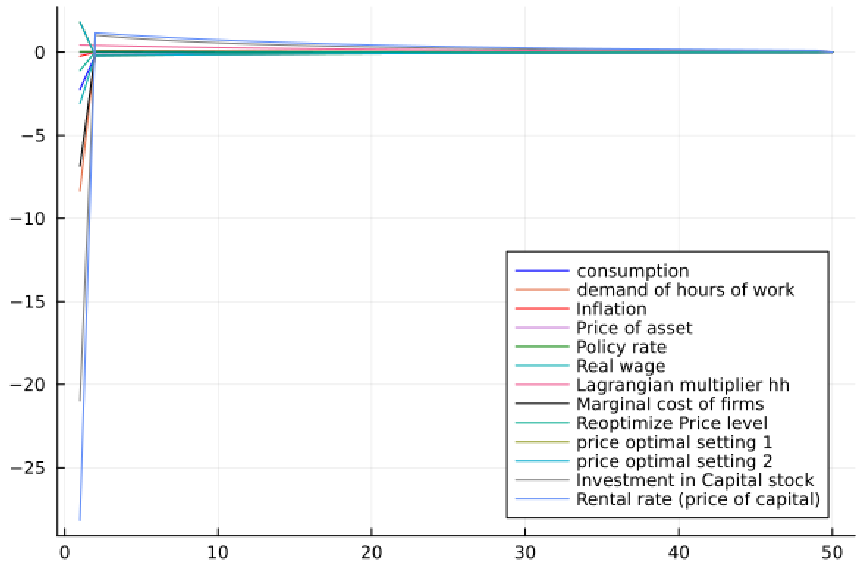
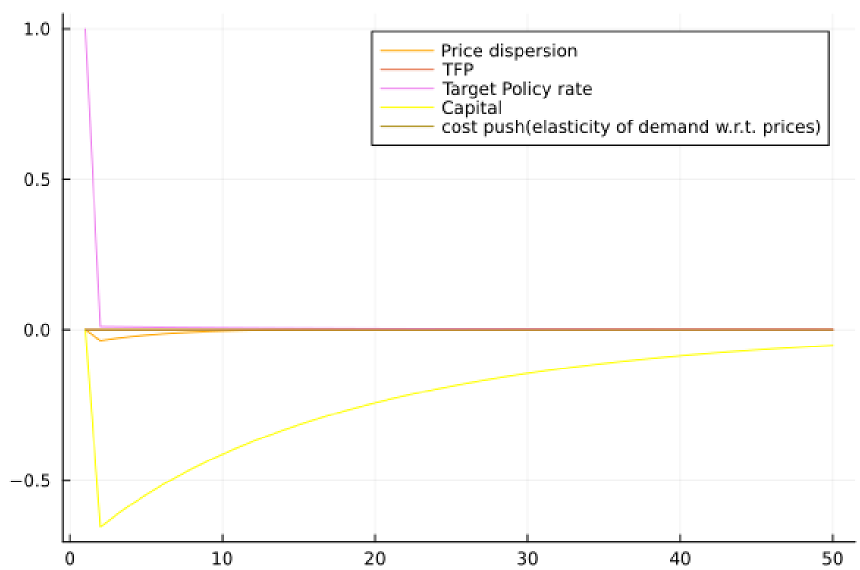


Figure 9: Monetary shock - Non predetermined variables



Periods

Figure 10: Monetary shock - Predetermined variables



Periods

Figure 11: Cost Push shock - Non predetermined variables

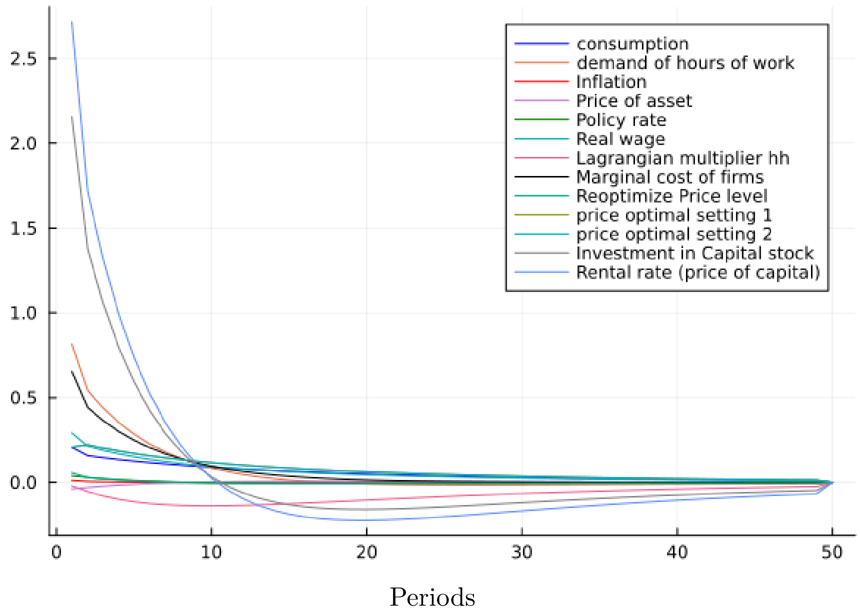


Figure 12: Cost Push shock - Predetermined variables