

Formulário

Equação não linear

Equação iterativa	
Método da secante	Método de Newton
$x_{k+1} = x_k - \frac{(x_k - x_{k-1})f(x_k)}{f(x_k) - f(x_{k-1})}, k = 2, 3, \dots$	$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}, k = 1, 2, \dots$
Critério de Paragem	
$\frac{ x_{k+1} - x_k }{ x_{k+1} } \leq \epsilon_1$ e $ f(x_{k+1}) \leq \epsilon_2$	

Método de Newton para sistemas de equações não lineares

Equação iterativa	Jacobiano - J	Critério de Paragem
$x^{(k+1)} = x^{(k)} + \Delta x^{(k)}$ $J(x^{(k)})\Delta x^{(k)} = -f(x^{(k)})$	$\begin{pmatrix} \frac{\partial f_1(x_1, x_2, \dots, x_n)}{\partial x_1} & \dots & \frac{\partial f_1(x_1, x_2, \dots, x_n)}{\partial x_n} \\ \dots & \dots & \dots \\ \frac{\partial f_n(x_1, x_2, \dots, x_n)}{\partial x_1} & \dots & \frac{\partial f_n(x_1, x_2, \dots, x_n)}{\partial x_n} \end{pmatrix}$	$\frac{\ \Delta x^{(k)}\ }{\ x^{(k+1)}\ } \leq \epsilon_1$ e $\ f(x^{(k+1)})\ \leq \epsilon_2$

Polinómio interpolador de Newton

$p_n(x) = f_0 + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2] + \dots + (x - x_0)(x - x_1) \dots (x - x_{n-1})[x_0, \dots, x_n]$
Erro de truncatura $R_n(x) = (x - x_0)(x - x_1) \dots (x - x_n)[x_0, x_1, \dots, x_n, x_z] $

Diferenças divididas

$$[x_j, x_{j+1}] = \frac{f_j - f_{j+1}}{x_j - x_{j+1}}, \quad j = 0, \dots, n-1 \quad (\text{diferença dividida de ordem } 1 - dd1)$$

$$[x_j, x_{j+1}, x_{j+2}] = \frac{[x_j, x_{j+1}] - [x_{j+1}, x_{j+2}]}{x_j - x_{j+2}}, \quad j = 0, \dots, n-2 \quad (dd2)$$

$$[x_0, x_1, \dots, x_{n-1}, x_n] = \frac{[x_0, x_1, \dots, x_{n-2}, x_{n-1}] - [x_1, x_2, \dots, x_{n-1}, x_n]}{x_0 - x_n} \quad (ddn)$$

$$[x_0, x_1, \dots, x_n] = \frac{f^{(n)}(\xi)}{n!}$$

Mínimos Quadrados

Modelo polinomial	$p_n(x) = c_0 P_0(x) + c_1 P_1(x) + \dots + c_n P_n(x)$
Polinómios ortogonais	$P_{i+1}(x) = A_i(x - B_i)P_i(x) - C_i P_{i-1}(x), \quad P_0(x) = 1, \quad P_{-1}(x) = 0$
$A_i = 1$	$B_i = \frac{\sum_{j=1}^m x_j P_i^2(x_j)}{\sum_{j=1}^m P_i^2(x_j)} \quad C_0 = 0 \text{ e } C_i = \frac{\sum_{j=1}^m P_i^2(x_j)}{\sum_{j=1}^m P_{i-1}^2(x_j)}$
Coeficientes do modelo polinomial	
$c_j = \frac{\sum_{i=1}^m f_i P_j(x_i)}{\sum_{i=1}^m P_j^2(x_i)} \quad j = 0, \dots, n$	

Modelo linear não polinomial	$M(x; c_1, c_2, \dots, c_n) = c_1 \phi_1(x) + c_2 \phi_2(x) + \dots + c_n \phi_n(x)$
$\begin{pmatrix} \sum_{i=1}^m \phi_1^2(x_i) & \dots & \sum_{i=1}^m \phi_1(x_i) \phi_n(x_i) \\ \dots & \dots & \dots \\ \sum_{i=1}^m \phi_n(x_i) \phi_1(x_i) & \dots & \sum_{i=1}^m \phi_n^2(x_i) \end{pmatrix} \begin{pmatrix} c_1 \\ \dots \\ c_n \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^m f_i \phi_1(x_i) \\ \dots \\ \sum_{i=1}^m f_i \phi_n(x_i) \end{pmatrix}$	

Resíduo $\sum_{i=1}^m (f_i - M(x_i))^2$

SplinesExpressão do segmento i da *spline* cúbica

$$s_3^i(x) = \frac{M_{i-1}}{6(x_i - x_{i-1})}(x_i - x)^3 + \frac{M_i}{6(x_i - x_{i-1})}(x - x_{i-1})^3 + \left[\frac{f_{i-1}}{(x_i - x_{i-1})} - \frac{M_{i-1}(x_i - x_{i-1})}{6} \right] (x_i - x) + \left[\frac{f_i}{(x_i - x_{i-1})} - \frac{M_i(x_i - x_{i-1})}{6} \right] (x - x_{i-1}) \text{ para } i = 1, 2, \dots, n$$

Expressão para os pontos interiores (nó i)

$$(x_i - x_{i-1})M_{i-1} + 2(x_{i+1} - x_{i-1})M_i + (x_{i+1} - x_i)M_{i+1} = \frac{6}{(x_{i+1} - x_i)}(f_{i+1} - f_i) - \frac{6}{(x_i - x_{i-1})}(f_i - f_{i-1})$$

<i>Spline</i> Natural	<i>Spline</i> Completa - Expressão para os pontos exteriores (nó 0 e n)
$M_0 = 0$	$2(x_1 - x_0)M_0 + (x_1 - x_0)M_1 = \frac{6}{(x_1 - x_0)}(f_1 - f_0) - 6f'_0$
$M_n = 0$	$2(x_n - x_{n-1})M_n + (x_n - x_{n-1})M_{n-1} = 6f'_n - \frac{6}{(x_n - x_{n-1})}(f_n - f_{n-1})$

Erro de truncatura *spline* cúbica $|f(x) - s_3(x)| \leq \frac{5}{384}h^4M_4$ $|f'(x) - s'_3(x)| \leq \frac{1}{24}h^3M_4$ com

$$\max_{\xi \in [x_0, x_n]} |f^{(iv)}(\xi)| \leq M_4 \quad h = \max_{0 \leq i \leq n-1} (x_{i+1} - x_i)$$

Integração numérica

Fórmulas simples Newton-Cotes

Trapézio	$\int_a^b f(x)dx \approx \frac{(b-a)}{2} [f(a) + f(b)]$	$ET = - \frac{(b-a)^3}{12} f''(\xi) , \xi \in [a, b]$
Simpson	$\int_a^b f(x)dx \approx \frac{(b-a)}{6} [f(a) + 4f(\frac{a+b}{2}) + f(b)]$	$ET = - \frac{(b-a)^5}{2880} f^{(iv)}(\xi) , \xi \in [a, b]$
$(\frac{3}{8})$	$\int_a^b f(x)dx \approx \frac{(b-a)}{8} [f(a) + 3f(\frac{2a+b}{3}) + 3f(\frac{a+2b}{3}) + f(b)]$	$ET = - \frac{(b-a)^5}{6480} f^{(iv)}(\xi) , \xi \in [a, b]$

Fórmulas compostas Newton-Cotes

Trapézio	$\int_a^b f(x)dx \approx \frac{h}{2} [f_0 + 2f_1 + 2f_2 + 2f_3 + 2f_4 + \dots + 2f_{n-4} + 2f_{n-3} + 2f_{n-2} + 2f_{n-1} + f_n]$ $ET = - \frac{h^2}{12}(b-a)f''(\eta) , \eta \in [a, b]$
Simpson	$\int_a^b f(x)dx \approx \frac{h}{3} [f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + \dots + 4f_{n-5} + 2f_{n-4} + 4f_{n-3} + 2f_{n-2} + 4f_{n-1} + f_n]$ $ET = - \frac{h^4}{180}(b-a)f^{(iv)}(\eta) , \eta \in [a, b]$
$(\frac{3}{8})$	$\int_a^b f(x)dx \approx \frac{3h}{8} [f_0 + 3f_1 + 3f_2 + 2f_3 + 3f_4 + 3f_5 + 2f_6 + \dots + 2f_{n-3} + 3f_{n-2} + 3f_{n-1} + f_n]$ $ET = - \frac{h^4}{80}(b-a)f^{(iv)}(\eta) , \eta \in [a, b]$