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Design of Analog Gaussian Filter Used in Roughness Measuring Instrument

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***Abstract*—According to the rational function approximation method of Gaussian filter, the transfer function of an 8-order approximation filter is obtained. Based on value of the quality factor Q of the pole of the transfer function, analog cascade circuits are determined. The optimal cascading sequence and the principle of components are given. When selecting the cut-off frequency of 400 Hz, the experiment of the circuit in frequency domain and time domain is done. The result shows that the circuit achieves the desired target.**

Keywords-Gaussian Filter; Function Approximation; Analog Filter; Quality Factor

I. INTRODUCTION

Usually, in roughness measuring instrument based on inductance sensor, analog low-pass filters are used to separate roughness signal from carrier signal and high-frequency noise of filter to limit the width of signals before sampling. At the same time, ideally, to extract the roughness signal, filter need to own linear phase in pass band in order to avoid the signal distortion resulted from non-linear phase[1-2].

Gaussian filter is a special, ideal, optimal filter [3] with the minimum product of time duration and frequency bandwidth, which is much fit for emphasizing reducing signal distortion when the signals pass filter and ensuring attenuating effectively when signal frequency is larger than the cutoff frequency of filter . Therefore, Gaussian filter is an ideal filter in roughness measurement application [4]. International standard ISO11562 has already stipulated definitely that Gaussian filter mean line as an assessment mean line in measuring surface roughness is employed to replace the traditional 2RC filter median.

Based on rational function approximation of Gaussian filter, an 8-order analog Gaussian filter is designed in this paper. According to the size of quality factor of pole of transfer function, the circuit is decided. The formulas of each component value are given too.

II. GAUSSIAN FILTER AND RATIONAL FUNCTION APPROXIMATION

Without loss of generality, we define the amplitude-frequency response of Gaussian filter as:

$$\mathrm{G}(j\Omega) = e^{-\alpha^2\Omega^2} \tag{1}$$

Here, α is a const related to the bandwidth of filter.

Formula (1) is a theoretical mathematical model. When squaring the amplitude of analog filter, we can obtain [4-6]:

$$\left|\mathrm{G(j\Omega)}\right|^2 = G(j\Omega)G(-j\Omega) = e^{-2\alpha^2\Omega^2} = \frac{1}{e^{2\alpha^2\Omega^2}} \tag{2}$$

When expanding the denominator of (2) with 16-order, Taylor series, (3) can be obtained [7].

$$\left|G(j\Omega)\right|^2 \approx \frac{1}{1+\sum_{i=1}^{8}\frac{2^i}{i!}(\alpha\Omega)^{2i}} \tag{3}$$

Here, we make $s = j\Omega$, $\Omega^2 = -s^2$, then (3) can be turned to (4).

$$\left|G(s)\right|^2 \approx \frac{1}{1+\sum_{i=1}^{8}(-1)^i\frac{2^i}{i!}(\alpha s)^{2i}} \approx G_p(s)G_p(-s) \tag{4}$$

Here, $G_p(s)$ is a system function of Gaussian approximation filter.

Calculate the poles of (4) and only poles that are in left-half plane of the complex plane are used to constitute the system function $G_p(s)$. The poles are shown in TABLE Ⅰ. The expression of $G_p(s)$ is as following:

$$G_p(s) = \frac{k_{p1}}{(s-s_1)(s-s_3)} * \frac{k_{p2}}{(s-s_5)(s-s_7)} * \frac{k_{p3}}{(s-s_9)(s-s_{11})} * \frac{k_{p4}}{(s-s_{13})(s-s_{15})} \tag{5}$$

Here, k_{pi} (i=1,2,3,4), a const, is the gain of system. We substitute the poles to (5) and then according to the gain condition of $G_p(s)\big|_{s=0} = G_p(j\Omega)\big|_{\Omega=0}$, we can get the gain const k_{pi}. At last, $G_p(s)$ can be expressed as:

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$$G_p(s)=\frac{\frac{2.57034429}{\alpha^2}}{s^2+\frac{1.7610}{\alpha}s+\frac{2.57034429}{\alpha^2}}*\frac{\frac{1.92672538}{\alpha^2}}{s^2+\frac{2.1546}{\alpha}s+\frac{1.92672538}{\alpha^2}}*\frac{\frac{1.64928817}{\alpha^2}}{s^2+\frac{2.3632}{\alpha}s+\frac{1.64928817}{\alpha^2}}*\frac{\frac{1.53655112}{\alpha^2}}{s^2+\frac{2.4572}{\alpha}s+\frac{1.53655112}{\alpha^2}} \quad (6)$$

Formula (6) is the transfer function of 8-order Gaussian approximation filter.

TABLE Ⅰ. THE POLES OF THE 8-ORDER GAUSSIAN APPROXIMATION FILTER AND VALUE OF Q

Poles	Value of poles	Value of Q
$s_{1,3}$	$(-0.8805\pm1.3398j)/\alpha$	0.910
$s_{5,7}$	$(-1.0773\pm0.8753j)/\alpha$	0.644
$s_{9,11}$	$(-1.1816\pm0.5031j)/\alpha$	0.543
$s_{13,15}$	$(-1.2286\pm0.1646j)/\alpha$	0.504

III. REALIZATION OF ANALOG FILTER

The realization of analog filter is a process of turning the system transfer function to electronic circuit. The interested thing is that simple, financial, low noise and distortion, high dynamic range realization of filter [8]. And the filter also requests that the tiny change of component values will not influence the filter seriously. In order to reduce the sensitivity of system function to the deviation of component value, and simplify the design process of filter, cascade realization of one-order and two-order filter is usually employed. Cascade method consists of implementing each double two-order system function with circuit and connecting them by cascading way. So each low-pass filter is independent and any change of one of filters will not influence any other cascading filters.

A. Implementation of circuit

For implementing the double two-order system function, suitable circuit is selected according to the quality factor Q of pole. The calculating formula of Q is as following (7):

$$Q=-\frac{|s|}{2\,\mathrm{Re}(s)} \quad (7)$$

Here, |s| is the amplitude of pole, and Re(s) is the real part of pole. So the quality factor of pole of the 8-order approximation filter can be obtained in TABLE Ⅰ. Since Q≤2, Sallen-Key filter with unity gain is selected to carry out the above filter [9]. The circuit is shown as Fig.1.

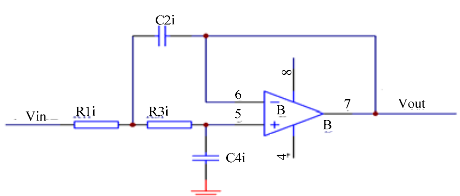

Fig. 1 a two-order Sallen-Key filter with unity gain

From the formula (2), the relation of α and cut-off frequency of logarithmic amplitude-frequency characteristics at the-3db is as (8):

$$\alpha=\frac{\sqrt{\ln 2}}{\sqrt{2}\Omega_c} \quad (8)$$

By deciding the value of Ωc, the value of α can be gotten and then the transfer function of low-pass filter for specified cut-off frequency can be obtained from (6) too. According to the reference [8] which introduced a formula to calculate component values of filter parameters, each parameter component values of the circuit in Fig. 1 can be computed.

B. Determination of cascading order

From the mathematical point of view, the order of the various parts of the cascade doesn't really matter. However, in practice, there may exist signal clamp in high Q node, so to avoid the loss of dynamic range and the reduction of filter accuracy, each section can be cascaded according to the order of increasing Q value, that is, lower Q value part is put on the first-class of the signal pathway. But the thing that the higher Q value part may influence concerned internal noise is not considered in this kind of cascading order. Any noise falling over resonance peak in higher Q value module can be significantly enlarged. So, higher Q value part should be placed in the forefront of cascade sequence to reduce the noise. Generally, the optimal cascading sequence is obtained based on input signal spectrum, filter type and the noise characteristics of the various parts [10].

In the roughness instrument applications, the signal into the filter is sine wave amplitude modulated signal whose amplitude is about tens of mill volts. The dynamic range of filter will not be a problem. Therefore, the cascading sequence which is advantageous to reduce the noise becomes the main factor to consider. The Fig. 2 shows a four two-order filter with high-Q two-order section in front and low-Q two-order section in the post.

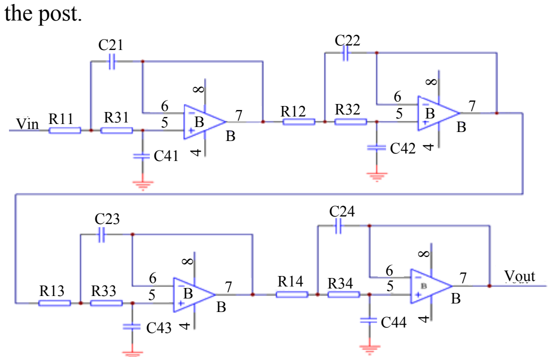

Fig. 2 an 8-order Gaussian approximation filter

C. Selection of components

The suitable selection of capacitance and resistance which are basic components of active filter is a critical factor of success of actual design. In all types of fixed resistors, thick and thin film chip resistors are very suitable for application in the filter, so a kind of thick-film resistor which has a positive temperature coefficient is chosen in this paper. And NPO-type ceramic capacitor which has a negative temperature coefficient is selected to compensate the positive temperature coefficient of resistor, which can reduce the sensitivity of active filters to resistance and capacitance change [11-12].

In active filters, the role of the operational amplifier is to provide amplification and isolation. Its closed-loop bandwidth has to be higher than the cutoff frequency of the filter at least 100 times and slew rate should be larger than the product of cut-off angular frequency and output voltage peak – peak, which can guarantee that there is no signal distortion in filter. Input bias current should be in the pa level in order to decrease the input offset voltage, which is also advantageous to selecting the component parameters of resistors and capacitors [13-14].

D. *Experiments & Result*

With cut-off frequency of 400 Hz in the 8-order filter as an example simulation experiment, amplitude-frequency response is obtained as Fig. 3 shown. The red and blue curves are the amplitude-frequency characteristics of Gaussian filter and its 8-order rational approximation filters respectively. And the amplitude-frequency characteristic deviation of rational approximation filters from the Gaussian filter is also shown as Fig.4. We can see that the maximum deviation is about 0.58%, which to meets the design requirements well.

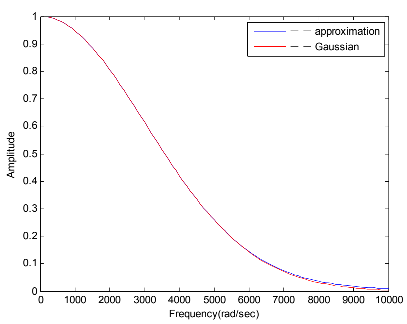

Fig. 3 Amplitude-frequency characteristics of Gaussian filter and its rational approximation filters

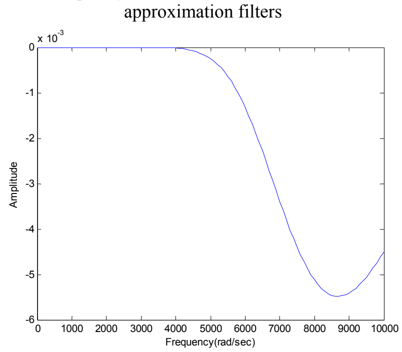

Fig.4 Amplitude-frequency characteristic deviations of rational approximation filters

IV. CONCLUSION

Basing on the rational function approximation method of Gaussian filter, an 8-order analog Gaussian filter is designed in this paper. According to the size of quality factor of transfer function poles, the implementation circuit is determined. The formula of each component value and the cascading sequence are given in accordance with the principle of maximum SNR (signal to noise ratio). The experiments to the circuit by means of frequency-domain analysis and time analysis show that the circuit can achieve the desired design target.

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