1. $\vec{r} = (2t^3-3)\vec{i} + (t^2+2t)\vec{j} + t^4\vec{k}$ (m)

a) Desplazamiento $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$t=1\,s$ $\vec{r}_1 = (2\cdot1^3-3)\vec{i} + (1^2+2\cdot1)\vec{j} + 1^4\vec{k} = -\vec{i} + 3\vec{j} + \vec{k}$ (m)

$t=2\,s$ $\vec{r}_2 = (2\cdot2^3-3)\vec{i} + (2^2+2\cdot2)\vec{j} + 2^4\vec{k} = 13\vec{i} + 8\vec{j} + 16\vec{k}$ (m)

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (13+1)\vec{i} + (8-3)\vec{j} + (16-1)\vec{k}$ (m)

$\Delta\vec{r} = \underline{14\vec{i} + 5\vec{j} + 15\vec{k}\ (m)}$

b) Aceleración media $\vec{a}_m = \dfrac{\Delta\vec{v}}{\Delta t}$

$\vec{v} = \dfrac{d\vec{r}}{dt} = 6t^2\vec{i} + (2t+2)\vec{j} + 4t^3\vec{k}$ (m/s)

$t=1\,s$ $\vec{v}_1 = 6\cdot1^2\vec{i} + (2\cdot1+2)\vec{j} + 4\cdot1^3\vec{k}$ (m/s)

$t=2\,s$ $\vec{v}_2 = 6\cdot2^2\vec{i} + (2\cdot2+2)\vec{j} + 4\cdot2^3\vec{k}$ (m/s)

$\vec{v}_1 = 6\vec{i} + 4\vec{j} + 4\vec{k}$ (m/s) ; $\vec{v}_2 = 24\vec{i} + 6\vec{j} + 32\vec{k}$ (m/s)

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = (24-6)\vec{i} + (6-4)\vec{j} + (32-4)\vec{k}$

$\vec{a}_m = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{18\vec{i} + 2\vec{j} + 28\vec{k}}{2-1} = \underline{18\vec{i} + 2\vec{j} + 28\vec{k}\ (m/s^2)}$

2. $\vec{v} = (2t+1)\vec{i} + (t^2-t)\vec{j}$ (m/s)

a) $t=1\,s$ $\vec{v}_1 = (2\cdot1+1)\vec{i} + (1^2-1)\vec{j}$ (m/s)

$t=3\,s$ $\vec{v}_2 = (2\cdot3+1)\vec{i} + (3^2-3)\vec{j}$ (m/s)

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = (7-3)\vec{i} + (6-0)\vec{j} = 4\vec{i} + 6\vec{j}$ (m/s)

$\vec{a}_m = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{4\vec{i} + 6\vec{j}}{3-1} = 2\vec{i} + 3\vec{j}$ (m/s²)

b) Aceleración normal en $t = 2\,s$

$$a_n = \sqrt{a^2 - a_t^2} \;;\; a = \sqrt{a_x^2 + a_y^2} \;;\; a_t = \frac{d|\vec{v}|}{dt}$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(2t+1)^2 + (t^2-t)^2} = \sqrt{t^4 - 2t^3 + 5t^2 + 4t + 1}\ \frac{m}{s}$$

$$a_t = \frac{d|\vec{v}|}{dt} = \frac{2t^3 - 3t^2 + 5t + 2}{\sqrt{t^4 - 2t^3 + 5t^2 + 4t + 1}} \;;\; a_t(2s) = \frac{16}{\sqrt{29}} = 2{,}97\ m/s^2$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 2\,\vec{i} + (2t-1)\,\vec{j}\ (m/s^2) \;;\; \vec{a}(2s) = 2\,\vec{i} + 3\,\vec{j}\ (m/s^2)$$

$$a = \sqrt{2^2 + 3^2} = \sqrt{13}\ m/s^2 \;;\; a_n = \sqrt{a^2 - a_t^2}$$

$$a_n = \sqrt{(\sqrt{13})^2 - \left(\frac{16}{\sqrt{29}}\right)^2} = 2{,}04\ m/s^2$$

3. $\vec{r} = 3t^2\,\vec{i} - t^3\,\vec{j} + (2t+2)\,\vec{k}\ (m) \;;\; a_n = \frac{v^2}{R}$

$$\vec{v} = \frac{d\vec{v}}{dt} = 6t\,\vec{i} - 3t^2\,\vec{j} + 2\,\vec{k}\ (m/s) \;;\; \vec{v}(1s) = 6\vec{i} - 3\vec{j} + 2\vec{k}\ \left(\frac{m}{s}\right)$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{6^2 + (-3)^2 + 2^2} = 7\ m/s$$

$$|\vec{v}(t)| = \sqrt{(6t)^2 + (-3t^2)^2 + 2^2} = \sqrt{9t^4 + 36t^2 + 4}\ (m/s)$$

$$a_t = \frac{d|\vec{v}|}{dt} = \frac{18t^3 + 36t}{\sqrt{9t^4 + 36t^2 + 4}} \;;\; a_t(1s) = \frac{54}{7} = 7{,}71\ m/s^2$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 6\vec{i} - 6t\,\vec{j}\ (m/s^2) \;;\; \vec{a}(1s) = 6\vec{i} - 6\vec{j}\ (m/s^2)$$

$$|\vec{a}| = \sqrt{6^2 + (-6)^2} = \sqrt{72} = 8{,}49\ m/s^2$$

$$a_n = \sqrt{a^2 - a_t^2} = \sqrt{(\sqrt{72})^2 - \left(\frac{54}{7}\right)^2} = 3{,}53\ m/s^2$$

$$R = \frac{v^2}{a_n} = \frac{7^2}{3{,}53} = \underline{13{,}9\ m}$$

4. $a_t = \frac{d|\vec{v}|}{dt}$; $\vec{a}_t = a_t \cdot \vec{\tau}$

$$|\vec{v}| = \sqrt{(4t+1)^2 + (t^2)^2 + 2^2} = \sqrt{t^4 + 16t^2 + 8t + 5} \text{ m/s}$$

$$a_t = \frac{d|\vec{v}|}{dt} = \frac{2t^3 + 16t + 4}{\sqrt{t^4 + 16t^2 + 8t + 5}}$$

5. $\vec{r} = 6t\,\vec{i} + (5t^2 - 2t)\vec{j} + (t^2 - 3)\vec{k}$ (m)

a) $\vec{v}_m = \frac{\Delta\vec{r}}{\Delta t}$; $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$t = 1\,s$ $\quad \vec{r}_1 = 6 \cdot 1\,\vec{i} + (5 \cdot 1^2 - 2 \cdot 1)\,\vec{j} + (1^2 - 3)\vec{k}$ (m)

$t = 4\,s$ $\quad \vec{r}_2 = 6 \cdot 4\,\vec{i} + (5 \cdot 4^2 - 2 \cdot 4)\vec{j} + (4^2 - 3)\vec{k}$ (m)

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (24 - 6)\,\vec{i} + (72 - 3)\,\vec{j} + (13 + 2)\,\vec{k}$ (m)

$\Delta\vec{r} = 18\,\vec{i} + 69\,\vec{j} + 15\,\vec{k}$ (m)

$$\vec{v}_m = \frac{\Delta\vec{r}}{\Delta t} = \frac{18\vec{i} + 69\vec{j} + 15\vec{k}}{4-1} = 6\vec{i} + 23\vec{j} + 5\vec{k} \text{ (m/s)}$$

b) $\vec{v} = \frac{d\vec{r}}{dt} = 6\,\vec{i} + (10t - 2)\,\vec{j} + 2t\,\vec{k}$ (m/s)

$\vec{v}(t = 2s) = 6\vec{i} + 18\,\vec{j} + 4\,\vec{k}$ (m/s)

$|\vec{v}| = \sqrt{6^2 + 18^2 + 4^2} = \sqrt{376} \simeq 19{,}39$ m/s

6. $\vec{r} = (t^2+1)\,\vec{i} + t^3\,\vec{j} + (2t^2 + t)\,\vec{k}$ (m); $\vec{r}(1s) = 2\,\vec{i} + \vec{j} + 3\,\vec{k}$ (m)

$\vec{v} = \frac{d\vec{r}}{dt} = 2t\,\vec{i} + 3t^2\vec{j} + (4t+1)\,\vec{k}$ (m/s); $\vec{v}(1s) = 2\vec{i} + 3\,\vec{j} + 5\,\vec{k}$ (m/s)

a) $|\vec{r}(1s)| = \sqrt{2^2 + 1^2 + 3^2} = \sqrt{14}$ m $\quad \vec{u}_r = \frac{\vec{r}}{r} = \frac{2\vec{i} + \vec{j} + 3\vec{k}}{\sqrt{14}}$;

$$\vec{u}_r = \frac{2}{\sqrt{14}}\,\vec{i} + \frac{1}{\sqrt{14}}\,\vec{j} + \frac{3}{\sqrt{14}}\,\vec{k}$$

b) $\vec{v}(1s) = 2\vec{i} + 3\vec{j} + 5\vec{k}$ (m/s)

$$|\vec{v}(1s)| = \sqrt{2^2+3^2+5^2} = \sqrt{38} \text{ m/s}$$

$$\vec{\tau} = \frac{\vec{v}}{v} = \frac{2\vec{i}+3\vec{j}+5\vec{k}}{\sqrt{38}} = \frac{2}{\sqrt{38}}\vec{i} + \frac{3}{\sqrt{38}}\vec{j} + \frac{5}{\sqrt{38}}\vec{k}$$

c) $\vec{n} = \frac{\vec{a}_n}{a_n}$; $\vec{a}_n = \vec{a} - \vec{a}_t$; $\vec{a}_t = a_t \cdot \vec{\tau}$

$$\vec{a} = \frac{d\vec{v}}{dt} = 2\vec{i} + 6t\vec{j} + 4\vec{k} \text{ (m/s}^2)$$

$$\vec{a}(1s) = 2\vec{i} + 6\vec{j} + 4\vec{k} \text{ (m/s}^2)$$

$$a_t = \frac{d|\vec{v}|}{dt} \; ; \; |\vec{v}| = \sqrt{(2t)^2+(3t^2)^2+(4t+1)^2} \text{ m/s}$$

$$|\vec{v}| = \sqrt{9t^4+20t^2+8t+1} \text{ m/s}$$

$$a_t = \frac{d|\vec{v}|}{dt} = \frac{18t^3+20t+4}{\sqrt{9t^4+20t^2+8t+1}} \text{ m/s}^2 ; \; a_t(1s) = \frac{42}{\sqrt{38}} \text{ m/s}^2$$

$$\vec{a}_t = a_t \cdot \vec{\tau} = \frac{42}{\sqrt{38}}\frac{2}{\sqrt{38}}\vec{i} + \frac{42}{\sqrt{38}}\frac{3}{\sqrt{38}}\vec{j} + \frac{42}{\sqrt{38}}\frac{5}{\sqrt{38}}\vec{k} \text{ (m/s}^2)$$

$$\vec{a}_t = \frac{42}{19}\vec{i} + \frac{63}{19}\vec{j} + \frac{105}{19}\vec{k} \text{ (m/s}^2)$$

$$\vec{a}_n = \vec{a} - \vec{a}_t = 2\vec{i} + 6\vec{j} + 4\vec{k} - \frac{42}{19}\vec{i} - \frac{63}{19}\vec{j} - \frac{105}{19}\vec{k} =$$

$$= -\frac{4}{19}\vec{i} + \frac{51}{19}\vec{j} - \frac{29}{19}\vec{k} \text{ (m/s}^2)$$

$$\vec{n} = -\frac{4}{\sqrt{3458}}\vec{i} + \frac{51}{\sqrt{3458}}\vec{j} - \frac{29}{\sqrt{3458}}\vec{k}$$