

CINEMATICA

1. $\vec{v} = \frac{d\vec{r}}{dt} = 8t^3 \vec{i} + (-3t^2 - 1) \vec{j} + 2t \vec{k} \text{ (m/s)}$

$$|\vec{v}| = \sqrt{(8t^3)^2 + (-3t^2 - 1)^2 + (2t)^2} = \sqrt{64t^6 + 9t^4 + 10t^2 + 1} \text{ (m/s)}$$

2. $\vec{v} = \frac{d\vec{r}}{dt} = 4t \vec{i} + \vec{j} + 9t^2 \vec{k} \text{ (m/s)}$

$$\vec{a} = \frac{d\vec{v}}{dt} = 4 \vec{i} + 18t \vec{k} \text{ (m/s}^2\text{)}; \quad \vec{a}(2s) = 4 \vec{i} + 36 \vec{k} \text{ (m/s}^2\text{)}$$

$$|\vec{a}| = \sqrt{4^2 + 36^2} = \sqrt{1312} = 36,22 \text{ m/s}^2$$

3. $\vec{r}(1s) = (1^2 + 2 \cdot 1) \vec{i} + (3 \cdot 1^2 - 1) \vec{j} + (2 \cdot 1^2 - 3) \vec{k} = 3 \vec{i} + 2 \vec{j} - \vec{k} \text{ (m)}$

$$\vec{r}(3s) = (3^2 + 2 \cdot 3) \vec{i} + (3 \cdot 3^2 - 1) \vec{j} + (2 \cdot 3^2 - 3) \vec{k} = 15 \vec{i} + 26 \vec{j} + 15 \vec{k} \text{ (m)}$$

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1 = (15 - 3) \vec{i} + (26 - 2) \vec{j} + (15 + 1) \vec{k} = 12 \vec{i} + 24 \vec{j} + 16 \vec{k} \text{ (m)}$$

4. $\vec{r}(1s) = (1^3 + 1) \vec{i} + (1^2 - 2 \cdot 1) \vec{j} = 2 \vec{i} - \vec{j} \text{ (m)}$

$$|\vec{r}| = \sqrt{2^2 + (-1)^2} = \sqrt{5} \text{ m}$$

$$\vec{u}_r = \frac{\vec{r}}{|\vec{r}|} = \frac{2 \vec{i} - \vec{j}}{\sqrt{5}} = \frac{2}{\sqrt{5}} \vec{i} - \frac{1}{\sqrt{5}} \vec{j}$$

5. $\vec{a}_m = \frac{\Delta \vec{v}}{\Delta t}; \quad \Delta \vec{v} = \vec{v}_2 - \vec{v}_1$

$$\vec{v}(1s) = (1^2 + 2 \cdot 1 + 2) \vec{i} + (2 \cdot 1 + 3) \vec{j} + (3 \cdot 1^2 + 1) \vec{k} = 5 \vec{i} + 5 \vec{j} + 4 \vec{k} \text{ (m/s)}$$

$$\vec{v}(3s) = (3^2 + 2 \cdot 3 + 2) \vec{i} + (2 \cdot 3 + 3) \vec{j} + (3 \cdot 3^2 + 1) \vec{k} = 17 \vec{i} + 9 \vec{j} + 28 \vec{k} \text{ (m/s)}$$

$$\Delta \vec{v} = (17 - 5) \vec{i} + (9 - 5) \vec{j} + (28 - 4) \vec{k} = 12 \vec{i} + 4 \vec{j} + 24 \vec{k} \text{ (m/s)}$$

$$\vec{a}_m = \frac{\Delta \vec{v}}{\Delta t} = \frac{12 \vec{i} + 4 \vec{j} + 24 \vec{k}}{3 - 1} = 6 \vec{i} + 2 \vec{j} + 12 \vec{k} \text{ (m/s}^2\text{)}$$

6. $\vec{r}(1s) = (1^2 + 2 \cdot 1 - 4) \vec{i} + 1^3 \vec{j} + (2 \cdot 1 + 1) \vec{k} = -\vec{i} + \vec{j} + 3 \vec{k} \text{ (m)}$

$$\vec{r}(2s) = (2^2 + 2 \cdot 2 - 4) \vec{i} + 2^3 \vec{j} + (2 \cdot 2 + 1) \vec{k} = 4 \vec{i} + 8 \vec{j} + 5 \vec{k} \text{ (m)}$$

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1 = (4 + 1) \vec{i} + (8 - 1) \vec{j} + (5 - 3) \vec{k} = 5 \vec{i} + 7 \vec{j} + 2 \vec{k} \text{ (m)}$$

$$\vec{v}_m = \frac{\Delta \vec{r}}{\Delta t} = \frac{5 \vec{i} + 7 \vec{j} + 2 \vec{k}}{2 - 1} = 5 \vec{i} + 7 \vec{j} + 2 \vec{k} \text{ (m/s)}$$

$$b) \vec{v} = \frac{d\vec{r}}{dt} = (2t+2)\vec{i} + 3t^2\vec{j} + 2\vec{k} \text{ (m/s)}$$

$$\vec{v}(1s) = 4\vec{i} + 3\vec{j} + 2\vec{k} \text{ (m/s)}$$

$$|\vec{v}| = \sqrt{4^2 + 3^2 + 2^2} = \sqrt{29} \text{ m/s}$$

$$7. a) \vec{v} = \frac{d\vec{r}}{dt} = 2t\vec{i} + \vec{j} + (2t+1)\vec{k} \text{ (m/s)}$$

$$\vec{v}(2s) = 4\vec{i} + \vec{j} + 5\vec{k} \text{ (m/s)}$$

$$b) \vec{a} = \frac{d\vec{v}}{dt} = 2\vec{i} + 2\vec{k} \text{ (m/s}^2\text{)}$$