# Bayesian calibration for model parameter uncertainty

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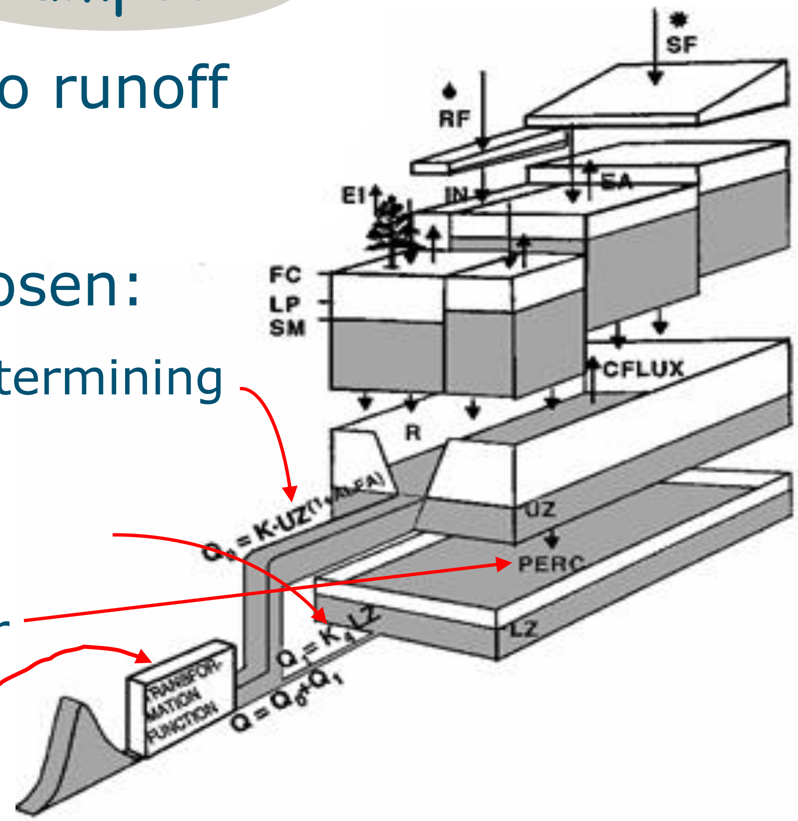
# TUWmodel: conceptual rainfall runoff model

lumped

- Components contributing to runoff
- Many parameters
- For calibration we have chosen:
  - lsuz: storage threshold determining very fast response [mm]
  - k1: fast component [day]
  - cperc: percolation to lower zone [mm/day]
  - croute: free parameter used for spreading outflow over time [day²/mm]
- Remaining structural (2 pars) + discharge measurement uncertainty (1 par.)




$\theta$

# Activity today's afternoon

- Run TUWmodel using a default set of parameter values
- Plot given prior distributions to express a priori belief about possible values for selected model parameters
- Update these prior distributions by MCMC using a time series of measured outflows
- Obtained posterior distributions *and* residual variance are measures of parameter uncertainty and structural model uncertainty
- Uncertainty analysis by simultaneously considering parameter, model-structural and observational error in the discharge;
- Assess whether parameter distributions obtained from Bayesian calibration improved model fit

# Model structural uncertainty and discharge observation uncertainty

- Assume that these sources of uncertainty are multiplicative:

$$Y = H \cdot \boxed{e^{\varepsilon}} \cdot \boxed{e^{\eta}}$$

where $Y$ is the measured discharge, $H$ is the TUWmodel output and the means of $e^{\varepsilon}$ and $e^{\eta}$ are forced to one.

- Log-transformation gives:

$$\log(Y) = \log(H) + \varepsilon + \eta$$

where:

$$\varepsilon(t) = \beta_0 + \beta_1 \cdot \varepsilon(t-1) + \delta(t)$$

$$\delta(t) \sim N\left(0, \sigma_\delta^2\right)$$

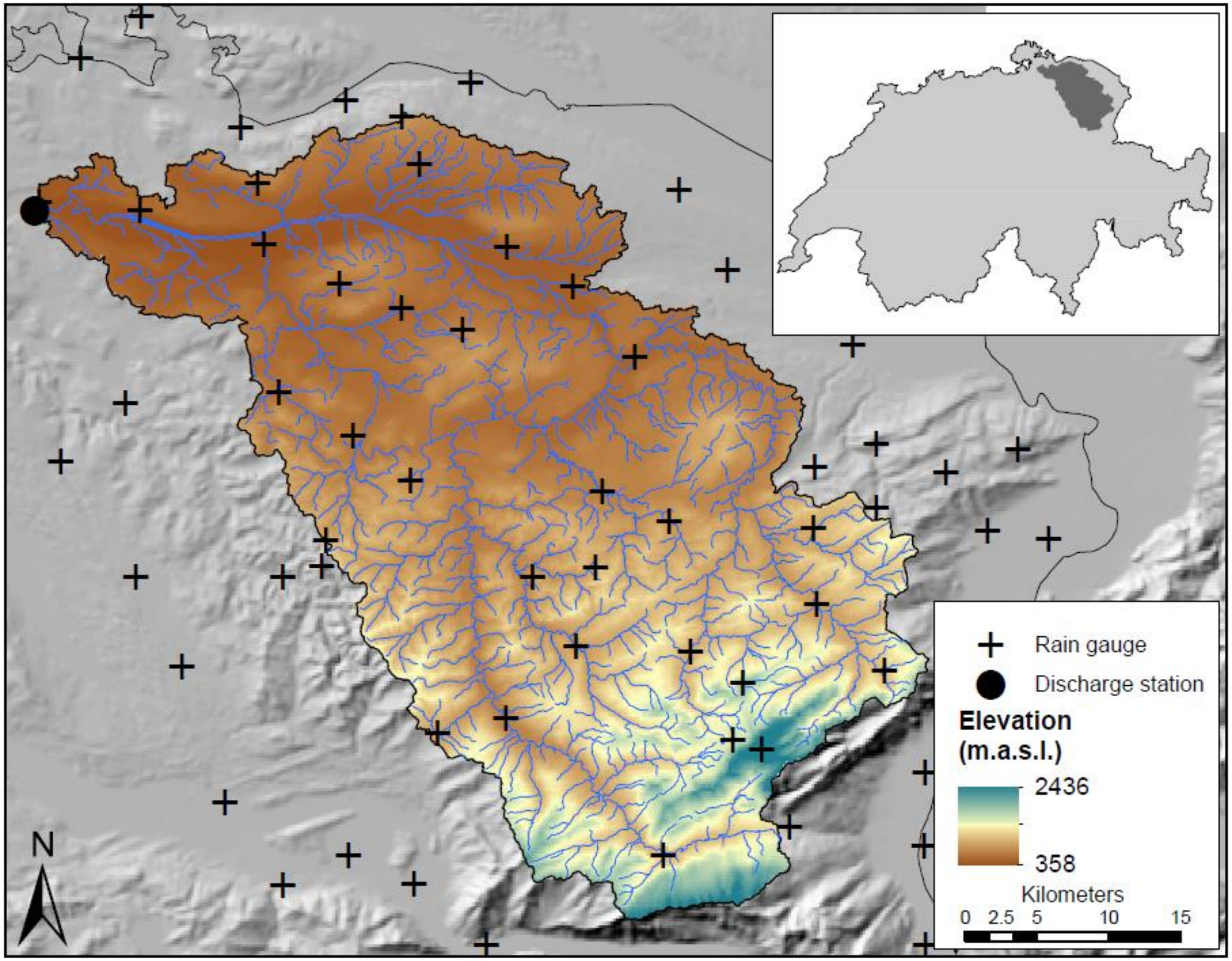
$$\eta(t) \sim N\left(\mu_\eta, \sigma_\eta^2\right)$$

$$\beta_0 = -\frac{1}{2} \cdot \frac{1-\beta_1}{1-\beta_1^2} \cdot \sigma_\delta^2$$

$$\mu_\eta = -\frac{1}{2}\sigma_\eta^2$$

For more details see Wadoux et al. (2020) in the literature folder

# Thur river basin (same as Wadoux et al. 2020)



# Provided data

| | |
|---|---|
| `agERA5prec.txt` | Spatially aggregated daily precipitation over the study area retrieved from agERA5 |
| `evap.txt` | Spatially aggregated daily evaporation over the study area (MeteoSwiss) |
| `temp.txt` | Spatially aggregated daily mean temperature (MeteoSwiss) |
| `runoff.txt` | Daily cumulative discharge data for the period 2004-2011 from the Swiss Federal Office for the Environment (FOEN). |
| `dates.txt` | Date strings for the period 01/01/2004 – 31/12/2011 |

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# Bayesian calibration for model parameter uncertainty

## Uncertainty propagation in spatial environmental modelling

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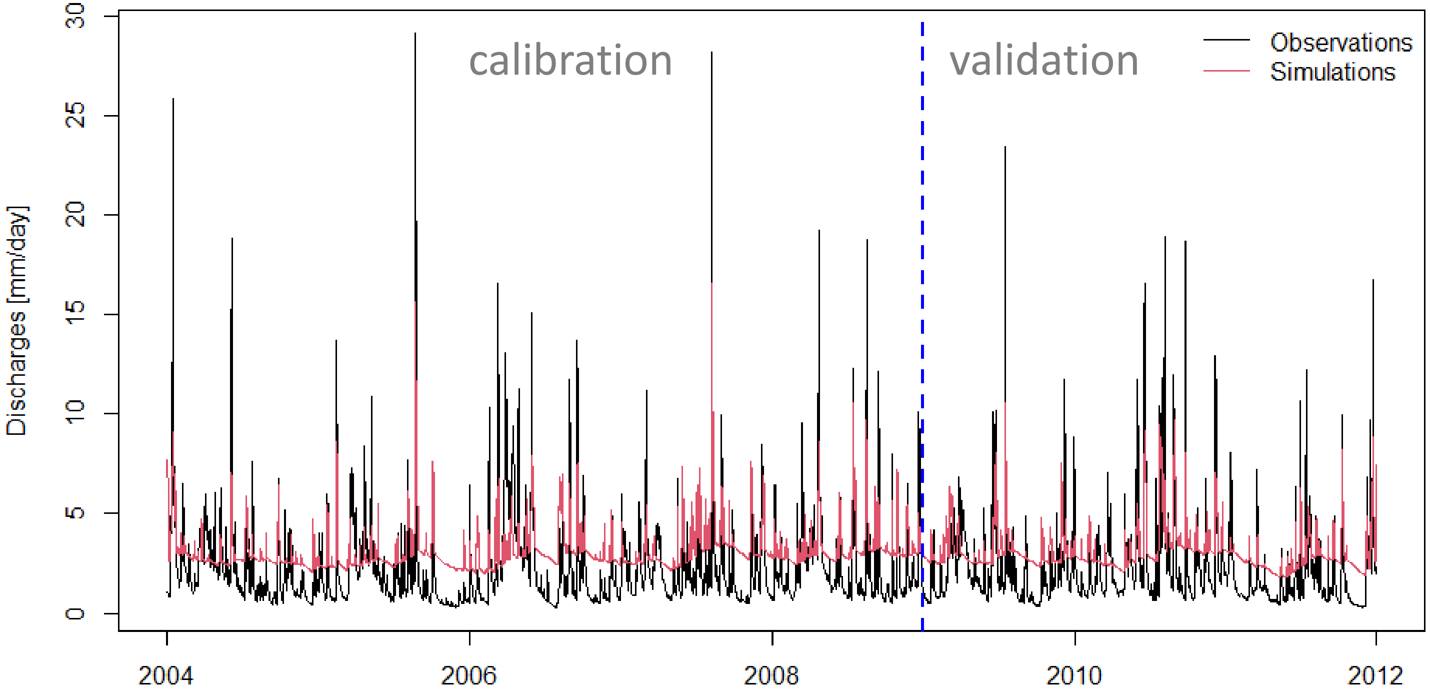
# Q1) Are the discharges predicted by the model biased?




```
summary(relResidsDflt)
Min.   :-74.38
1st Qu.: 24.98
Median : 93.34
Mean   :124.02
3rd Qu.:196.55
Max.   :786.07
```

The model systematically over predicts outflow. Potential causes are: water leaving the basin differently, overestimated precipitation within the catchment, or both.

# Priors for parameters provided in Parajka, et al. 2007

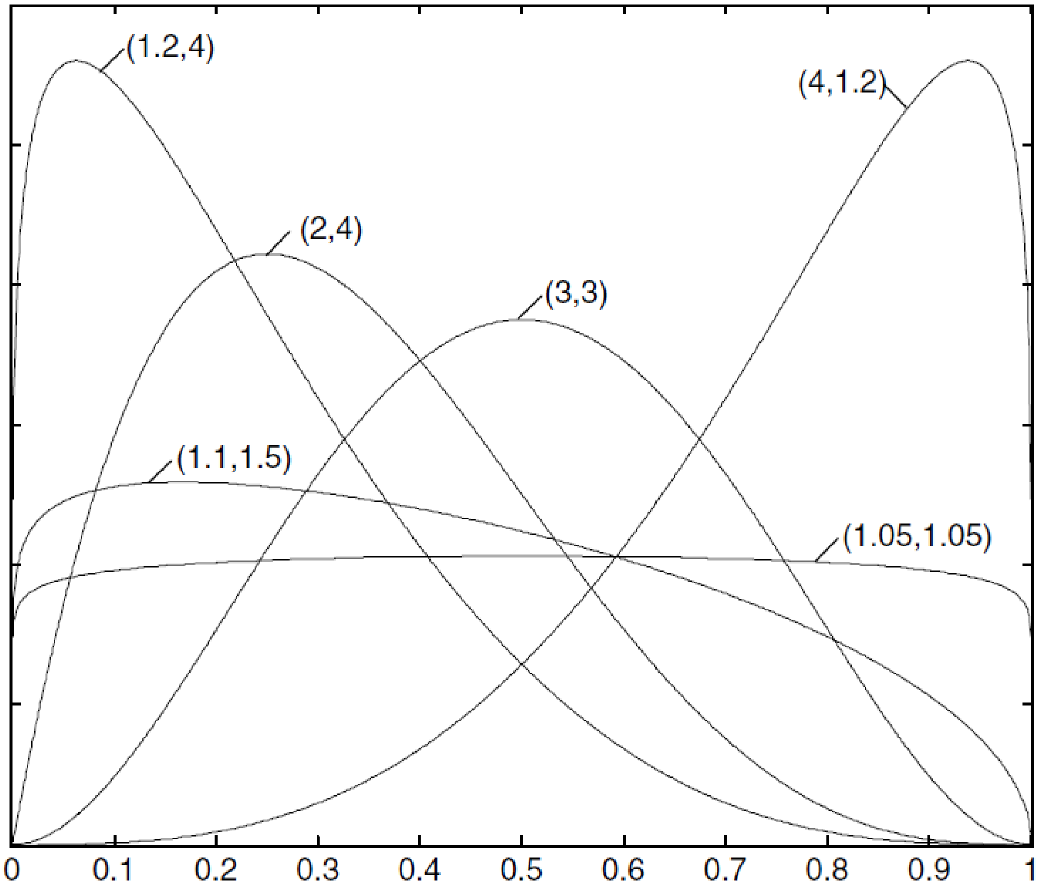
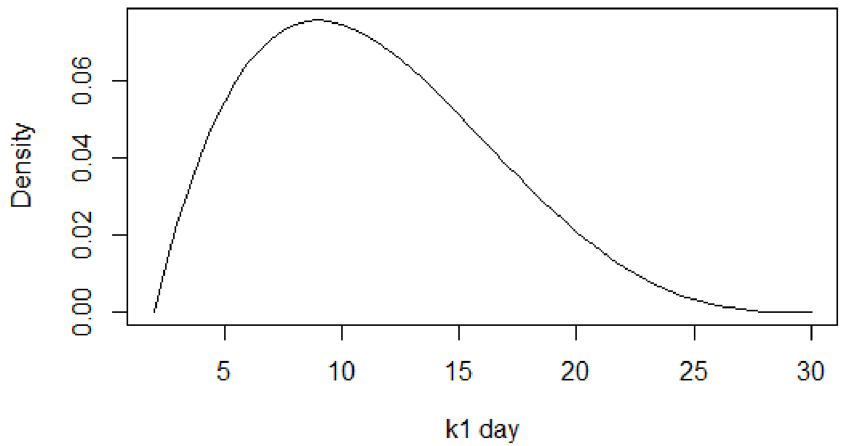
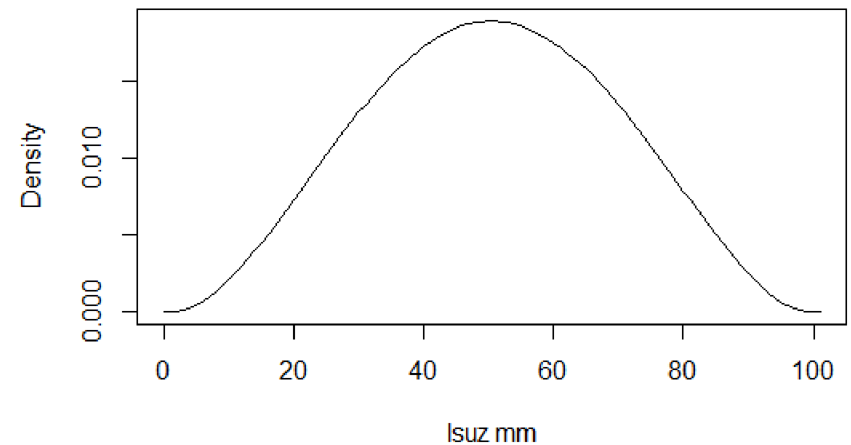


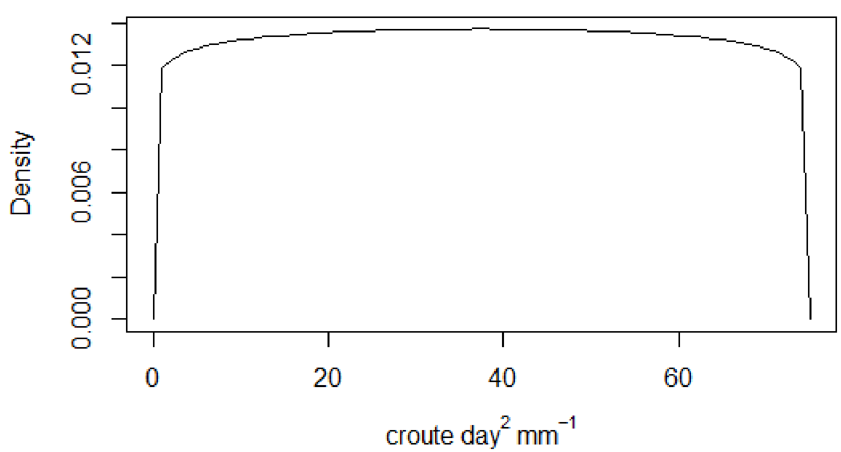

Figure 2. Shapes of the Beta functions used for defining the *a priori* distributions of the model parameters. Number in parentheses are $u$ and $v$ (see Table I)

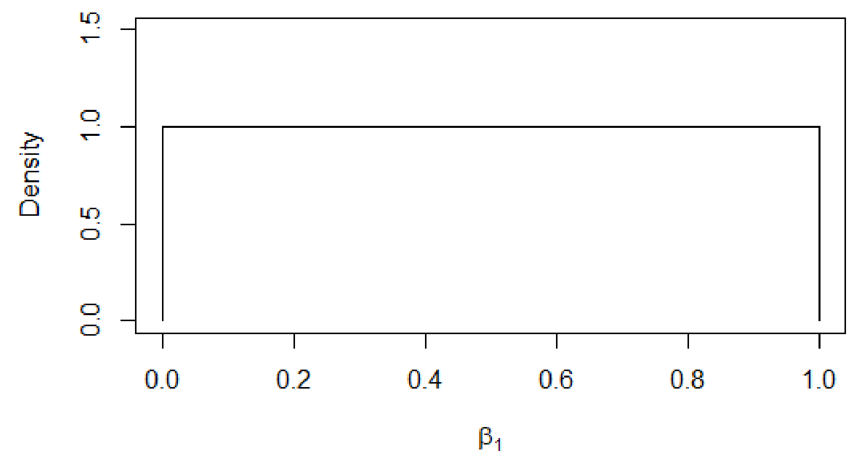
Aim: update priors using observed discharges

# Q2) Which priors seem to be most informative?




The "peaked" priors of k1, lsuz and cperc are more informative than the flat priors of croute and $\beta_1$



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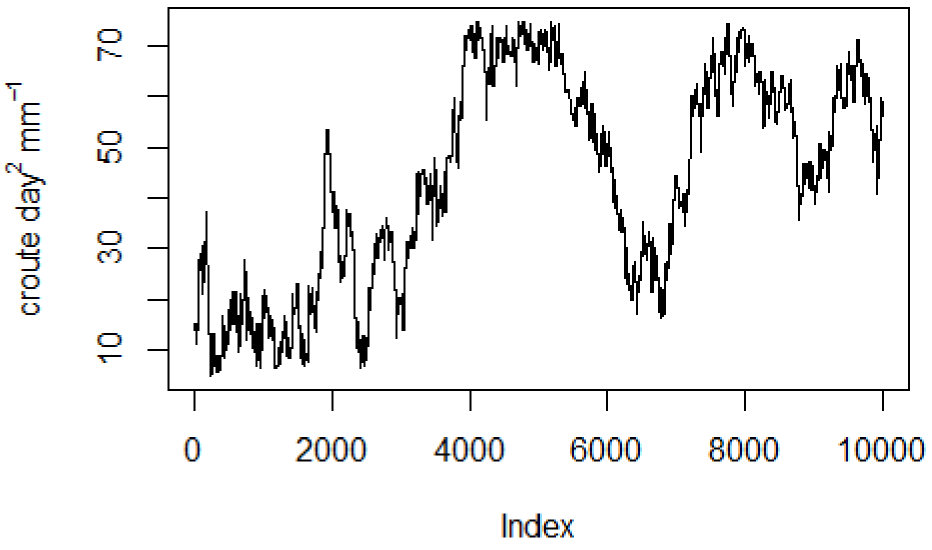
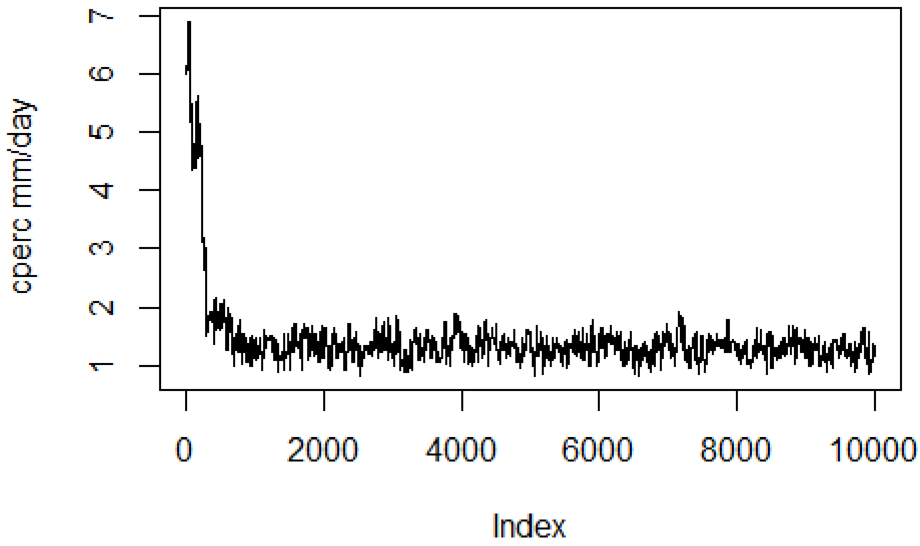
# Q3) Striking behaviour in trace plots?



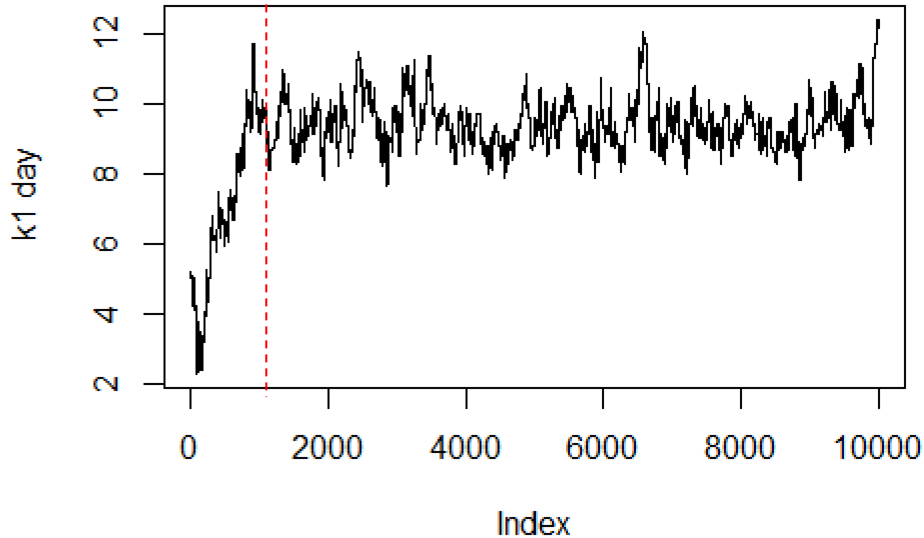

*Serial correlation in for example croute*

# Q3) Striking behaviour in trace plots?




*The initial proposal for cperc was way off (for example), also note “platforms, representing a sequences of runs in which new proposals were not accepted*

# Q4) Why subsample?




*First, a burn-in sequence is removed to circumvent the effect of the starting values of the initial proposal. Next, the sample is thinned at an interval of 100 to get around the sequential correlation effect. The latter is important if you want to use the output in an uncertainty propagation analysis.*

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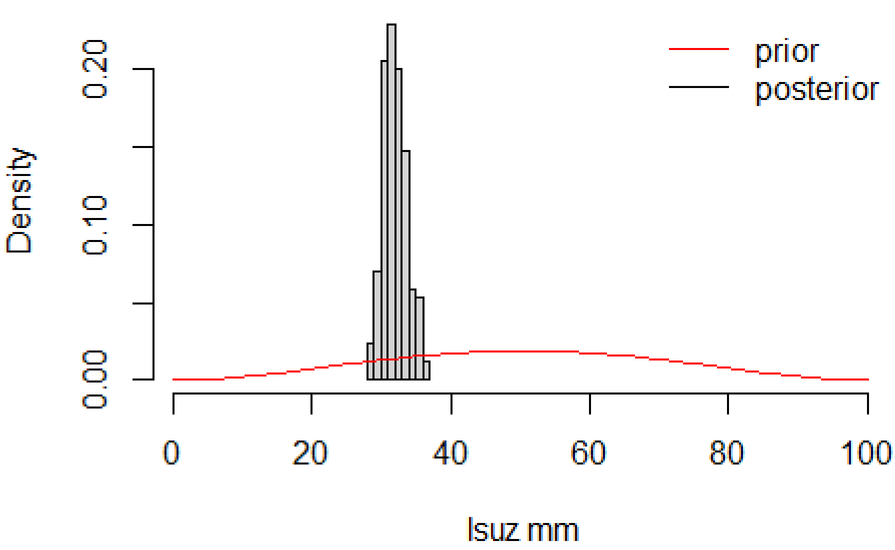
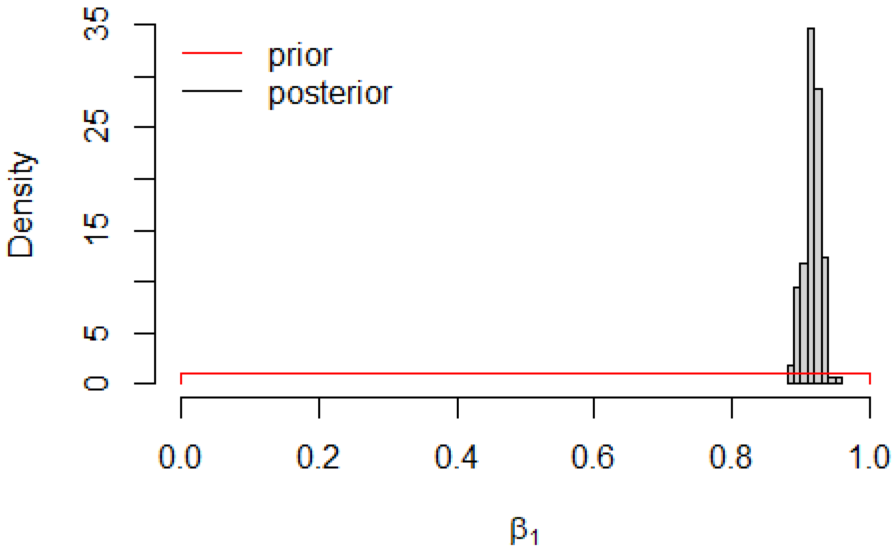
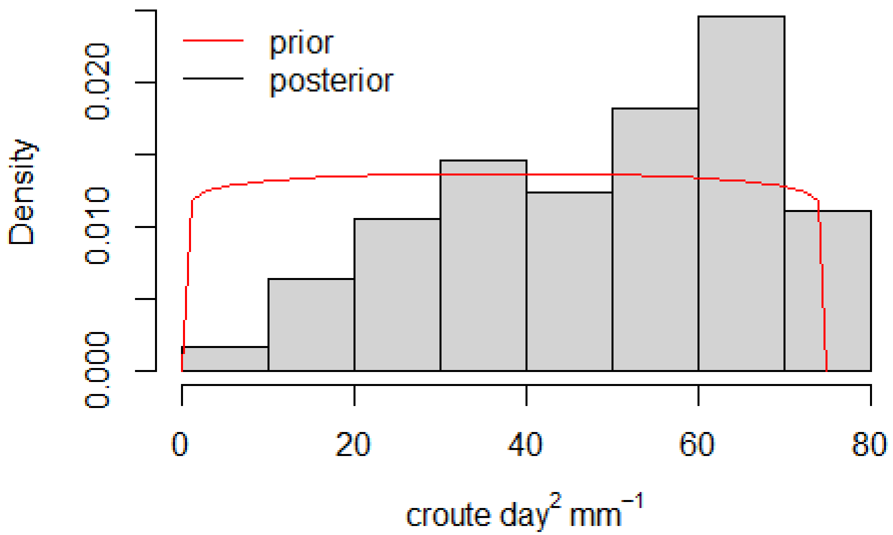
# Which priors were most influenced by Bayesian calibration?



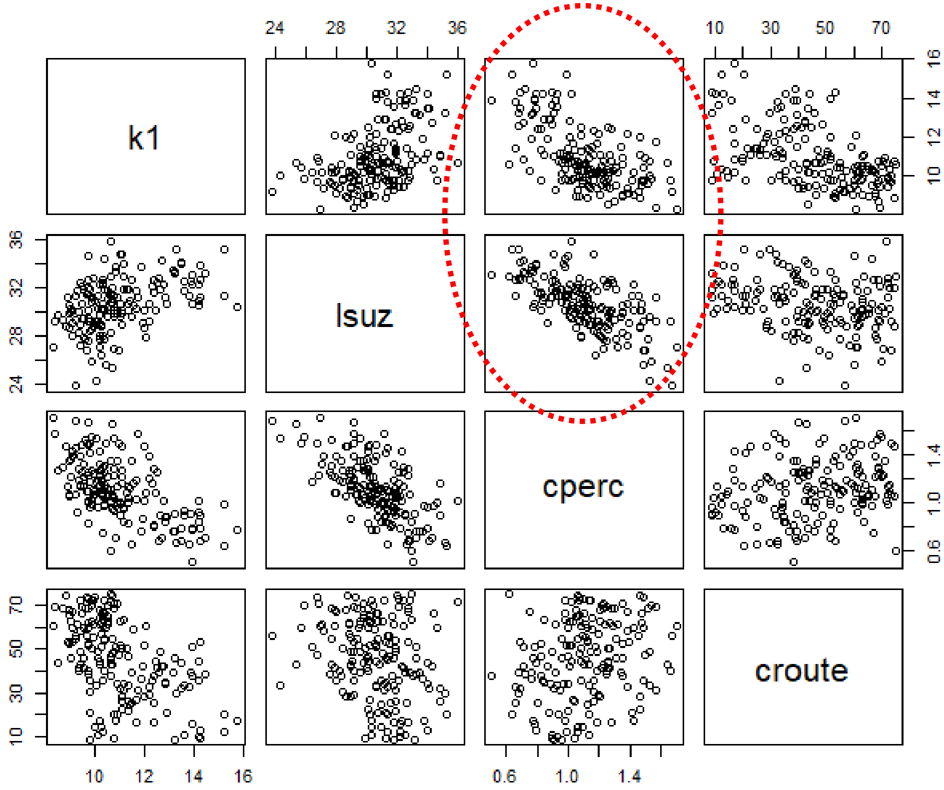

*k1, lsuz, cperc, beta1, sigma.delta and sigma.eta were most influenced, see e.g. plot for lsuz and beta1*

# The parameter “croute” remained very uncertain, see below




*The plot above suggests the upper boundary of the prior for “croute” is restrictive. Increasing the upper boundary would reduce the time period over which surface flow is smoothed out, so that the model gets more responsive to smaller peaks. This could make sense; I don’t think there is a physical reason not to do so (actually we already set a larger range than suggested in the help file.*

# Q5) Correlation plots sampled parameters



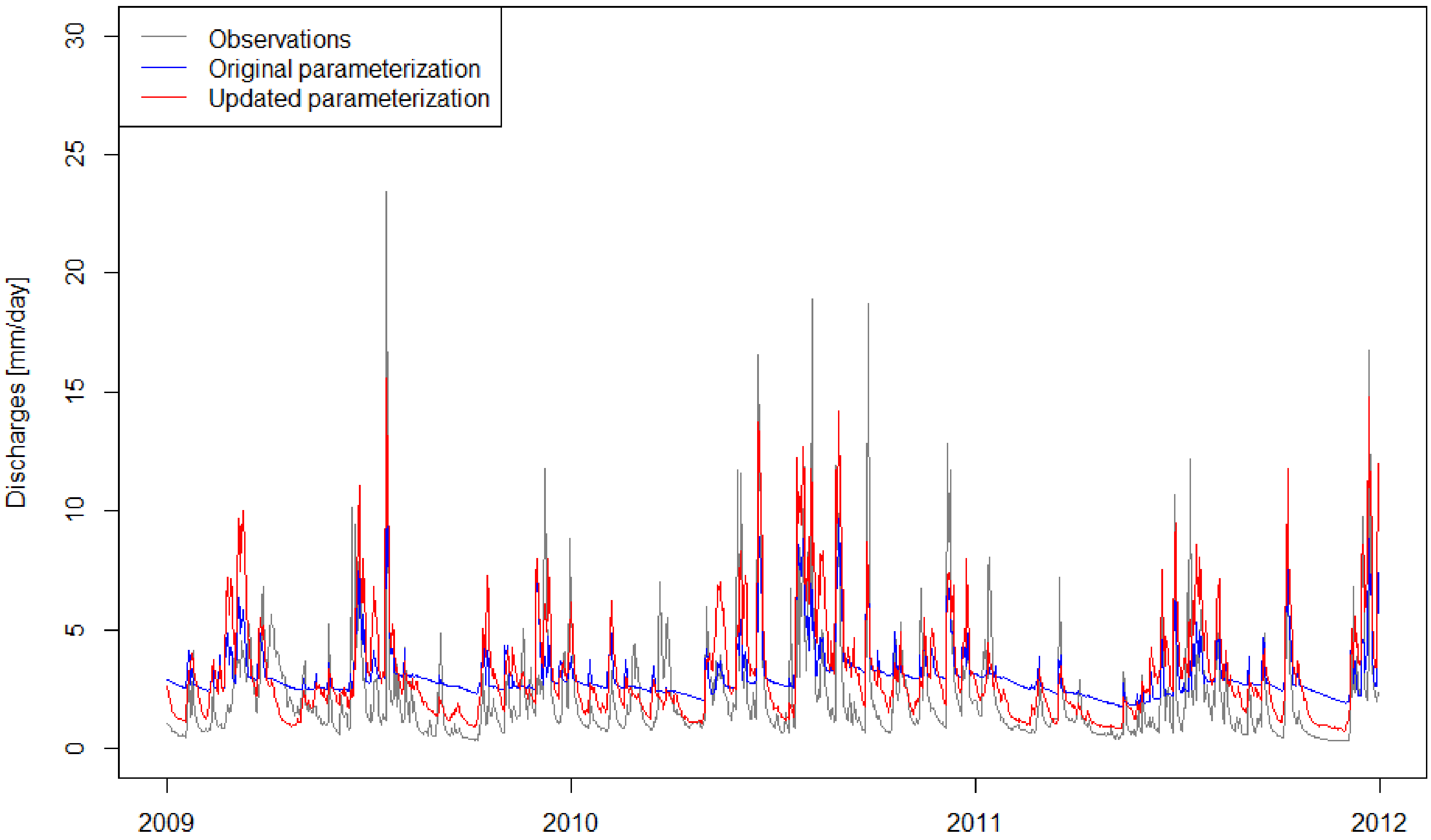

*Perhaps the clearest result is that k1 and lsuz are negatively correlated with cperc; the (other) correlations are rather low.*

# Interpretation of negative correlation

upper and a lower soil reservoir. Excess rainfall enters the upper zone reservoir and leaves this reservoir through three paths: outflow from the reservoir based on a fast storage coefficient $K_1$; percolation to the lower zone with a constant percolation rate $C_P$; and, if a threshold of the storage state $LS_{UZ}$ is exceeded, through an additional outlet based on a very fast storage coefficient $K_0$. Water leaves the lower zone based on a slow storage coefficient $K_2$. The outflow from both reservoirs is then routed by a triangular transfer function representing runoff routing in the streams, where $C_R$ is a free parameter. More details on the model are given in Appendix A.

*If more water leaves the system through a fast outflow, there is less water for percolation*

# Q6) Did Bayesian calibration improve the model predictions?




RMSE(relative residuals) calibrated parameters: 122%

RMSE(relative residuals) default run: 181%

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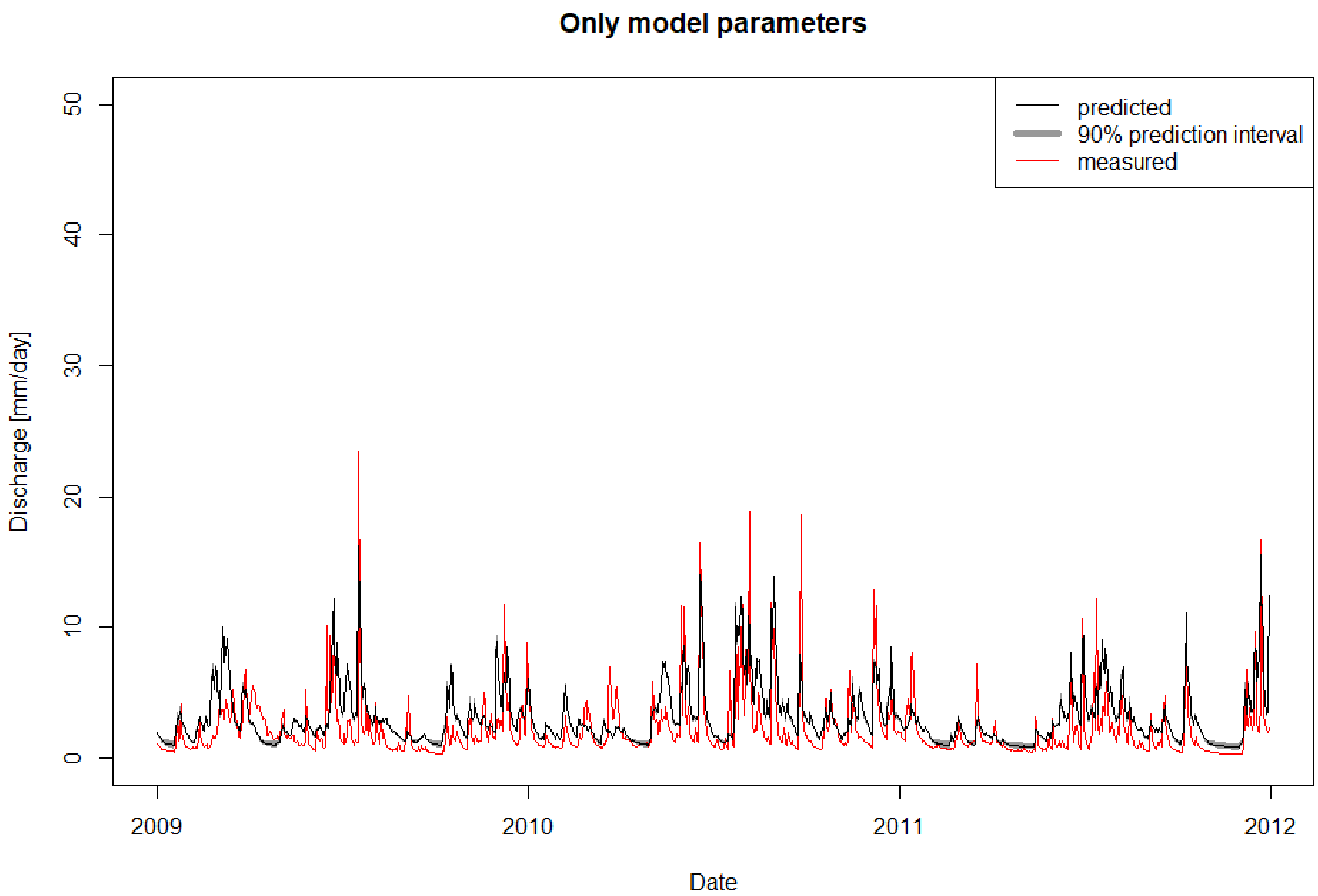
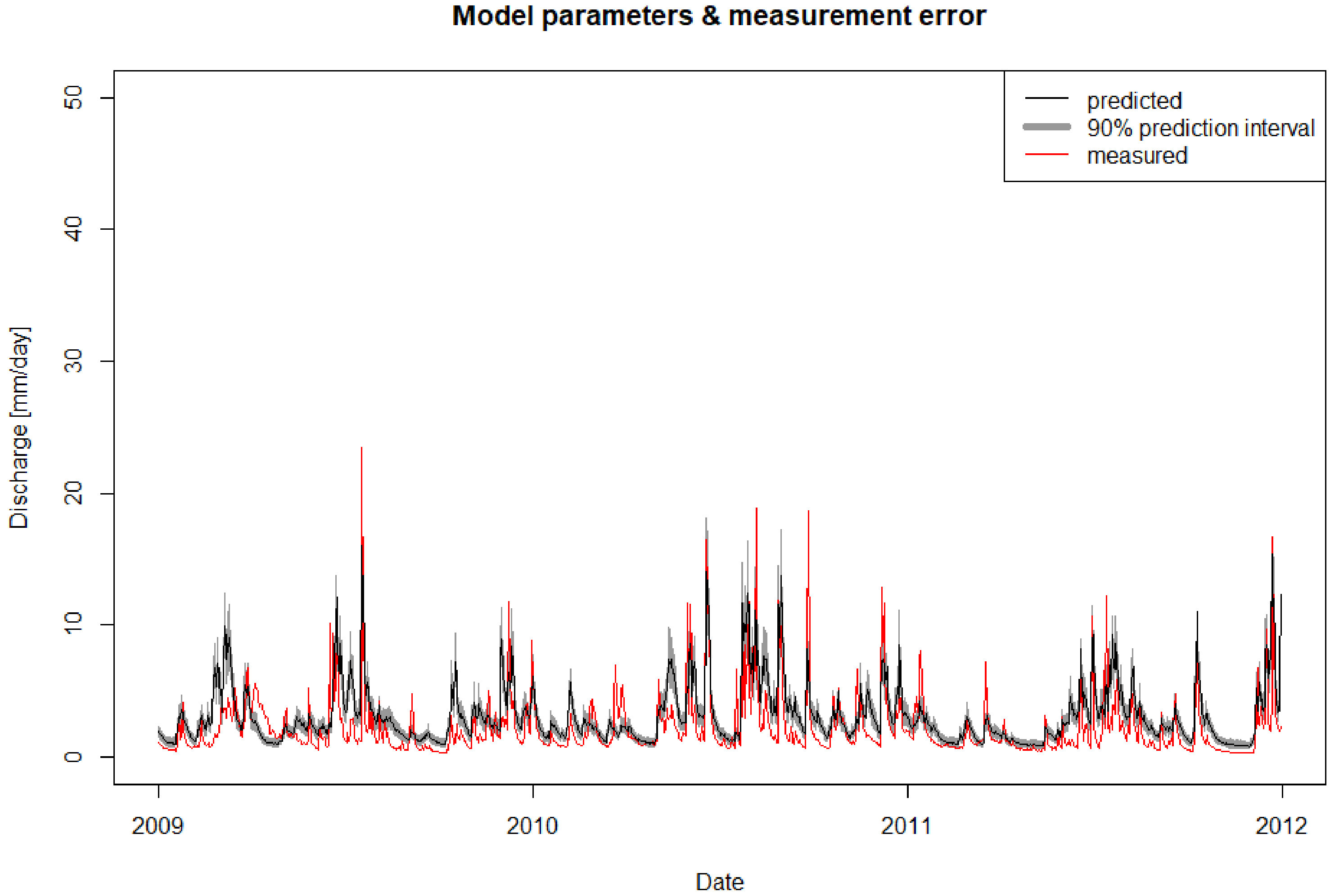
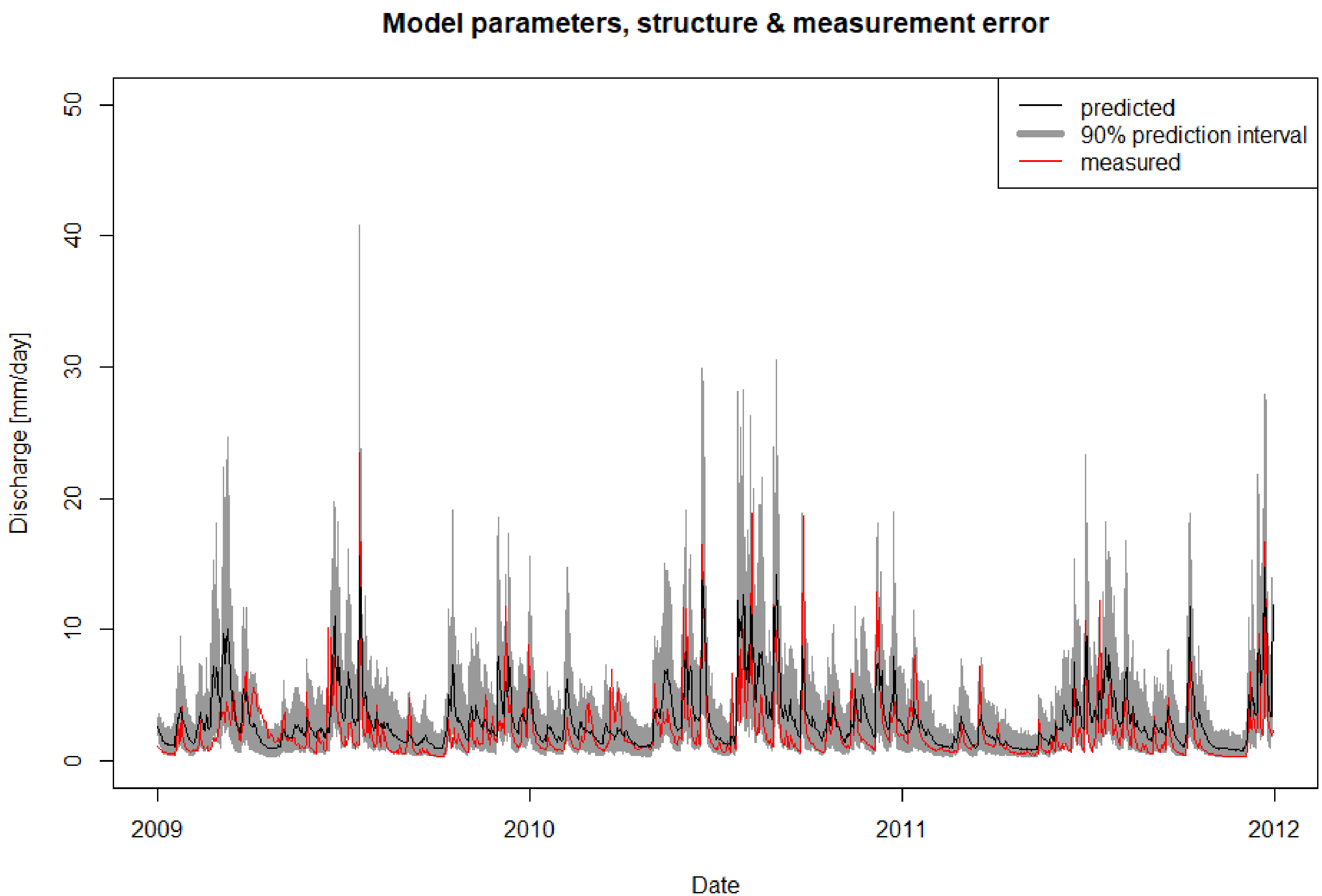
# Q7) Uncertainty component contribution-1



# Q7) Uncertainty component contribution-2



# Q7) Uncertainty component contribution-3




Does the 90% prediction interval cover most of the observed discharges?

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# Q8) Did we forget some contribution to total uncertainty?

- Think of the Wadoux et al. paper
- Or is it somehow covered?
- How can the approach be improved?

# Take home

- MCMC implementation of Bayes approach allows accounting for parameter uncertainty, uncertainty owing to model structural error, input uncertainty *and* output measurement uncertainty
- You need prior beliefs about the distributions of parameters
- It is computationally intensive
- There are alternatives (e.g., INLA); however, we did not demonstrate them