Vegetation Indices – 1st order Taylor

Uncertainty propagation in spatial environmental modelling

2024, Sytze de Bruin

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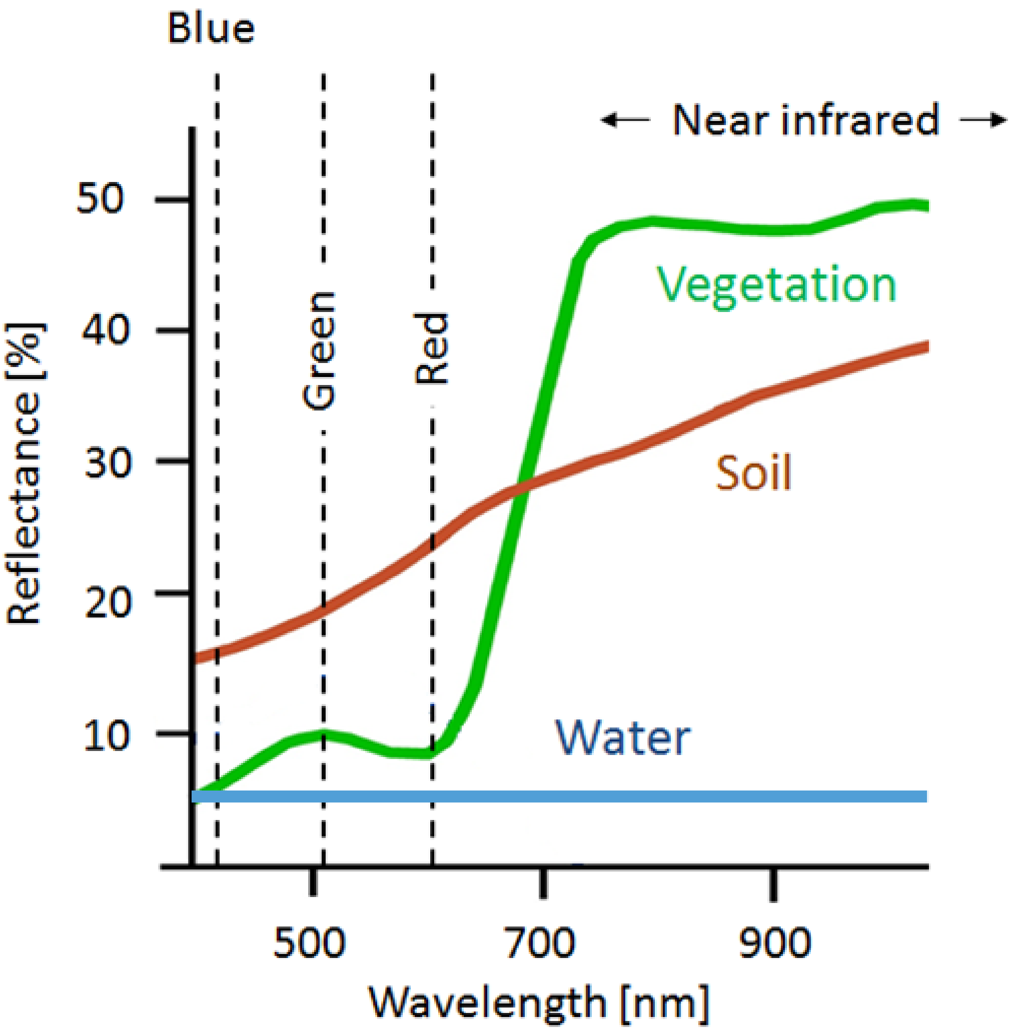
Background: spectral signatures land cover



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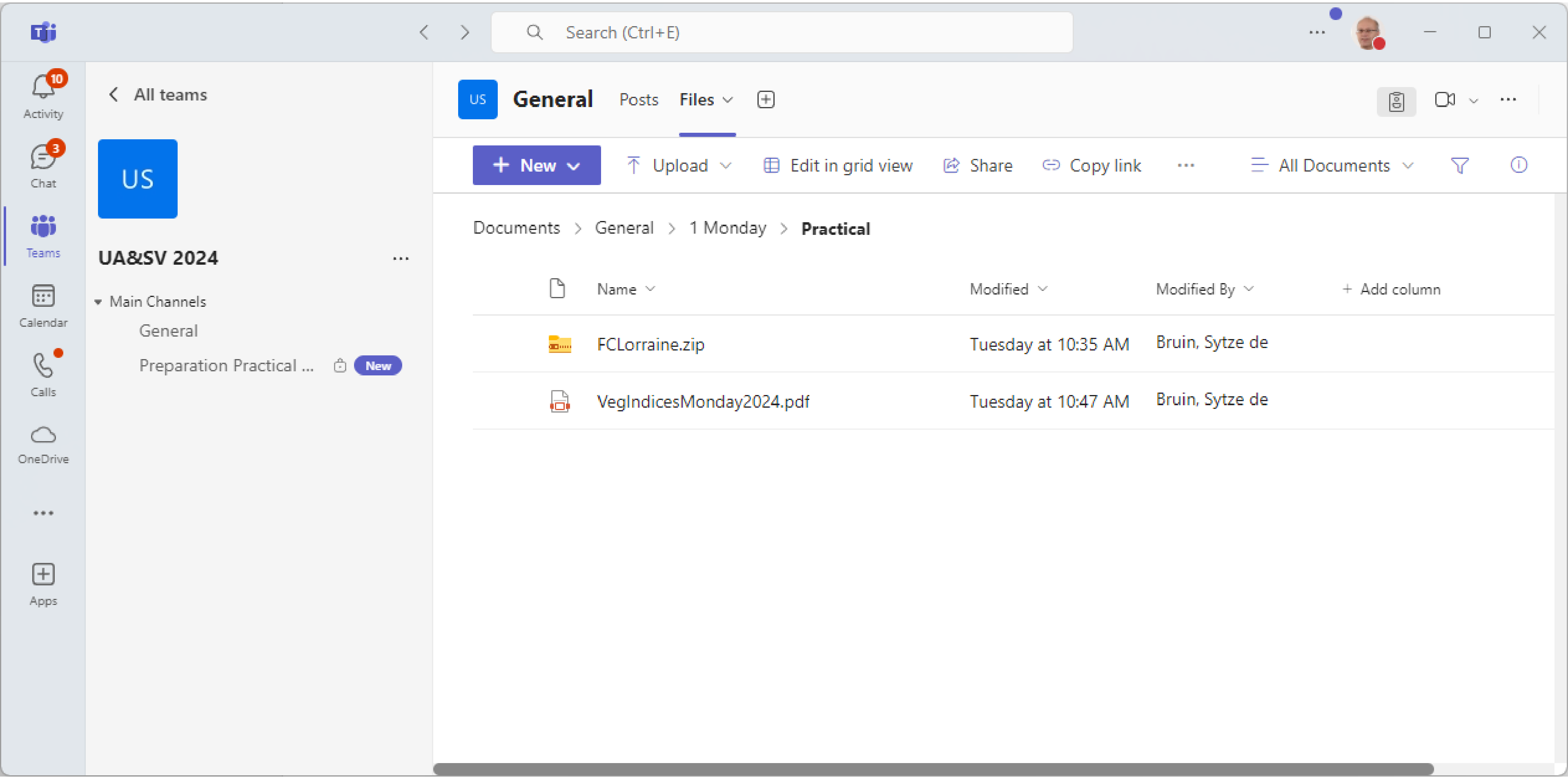
Vegetation indices

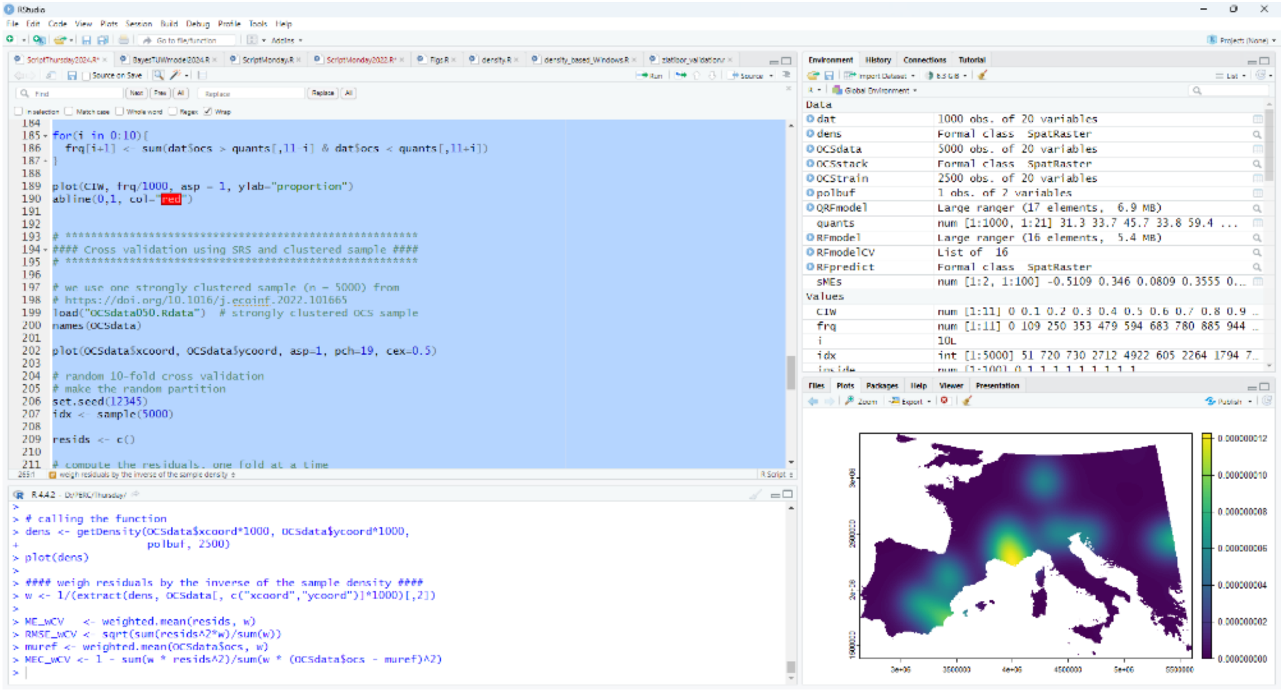
- Use the jump around the "red edge" to characterize vegetation
- There are many of them, e.g.:
  - NDVI = (NIR - RED)/ (NIR + RED)
  - SR = NIR/RED
- They are frequently used in vegetation mapping
- Errors in the inputs propagate into the indices
- Demonstrated with case study
- Today: 1^{st} order Taylor Series method

Microsoft Teams



R and RStudio

- Make and save script file
- Complete script will be provided at end of the day



The practical / tutorial

- Instructions in file
    - Code block
    - Snippets
    - Complete code (Wednesday – Friday)
- Go through the tutorial & interpret results
- Questions are to help thinking about the matter
- At ~ 15:45 Feedback
- Ask us if something is unclear / does not work
- Any Mac users? → https://www.xquartz.org/index.html

1) Derivation Tau(SR)

From chapter 14: $\tau^2 \approx \sum_{i=1}^{m} \sum_{j=1}^{m} \rho_{ij} \sigma_i \sigma_j g_i'(\bar{b})\, g_j'(\bar{b})$

SR = NIR/RED

f'_{NIR}= 1/RED (multiplication by constant)

f'_{RED}= -NIR/RED^2 (power rule, if f(x) = x^n , then derivative f'(x) = nx^{n-1})

Fill in: $\tau^2 \approx \sum_{i=1}^{m} \sum_{j=1}^{m} \rho_{ij} \sigma_i \sigma_j g_i'(\bar{b})\, g_j'(\bar{b})$

$$\tau_{SR}^2 \approx \frac{\sigma_{NIR}^2}{RED^2} + \sigma_{RED}^2 \cdot \frac{NIR^2}{RED^4} - 2\rho \cdot \sigma_{NIR} \cdot \sigma_{RED} \cdot \frac{NIR}{RED^3}$$

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2) Derivation Tau(NDVI)

NDVI = (NIR - RED)/ (NIR + RED)

Quotient rule, if f(x) = g(x)/h(x) and h(x) ≠ 0, then the derivative

$$f'(x) = \frac{h(x) \cdot g'(x) - h'(x) \cdot g(x)}{(h(x))^2}$$

$$f'_{NIR} = \frac{1 \cdot (NIR + RED) - (NIR - RED) \cdot 1}{(NIR + RED)^2} = \frac{2\,RED}{(NIR + RED)^2}$$

$$f'_{RED} = \frac{-1 \cdot (NIR + RED) - (NIR - RED) \cdot 1}{(NIR + RED)^2} = \frac{-2\,NIR}{(NIR + RED)^2}$$

Fill in: $\tau^2 \approx \sum_{i=1}^{m} \sum_{j=1}^{m} \rho_{ij} \sigma_i \sigma_j g_i'(\bar{b})\, g_j'(\bar{b})$

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NDVI (continued)

$$\tau_{NDVI}^2 \approx \sigma_{NIR}^2 \cdot \left(\frac{2\ RED}{(NIR + RED)^2}\right)^2 + \ \sigma_{RED}^2 \cdot \left(\frac{-2\ NIR}{(NIR + RED)^2}\right)^2$$

$$+2\rho \cdot \sigma_{NIR} \cdot \sigma_{RED} \cdot \frac{-2\ NIR}{(NIR + RED)^2} \cdot \frac{2\ RED}{(NIR + RED)^2}$$

$$= \sigma_{NIR}^2 \cdot \frac{4\ RED^2}{(NIR + RED)^4} + \ \sigma_{RED}^2 \cdot \frac{4\ NIR^2}{(NIR + RED)^4}$$

$$-2\rho \cdot \sigma_{NIR} \cdot \sigma_{RED} \cdot \frac{4NIR \cdot RED}{(NIR + RED)^4}$$

$$= \frac{4(\sigma_{NIR}^2 \cdot RED^2 + \sigma_{RED}^2 \cdot NIR^2) - 8\rho \cdot \sigma_{NIR} \cdot \sigma_{RED} \cdot NIR \cdot RED}{(NIR + RED)^4}$$

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4) Function in R

```
# ====================== Exercise 4 ========================
# Uncertainty NDVI by Taylor series method, returns tau.
# Inputs as TaylorSR.
TaylorNDVI <- function(red, nir, s_red, s_nir, rho) {
  tau_sq <- 4*(s_nir^2*red^2+s_red^2*nir^2 -
                 2*rho*nir*red*s_red*s_nir)/(nir+red)^4
  tau_sq[tau_sq < 0] <- 0
  sqrt(tau_sq)
}
# ==========================================================
```

Download from: MS Teams

$$\frac{4(\sigma_{NIR}^2 \cdot RED^2 + \sigma_{RED}^2 \cdot NIR^2) - 8\rho \cdot \sigma_{NIR} \cdot \sigma_{RED} \cdot NIR \cdot RED}{(NIR + RED)^4}$$

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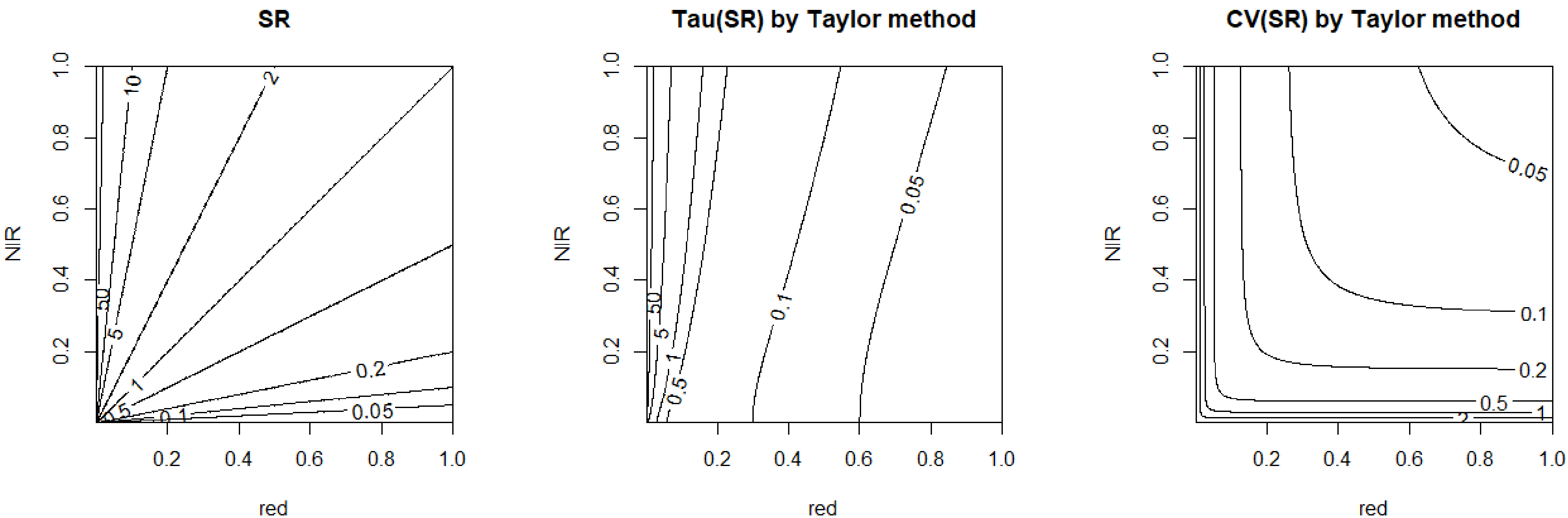
5) Plots τ_{SR}, 1st order Taylor, $\rho = 0$



$$\tau_{SR}^2 \approx \frac{\sigma_{NIR}^2}{RED^2} + \sigma_{RED}^2 \cdot \frac{NIR^2}{RED^4} - 2\rho \cdot \sigma_{NIR} \cdot \sigma_{RED} \cdot \frac{NIR}{RED^3}$$

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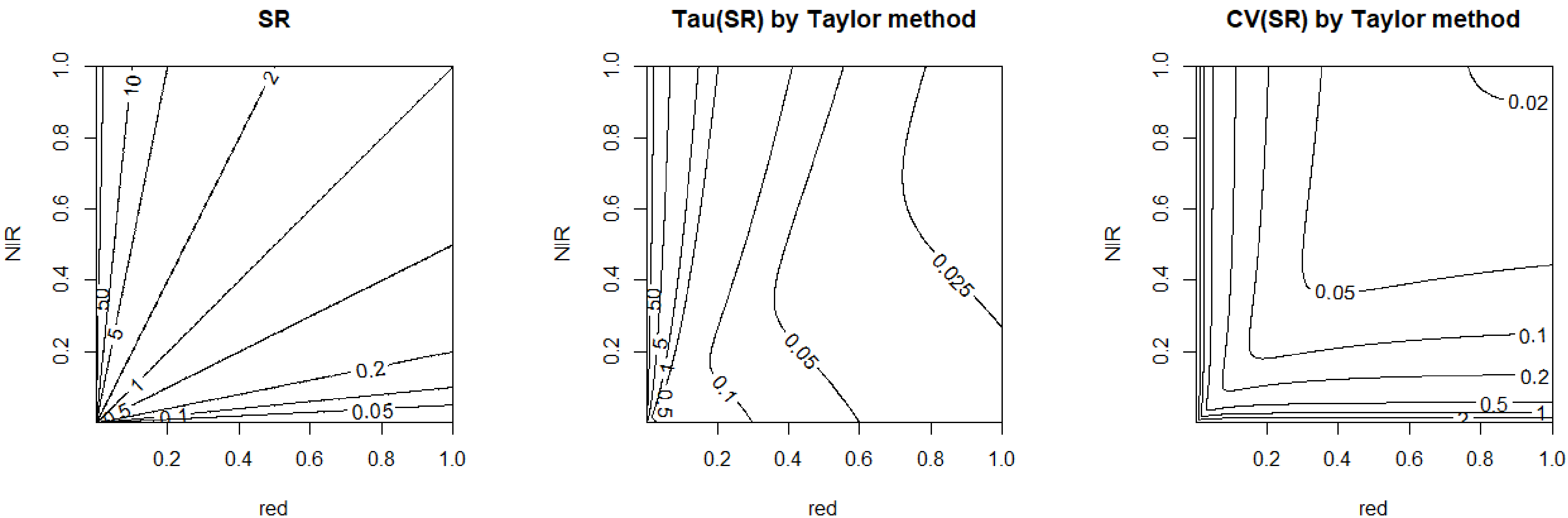
5) Plots τ_{SR}, 1st order Taylor, ρ = 0.8



$$\tau_{SR}^2 \approx \frac{\sigma_{NIR}^2}{RED^2} + \sigma_{RED}^2 \cdot \frac{NIR^2}{RED^4} - 2\rho \cdot \sigma_{NIR} \cdot \sigma_{RED} \cdot \frac{NIR}{RED^3}$$

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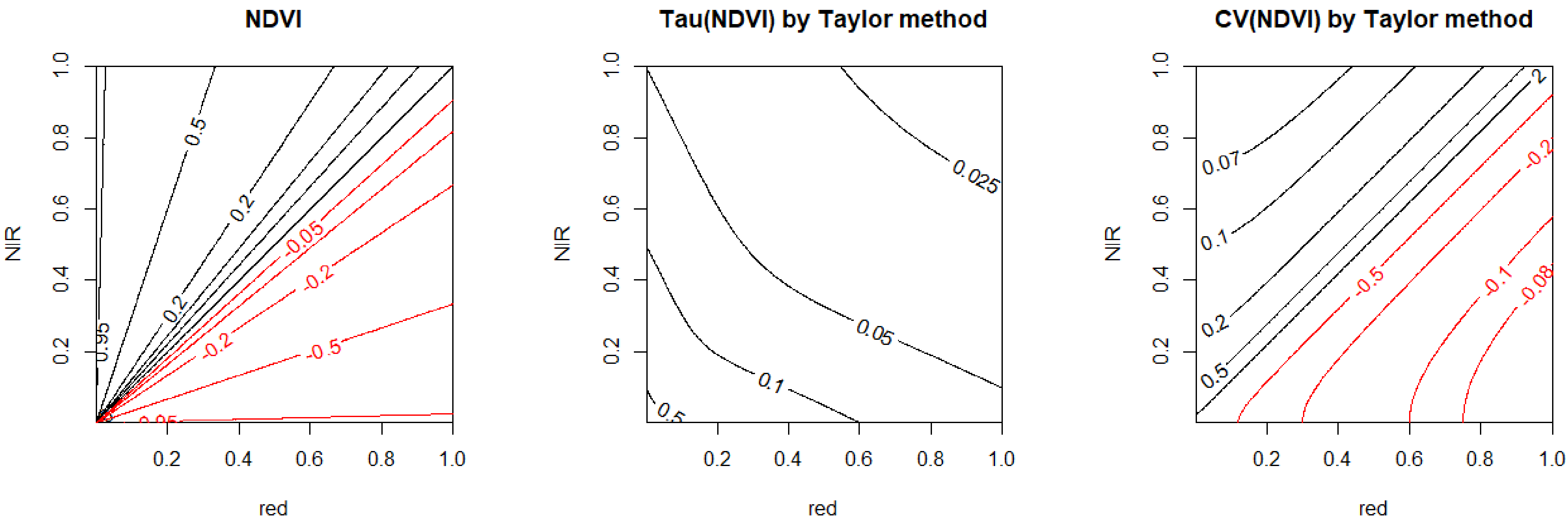
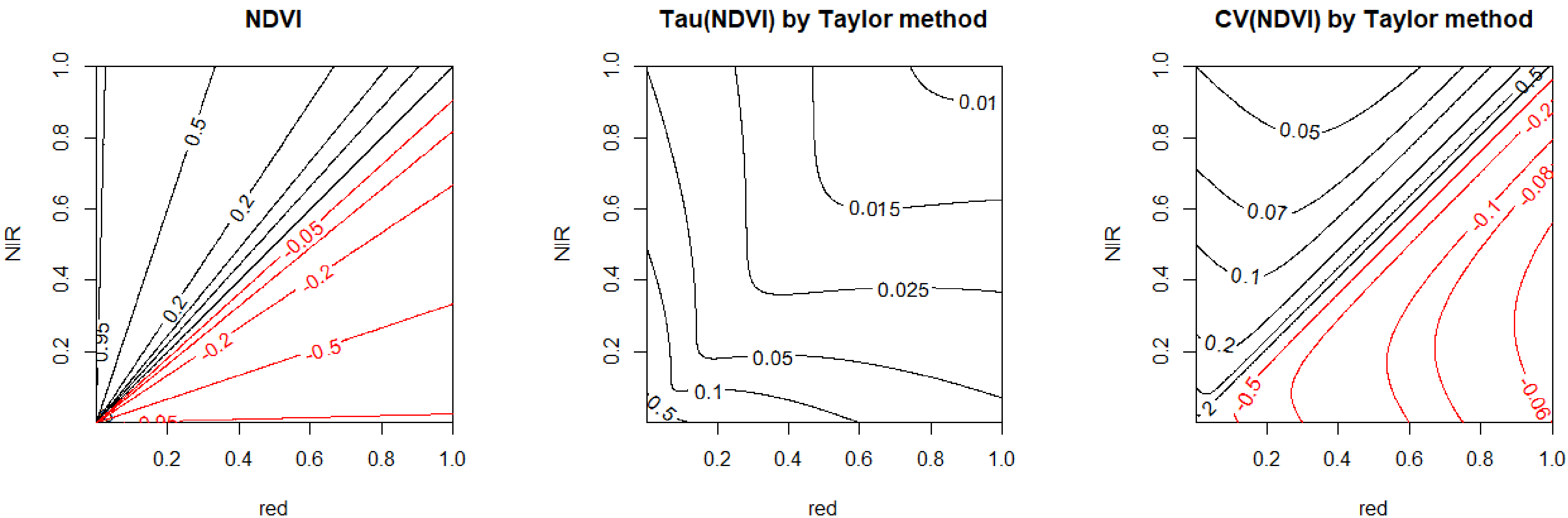
6) Plots τ_{NDVI}- 1^{st} order Taylor, $\rho = 0$



$$\frac{4(\sigma_{NIR}^2 \cdot RED^2 + \sigma_{RED}^2 \cdot NIR^2) - 8\rho \cdot \sigma_{NIR} \cdot \sigma_{RED} \cdot NIR \cdot RED}{(NIR + RED)^4}$$

6) Plots τ_{NDVI}- 1st order Taylor, $\rho = 0.8$



$$\frac{4(\sigma_{NIR}^2 \cdot RED^2 + \sigma_{RED}^2 \cdot NIR^2) - 8\rho \cdot \sigma_{NIR} \cdot \sigma_{RED} \cdot NIR \cdot RED}{(NIR + RED)^4}$$

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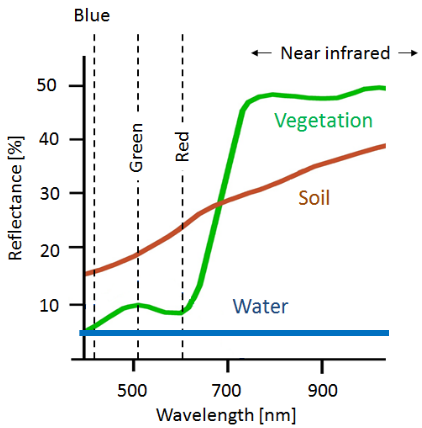
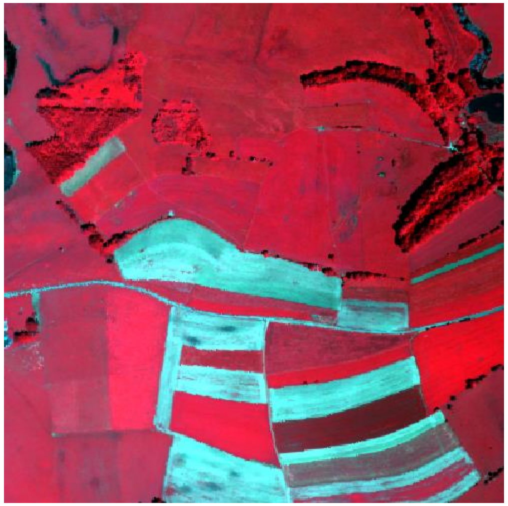
7) Remotely sensed image

```
> plotRGB(false_color, 3, 2, 1, stretch='lin')
```





```
>summary(false_color, size=2e5)
    Layer_1           Layer_2           Layer_3
 Min.   :0.02136   Min.   :0.02699   Min.   :0.1069
 1st Qu.:0.08065   1st Qu.:0.07160   1st Qu.:0.5246
 Median :0.09480   Median :0.08420   Median :0.6137
 Mean   :0.10701   Mean   :0.10806   Mean   :0.5911
 3rd Qu.:0.11412   3rd Qu.:0.10425   3rd Qu.:0.6579
 Max.   :0.32644   Max.   :0.35923   Max.   :0.8906
```

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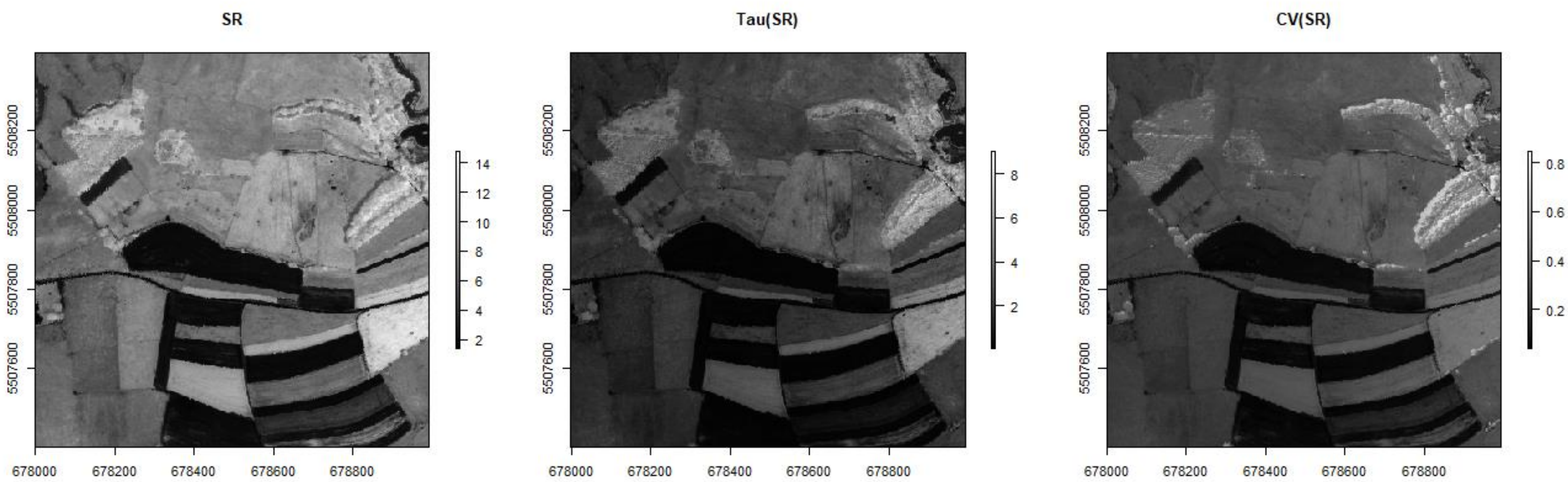
8) Coefficient of variation SR

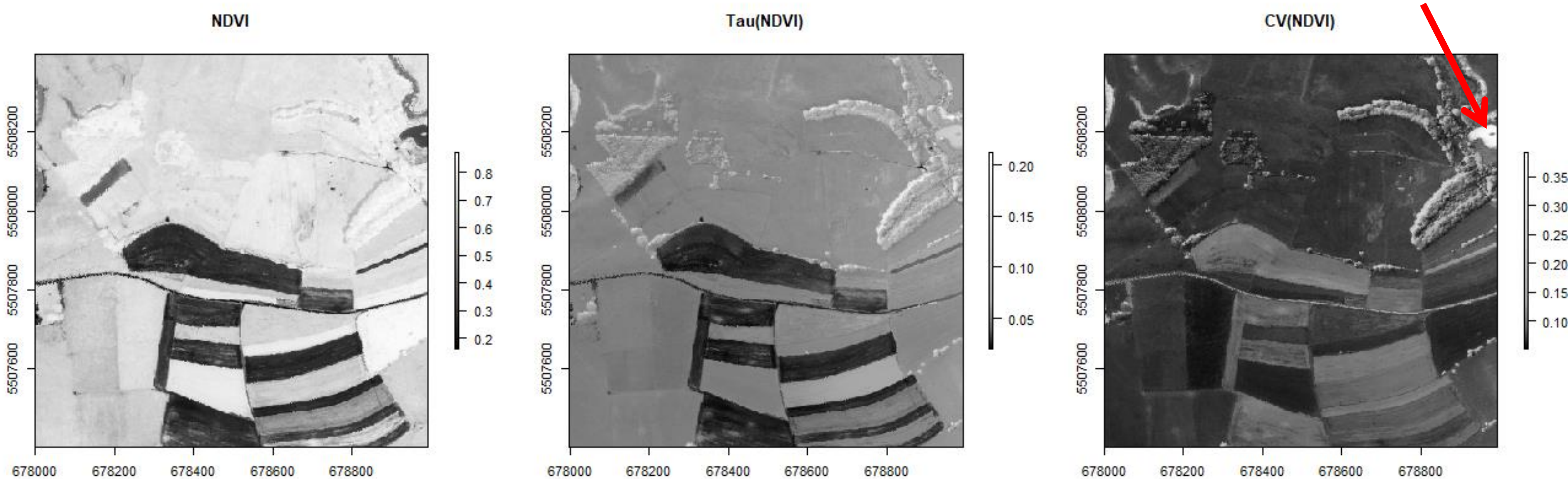
Apply a function to the cells of a SpatRaster

```
CV_SR <- app(c(tau_SR, SRim), fun=function(x) {x[1]/x[2]},
                  filename="CV_SR.tif", overwrite=T)

plot(CV_SR, col=gray(1:100/100), main= "CV(SR)", cex=1)
```

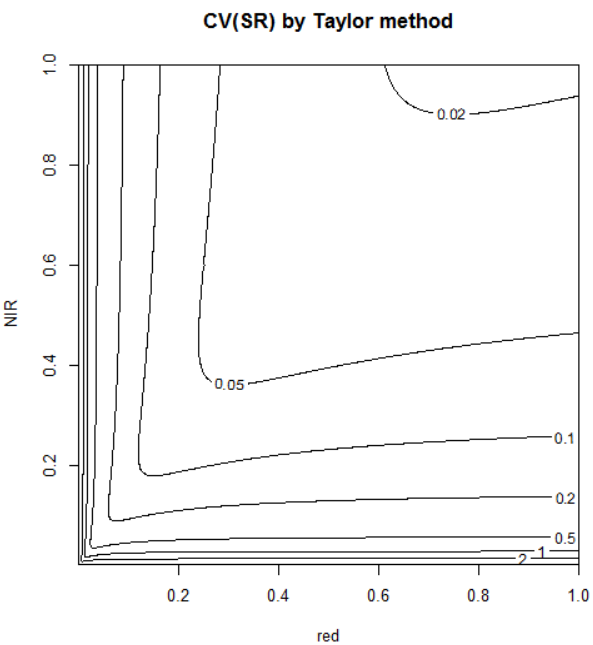


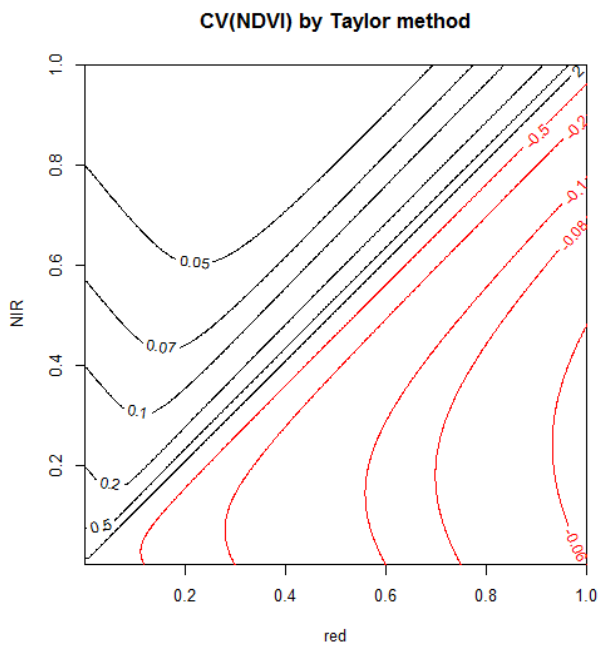
9) NDVI, Tau_NDVI and CV_NDVI

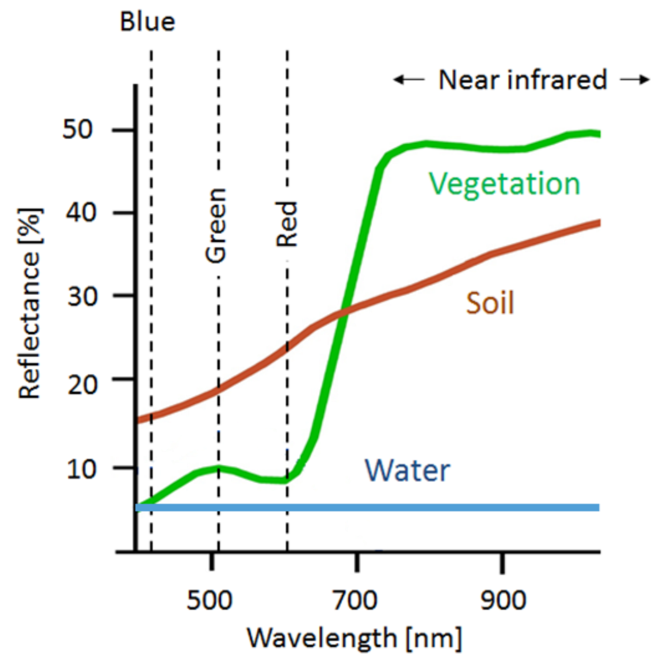
```
NDVIim <- app(false_color, fun=function(x) (x[3]-x[2])/(x[3]+x[2]),
               filename="NDVI.tif", overwrite = T)
plot(NDVIim, col=gray(1:100/100), main="NDVI")

tau_NDVI <- app(false_color, fun=function(x)
  TaylorNDVI(x[2], x[3], 0.025, 0.03, 0.8),
  filename="tau_NDVI.tif", overwrite = T)
```



The bright white spot in previous slide







Must be water!

https://www.google.nl/maps/@49.6987691,5.4824111,515m/data=!3m1!1e3

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Take home

- 1^{st} order Taylor series approximation gives a sum of terms, which can be easily interpreted
- Requires partial differentiation with respect to each considered uncertain input
- Introduces an approximation error in case of non-linear models
- Can be applied on spatial data
- It is fast!

Course dinner

- H41, Herenstraat 41
- 18:30

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