Killing fire ants

An exercise with the ANOVA method

PhD Course Mixed models PERC

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Quantitative methods brought to life

Killing fire ants
an ANOVA exercise

y = number of ants killed (in thousands) out of 50000

A = four chemicals (to be compared),

B = five locations (to represent possible conditions)

40 artificial mounds of ants, two mounds per combination of chemical and location.

Identify fixed and/or random effects that should be in the model and motivate your choice.

Any need for interaction terms (random or fixed)?

What is the practical interpretation of the different effects?

Killing fire ants – towards the model

Chemicals are represented by fixed effects.

Locations (as if sampled from a population of possible locations) are represented by random effects.

Interaction between chemicals and locations is represented by random effects (interaction between fixed and random is random).

Finally, there are individual error terms for the 40 mounts.

Killing fire ants – the model

$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \epsilon_{ijk}$,

for the number killed by chemical i, at location j, for mound k

μ is the expected number killed by chemical 1 (the reference) at a random location, for a random mound,

α_i is the fixed effect of the i-th chemical, $\alpha_1 = 0$

β_j is the random effect of the j-th location, $\beta_j \sim N(0, \sigma_\beta^2)$

$\alpha\beta_{ij}$ is the random interaction effect, $\alpha\beta_{ij} \sim N(0, \sigma_{\alpha\beta}^2)$

ϵ_{ijk} is the random error term for the i-th chemical at the j-th location and the k-th mound at that location $\epsilon_{ijk} \sim N(0, \sigma_\epsilon^2)$

$\beta_j, \alpha\beta_{ij}, \epsilon_{ijk}$ mutually independent.

Killing fire ants - Expected mean squares

- Quadratic function *Q* for fixed effect (chemicals)
- Coefficient of component is number of times random effect occurs in the data set
- Interaction involves differences between chemicals at one location versus differences between chemicals at another location
- So, main effects chemicals and locations cancel in *MS* for interaction

$$E(MSchemical) = \sigma_\epsilon^2 + 2\sigma_{loc*chem}^2 + Q_{chem}$$
$$E(MSlocation) = \sigma_\epsilon^2 + 2\sigma_{loc*chem}^2 + 8\sigma_{loc}^2$$
$$E(MSloc * chem) = \sigma_\epsilon^2 + 2\sigma_{loc*chem}^2$$
$$E(MSE) = \sigma_\epsilon^2$$

Killing fire ants - F tests

$$
\begin{aligned}
E(MSchemical) &= \sigma_\epsilon^2 + 2\sigma_{loc*chem}^2 + Q_{chem} \\
E(MSlocation) &= \sigma_\epsilon^2 + 2\sigma_{loc*chem}^2 + 8\sigma_{loc}^2 \\
E(MSloc*chem) &= \sigma_\epsilon^2 + 2\sigma_{loc*chem}^2 \\
E(MSE) &= \sigma_\epsilon^2
\end{aligned}
$$

F test interaction location * chemical (H_0: variance component interactions = 0):

*F = MSloc*chem / MSE*

F test main effect chemicals (H_0: same expected values):

*F = MSchem / MSloc*chem*

F test main effect locations (H_0: variance component locations = 0):

*F = MSloc / MSloc*chem*