Formulário

Equação não linear

Equação iterativa	
Método da secante	Método de Newton
$x_{k+1} = x_k - \dfrac{(x_k - x_{k-1})f(x_k)}{f(x_k) - f(x_{k-1})}$, $k = 2, 3, ...$	$x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$, $k = 1, 2, ...$

Critério de Paragem
$\dfrac{\lvert x_{k+1} - x_k\rvert}{\lvert x_{k+1}\rvert} \leq \epsilon_1$ e $\lvert f(x_{k+1})\rvert \leq \epsilon_2$

Método de Newton para sistemas de equações não lineares

Equação iterativa	Jacobiano - J	Critério de Paragem
$x^{(k+1)} = x^{(k)} + \Delta x^{(k)}$ $J(x^{(k)})\Delta x^{(k)} = -f(x^{(k)})$	$\left(\begin{array}{ccc} \frac{\partial f_1(x_1,x_2,\ldots,x_n)}{\partial x_1} & \ldots & \frac{\partial f_1(x_1,x_2,\ldots,x_n)}{\partial x_n} \\ \ldots & & \ldots \\ \frac{\partial f_n(x_1,x_2,\ldots,x_n)}{\partial x_1} & \ldots & \frac{\partial f_n(x_1,x_2,\ldots,x_n)}{\partial x_n} \end{array}\right)$	$\dfrac{\lVert\Delta x^{(k)}\rVert}{\lVert x^{(k+1)}\rVert} \leq \epsilon_1$ e $\lVert f(x^{(k+1)})\rVert \leq \epsilon_2$

Polinómio interpolador de Newton

$$\boxed{p_n(x) = f_0 + (x - x_0)\,[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2] + \cdots + (x - x_0)(x - x_1)\cdots(x - x_{n-1})\,[x_0, \ldots, x_n]}$$

Erro de truncatura $\boxed{R_n(x) = (x - x_0)(x - x_1)\cdots(x - x_n)[x_0, x_1, \ldots, x_n, x_z]}$

Diferenças divididas

$$[x_j, x_{j+1}] = \frac{f_j - f_{j+1}}{x_j - x_{j+1}}, \qquad j = 0, \ldots, n-1 \qquad \text{(diferença dividida de ordem } 1 - dd1)$$

$$[x_j, x_{j+1}, x_{j+2}] = \frac{[x_j, x_{j+1}] - [x_{j+1}, x_{j+2}]}{x_j - x_{j+2}}, \qquad j = 0, \ldots, n-2 \qquad (dd2)$$

$$[x_0, x_1, \ldots, x_{n-1}, x_n] = \frac{[x_0, x_1, \ldots, x_{n-2}, x_{n-1}] - [x_1, x_2, \ldots, x_{n-1}, x_n]}{x_0 - x_n} \qquad (ddn)$$

$$[x_0, x_1, \ldots, x_n] = \frac{f^{(n)}(\xi)}{n!}$$

Mínimos Quadrados

Modelo polinomial $\boxed{p_n(x) = c_0P_0(x) + c_1P_1(x) + \ldots + c_nP_n(x)}$

Polinómios ortogonais $\boxed{P_{i+1}(x) = A_i(x - B_i)P_i(x) - C_iP_{i-1}(x)}$, $\boxed{P_0(x) = 1}$, $\boxed{P_{-1}(x) = 0}$

$$\boxed{A_i = 1} \quad \boxed{B_i = \frac{\sum_{j=1}^m x_jP_i^2(x_j)}{\sum_{j=1}^m P_i^2(x_j)}} \quad \boxed{C_0 = 0 \text{ e } C_i = \frac{\sum_{j=1}^m P_i^2(x_j)}{\sum_{j=1}^m P_{i-1}^2(x_j)}}$$

Coeficientes do modelo polinomial

$$\boxed{c_j = \frac{\sum_{i=1}^m f_iP_j(x_i)}{\sum_{i=1}^m P_j^2(x_i)} \quad j = 0, \ldots, n}$$

Modelo linear não polinomial $\boxed{M(x; c_1, c_2, \ldots, c_n) = c_1\phi_1(x) + c_2\phi_2(x) + \ldots + c_n\phi_n(x)}$

$$\left(\begin{array}{lll} \sum_{i=1}^m \phi_1^2(x_i) & \ldots & \sum_{i=1}^m \phi_1(x_i)\phi_n(x_i) \\ \ldots & \ldots & \ldots \\ \sum_{i=1}^m \phi_n(x_i)\phi_1(x_i) & \ldots & \sum_{i=1}^m \phi_n^2(x_i) \end{array}\right) \left(\begin{array}{c} c_1 \\ \ldots \\ c_n \end{array}\right) = \left(\begin{array}{l} \sum_{i=1}^m f_i\phi_1(x_i) \\ \ldots \\ \sum_{i=1}^m f_i\phi_n(x_i) \end{array}\right)$$

Resíduo $\boxed{\sum_{i=1}^m (f_i - M(x_i))^2}$

Splines

Expressão do segmento i da *spline* cúbica

$$s_3^i(x) = \frac{M_{i-1}}{6(x_i-x_{i-1})}(x_i-x)^3 + \frac{M_i}{6(x_i-x_{i-1})}(x-x_{i-1})^3 + \left[\frac{f_{i-1}}{(x_i-x_{i-1})} - \frac{M_{i-1}(x_i-x_{i-1})}{6}\right](x_i-x) + \left[\frac{f_i}{(x_i-x_{i-1})} - \frac{M_i(x_i-x_{i-1})}{6}\right](x-x_{i-1}) \text{ para } i=1,2,\dots,n$$

Expressão para os pontos interiores (nó i)

$$(x_i-x_{i-1})M_{i-1} + 2(x_{i+1}-x_{i-1})M_i + (x_{i+1}-x_i)M_{i+1} = \frac{6}{(x_{i+1}-x_i)}(f_{i+1}-f_i) - \frac{6}{(x_i-x_{i-1})}(f_i-f_{i-1})$$

Spline Natural	*Spline* Completa - Expressão para os pontos exteriores (nó 0 e n)
$M_0 = 0$	$2(x_1-x_0)M_0 + (x_1-x_0)M_1 = \frac{6}{(x_1-x_0)}(f_1-f_0) - 6f_0^{'}$
$M_n = 0$	$2(x_n-x_{n-1})M_n + (x_n-x_{n-1})M_{n-1} = 6f_n^{'} - \frac{6}{(x_n-x_{n-1})}(f_n-f_{n-1})$

Erro de truncatura *spline* cúbica $\boxed{|f(x)-s_3(x)| \le \frac{5}{384}h^4M_4}$ $\boxed{\left|f'(x)-s_3^{'}(x)\right| \le \frac{1}{24}h^3M_4}$ com

$$\max_{\xi\in[x_0,x_n]} |f^{(iv)}(\xi)| \le M_4 \qquad h = \max_{0\le i\le n-1}(x_{i+1}-x_i)$$

Integração numérica

Fórmulas simples Newton-Cotes		
Trapézio	$\int_a^b f(x)dx \approx \frac{(b-a)}{2}[f(a)+f(b)]$	$ET = \lvert -\frac{(b-a)^3}{12}f''(\xi)\rvert,\ \xi\in[a,b]$
Simpson	$\int_a^b f(x)dx \approx \frac{(b-a)}{6}\left[f(a)+4f(\frac{a+b}{2})+f(b)\right]$	$ET = \lvert -\frac{(b-a)^5}{2880}f^{(iv)}(\xi)\rvert,\ \xi\in[a,b]$
$(\frac{3}{8})$	$\int_a^b f(x)dx \approx \frac{(b-a)}{8}\left[f(a)+3f(\frac{2a+b}{3})+3f(\frac{a+2b}{3})+f(b)\right]$	$ET = \lvert -\frac{(b-a)^5}{6480}f^{(iv)}(\xi)\rvert,\ \xi\in[a,b]$

Fórmulas compostas Newton-Cotes	
Trapézio	$\int_a^b f(x)dx \approx \frac{h}{2}[f_0+2f_1+2f_2+2f_3+2f_4+...+2f_{n-4}+2f_{n-3}+2f_{n-2}+2f_{n-1}+f_n]$ $ET = \lvert -\frac{h^2}{12}(b-a)f^{''}(\eta)\rvert,\ \eta\in[a,b]$
Simpson	$\int_a^b f(x)dx \approx \frac{h}{3}[f_0+4f_1+2f_2+4f_3+2f_4+...+4f_{n-5}+2f_{n-4}+4f_{n-3}+2f_{n-2}+4f_{n-1}+f_n]$ $ET = \lvert -\frac{h^4}{180}(b-a)f^{(iv)}(\eta)\rvert,\ \eta\in[a,b]$
$(\frac{3}{8})$	$\int_a^b f(x)dx \approx \frac{3h}{8}[f_0+3f_1+3f_2+2f_3+3f_4+3f_5+2f_6+...+2f_{n-3}+3f_{n-2}+3f_{n-1}+f_n]$ $ET = \lvert -\frac{h^4}{80}(b-a)f^{(iv)}(\eta)\rvert,\ \eta\in[a,b]$