Vegetation Indices – Monte Carlo

Uncertainty propagation in spatial environmental modelling

2024, Sytze de Bruin & Gerard Heuvelink

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Ex. 1 - With which *n* SR results are stable?

```
> set.seed(1234567)
> n <- 200       # sample size
> Z <- matrix(rnorm(2*n),2,n) # 2 rows, n columns; independent draws
> #                             from standard normal distribution
> devs <- t(M %*% Z)
> SRsamp <- MC_SR(0.1, 0.6, devs)
> sd(SRsamp)
[1] 2.837269
> mean(SRsamp)
[1] 6.441485
> Z <- matrix(rnorm(2*n),2,n)
> devs <- t(M %*% Z)
> SRsamp <- MC_SR(0.1, 0.6, devs)
> sd(SRsamp)
[1] 1.559619
> mean(SRsamp)
[1] 6.38809
```

2 replications of a MC experiment producing very different results

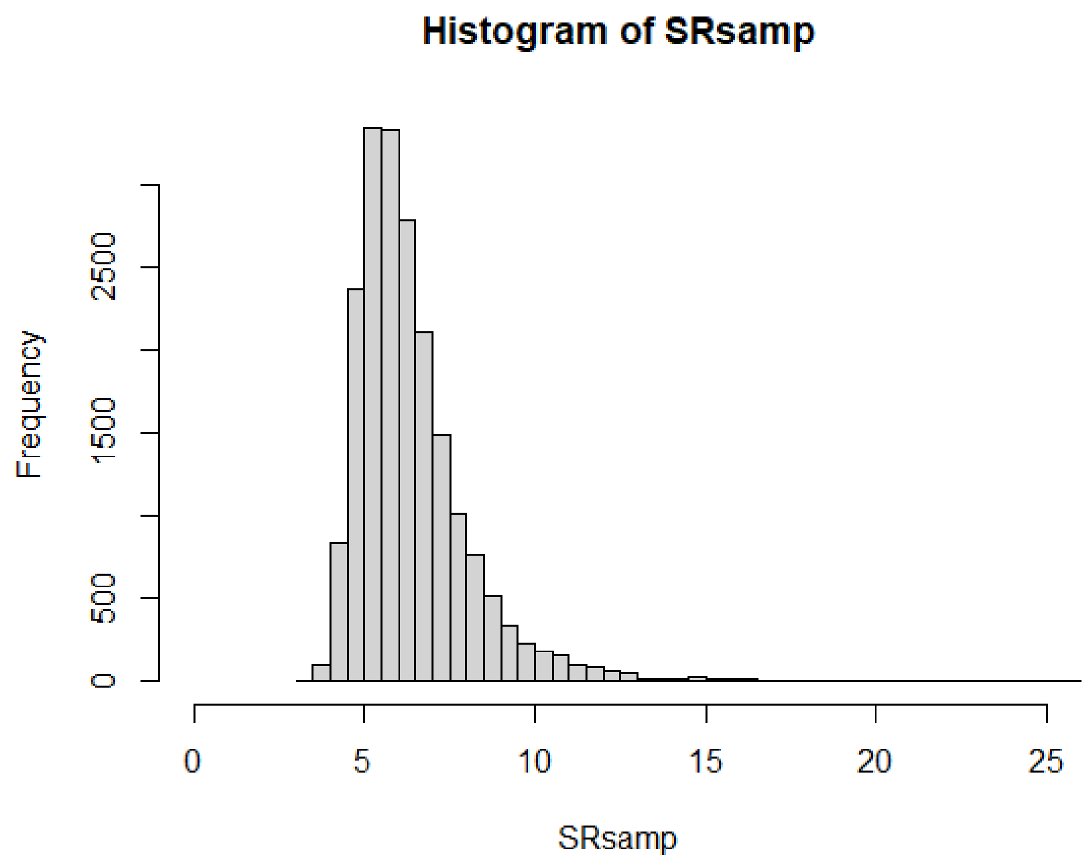
Ex. 1 - Continued

```
> set.seed(1234567)
> n <- 20000     # new sample size
> Z <- matrix(rnorm(2*n),2,n) # 2 rows, n columns; independent draws
> #                                from standard normal distribution
> devs <- t(M %*% Z)
> SRsamp <- MC_SR(0.1, 0.6, devs)
> sd(SRsamp)
[1] 1.97321
> mean(SRsamp)
[1] 6.425931
> Z <- matrix(rnorm(2*n),2,n)
> devs <- t(M %*% Z)
> SRsamp <- MC_SR(0.1, 0.6, devs)
> sd(SRsamp)
[1] 1.931545
> mean(SRsamp)
[1] 6.39086
```

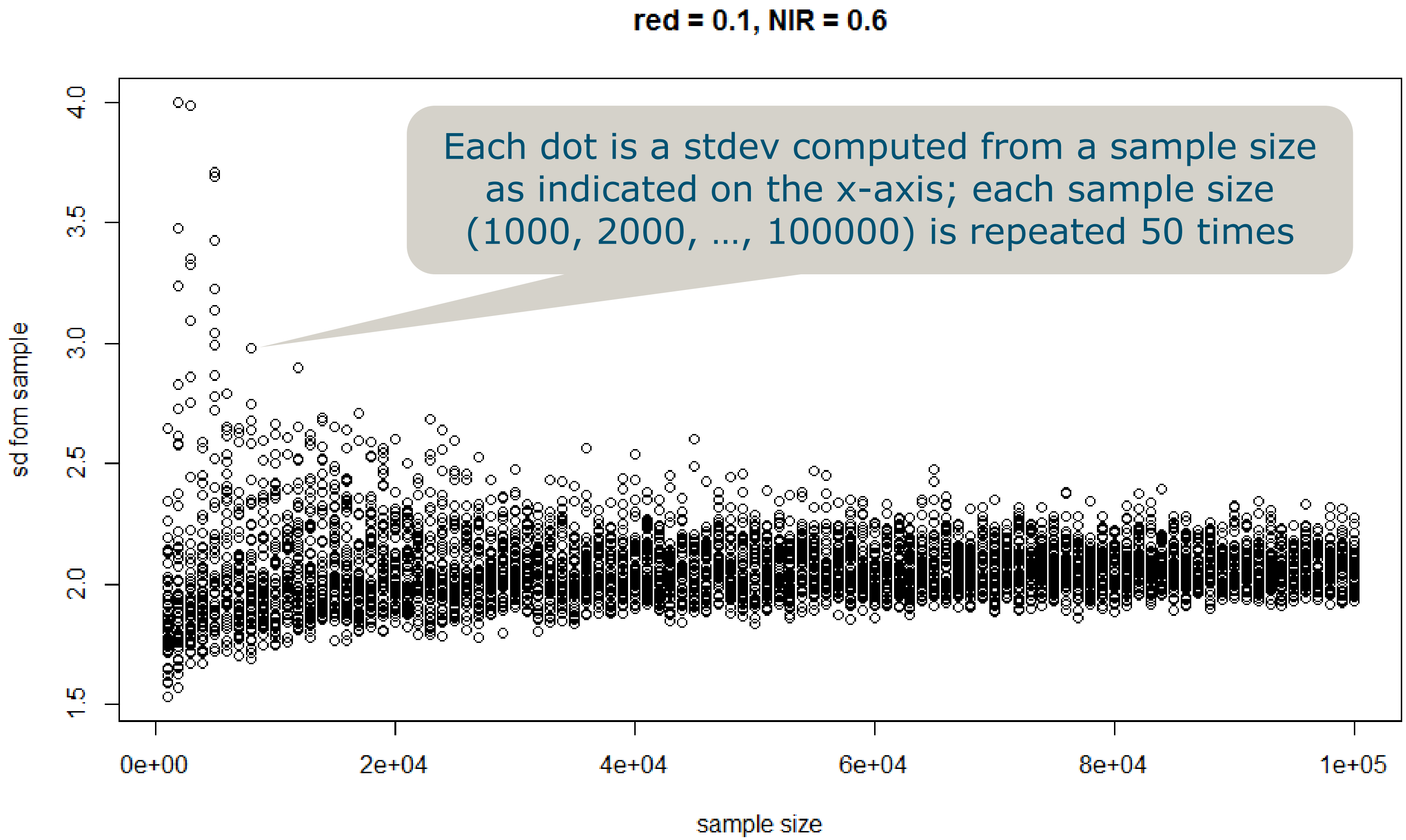
Standard deviation stabilizes

Ex. 1 – distribution of uncertain SR (n=20000)

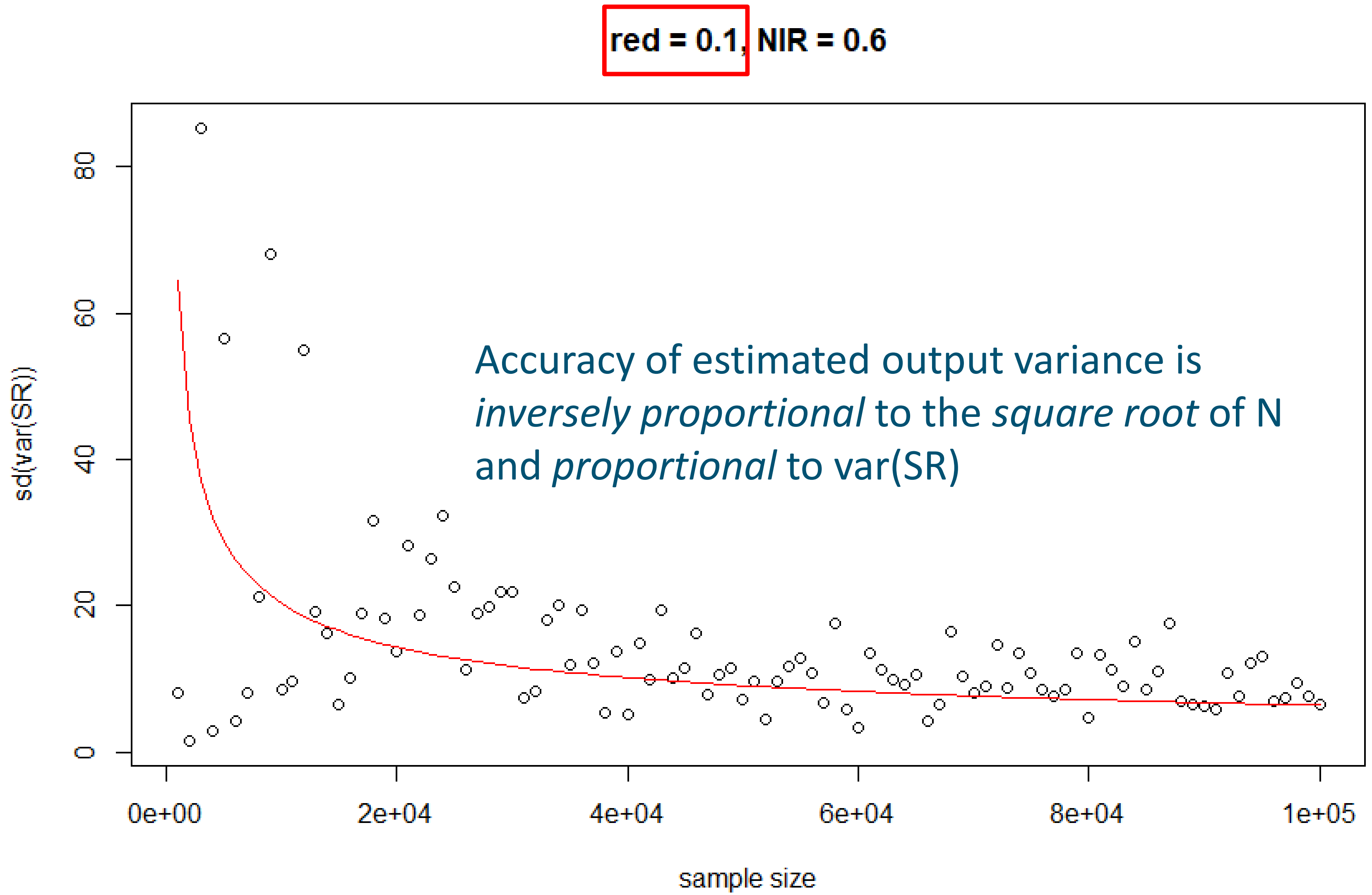
```
hist(SRsamp, breaks = 100, xlim=c(0, 25))
```



Ex. 2 – Plot many stdevs. versus sample size



Ex. 2 – sd(var(SR)) versus sample size



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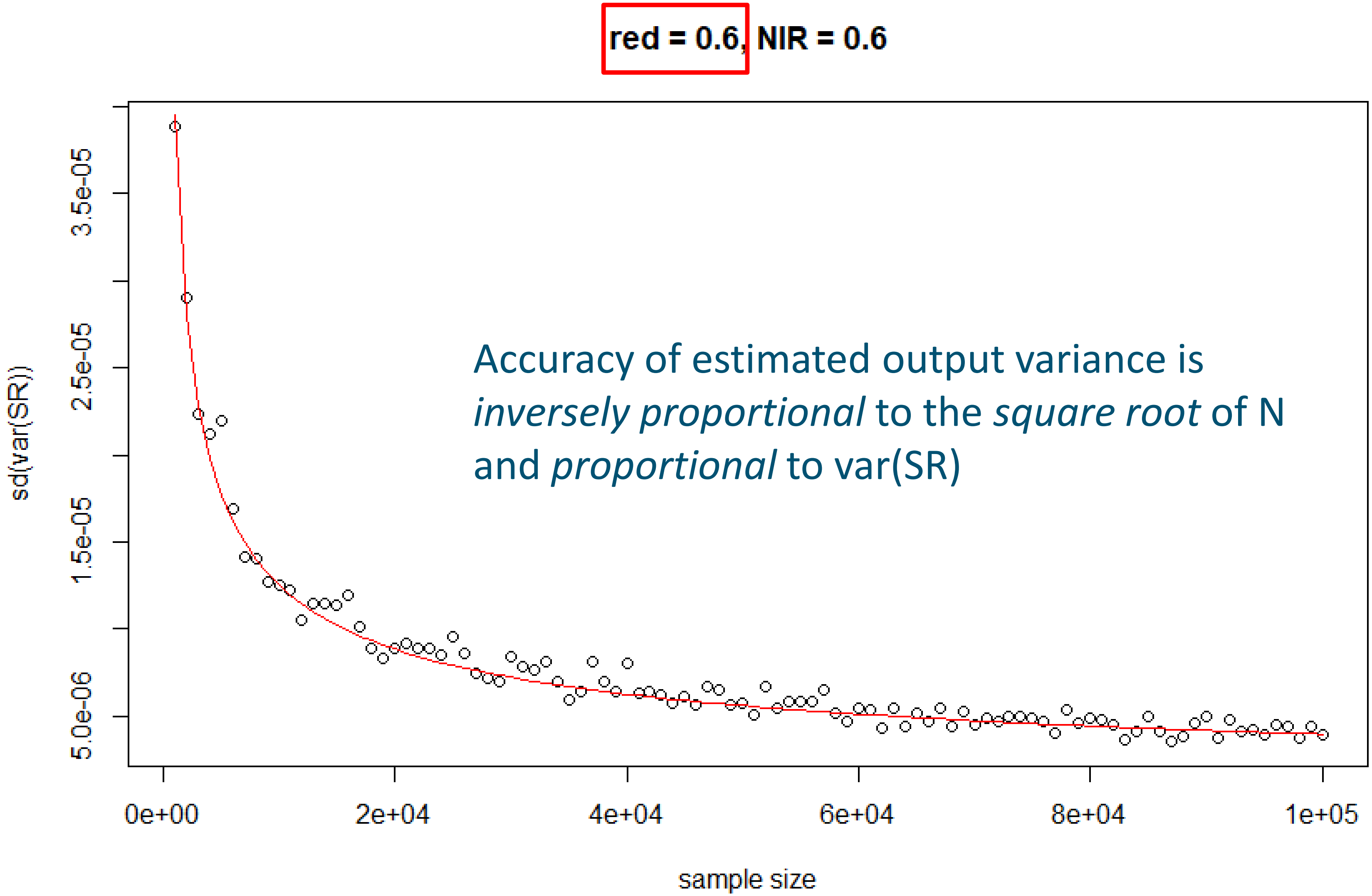
Ex. 2 – sd(var(SR)) versus sample size



Accuracy of estimated output variance is *inversely proportional* to the *square root* of N and *proportional* to var(SR)

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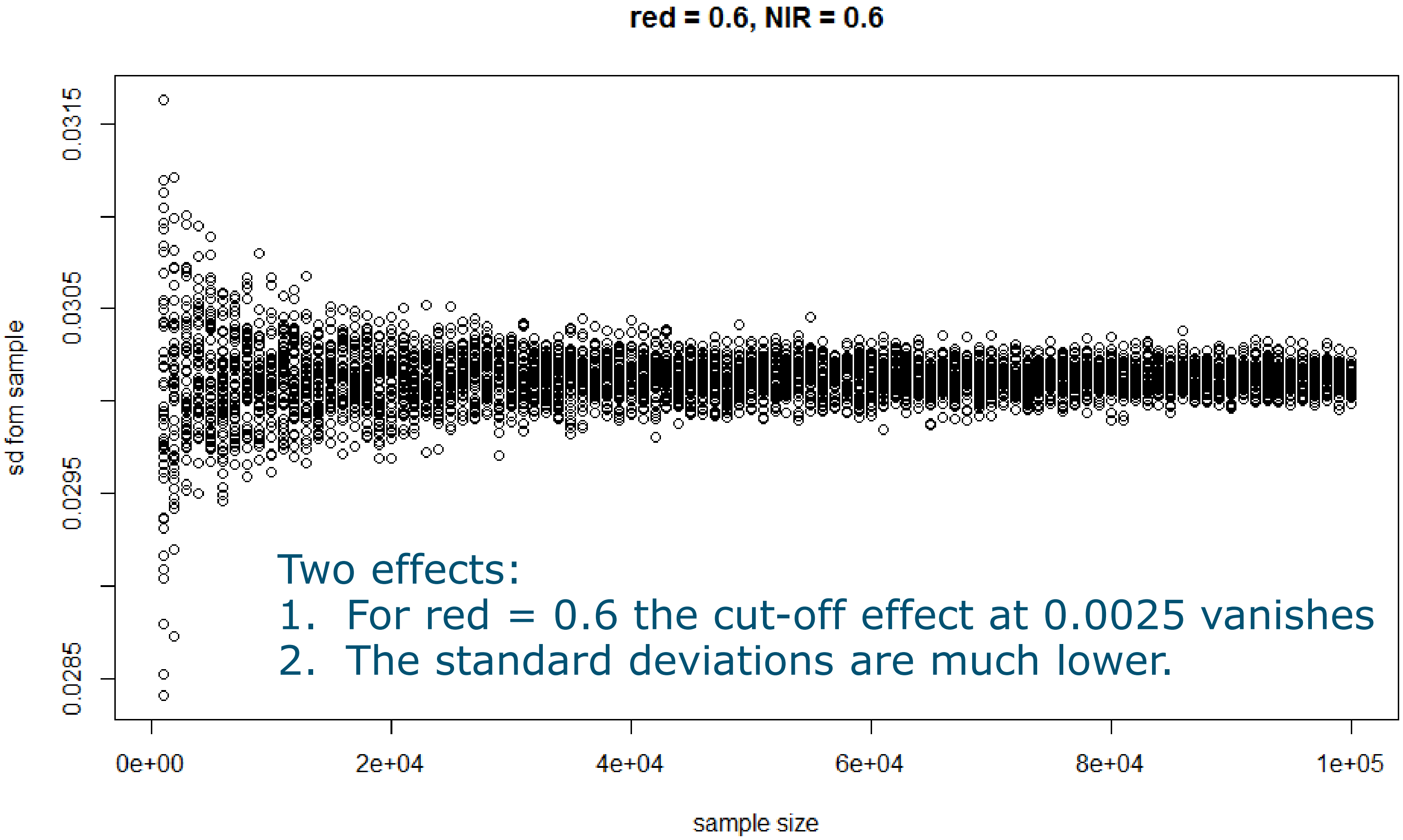
Ex. 2 – Plot many stdevs. versus sample size

Is the number of required runs affected when using higher (e.g. 0.6) values for the reflectance in the red band? Explain.



Two effects:

1. For red = 0.6 the cut-off effect at 0.0025 vanishes
2. The standard deviations are much lower.

Exercise 3 - Function for NDVI

```
MC_sd_NDVI <- function(x) {
  RED <- x[1]+devs[,1]
  NIR <- x[2]+devs[,2]
  Den <- RED + NIR
  # ignore sampled denominators near zero
  Den[abs(Den) < 0.0025] <- NA
  samp <- (NIR-RED)/Den
  return(sd(samp, na.rm=T))
}

MC_sd_NDVI(c(0.1, 0.6))
[1] 0.053
```

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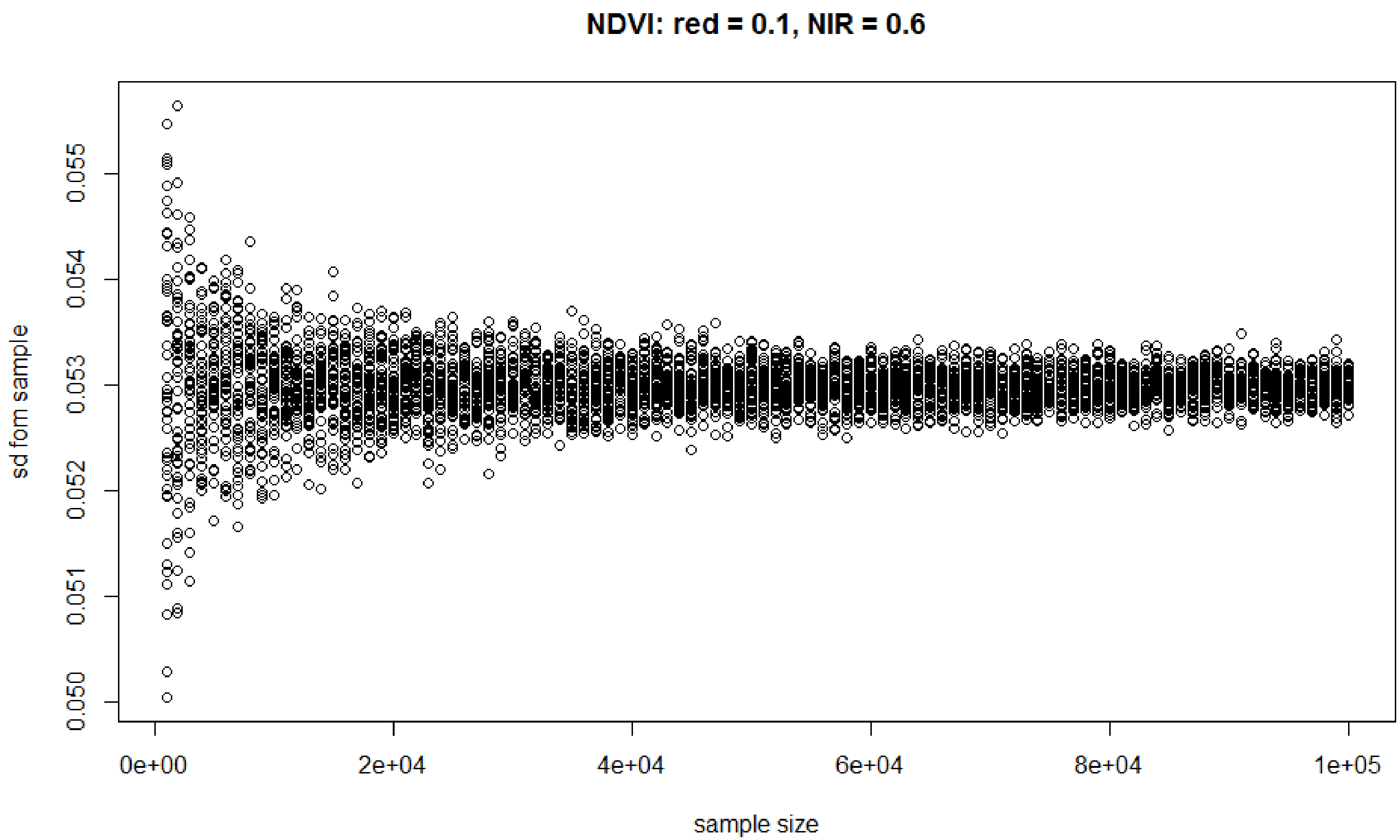
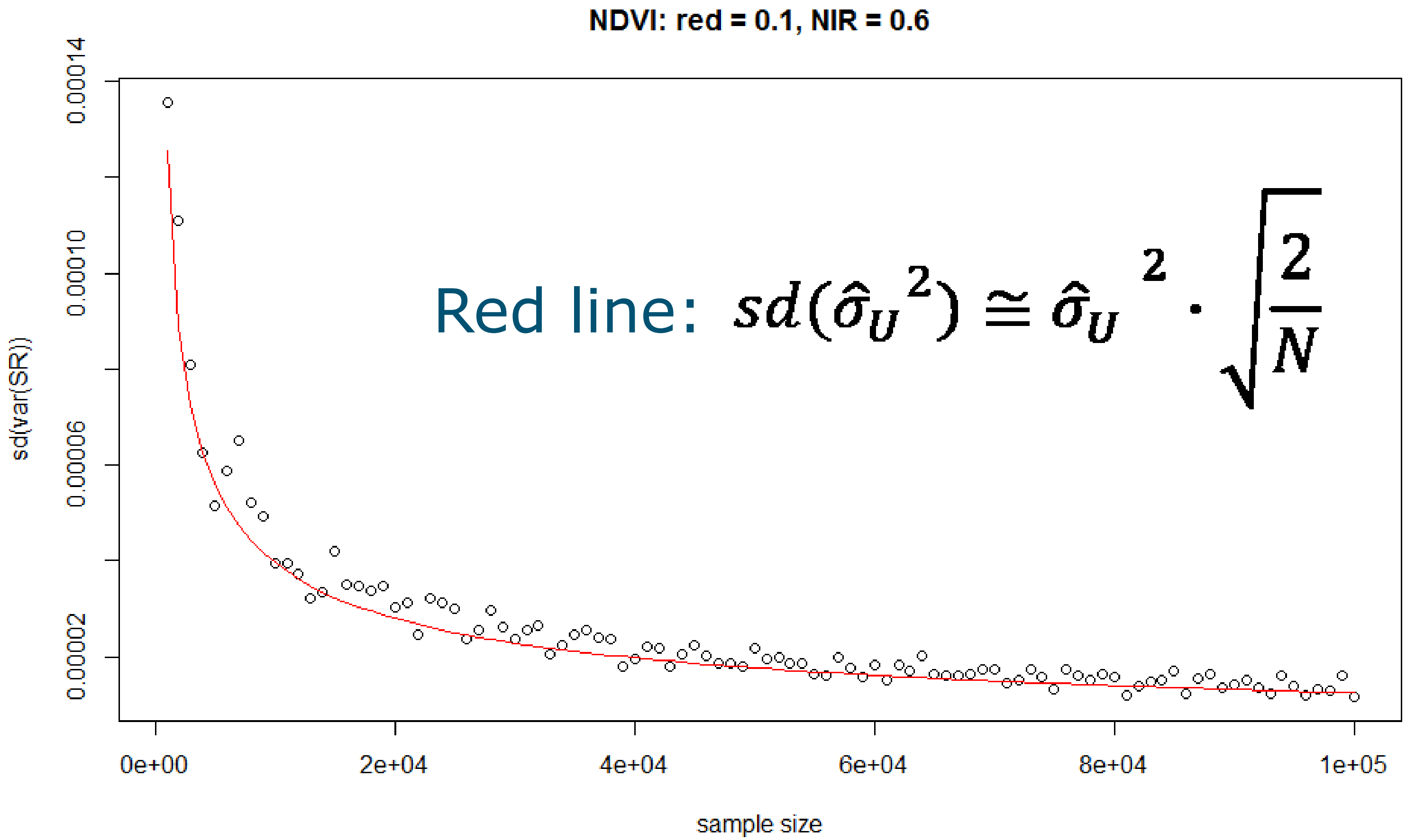
Ex. 4 – Plot many stdevs. versus sample size



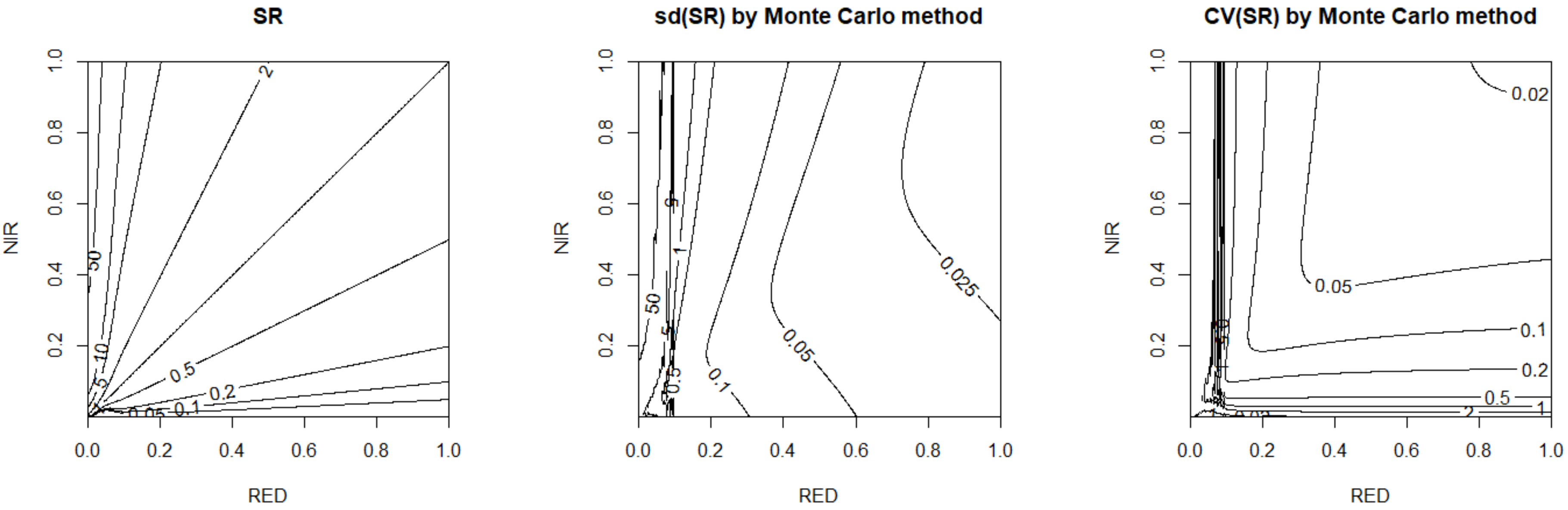
Ex. 4 - sd(var(SR)) versus sample size



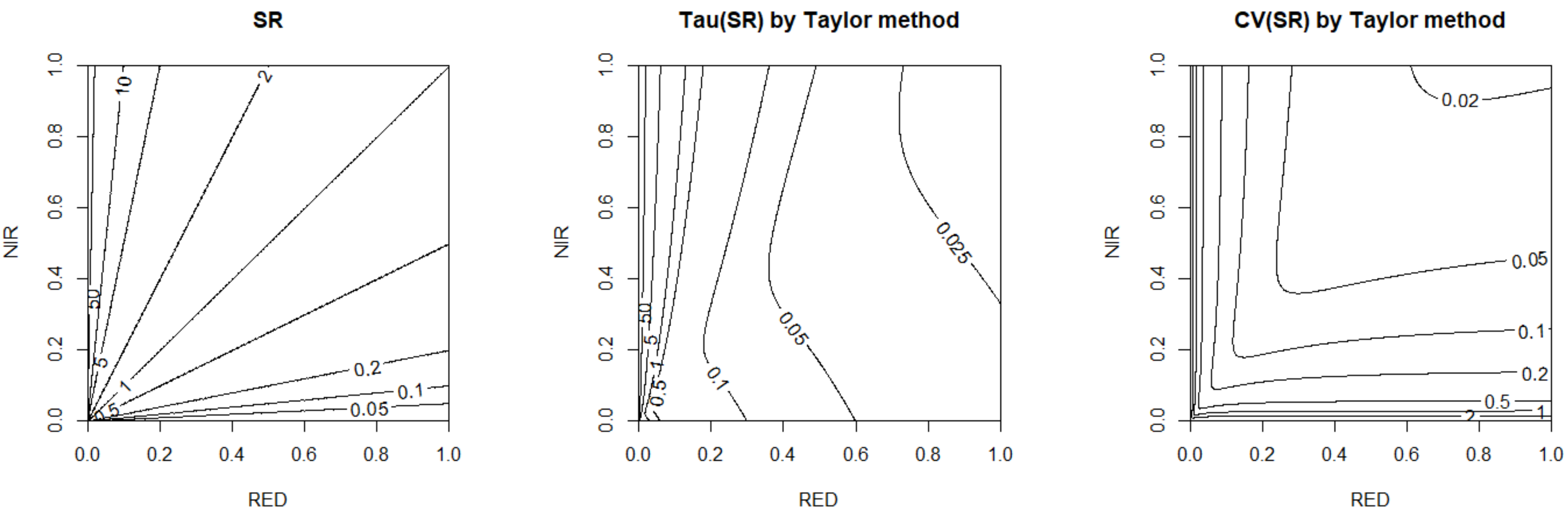
Red line: $sd(\hat{\sigma}_U{}^2) \cong \hat{\sigma}_U{}^2 \cdot \sqrt{\frac{2}{N}}$

For approximately normally distributed variable

Ex. 5 - Contour plot SR – Monte Carlo (n = 20000)



Ex. 5 - Contour plot SR – 1st order Tayor



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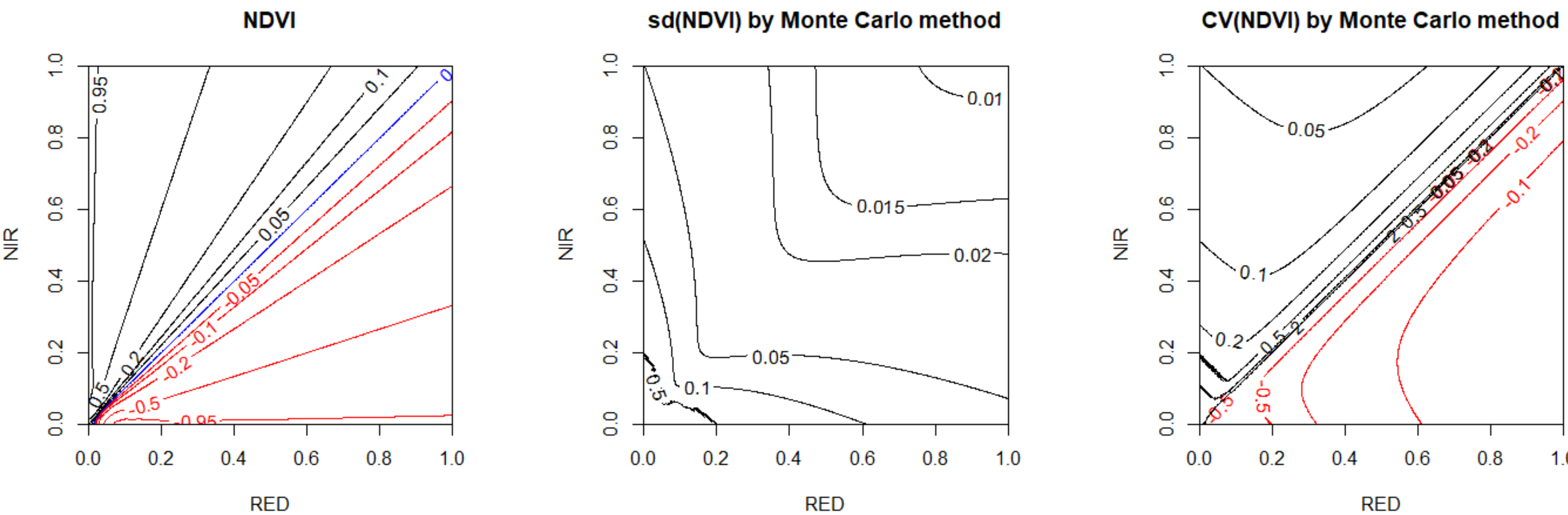
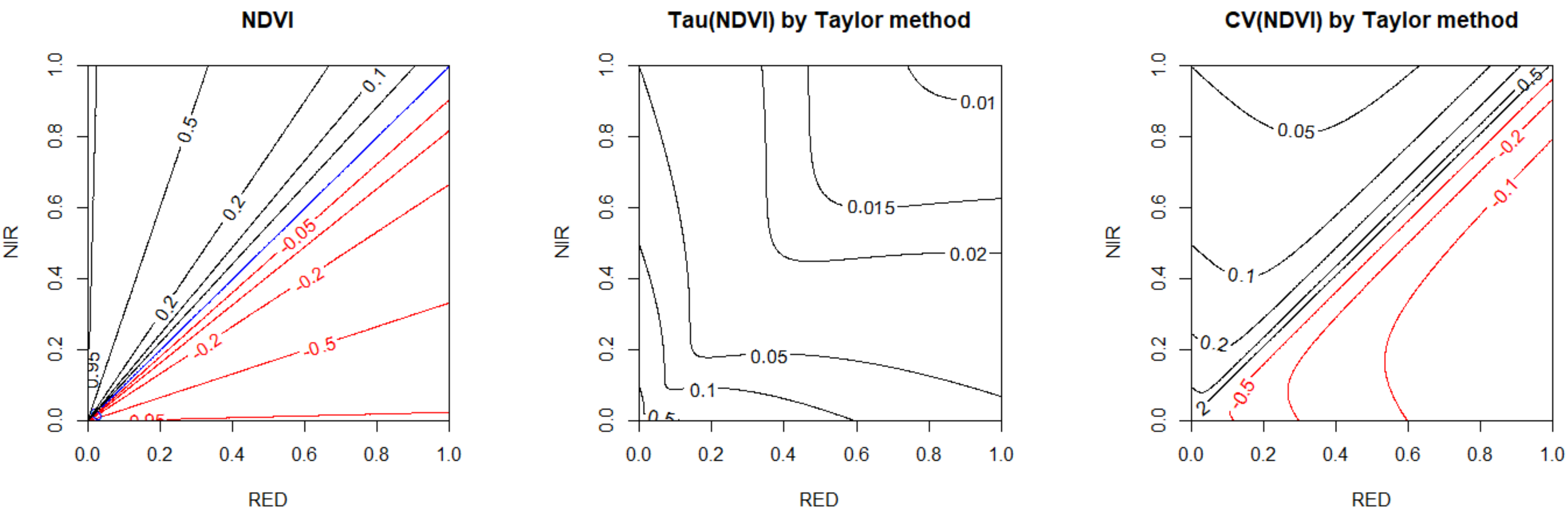
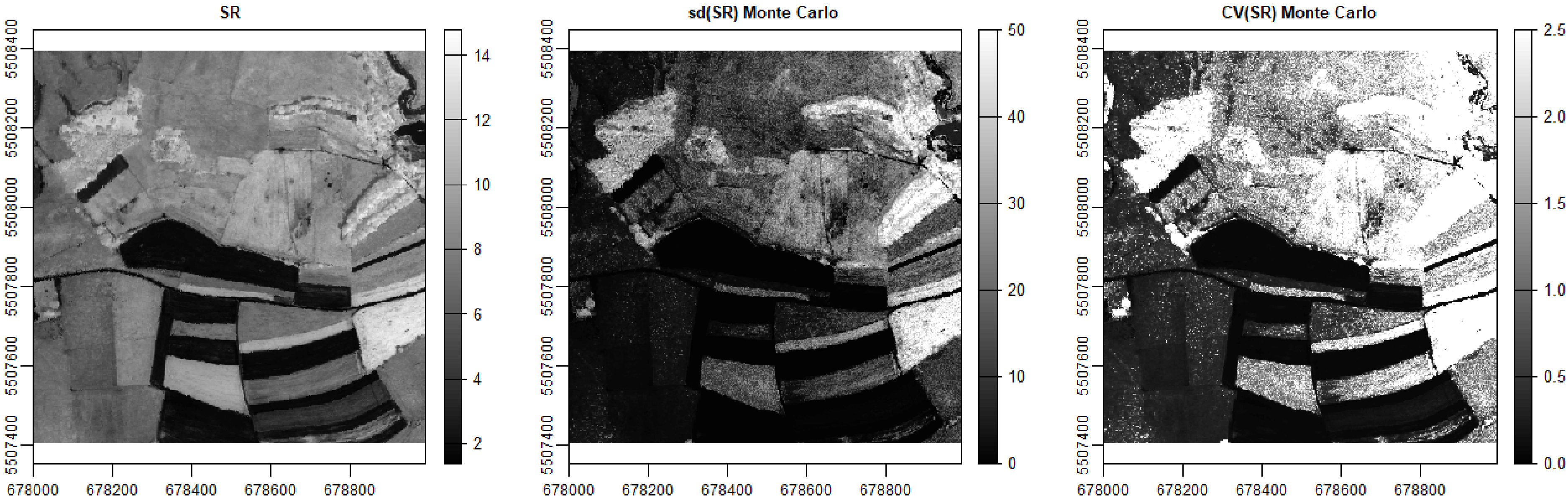
Ex. 5 - Contour plot NDVI – Monte Carlo (n = 20000)



Ex. 5 - Contour plot NDVI – 1st order Tayor



Ex. 6 - SR: Monte Carlo (sd)



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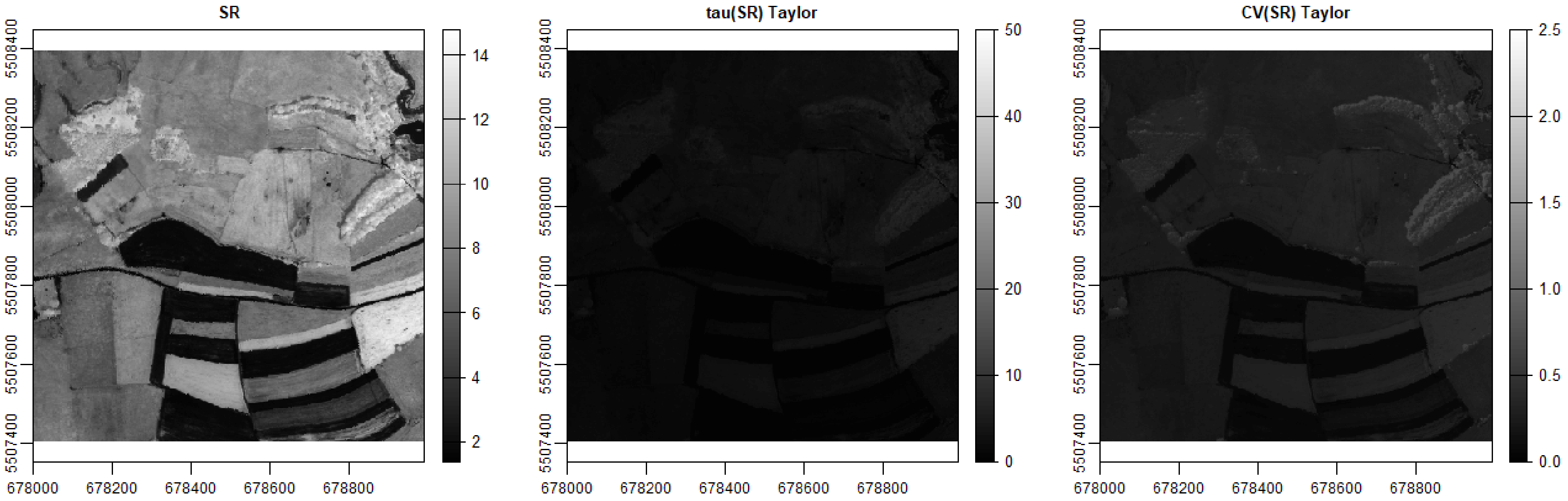
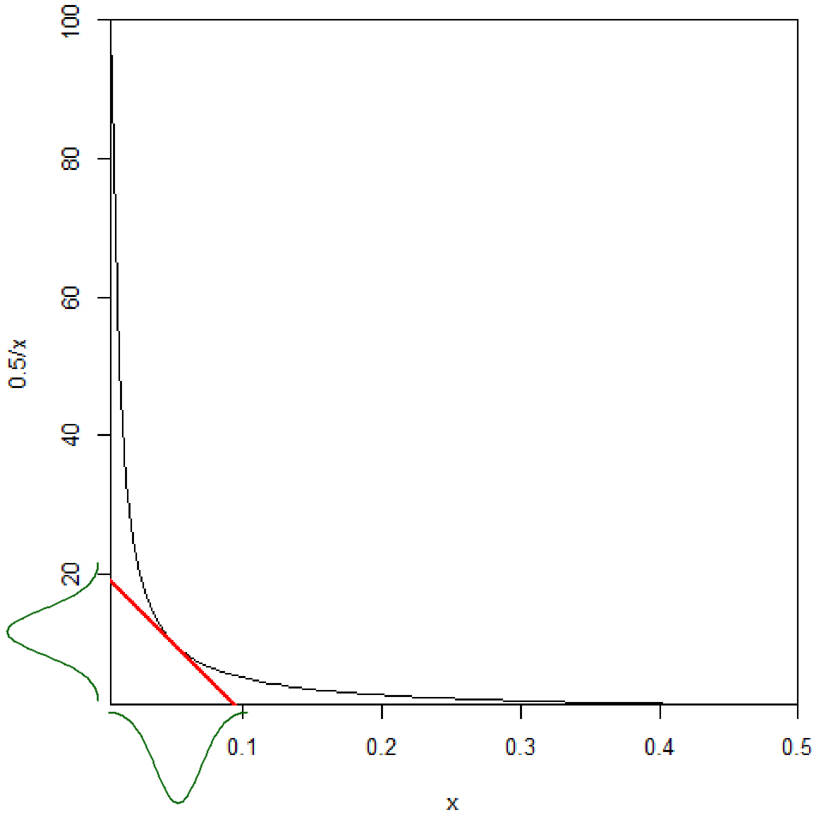
Ex. 6 - SR: 1st order Taylor (tau)



Ex. 6 – Partial explanation



Plot of 0.5/x in the range 0 – 0.5

The larger part of the image is covered by vegetation with low reflectance in the red band (the denominator). Linearization of SR in this region underestimates uncertainty propagation.

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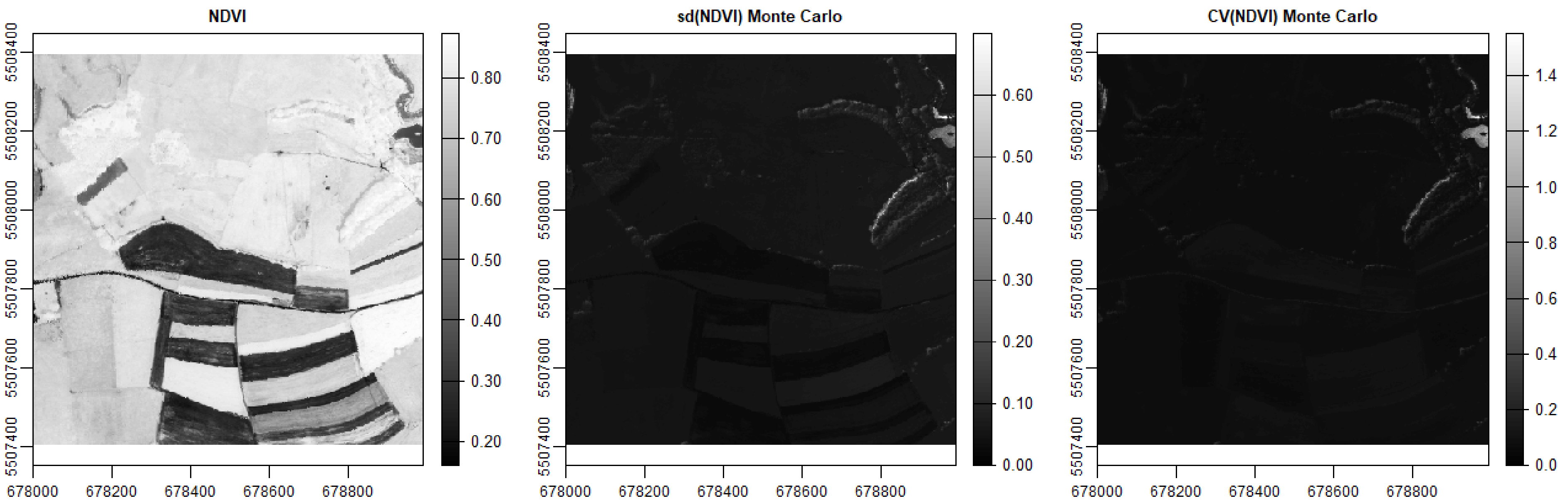
Ex. 6 – NDVI: Monte Carlo (sd)



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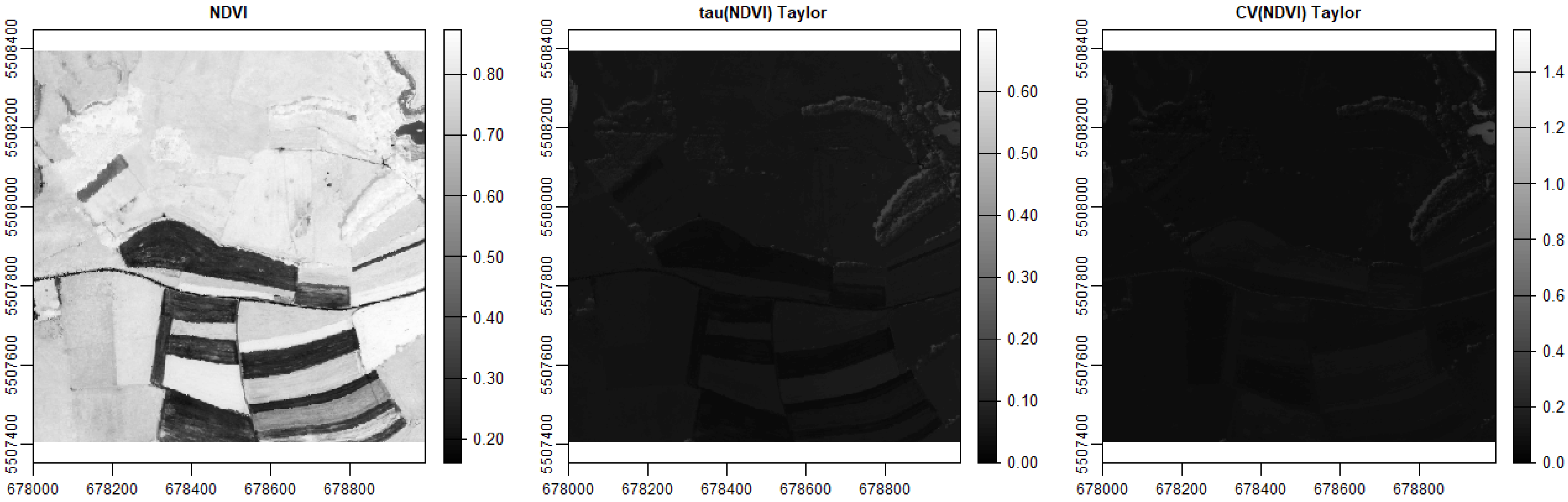
Ex. 6 – NDVI: 1st order Taylor (tau)



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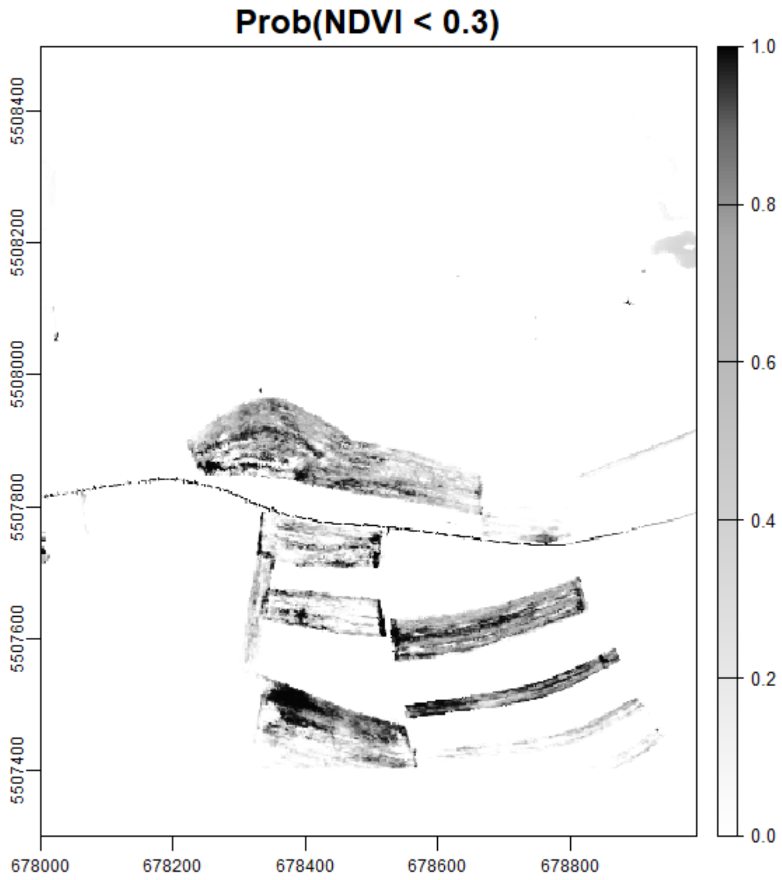
Ex. 7 – Threshold on probability – Monte Carlo

```
prob03MC <- app(false_color, fun = function(x)
  mean(ifelse(MC_NDVI(x[2], x[3], devs) < 0.3,1, 0)),
  filename="prob03MC.tif", overwrite = T)
```



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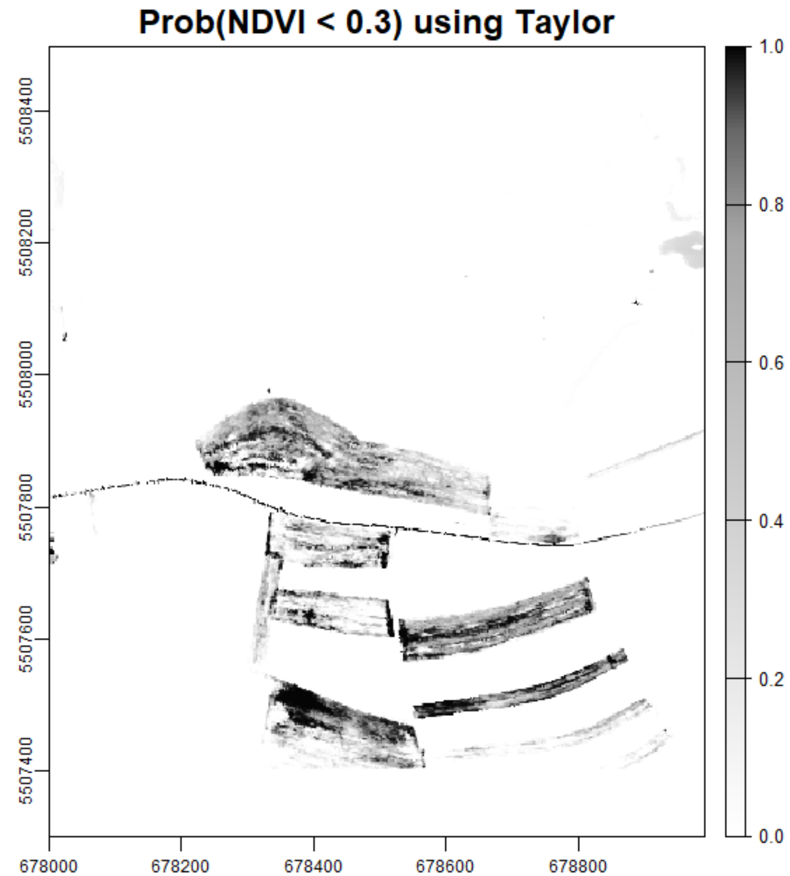
6) Taylor: assume normal distribution

```
NDVIim <- rast("NDVI.tif")
tau_NDVI <- rast("tau_NDVI.tif")
TaylorProb <- app(c(NDVIim, tau_NDVI),
                  fun = function(x) pnorm(0.3, x[1], x[2]))
```



But NDVI is no linear function of the inputs

Take home

- Monte Carlo works for any model, including black box models
- Produces sample of the *output distribution* (not just variance)
- The “approximation error” can be reduced by increasing the sample size
- It is computationally demanding; particularly for complex models
- It can be applied on spatial data; so far we did not consider spatial correlation (recall discussion @morning lecture)
- There are smarter sampling methods than random sampling (e.g., stratified random sampling, Latin hypercube sampling)