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A new S^2 control chart using repetitive sampling

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A new S^2 control chart is presented for monitoring the process variance by utilizing a repetitive sampling scheme. The double control limits called inner and outer control limits are proposed, whose coefficients are determined by considering the average run length (ARL) and the average sample number when the process is in control. The proposed control chart is compared with the existing Shewhart S^2 control chart in terms of the ARLs. The result shows that the proposed control chart is more efficient than the existing control chart in detecting the process shift.

Keywords: repetitive sampling; control chart; average run length; process shift

1. Introduction

The control charts are the statistical tools widely used in industries to monitor the manufacturing process. Although the latest machinery is available to a manufacturer, there is a chance of variation in the process due to several factors. When the process is out of control, industrial engineers are required to remove the cause of the variation in time. Therefore, researchers are trying to explore new control charts so that the out-of-control signal can be given as soon as the process is going out of control. Shewhart originally developed the control chart schemes to monitor the process, but the Shewhart control charts are known to be useful to detect large shifts quickly.

Traditionally, the Shewhart-type control charts such as R, S and S^2 control charts are popularly used for monitoring the process variance. Research on improving the performance of these control charts is still ongoing. Khoo [12] proposed an S^2 control chart based on the double sampling approach. Zhang *et al.* [26] presented the statistical design of the S^2 chart. Lee [15] designed the adaptive R charts by considering the variable parameters of the chart. Lee *et al.* [16] presented the designing method of S control charts by combining double sampling and variable sampling

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interval (VSI) schemes. Guo and Wang [9] presented the designing of the S^2 chart providing an unbiased estimator of the average run length (ARL) when in-control variance is estimated.

Lots of other types of control charts for controlling the variability have been developed. Chen [7] discussed the ARL distributions of different variability charts. David [8] discussed early sample measures of variability. Khoo [11] proposed the modified S chart for process variance. Huang and Chen [10] designed a synthetic control chart for monitoring the process standard deviation. Riaz [20] proposed a dispersion control chart called 'Q-chart'. Riaz and Saghir [21] designed a mean-deviation-based control chart. Mahmoud *et al.* [17] discussed the estimation of standard deviation in control chart applications. Rakitzis and Antzoulakos [19] worked on the improvement of the one-sided S control chart. Schoonhoven *et al.* [22] designed a control chart for a process standard deviation when the estimation of parameters is needed. Kuo and Lee [13] developed the design of an adaptive standard deviation control chart. Zafar *et al.* [24] proposed the progressive variance control chart for monitoring process dispersion. Zhang [25] designed the improved R and S control chart for monitoring process variance. Ahmad *et al.* [1,2] developed an efficient median chart.

The main purpose of this paper is to introduce the repetitive sampling approach to developing a new control chart. Repetitive sampling was originally proposed by Sherman [23] for developing an attributes acceptance sampling plan. The operation of the repetitive sampling is similar to the sequential sampling scheme. Later on, according to Balamurali and Jun [6], repetitive sampling is applied to develop more efficient variables acceptance sampling plans. The idea of repetitive sampling is different from the double sampling approach or the VSI approach. In a double sampling or a repetitive sampling scheme, a second sample is taken if the decision cannot be made on the first sample. Usually, the double sampling scheme is based on four parameters and so it is difficult to apply from an administrative point of view. On the other hand, the repetitive sampling scheme is simpler to apply because it is based on two parameters. The repetitive sampling scheme is also different from the VSI scheme in that the latter adjusts the sampling interval instead of the control limits. Therefore, the control charts based on the repetitive sampling scheme will be different from the existing control charts in the literature. Recently, Aslam *et al.* [3,4] proposed a t-control chart using repetitive sampling and Ahmad *et al.* [1,2] designed the X-bar control chart based on the process capability index using repetitive sampling and proved its efficiency. Azam *et al.* [5] designed a hybrid EWMA chart using repetitive sampling for normal distribution. Lee *et al.* [14] proposed a control chart using an auxiliary variable and repetitive sampling to detect the process mean. Aslam *et al.* [3,4] designed some attribute and variable control charts using repetitive sampling for monitoring the process mean.

By exploring the literature, we note that S or S^2 control charts are available based on the single and double sampling approaches. No attempt has been made to propose S or S^2 control charts using the repetitive sampling scheme. In this paper, we expect that the S^2 control chart using repetitive sampling will be more efficient than the S^2 control chart based on the single sampling scheme. The rest of the paper is organized as follows: the designing of the proposed control chart is given in Section 2. The performance comparison with the Shewhart S^2 chart is given in Section 3 and a simulation study using synthetic data is given in Section 4. Some concluding remarks are given in the last section.

2. Designing of new S^2 control chart

We proposed the following S^2 control chart using repetitive sampling.

Step 1 Take a sample of size n. Calculate S^2 from the sample data.

Step 2 Declare the process as out-of-control if $S^2 \geq \mathrm{UCL}_1$ or $S^2 \leq \mathrm{LCL}_1$ (UCL_1 and LCL_1 are called the outer control limits). Declare the process as in-control if $\mathrm{LCL}_2 \leq S^2 \leq \mathrm{UCL}_2$

(UCL_2 and LCL_2 are called the inner control limits). Otherwise, go to Step 1 and repeat the process.

The rationale of the proposed control chart is that the decision can be made from the first sample when the evidence is sufficient but it may be better to postpone when the evidence is not enough.

These four control charts' limits for the proposed control chart are given as follows when σ^2 is known. The outer control limits are given by

$$\mathrm{UCL}_1 = \sigma^2 + k_1\sqrt{\frac{2(\sigma^2)^2}{(n-1)}}, \quad \mathrm{LCL}_1 = \sigma^2 - k_1\sqrt{\frac{2(\sigma^2)^2}{(n-1)}},$$

where k_1 is a constant to be determined. The inner control limits are

$$\mathrm{UCL}_2 = \sigma^2 + k_2\sqrt{\frac{2(\sigma^2)^2}{(n-1)}}, \quad \mathrm{LCL}_2 = \sigma^2 - k_2\sqrt{\frac{2(\sigma^2)^2}{(n-1)}},$$

where k_2 is a constant to be determined.

The probability that the process is declared to be out of control based on a single sample is

$$P_{\mathrm{out}}^{(1)} = P(S^2 \geq \mathrm{UCL}_1) + P(S^2 \leq \mathrm{LCL}_1). \tag{1}$$

Note that $(n-1)S^2/\sigma^2$ follows the chi-square distribution with degree of freedom $n-1$ if the process is in control. Let G be the distribution function (df) of such a distribution. Then, when the process is in control,

$$P(S^2 \geq \mathrm{UCL}_1) = 1 - G\left(\frac{(n-1)\mathrm{UCL}_1}{\sigma^2}\right) = 1 - G\left((n-1)\left(1 + k_1\sqrt{\frac{2}{n-1}}\right)\right).$$

Similarly,

$$P(S^2 \leq \mathrm{LCL}_1) = G\left((n-1)\left(1 - k_1\sqrt{\frac{2}{n-1}}\right)\right).$$

So, finally Equation (1) can be rewritten as follows when the process is in control,

$$P_{\mathrm{out}}^{(1)} = 1 + G\left((n-1)\left(1 - k_1\sqrt{\frac{2}{n-1}}\right)\right) - G\left((n-1)\left(1 + k_1\sqrt{\frac{2}{n-1}}\right)\right). \tag{2}$$

The probability that the process is declared to be in-control based on a single sample can be calculated as follows:

$$P_{\mathrm{in}}^{(1)} = P(\mathrm{LCL}_2 \leq S^2 \leq \mathrm{UCL}_2) = P(S^2 \leq \mathrm{UCL}_2) - P(S^2 \leq \mathrm{LCL}_2),$$

and when the process is in control

$$P_{\mathrm{in}}^{(1)} = G\left((n-1)\left(1 - k_2\sqrt{\frac{2}{n-1}}\right)\right) - G\left((n-1)\left(1 + k_2\sqrt{\frac{2}{n-1}}\right)\right). \tag{3}$$

The probability of repetition can be calculated as follows:

$$P_{\mathrm{rep}} = P\{\mathrm{LCL}_1 \leq S^2 < \mathrm{LCL}_2\} + P\{\mathrm{UCL}_2 < S^2 \leq \mathrm{UCL}_1\}.$$

So, when the process is in control

$$P_{\text{rep}} = \left\{G\left((n-1)\left(1-k_1\sqrt{\frac{2}{n-1}}\right)\right) - G\left((n-1)\left(1-k_2\sqrt{\frac{2}{n-1}}\right)\right)\right\} + \left\{G\left((n-1)\left(1+k_2\sqrt{\frac{2}{n-1}}\right)\right) - G\left((n-1)\left(1+k_1\sqrt{\frac{2}{n-1}}\right)\right)\right\}. \tag{4}$$

Therefore, the probability the process is declared to be out of control for the proposed plan when the process is actually in control is given as follows:

$$P_{\text{out}} = \frac{1 + G((n-1)\ (1-k_1\sqrt{2/n-1})) - G((n-1)(1+k_1\sqrt{2/n-1}))}{1 - [\{G((n-1)(1-k_1\sqrt{2/\text{n}-1})) - G((n-1)(1-k_2\sqrt{2/n-1}))\} + \{G((n-1)(1+k_2\sqrt{2/n-1})) - G((n-1)(1+k_1\sqrt{2/\text{n}-1}))\}]}. \tag{5}$$

The average sample number (ASN) for the proposed control chart when the process is in control is given by

$$\text{ASN}_0 = \frac{n}{1 - [\{G((n-1)(1-k_1\sqrt{2/\text{n}-1})) - G((n-1)(1-k_2\sqrt{2/n-1}))\} + \{G((n-1)(1+k_2\sqrt{2/n-1})) - G((n-1)(1+k_1\sqrt{2/n-1}))\}]}. \tag{6}$$

Now we derive the equations when the process is shifted. Suppose that the process variance has been shifted from σ^2 to $\sigma_1^2 = c\sigma^2$ for a constant c while the mean remains the same. Now, $(n-1)S^2/(c\sigma^2)$ follows the chi-square distribution with degree of freedom $n-1$.

The probability that the process is declared to be out of control under variance shift is derived as follows.

$$\begin{aligned} P_{\text{out,shift}}^{(1)} &= P(S^2 \geq \text{UCL}_1|\sigma_1^2) + P(S^2 \leq \text{LCL}_1|\sigma_1^2), \\ &= 1 + G\left(\frac{n-1}{c}\left(1-k_1\sqrt{\frac{2}{n-1}}\right)\right) - G\left(\frac{n-1}{c}\left(1+k_1\sqrt{\frac{2}{n-1}}\right)\right). \end{aligned} \tag{7}$$

Hence, the probability that the process is declared as out-of-control when the process is shifted can be obtained as follows:

$$P_{\text{out,shift}} = \frac{1 + G((n-1/c)(1-k_1\sqrt{2/n-1})) - G((n-1/c)(1+k_1\sqrt{2/n-1}))}{1 - [\{G((n-1/c)(1-k_1\sqrt{2/n-1})) - G((n-1/c)(1-k_2\sqrt{2/n-1}))\} + \{G((n-1/c)(1+k_2\sqrt{2/n-1})) - G((n-1/c)(1+k_1\sqrt{2/\text{n}-1}))\}]}. \tag{8}$$

The ARL when the process is under control is given as

$$\text{ARL}_0 = \frac{1}{P_{\text{out}}}. \tag{9}$$

The ARL when the process is out of control is given as

$$\text{ARL}_1 = \frac{1}{P_{\text{out,shift}}}. \tag{10}$$

Table 1. Average run lengths of the proposed control charts when $ARL_0 = 200$.

c	$n = 4$		$n = 5$		$n = 6$		$n = 7$	
c	$k_1 = 4.03985$ $k_2 = 2.39055$		$k_1 = 3.91435$ $k_2 = 1.39822$		$k_1 = 3.79672$ $k_2 = 1.38838$		$k_1 = 3.6298$ $k_2 = 2.7954$	
c	ARL	ASN	ARL	ASN	ARL	ASN	ARL	ASN
1.0	200.00	4.11	200.00	5.49	200.00	6.68	200.00	7.08
1.1	114.78	4.15	106.35	5.66	101.50	6.90	101.18	7.12
1.2	72.27	4.20	62.80	5.84	57.74	7.14	57.87	7.18
1.3	48.86	4.25	40.21	6.02	35.86	7.40	36.34	7.25
1.4	34.94	4.30	27.44	6.21	23.88	7.66	24.54	7.32
1.5	26.13	4.35	19.73	6.39	16.82	7.91	17.56	7.39
1.6	20.28	4.40	14.80	6.57	12.42	8.15	13.16	7.46
1.7	16.22	4.45	11.51	6.73	9.53	8.38	10.25	7.53
1.8	13.30	4.50	9.22	6.88	7.56	8.58	8.24	7.60
1.9	11.15	4.54	7.57	7.02	6.17	8.76	6.80	7.66
2	9.51	4.58	6.36	7.13	5.16	8.90	5.75	7.71
3	3.60	4.81	2.34	7.49	1.94	9.11	2.21	7.89
4	2.31	4.82	1.60	7.14	1.39	8.40	1.53	7.75

Table 2. Average run lengths of the proposed control charts when $ARL_0 = 300$.

	$n = 4$		$n = 5$		$n = 6$		$n = 7$	
	$k_1 = 4.40671$ $k_2 = 2.09285$		$k_1 = 4.18449$ $k_2 = 2.40599$		$k_1 = 4.05323$ $k_2 = 1.95393$		$k_1 = 3.93845$ $k_2 = 2.04327$	
c	ARL	ASN	ARL	ASN	ARL	ASN	ARL	ASN
1	300.00	4.17	300.00	5.13	300.00	6.28	300.00	7.28
1.1	164.55	4.23	156.96	5.19	148.46	6.41	142.78	7.43
1.2	99.67	4.30	91.68	5.27	82.84	6.57	77.32	7.61
1.3	65.16	4.38	58.26	5.35	50.68	6.73	46.21	7.80
1.4	45.24	4.45	39.55	5.43	33.32	6.90	29.84	8.01
1.5	32.96	4.53	28.31	5.52	23.22	7.08	20.50	8.22
1.6	24.98	4.60	21.16	5.60	16.97	7.25	14.81	8.43
1.7	19.56	4.68	16.38	5.68	12.90	7.42	11.16	8.63
1.8	15.75	4.75	13.07	5.76	10.13	7.58	8.72	8.81
1.9	12.97	4.81	10.69	5.84	8.19	7.72	7.01	8.98
2	10.90	4.87	8.94	5.91	6.78	7.85	5.79	9.13
3	3.76	5.23	3.07	6.27	2.32	8.30	2.02	9.50
4	2.33	5.25	1.95	6.23	1.55	7.95	1.41	8.96

The coefficients k_1 and k_2 for the proposed control chart can be determined by considering the target ARL_0, when the process is in control while keeping the ASN_0 in Equation (6) as minimal as possible.

We presented the control coefficients in Table 1 for various sample sizes (n) when $ARL_0 = 200$, where the ARLs as well as ASNs are reported according to various shift constants (c's). We used $n = 4$, 5, 6, 7 and c from 1.0 to 2.0 with the increment of 0.1. Table 2 is the case when $ARL_0 = 300$ and Table 3 is the case when $ARL_0 = 375$.

From Tables 1–3, we note the following trends in the proposed control charts:

(1) When c increases from 1.1 to 2.0, we note the decreasing trend in ARL_1 in Tables 1–3.
(2) As n changes from 4 to 7 while other values remain unchanged, we note the decreasing trend in ARL_1.

Table 3. Average run lengths of the proposed control charts when $\mathrm{ARL}_0 = 370$.

	$n = 4$		$n = 5$		$n = 6$		$n = 7$	
	$k_1 = 4.57769$ $k_2 = 2.43202$		$k_1 = 4.37021$ $k_2 = 1.92006$		$k_1 = 4.19825$ $k_2 = 2.24743$		$k_1 = 4.09419$ $k_2 = 1.8737$	
c	ARL	ASN	ARL	ASN	ARL	ASN	ARL	ASN
1	370.00	4.11	370.00	5.26	370.00	6.19	370.00	7.36
1.1	199.90	4.16	187.55	5.36	180.28	6.29	171.14	7.55
1.2	119.63	4.21	106.51	5.48	99.40	6.41	90.40	7.76
1.3	77.45	4.27	66.01	5.62	60.23	6.54	52.86	8.01
1.4	53.33	4.33	43.81	5.75	39.30	6.67	33.47	8.26
1.5	38.59	4.39	30.73	5.89	27.20	6.82	22.59	8.52
1.6	29.08	4.45	22.55	6.03	19.76	6.96	16.07	8.78
1.7	22.65	4.51	17.17	6.16	14.94	7.10	11.94	9.03
1.8	18.14	4.57	13.50	6.29	11.68	7.24	9.21	9.27
1.9	14.88	4.63	10.90	6.41	9.39	7.36	7.33	9.48
2	12.46	4.68	9.01	6.52	7.74	7.48	6.00	9.66
3	4.16	5.01	2.91	7.04	2.53	7.97	2.00	10.11
4	2.52	5.07	1.84	6.91	1.65	7.74	1.38	9.40

(3) As ARL_0 changes from 200 to 375, we note the increasing trend in ARL_1.
(4) The values of ASN are almost similar to the sample size in all cases.

Example. Let us consider an example for automobile engine piston rings taken from Montgomery [18]. Each subgroup consists of five piston rings. Suppose that the proposed control chart is used by specifying $\mathrm{ARL}_0 = 370$. Then, from Table 3 we determined the control chart coefficients as $k_1 = 4.37021$ and $k_2 = 1.92006$. The value of S^2 is 0.000100627. The four control limits using sample estimates of the proposed chart for automobile engine piston rings data are computed as follows. First, the outer control limits are given by

$$\begin{aligned}
\mathrm{UCL}_1 &= S^2 + k_1\sqrt{2(S^2)^2/(n-1)} \\
&= 0.000100627 + 4.37021\ \sqrt{2(0.000100627)^2/4} = 0.000411584, \\
\mathrm{LCL}_1 &= S^2 - k_1\sqrt{2(S^2)^2/(n-1)} \\
&= 0.000100627 - 4.37021\ \sqrt{2(0.000100627)^2/4} = -0.000210331, \\
\mathrm{UCL}_2 &= S^2 + k_2\sqrt{2(S^2)^2/(n-1)} \\
&= 0.000100627 + 1.92006\sqrt{2(0.000100627)^2/4} = 0.000237247, \\
\mathrm{LCL}_2 &= S^2 - k_2\sqrt{2(S^2)^2/(n-1)} \\
&= 0.000100627 - 1.92006\sqrt{2(0.000100627)^2/4} = -0.00035992.
\end{aligned}$$

The proposed control for this data is shown in Figure 1.

The values of S^2 are plotted along with four control limits in Figure 1. From Figure 1, we see that there have been several cases requiring repetitive sampling such as the subgroups 3 and 6. But it turns out that the process is in control.

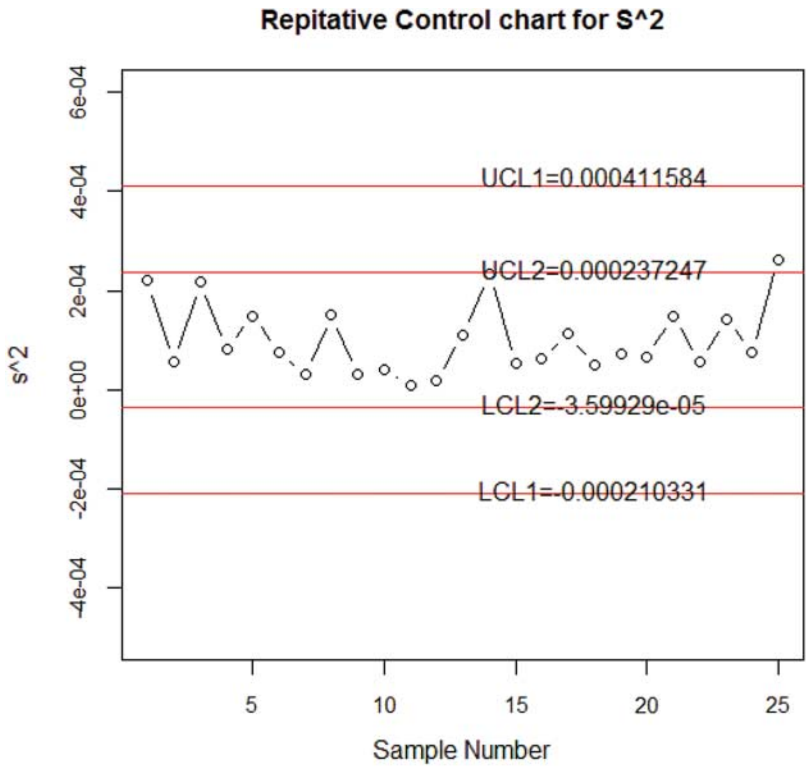

Figure 1. The control chart for real data.

3. Comparative study

In this section, we will compare the efficiency of the proposed control chart with the existing S^2 control charts. The control chart with smaller values of ARLs is said to be more efficient. So, the efficiency of charts will be discussed in terms of ARLs.

3.1 *Proposed S^2 control chart versus Shewhart S^2 chart*

The traditional Shewhart S^2 control chart is based on single sampling and has two control limits. The proposed S^2 control chart is the extension of the Shewhart S^2 control chart because the proposed chart becomes the Shewhart S^2 control chart when $k_1 = k_2$. We presented two tables for the comparison purpose. Table 4 is given when $\text{ARL}_0 = 300$ and Table 5 is given when $\text{ARL}_0 = 375$. We selected these values of ARL_0 as commonly used in the area of quality control.

From Tables 4 and 5, we note that the values of ARL_1 are smaller in the proposed control charts as compared to the Shewhart S^2 control chart. For example, when $\text{ARL}_0 = 370$, $n = 5$ and $c = 1.5$ from Table 5, we note $\text{ARL}_1 = 30.7$ from the proposed control chart while $\text{ARL}_1 = 35.1$ from the existing Shewhart control chart. It is noted that the difference between the proposed chart and the Shewhart S^2 control chart is increased as n is increased. The difference of the ARL_1 values between two control charts increases when the process shifts increase.

3.2 *Proposed chart versus chart by Zhang et al. [26]*

Now, the ARL comparison of the proposed chart and the chart by Zhang *et al.* [26] is presented. Again, the same specified parametric values are taken for the comparison purpose. It can be

Table 4. Comparison of the proposed control chart with Shewhart S^2 chart when $\text{ARL}_0 = 300$.

	$n = 4$		$n = 5$		$n = 6$		$n = 7$	
c	Proposed with $k_1 = 4.406$ $k_2 = 2.092$	Shewhart with $k = 4.370$	Proposed with $k_1 = 4.184$ $k_2 = 2.405$	Shewhart with $k = 4.163$	Proposed with $k_1 = 4.053$ $k_2 = 1.953$	Shewhart with $k = 4.019$	Proposed with $k_1 = 3.938$ $k_2 = 2.043$	Shewhart with $k = 3.910$
1	300.00	300.00	300.00	300.00	370.00	300.00	300.00	300.00
1.1	164.55	167.74	156.96	159.30	180.28	152.31	142.78	146.33
1.2	99.67	103.63	91.68	94.55	99.40	87.35	77.32	81.40
1.3	65.16	69.11	58.26	61.10	60.23	54.97	46.21	50.05
1.4	45.24	48.93	39.55	42.20	39.30	37.19	29.84	33.26
1.5	32.96	36.34	28.31	30.72	27.20	26.64	20.50	23.51
1.6	24.98	28.05	21.16	23.34	19.76	19.98	14.81	17.45
1.7	19.56	22.35	16.38	18.36	14.94	15.57	11.16	13.49
1.8	15.75	18.29	13.07	14.87	11.68	12.51	8.72	10.77
1.9	12.97	15.29	10.69	12.34	9.39	10.32	7.01	8.85
2	10.90	13.03	8.94	10.44	7.74	8.70	5.79	7.43
3	3.76	4.85	3.07	3.82	2.53	3.17	2.02	2.72
4	2.33	3.03	1.95	2.42	1.65	2.04	1.41	1.79

Table 5. Comparison of the proposed control chart with Shewhart S^2 chart when $\text{ARL}_0 = 370$.

	$n = 4$		$n = 5$		$n = 6$		$n = 7$	
c	Proposed with $k_1 = 4.57$ $k_2 = 2.432$	Shewhart with $k = 4.553$	Proposed with $k_1 = 4.370$ $k_2 = 1.920$	Shewhart with $k = 4.330$	Proposed with $k_1 = 4.198$ $k_2 = 2.247$	Shewhart with $k = 4.175$	Proposed with $k_1 = 4.094$ $k_2 = 1.873$	Shewhart with $k = 4.05862$
1	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
1.1	199.90	202.74	187.55	192.35	180.28	183.76	171.14	176.40
1.2	119.63	123.17	106.51	112.18	99.40	103.48	90.40	96.29
1.3	77.45	80.98	66.01	71.42	60.23	64.12	52.86	58.27
1.4	53.33	56.65	43.81	48.71	39.30	42.81	33.47	38.20
1.5	38.59	41.63	30.73	35.07	27.20	30.32	22.59	26.68
1.6	29.08	31.84	22.55	26.39	19.76	22.52	16.07	19.61
1.7	22.65	25.17	17.17	20.59	14.94	17.39	11.94	15.02
1.8	18.14	20.44	13.50	16.55	11.68	13.87	9.21	11.90
1.9	14.88	16.99	10.90	13.64	9.39	11.36	7.33	9.70
2	12.46	14.39	9.01	11.48	7.74	9.52	6.00	8.10
3	4.16	5.16	2.91	4.05	2.53	3.34	2.00	2.85
4	2.52	3.17	1.84	2.51	1.65	2.11	1.38	1.85

seen that the proposed control chart provides a smaller value of ARLs as compared to the chart by Zhang *et al.* [26]. For example, when $c = 1.5$, $r_0 = 370$ and $n = 5$, from Table 3, we note $\text{ARL}_1 = 30.70$ from the proposed control chart while $\text{ARL}_1 = 70$ from Zhang *et al.* [26]. Similarly, when $c = 2$, $r_0 = 370$ and $n = 5$, from Table 3, we note $\text{ARL}_1 = 9$ from the proposed control chart while $\text{ARL}_1 = 13$ from the Zhang *et al.* [26] chart. So, the proposed chart is more efficient than the Zhang *et al.* [26] chart in detecting the shift in the process.

4. Simulation study

In this section, the performance of the proposed control chart is demonstrated with the help of simulated data. The in-control process is assumed to follow the normal distribution with $\mu = 0$

Table 6. The simulated data.

Subgroup no.	Sample 1	2	3	4	5	$\bar{X}$	S^2
1	0.822203	−2.90516	−2.77059	2.698414	3.342245	0.237422	8.740587
2	3.488448	0.747544	0.803628	−0.91268	0.45191	0.915769	2.55449
3	1.384665	0.837499	−0.86015	2.304064	−2.04822	0.323571	3.08475
4	1.37076	0.46269	2.25603	2.31747	−1.67078	0.947234	2.71467
5	1.79087	−5.55334	−3.08325	3.88745	0.104013	−0.57085	14.26173
6	−0.60775	2.014371	2.564402	0.558486	1.959444	1.29779	1.683309
7	−1.19902	0.826443	1.940512	3.911588	−0.50526	0.994854	4.123343
8	0.214864	0.104219	−1.43745	−1.14172	0.392258	−0.37356	0.720726
9	0.271539	−0.73651	2.8615	0.843192	0.939923	0.83593	1.726222
10	−0.7972	−0.51352	0.986834	2.304587	2.662675	0.928675	2.490463
11	1.468094	0.719437	−4.39037	−2.9949	−1.40409	−1.32036	6.043072
12	−0.48791	0.463019	3.000626	4.350904	−0.68374	1.32858	5.00711
13	−4.53169	2.79933	1.41939	−2.26276	−1.66621	−0.84839	8.671158
14	−2.50991	−0.53077	0.161477	−1.81875	1.917606	−0.55607	3.011613
15	−0.89232	−1.56657	0.547575	3.241542	6.672298	1.600505	11.44232
16	−0.95952	1.36188	−0.30387	−1.94316	0.491244	−0.27069	1.629648
17	−0.95196	−4.45544	2.858189	1.730991	−1.22111	−0.40786	8.145512
18	−2.71638	−1.20544	−4.42638	3.2106	−2.03404	−1.43433	8.145861
19	−0.3692	−1.9095	1.412618	5.499141	−1.7966	0.567293	9.409297
20	−1.99763	3.468787	0.79211	−5.43292	0.189963	−0.59594	11.10685
21	−2.42559	2.948568	0.008251	1.097331	3.094154	0.944542	5.223687
22	1.73353	3.04432	1.59248	3.3143	−5.25616	0.885693	12.37464
23	−1.52097	1.422285	1.039017	−0.08855	2.272255	0.624808	2.158715
24	−0.95748	−0.8025	−0.22106	−0.31856	1.604996	−0.13892	1.047662
25	−0.06265	5.307733	0.594644	2.243393	−1.55092	1.306441	6.858445
26	−0.886	−3.0917	−3.41418	−1.04451	1.500224	−1.38723	3.929958
27	−1.89427	−3.43582	−0.88469	−5.634	−0.34561	−2.43888	4.57415
28	1.80112	1.22569	2.56498	−4.314	2.09117	0.673792	8.009777
29	−1.13508	1.174176	−0.13989	1.889322	1.739027	0.705511	1.697921
30	1.61897	−1.13753	5.00347	−3.97006	2.07253	0.717474	11.60817
31	3.30445	−4.44497	1.28714	−2.2146	−3.68088	−1.14977	11.04433
32	0.791115	−1.41377	6.180332	−1.50607	−3.19536	0.17125	13.28826
33	1.50802	5.520017	1.574205	−2.77276	0.656529	1.297202	8.732104
34	−1.74099	−1.486	1.5429	3.66359	−1.77234	0.041432	6.043566
35	−0.50634	−4.39024	2.841474	0.801154	2.02469	0.154147	8.056748
36	1.070506	−3.99678	3.701629	0.418484	5.162201	1.271207	12.39148
37	2.845333	2.038118	5.634573	−0.18415	3.983728	2.86352	4.725882
38	−1.31666	1.9464	−1.83967	−3.16129	1.97661	−0.47892	5.4151
39	−1.54519	−0.09658	−1.85159	4.265676	−1.23559	−0.09266	6.376273
40	3.51314	−3.78166	6.43307	1.91043	−2.73592	1.067814	18.36535

and $\sigma^2 = 4$. The first 10 observations are generated from the in-control process and the next 30 observations are generated with the shifted process, which follows a normal distribution with $\mu = 0$ and variance of $1.5\,\sigma^2 = 6$. The data are reported in Table 6.

For this simulation study, let $\text{ARL}_0 = 370$ and $n = 5$. The four control limits for the simulated data are as follows:

$$\text{UCL}_1 = 16.36, \quad \text{LCL}_1 = -8.36, \quad \text{UCL}_2 = 9.43 \quad \text{and} \quad \text{LCL}_2 = -1.43.$$

The values of S^2 are also reported in Table 6. The values of S^2 are plotted on the control chart in Figure 3. From Figure 2, it can be seen that the proposed chart detects the shift at the 39th value. The values of S^2 are also plotted on the traditional Shewhart S^2 control chart in Figure 3, which could not detect the shift in the process. So, the proposed control chart has the ability to detect shift quickly as compared to the existing control chart.

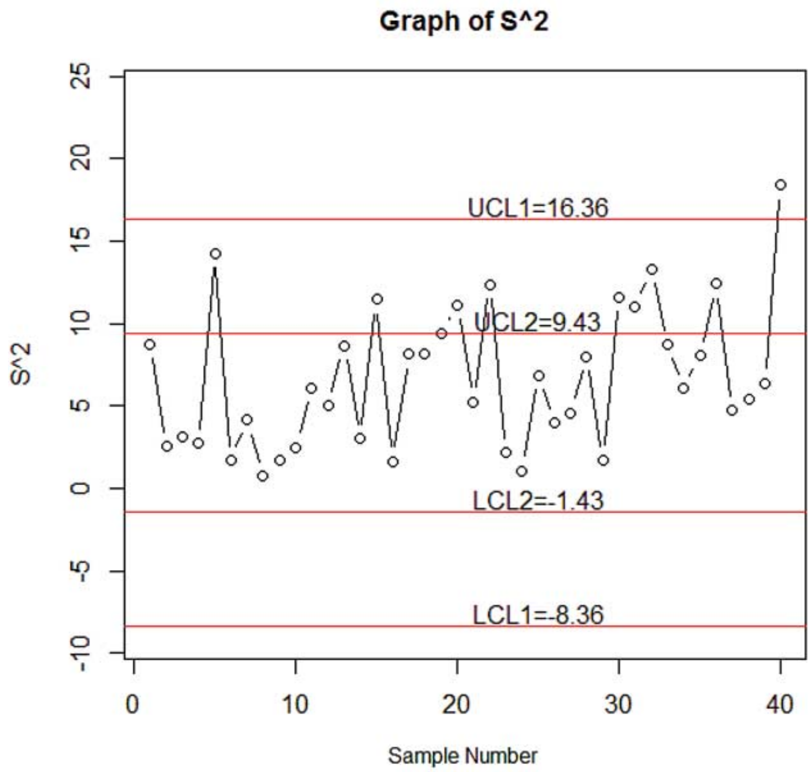

Figure 2. The proposed S^2 control chart for the simulated data.

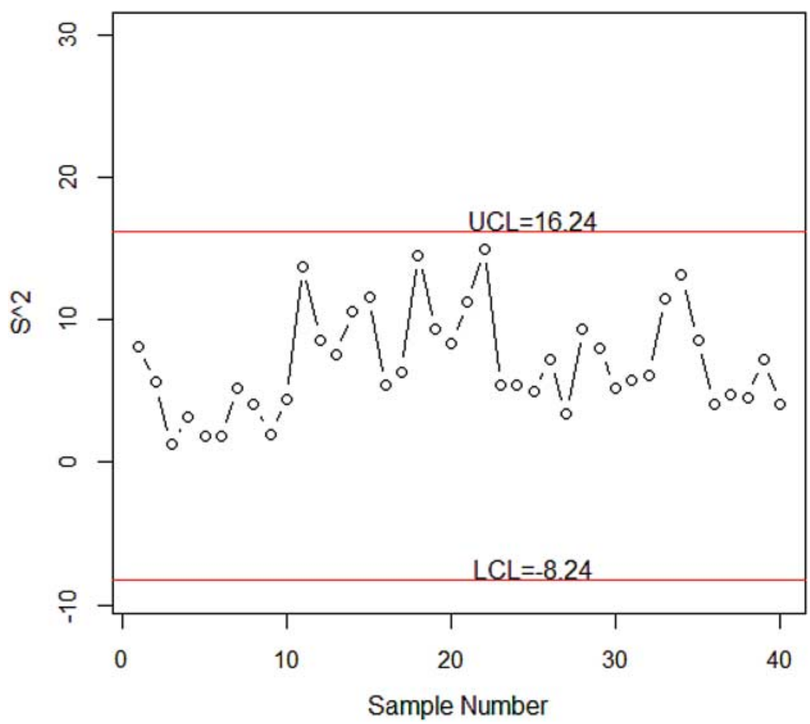

Figure 3. Shewhart S^2 control chart for the simulated data.

5. Conclusion

In this paper, a new S^2 control chart under the repetitive sampling scheme is presented for monitoring the process variance. The tables are given for the practical application of the proposed control chart in the industries. The proposed control chart is more efficient than the Shewhart S^2 control chart in terms of the ARL as the former declares out-of-control more quickly. It is strongly suggested to use the proposed control chart in the industry to monitor the process variability instead of the existing control charts because it is more efficient and easier to apply. The proposed repetitive sampling scheme can be applied to other types of control charts to improve the performance of early detection of the process shift. Still, there may a room to improve the proposed control chart by incorporating the exponentially weighted moving average or the cumulative sum. Furthermore, an extensive study of comparing the performance of the existing control charts in a fair ground is necessary.

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References

[1] L. Ahmad, M. Aslam, and C.-H. Jun, *Designing of X-bar control charts based on process capability index using repetitive sampling*, T. I. Meas. Control, 36(3) (2014), pp. 367–374.

[2] S. Ahmad, M. Riaz, S.A. Abbasi, and Z. Lin, *On efficient median control charting*, J. Chin. Inst. Eng. 37(3) (2014), pp. 358–375.

[3] M. Aslam, M. Azam, and C.H. Jun, *New attributes and variables control charts under repetitive sampling*, Ind. Eng. Manage. Syst. 13(1) (2014), pp. 101–106.

[4] M. Aslam, N. Khan, M. Azam, C.-H. Jun, *Designing of a new monitoring T-chart using repetitive sampling*, Inf. Sci. 269 (2014), pp. 210–216.

[5] M. Azam, M. Aslam, and C.-H. Jun, *Designing of a hybrid exponentially weighted moving average control chart using repetitive sampling*, Int. J. Adv. Manuf. Tech., 77 (2015), pp. 1927–1933.

[6] S. Balamurali, C.-H. Jun, *Repetitive group sampling procedure for variables inspection*, J. Appl. Stat. 33 (2006), pp. 327–338.

[7] G. Chen, *The run length distributions of the R, S and S2 control charts when is estimated*, Can. J. Stat. 26(2) (1998), pp. 311–322.

[8] H.A. David, *Early sample measures of variability*, Stat. Sci. 13(4) (1998), pp. 368–377.

[9] B. Guo and B.X. Wang, *The design of the ARL-unbiased S2 chart when the in-control variance is estimated*, Qual. Reliab. Eng. Int. 31 (2015), pp. 501–511.

[10] H.J. Huang and F.L. Chen, *A synthetic control chart for monitoring process dispersion with sample standard deviation*, Comput. Ind. Eng. 49(2) (2005), pp. 221–240.

[11] M.B. Khoo, *A modified s chart for the process variance*, Qual. Eng. 17(4) (2005), pp. 567–577.

[12] M.B.C. Khoo, *S^2 control chart based on double sampling*, Int. J. Pure Ap. Mat. 13(2) (2004), pp. 249–258.

[13] T.I. Kuo and P.H. Lee, *Design of adaptive s control charts*, J. Stat. Comput. Simul. 83(11) (2013), pp. 2002–2014.

[14] H. Lee, M. Aslam, Q.U.A. Shakeel, W. Lee, and C.H. Jun, *A control chart using an auxiliary variable and repetitive sampling for monitoring process mean*, J. Stat. Comput. Simul. doi:10.1080/00949655.2014.970553.

[15] P.H. Lee, *Adaptive R charts with variable parameters*, Comput. Stat. Data Anal. 55 (2011), pp. 2003–2010.

[16] P.H. Lee, Y.C. Chang, and C.C. Torng, *A design of s control charts with combined double sampling and variable sampling interval scheme*, Commun. Stat. A-Theor. 41 (2012), pp. 153–165.

[17] M.A. Mahmoud, G.R. Henderson, E.K. Epprecht, and W.H. Woodall, *Estimating the standard deviation in quality-control applications*, J. Qual. Technol. 42(4) (2010), pp. 348–357.

[18] D.C. Montgomery, *Introduction to Statistical Quality Control*, 6th ed., Wiley, New York, 2009.

[19] A.C. Rakitzis and D.L. Antzoulakos, *On the improvement of one-sided S control charts*, J. Appl. Stat. 38(12) (2011), pp. 2839–2858.

[20] M. Riaz, *A dispersion control chart*, Commun Stat B-Simul, 37(6) (2008), pp. 1239–1261.

[21] M. Riaz and A. Saghir, *A mean deviation-based approach to monitor process variability*, J. Stat. Comput. Simul. 79(10) (2009), pp. 1173–1193.

[22] M. Schoonhoven, M. Riaz, and R.J. Does, *Design and analysis of control charts for standard deviation with estimated parameters*, J. Qual. Technol. 43(4) (2011), pp. 307–333.

[23] R.E. Sherman, *Design and evaluation of repetitive group sampling plan*, Technometrics. 7 (1965), pp. 11–21.

[24] R.F. Zafar, N. Abbas, M. Riaz, and Z. Hussain, *Progressive variance control charts for monitoring process dispersion*, Commun. Stat. A-Theor. 43(23) (2014), pp. 4893–4907.

[25] G. Zhang, *Improved R and s control charts for monitoring the process variance*, J. Appl. Stat. 41(6) (2014), pp. 1260–1273.

[26] L. Zhang, M.S. Bebbington, C.D. Lai, and K. Govindaraju, *On statistical design of the S2 control chart*, Commun. Stat. A-Theor. 34(1) (2005), pp. 229–244.