Synthetic EEG Signal Generator of Morphologies Associated with Epileptogenic Events

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***Abstract*—Abstract—Electroencephalography (EEG) is crucial for understanding brain signals. This work presents a methodology for synthetic EEG signal generation to simulate frequency bands, incorporate noise, and emulate specific phenomena. A Python-based tool allows controlled signal generation and export in various formats, and features a visualizer for analyzing user-uploaded signals in TXT, CSV, or EDF formats. EEGLAB evaluation confirms the accuracy and consistency of the generated signals. The tool supports research, diagnosis, and data analysis models requiring large datasets, particularly in neurological diseases like epilepsy. Pending web hosting, the tool will enhance accessibility and usability for broader applications.**

***Index Terms*—EEG signals, spike wave, sharp wave, slow wave, spike-slow wave, sharp-slow wave, rhythm bands, Noise types.**

I. Introduction

The Electroencephalogram (EEG) is widely used to study brain activity, offering spatiotemporal insights into brain function and dysfunction [1]. It significantly aids in diagnosing and treating conditions like epilepsy by identifying abnormal rhythms and wave patterns in specific brain regions, crucial for locating epileptic lesions and informing treatment decisions [2]. Advances in digitization have enhanced EEG interpretative capabilities, allowing detailed analyses of intracranial sources and their interactions [2]. However, electrode dimensions and scalp connectivity can introduce distortions [3, 4], making EEG a complex signal to interpret. EEG readings exhibit morphological diversity influenced by factors such as age and specific cerebral activities, often perceived as stochastic signals with discernible patterns in pathological states [5].

Because of the inter- and intra-individual variances are crucial for algorithm training and validation in machine learning. A robust EEG dataset repository is essential for reliable data processing [5].

This study aims to develop an application for the synthetic generation of clinically relevant EEG signals, for augmentation of the repository for scientific research and educational purposes. This article is organized to first contextualize the significance of EEG in clinical diagnosis (Section 2), then to detail nature of EEG waves and their simulation (Section 3). In Section 4, the EEG signal synthesis methodology is described, using a web interface for the generation of these synthesized signals. The evaluation of the tool's efficacy using EEGLAB is presented in Section 5. Finally, the conclusions and future work are discussed in Section 6.

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II. EEG Signals in the Diagnosis of Neurological Disease

EEG signals are crucial in diagnosing various central nervous system (CNS) diseases by correlating specific patterns with CNS functionalities and pathologies. Clinically, EEG signals diagnose cerebral afflictions, including convulsive and metabolic disorders, which arise from the extracellular potentials generated by neuronal and glial activities. These pathologies are due to the malfunctioning of neuronal processes. These pathologies are often due to the malfunctioning of neuronal processes.

Understanding the origin of field potentials is essential for accurate clinical interpretation [2]. EEG readings are characterized by various wave patterns, such as spike waves, sharp waves, spike-slow wave complexes, and sharp wave-slow wave graph elements, critical for determining normality or pathology in EEG traces [6].

Manual interpretation of EEG signals is challenging, requiring a deep understanding of typical EEG activities based on the patient's age and clinical status. Identifying artifacts, technical anomalies, and borderline patterns demands specialized expertise, highlighting the rigorous efforts of EEG specialists in ensuring accurate diagnoses or ruling out specific medical hypotheses [6]. The time invested by these specialists can be extensive and tedious, which underscores the need for automated methods to assist in the determination of these morphologies. Developing such applications necessitates a large amount of data labeled by experts. To date, the authors have not found a system capable of emulating a dataset with the clinically studied morphological characteristics identified by physicians.

This study aims to address this gap by presenting a methodology for the automatic generation of synthetic EEG signals that mimic these clinically relevant features

A. Background

Within the field of biomedicine, various tools and simulators have been developed for generating and simulating biosignals, each with specific goals and methodologies. By comparing these tools, we can identify their strengths and limitations, which provides a foundation for understanding the novel contributions of this project.

Figure 1 displays outputs from different projects, demonstrating the capabilities of existing biosignal generation tools. Although this project focuses on generating synthetic EEG signals, insights and methodologies from existing work on ECG (Electrocardiogram) signal generation are also valuable. For example, the University of Calabria's synthetic ECG signal model uses mathematical techniques, such as trigonometric functions and Gaussian monopulses, to create artificial ECG signals from real data [7].

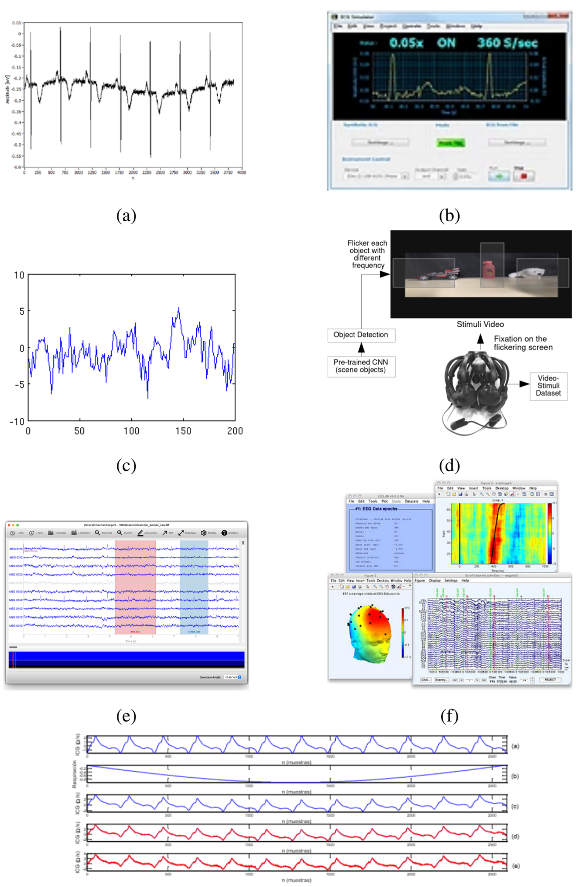


Fig. 1: a) Model for generating simple synthetic ECG signals [7]. b) ECG Simulator [8]. c) Simulated EEG data generator [9]. d) Simulating brain signals: creating synthetic EEG data via neural-based generative models for improved SSVEP classification [10]. e) MNE [11]. f) EEGLAB [12]. g) Generation of synthetic biosignals using time-varying Fourier series [13].

Similarly, within the context of constructing synthetic biomedical signal databases, researchers have synthetically generated impedance cardiography (ICG) signals by introducing variations over time using Fourier series, involving the analysis of real signals to build comprehensive datasets [13].

This methodology can be adapted for generating synthetic EEG signals using similar synthesis and mathematical modeling techniques. Additionally, the Data Science Automation (DSA) ECG simulator produces ECG signals compatible with NI-DAQmx hardware [8], emphasizing the potential of hardware-compatible signal generation tools.

The Simulated EEG data generator from the University of Oxford generates EEG data based on Event Related Potentials (ERP) theories but lacks features for specific morphologies [9]. Our methodology emphasizes the temporal integration of elements categorized in specific frequency bands that synergize to establish distinguishable patterns. This tactic is reminiscent of strategies employed in ECG [7] and the creation of synthetic biosignals reported by [13], grounding them in authentic signal paradigms.

The project leverages EEGLAB, an open-source, interactive environment for EEG data processing within MATLAB, and MNE, a comprehensive tool for analyzing and visualizing EEG data, to enhance signal analysis and validation, ensuring the generated signals resemble real EEG data [11, 12]. Additionally, MNE-style visuals and Plotly Dash (a Python library for creating interactive web applications) support user-specific visualizations. Diverging from certain research propositions in mathematical modeling, this project can be used in generative models powered by neural networks [10].

By integrating these advanced tools, the project aims to provide a robust platform for synthetic EEG signal generation, enhancing resources for scientific research and educational purposes.

III. EEG Signals: Main Characteristics and Synthetic Generation

EEG captures brain dynamics through electrodes placed on the scalp. Conventionally, these electrodes are organized according to the 10–20 system or its extension, the 10–10 system, as described by Oostenveld and Praamstra [14]. In specific circumstances, electrodes can be positioned on the cerebral cortex, termed ECoG, or intracranially implanted. This approach yields less filtered, localized, and higher quality data.

Nonetheless, being invasive entails greater associated risks. Both scalp EEG and intracranial signals are assessed for discernible patterns like epileptic spikes and ongoing background activity [15].

A. EEG Frequency Characteristics

Clinical neurophysiology categorizes EEG background activity into various rhythmic bands, from the Delta rhythm (0-4 Hz) to higher EEG frequency components [16]. These bands are:

Rhythmic Bands:

- **Delta Band (0.1-4 Hz, >75 µV) (δ):** Associated with children under three months and during Phase III sleep, characterized by regular waveforms.

- **Theta Band (4-7 Hz, 50-75 µV) (θ):** Predominantly fronto-central, associated with relaxation and meditative states.
- **Alpha Band (8-12 Hz, 25-50 µV) (α):** Associated with the occipital region, linked to tranquility and mental relaxation.
- **Beta Band (13-30 Hz, 10-25 µV) (β):** Found in frontal and central regions, associated with active cognition and focus.
- **Gamma Band (30-70 Hz) (γ):** Higher frequencies associated with perceptual processes and higher cognitive functions.

The 1/f Statistical Behavior of EEG and Its Types

EEG exhibits a 1/f statistical behavior, indicating complexity across temporal scales. Fourier analysis shows a power spectrum linear on a log-log chart, $A \approx 1/f^{a}$, reflecting the brain's internal noise. The cerebral cortex's complex architecture influences neuron behavior through noise [18], so in EEG three primary noise type are presented:

- **Pink Noise:** Adheres to a 1/f relation, between white and brown noise in predictability.
- **White Noise:** No correlation between frequency bands, flat spectrum $(1/f^0)$.
- **Brown Noise (Brownian Noise):** Power density diminishes with frequency $(1/f^2)$, showing strong correlations over shorter spans.

A.1 Implementation of Background Signals

Implementing background signals is crucial for simulating realistic EEG datasets. This section outlines generating various EEG frequency bands and integrating different noise types to mimic authentic EEG signals.

Table 1 provides the simulation parameters for EEG frequency bands and noise integration, Each band is simulated following [16], [19]. Highlighting the characteristics and generation methods for each frequency band and noise type.

TABLE I: Simulation Parameters for EEG Frequency Bands and Noise Integration

Frequency Band	Frequency Range (Hz)	Description
Delta Band	0-4	Profound sleep in adults
Theta Band	4-7	Shallow sleep, contemplative states
Alpha Band	8-12	Tranquility
Beta Band	13-30	Vigilant states
Gamma Band	30-70	Cognition and alertness

Noise Type	Generation Method	Description
White Noise	np.random.randn()	Constant spectral density
Pink Noise	Aggregated white noise	Frequency-dependent power spectrum
Brown Noise	Integrated white noise	Stochastic fluctuations over time

The pseudocode for conceptual understanding in Fig. 2 defines the generation of EEG frequency bands, noise integration based on user specifications, and the final preparation of the EEG signal. The Fig. 3, illustrates the synthetic generation of EEG frequency bands with the addition of noise. This visual representation helps to understand how different noise types are integrated into the EEG signal, enhancing its realism and utility for various applications.

```
Input: User definition in web App
Output: eeg_signal (synthesized EEG signal)

1: Define EEG frequency bands
  delta_band <-- [0, 4] Hz      ;   theta_band <-- [4, 8] Hz      ;   alpha_band <-- [8, 12] Hz
  beta_band  <-- [12, 30] Hz    ;   gamma_band <-- [30, 70] Hz

2: Set parameters
  duration    <-- 10 seconds    ;   sampling_freq <-- 500 Hz    ;
  num_samples <-- duration * sampling_freq
  time        <-- [0, duration) incremented by 1/sampling_freq

3: Generate band signals
  Define function generate_band(freq_range, amplitude, duration, sampling_freq):
     frequency <-- random_value(freq_range)
     phase     <-- random_value(0, 2*pi)
     return amplitude * sin(2*pi * frequency * time + phase)

  band_signals <-- {
    'delta': generate_band(delta_band, 5, duration, sampling_freq),
    'theta': generate_band(theta_band, 10, duration, sampling_freq),
    'alpha': generate_band(alpha_band, 20, duration, sampling_freq),
    'beta' : generate_band(beta_band, 30, duration, sampling_freq),
    'gamma': generate_band(gamma_band, 50, duration, sampling_freq)
  }

4: Generate noise
  pink_noise <-- scale_noise(generate_noise(num_samples), 0.1, 'pink')
  ... Scale  white and brown noise

5: Integrate noise into signals
  for each band_name, signal in band_signals:
     signal_with_noise <-- signal
     if user specifies pink_noise_amplitude:
        signal_with_noise += pink_noise * user_specified_value
    ... Condition repeats to white and brown noise
The resulting signal is ready to be used for any purpose: display, background implementation, etc.
```

Fig. 2: This algorithm defines the generation of various EEG frequency bands, the integration of different types of noise based on user specifications, and the final preparation of the EEG signal.

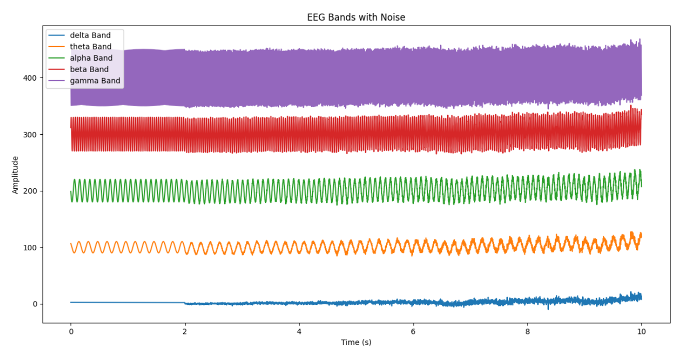


Fig. 3: Synthetic generation of frequency bands, gradually increasing noise from half of the signal. The noise intensity escalates from 0% to a maximum level determined by scaling factors for each noise type

B. Waves

In EEG recordings, the potential difference between two electrodes is termed a wave, representing shifts in cerebral electrical activity [16]. Proper EEG interpretation requires distinguishing between normal and abnormal characteristics, such as:

- **Spikes.** Transient waves lasting between 20 to 70 ms, characterized by high amplitude in contrast to the background and sharp morphology, indicative of synchronous neuronal discharge [2].
- **Sharp Waves.** Transient waves lasting 70 to 200 ms with sharp peaks. The Sharp Wave-Slow Wave complex, combining a sharp wave followed by a slow wave, is often seen in typical absences (petit mal) and other clinical conditions [2], [17].

- **Slow Waves.** Low-frequency, low-amplitude discharges lasting 200 to 500 ms, reflecting slow depolarization processes and cortical spread. Observed in deep sleep and certain seizure disorders [2].
- **Spike/Sharp-Slow Wave pattern.** Sequence of a spike wave followed by a slow one in the EEG, is often seen in typical absence seizures but can also be evident in other clinical and epileptologic situations. Sharp/Spike waves indicate a synchronous, brief neuronal discharge, whereas slow waves might suggest the spread of electrical activity through expansive cortical circuits [2].

Transient Events and Wave Complexes:

A complex is defined as ”a series of two or more waves with a characteristic or consistently repeated shape, distinct from the background activity” [17]. Complexes can be diphasic, triphasic, or polyphasic waves, typically with at least three distinctive waveform components. These complexes maintain a consistent overall waveform across occurrences, which makes them identifiable as a specific type of EEG activity. Unlike transient events, complexes do not necessarily disrupt the background activity but are distinguished by their recurring and consistent form within the EEG recording.

Transient events, on the other hand, are ”individual waves or complexes distinguishable from background activity,” marked by their clear disruption of the background activity and their duration [17]. Transient events are characterized by a distinct beginning and end, occurring as isolated phenomena within the EEG signal, thereby interrupting the ongoing background activity.

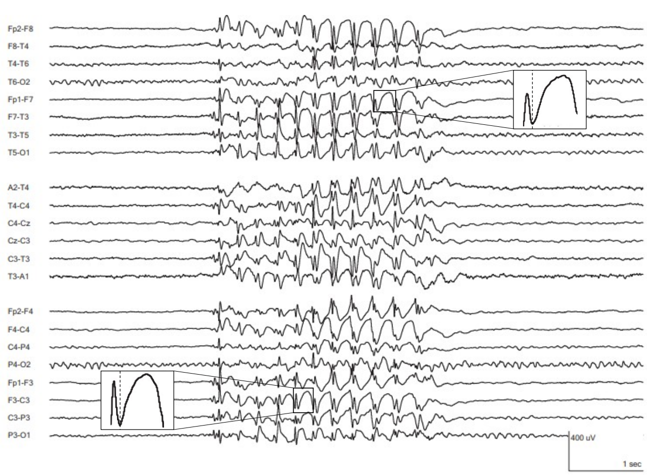


Fig. 4: Generalized interictal epileptiform discharges recorded from a 16-year-old who is seizure-free but has a history of absence seizures (LFF 1 Hz, HFF 70 Hz). Modified from [17].

Fig. 4 illustrates polyspikes preceding the initial spike and the ensuing slow wave complex, with subsequent complexes also showcasing initiating polyspikes in individual channels. The complexes have pronounced amplitudes (200 µV) with standard alpha rhythm and background amplitudes, the information and the original image were taken from the Atlas of EEG Patterns [17].

A.1 EEG Synthetic Signal Generator

The signal generator module is essential for synthesizing EEG datasets, enabling the generation of signals with unique characteristics.

The software design emphasizes three core elements: modularity, scalability, and consistency.Aligns with the Model-View-Controller (MVC) framework, where the UI definition serves as the View, the signal generation functions as the Model, and the callback management operates as the Controller. Ensuring clear separation of concerns and enhancing maintainability and scalability. The Python script, visualized in Figure 5, is designed with a clear separation of concerns for independent development, testing, and maintenance.

```
Input: User parameters
Output: EEG waveforms

1: Generate Sharp Wave
function generate_sharp_wave(amplitude, duration, sampling_freq, target_length=None):
amplitude = random_value(range)        ;        n_samples = duration * sampling_freq
sharp_wave = linear_rise_and_decay(amplitude, n_samples)
return pad_or_trim(sharp_wave, target_length)

2: Generate Spike
function generate_spike(amplitude, duration, sampling_freq, target_length=None):
amplitude = random_value(range)        ;        n_samples = duration * sampling_freq
spike = gaussian_curve(amplitude, n_samples)
return pad_or_trim(spike, target_length)

3: Generate Slow Wave
function generate_slow_wave(amplitude, duration, sampling_freq, frequency=1.0, cutoff_range=(-5,
2), target_length=None):
amplitude = random_value(range)        ;        period = 1 / frequency
full_wave = amplitude * sin_wave(frequency, n_samples)
cutoff_index = find_cutoff(full_wave, cutoff_range)
return pad_or_trim(full_wave[
], target_length)

4, 5: To join the peak/sharp waves with the slow waves, in 2 different functions according to their
characteristics

5: Generate Repeated Events
function generate_repeated_events(n_events, times, sampling_freq, baseline, generate_wave,
amplitude_range, duration_range, mode='transient', noise_params=None):
channel_data = copy(baseline)
event_length = mean(duration_range) * sampling_freq
  if mode == 'transient':
    start_index = random_index_within_bounds(channel_data, event_length)
    for _ in range(n_events):
      event = generate_wave(amplitude_range, duration_range, sampling_freq)
      add_event_to_channel(channel_data, event, start_index)
      start_index = update_start_index(start_index, event_length, random_separation)

  if mode == 'complex':
    total_event_length = n_events * event_length
    start_index = (len(channel_data) - total_event_length) // 2
    for _ in range(n_events):
      event = generate_wave(amplitude_range, duration_range, sampling_freq)
      add_event_to_channel(channel_data, event, start_index)
      start_index += event_length

  return channel_data
The resulting waves are ready for integration into EEG signals or plotting.
```

Fig. 5: Algorithm for generating various EEG waveforms based on user-defined parameters (extract of the module signal_generator)

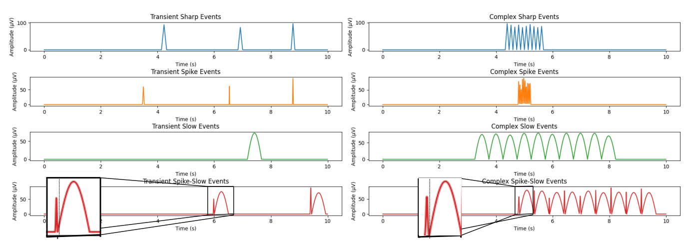


Fig. 6: Visualization of the generated EEG Waves and Events. Transient events (left) and complex events (right) showcase various EEG waveforms with amplitude (µV) over time (s). NOTE: The generated waves do not contain background signals for EEG generation.

The EEG Synthetic Signal Generator module effectively simulates various EEG waveforms through its modular design, enabling the generation of transient and complex events.

Fig. 6, presents simulated EEG signals, illustrating both transient and complex events. Transient events, characterized by isolated occurrences, include sharp, spike, slow, and spike-slow waves. Complex events, characterized by repetitive patterns, include complex sharp, spike, slow, and spike-slow waves.

IV. Methodology

Electroencephalography (EEG) has significantly advanced the understanding of cerebral signals, leading to the development of tools for simulating and analyzing EEG data.

These simulations are essential when acquiring authentic signals is challenging. EEG waves, resulting from cerebral electrical activity, are represented as voltage discrepancies between electrodes, offering insights into cerebral dynamics. Balancing precision and realism in simulations is crucial, and collaboration with EEG specialists ensures accuracy and iterative feedback.

A. *Synthesis of EEG Signals*

This section details the methodology for generating synthetic EEG signals, emphasizing their relevance, techniques, and applications, and evaluating their accuracy compared to genuine counterparts.

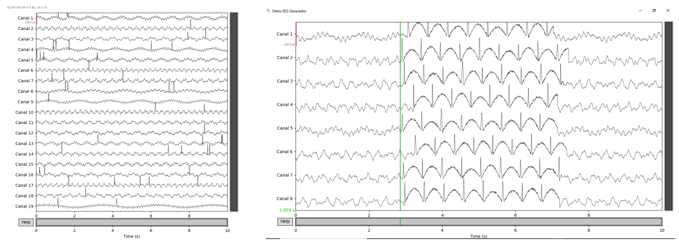
Recognizing and emulating distinctive EEG patterns, especially those linked to clinical conditions, is essential. Figure 6 illustrates the pseudocode for generating various EEG waveforms based on user-defined parameters, highlighting its utility in training and research.

B. *Application Overview*

This project focuses on the synthetic generation of EEG signals with distinct morphologies such as spike-waves, slow-waves, and spike-slow wave combinations. The application offers two primary operational modes and one visualisation (Evaluation section):

- **Default Generation:** User selectes from available waveform types, considering number of wave per channel,number of channels and the 10 s-pages number (see Fig.7). Export options include ".txt", ".edf", and ".png". An integrated visualization provides insights before exportation. Examples of generated images are shown in Fig. 8.

Fig. 7: Default generation interface, showing the generation of 5 transient channels of sharp waves and the download buttons of the 2 user-specified sheets (complex and transient signal).



(a) (b)

Fig. 8: Sample visual outputs in ".png" format, illustrating a) spike-wave transient formations and b) spike-slow complexes waves configurations in an optimized MNE format.

- **Detailed Generation:** This mode offers intricate control over all available parameters: such as waveform amplitude, duration, sampling rate, and noise infusion. Users can define frequency bands (delta, theta, alpha, beta, gamma), noise conditions, and EEG amplitude. An interactive dashboard provides visual feedback, displaying aggregated and channel-specific views, with outputs formatted in MNE for user convenience (see Fig. 9).

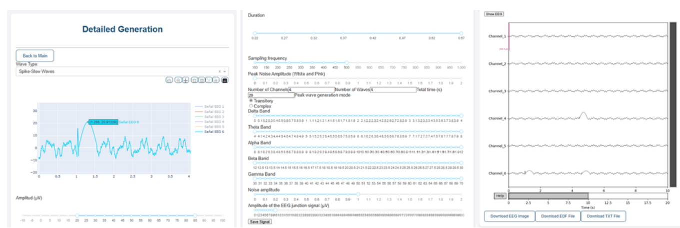


Fig. 9: Signal customization interface, trimming a segment from 20s to 4s for better visualization of the transient event with peak-slow wave morphologies.

V. Evaluation

EEGLAB, a prominent tool in neuroscience, offers extensive functionalities for EEG processing [12]. To verify the compatibility and usability of the synthetically generated signals, EEGLAB was used for rigorous testing.

In Fig. 10 the initial three illustrations (a-c) are derived from EEGLAB, show casing the integration and analysis capabilities of this software with the signals generated by the "EEG signal generation tool for epiloptogenic morphologies". These sections demonstrate how the ".edf" file, synthesized by our application, is incorporated into EEGLAB, followed by the display of temporal channel anal yses and the validation of signal authenticity via coherence analysis.

The illustrations (d-f) represent aspects of the "EEG Synthetic Signal Generator" application. They include the representation of spike waveforms within a specific time frame, analyzed in EEGLAB to note visual similarities. Additionally, the logging/register proposal for the production page and the initial menu leading to the "Viewing EEG files" page are shown. On this page, the application supports text files ".txt", EDF format, and excel ".csv" files, reading columns as channels and rows as signal over time. Users can capture specific segments, save images, scroll temporally, and focus the signal for better visual annotation.

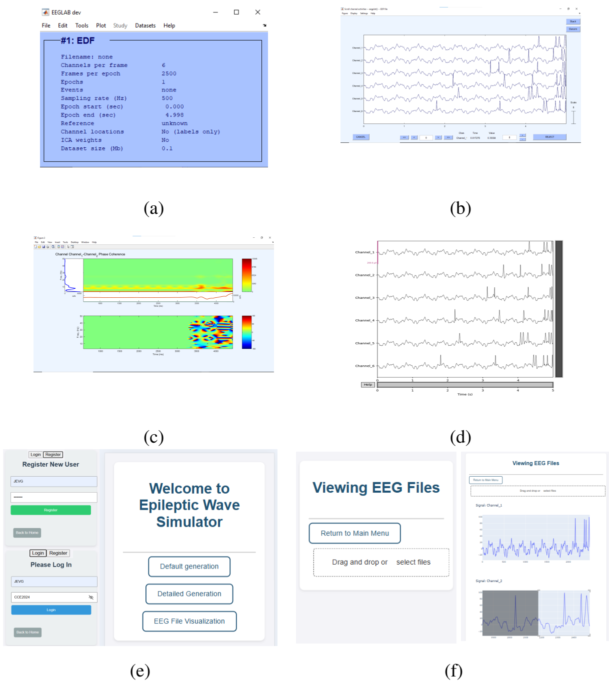


Fig. 10: a) Integration of the ".edf" file synthesized in EEGLAB. b) Temporal channel display in EEGLAB. c) Coherence analysis for signal authenticity validation in EEGLAB. d) Spike waveforms within a 6-second window. e) Initial interface of the application after logging in f) Display page, and selection of a signal fragment from channel 2 to maximise its size and better analyse it.

Signal Generation Precision

Comparison of generated signals with those in EEGLAB shows notable uniformity, validating their precision and compatibility. Integration with external tools like EEGLAB confirms the simulator's consistency and interoperability with industry standards. This facilitates the creation of datasets that drive the development of algorithms in EEG signal analysis.

Figure 11 illustrates the tool's ability to reproduce realistic EEG signal patterns, comparing a synthetically generated signal with a real one, with small random differences.

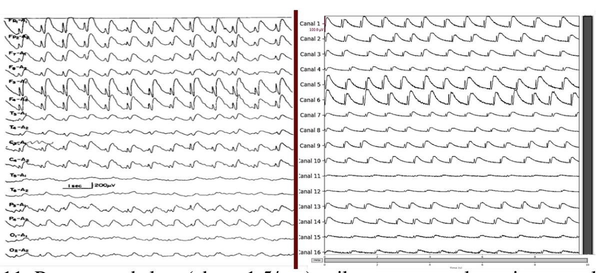


Fig. 11: Pronounced slow (about 1.5/sec) spike wave complexes in generalized synchrony, Extracts of [2] (Left), and synthetic EEG signal generated by the algorithm by modifying its parameters to emulate the real signal. (Right))

VI. Conclusions

Electroencephalography (EEG) remains crucial for understanding cerebral dynamics. Advances in analytical methodologies and integrated tools have enhanced EEG's relevance. This project addresses the gap in synthetic EEG signal generation, creating waveforms that mirror genuine EEG morphologies.

Using mathematical models and Python programming, the tool simulates various EEG attributes, including diverse frequency bands and specific events like spike waveforms.

EEG simulation requires noise infusion to ensure realistic data representation. The modular code architecture enhances adaptability and scalability, facilitating future advancements in biosignal research. Compatibility with tools like EEGLAB underscores its potential for multi-platform integration and collaboration. The generation of synthetic EEG signals significantly benefits machine learning, providing large, controlled datasets crucial for training robust models, particularly where real EEG data is scarce or difficult to obtain.

This project augments existing platforms while introducing unique features to revolutionize EEG signal synthesis and simulation. Future directions include presenting the project to the medical community for feedback to improve signal precision and utility. Potential extensions could offer default configurations tailored to medical needs, making this application a valuable tool for EEG researchers and advancing EEG signal synthesis and simulations.

References

[1] C.M. Michel and M.M. Murray, "Towards the utilization of EEG as a brain imaging tool," NeuroImage, vol. 61, pp. 371–385, 2012.

[2] E. Niedermeyer and F.L. da Silva, Electroencephalography: Basic Principles, Clinical Applications, and Related Fields, 5th ed. Baltimore, MD, USA: Lippincott Williams & Wilkins, 2005.

[3] P.L. Nunez and V. Ramachandran, "Electroencephalography (EEG)," in Reference Module in Neuroscience and Biobehavioral Psychology, 2002.

[4] C. Brunner, M. Billinger, M. Seeber, T.R. Mullen, and S. Makeig, "Volume conduction influences scalp-based connectivity estimates," Frontiers in Computational Neuroscience, vol. 10, p. 121, 2016.

[5] N. Bigdely-Shamlo, T. Mullen, C. Kothe, K.M. Su, and K.A. Robbins, "The PREP pipeline: standardized preprocessing for large-scale EEG analysis," Frontiers in Neuroinformatics, vol. 9, p. 16, 2015.

[6] M.I. Martínez Barros and G. Trout Guardiola, "Conceptos básicos de electroencefalografía," Duazary, vol. 3, pp. 18–23, 2006.

[7] D. Pavol, A. Imrich, M. Linus, and D. Grimaldi, "Model for generating simple synthetic ECG signals," Acta Electrotechnica et Informatica, vol. 18, p. 5, 2018.

[8] National Instruments, "ECG Simulator," Data Science Automation (DSA), 2012.

[9] N. Yeung, R. Bogacz, C. Holroyd, S. Nieuwenhuis, and J. Cohen, "MRC Brain Network Dynamics Unit," 2004.

[10] K. Nik Aznan, A. Atapour-Abarghouei, S. Bonner, J.D. Connolly, N. Noura, and T.P. Breckon, "Simulating brain signals: Creating synthetic EEG data via neural-based generative models for improved SSVEP classification," 2019.

[11] MNE Developers, "Open-source Python package for exploring, visualizing, and analyzing human neurophysiological data: MEG, EEG, sEEG, ECoG, NIRS, and more," 2012.

[12] Swartz Center for Computational Neuroscience, "What is EEGLAB?," 2004.

[13] J. Escrivá Muñoz, A. Galea Cazorla, C. González Pijuan, and M. Vallverdú, "Generación de bioseñales sintéticas mediante series de Fourier variantes en el tiempo," 2016.

[14] R. Oostenveld and P. Praamstra, "The five percent electrode system for high resolution EEG and ERP measurements," 2001.

[15] W. van Drongelen, Signal Processing for Neuroscientists: An Introduction to the Analysis of Physiological Signals. Elsevier Inc, 2007.

[16] C.A. Ramírez Fuentes, "Estudio de la señal EEG para la identificación de patrones precursores de crisis epilépticas," Master's thesis, UPIITA, Ciudad de México, 2016.

[17] M.J. Stern, Atlas of EEG Patterns, 2nd ed. Philadelphia: Lippincott Williams & Wilkins, a Wolters Kluwer Business, 2001.

[18] G. Buzsáki, Rhythms of the Brain. New York: Oxford University Press, 2006.