

## **Circuit Theory and Electronics Fundamentals**

Álvaro Dias Patrício, 95768

Inês de Sousa Afonso, 95797

José Severino Medeiro, 95811

Integrated Master in Aerospace Engineering, Técnico, University of Lisbon

Laboratory 1 Report

Group 8

February 25, 2021

# Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
<b>2</b>	<b>Theoretical Analysis</b>	<b>4</b>
2.1	Step 1: Solution for $t < 0$	4
2.2	Step 2: Solution for $t = 0$ and $R_{eq}$	5
2.3	Step 3: Natural Solution for $t > 0$	7
2.4	Step 4: Forced Solution for $t > 0$	8
2.5	Step 5: Total Solution for $t \in [-5, 20]$ ms	9
2.6	Step 6: Frequency Response of $v_6$ and $V_c$	10
<b>3</b>	<b>Simulation Analysis</b>	<b>11</b>
3.1	Operating Point Analysis for $t < 0$	11
3.2	Operating Point Analysis for $t = 0$	11
3.3	Natural Solution for $V_6$	12
3.4	Total Solution for $V_6$	13
3.5	Frequency Response	14
<b>4</b>	<b>Conclusion</b>	<b>15</b>

# 1 Introduction

The objective of this laboratory assignment is to study a circuit with four meshes containing a capacitor, a sinusoidal voltage source  $v_s$  and two linearly dependent sources: a voltage controlled current source  $I_b$  and a current controlled voltage source  $V_d$ . The circuit also contains seven resistors from  $R_1$  to  $R_7$  as it is shown in Figure 1. The values for the characteristics of this components, apart from  $I_d$  and  $V_b$  and including  $v_s$ ,  $C$ ,  $K_b$  and  $K_d$ , are given by Python and are:

Name	Value [A, V, S, $\Omega$ or F]
# $R_1$	1.032580e+03
# $R_2$	2.058543e+03
# $R_3$	3.056589e+03
# $R_4$	4.120838e+03
# $R_5$	3.102237e+03
# $R_6$	2.099094e+03
# $R_7$	1.015699e+03
$V_s$	5.198324e+00
& $C$	1.047395e-06
§ $K_b$	7.074481e-03
# $K_d$	8.223457e+03

Table 1: Variables in the Nodal Method. A variable preceded by @ is of type *current* and expressed in Ampere; variables preceded by # is of type *resistance* and expressed in Ohm; variables preceded by § is of type *conductance* and expressed in Seimens; variables preceded by & is of type *capacitance* and expressed in Farad; other variables are of type *voltage* and expressed in Volt.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical ones obtained in Section 2. The conclusions of this study are outlined in Section 4.

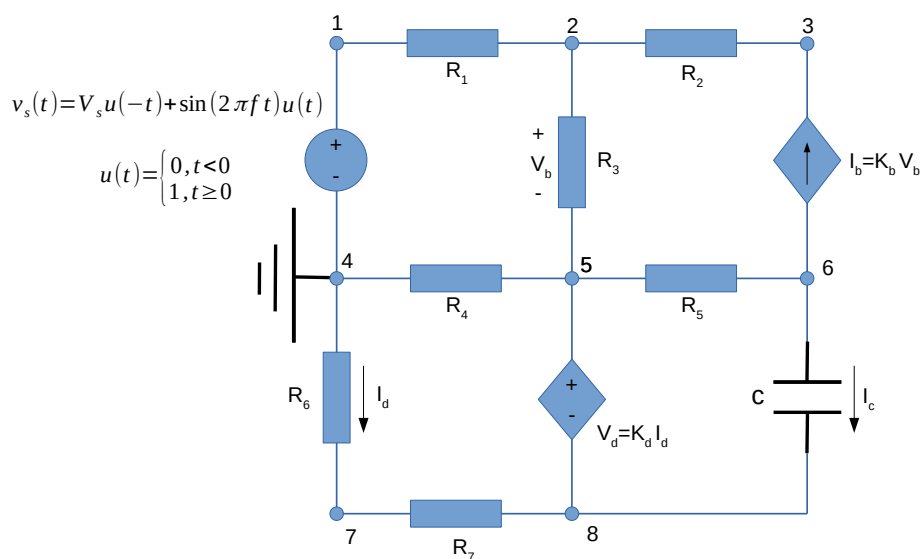


Figure 1: Circuit analysed.

## 2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analyzed theoretically with the Nodal Method and General Solution Method in order to characterize it in the interval  $[-5, 20[$  ms.

### 2.1 Step 1: Solution for $t < 0$

When  $t < 0$  we consider that the circuit is in equilibrium and, because of that, no current passes through the capacitor, since the relation between current and voltage in this component is:

$$\frac{dV}{dt} = \frac{I}{C} \Rightarrow I = 0. \quad (1)$$

Hence, we can simplify the circuit:

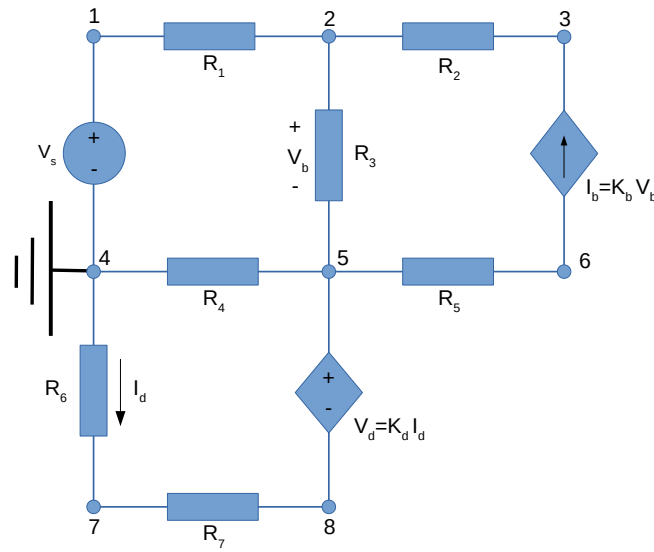


Figure 2: Simplified circuit for Step 1.

Applying the Nodal Method to this circuit leads to the following equations:

$$\left\{ \begin{array}{lcl} v_1 - v_4 & & = V_s; \\ \frac{1}{R_1} v_1 - (\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_3}) v_2 + (\frac{1}{R_2}) v_3 + \frac{1}{R_3} v_5 & & = 0; \\ \frac{1}{R_2} v_2 - \frac{1}{R_2} v_3 + I_b & & = 0; \\ -\frac{1}{R_1} v_1 + \frac{1}{R_1} v_2 - (\frac{1}{R_4} + \frac{1}{R_6}) v_4 + \frac{1}{R_4} v_5 + \frac{1}{R_6} v_7 & & = 0; \\ v_5 - v_8 - V_d & & = 0; \\ \frac{1}{R_5} v_5 - \frac{1}{R_5} v_6 - I_b & & = 0; \\ -(\frac{1}{R_7} + \frac{1}{R_6}) v_7 + \frac{1}{R_7} v_8 + \frac{1}{R_6} v_4 & & = 0; \\ v_2 - v_3 - V_b & & = 0; \\ \frac{1}{R_6} v_4 - \frac{1}{R_6} v_7 - I_d & & = 0; \\ v_4 & & = 0; \\ -K_b V_b + I_b & & = 0; \\ V_d - K_d I_c & & = 0. \end{array} \right. \quad (2)$$

Where the  $2^{nd}$ ,  $3^{rd}$ ,  $6^{th}$  and  $7^{th}$  are the equations in the respective nodes; the  $1^{st}$  and  $5^{th}$  are the relations imposed by the voltages sources in those nodes; the  $4^{th}$  is the supernode

that bypasses  $V_s$ ; the  $8^{th}$  to  $10^{th}$  are the relations between the circuit and, in order,  $V_b$ ,  $I_d$  and  $v_4$ ; the final two equations are the ones that describe  $I_b$  and  $V_d$ , respectively.

The solution to this linear system of equations is determined by Octave, plus the currents flowing in the circuit using Ohm's Law to the resistor's branches and Kirchhoff Current Law (KCL) to the branches with voltage sources:

Name	Value [A or V]
@ $I_c$	0.000000e+00
@ $I_b$	-2.455590e-04
@ $I_1$	2.342031e-04
@ $I_2$	2.455590e-04
@ $I_3$	-1.135597e-05
@ $I_4$	-1.211210e-03
@ $I_5$	-2.455590e-04
@ $I_6$	9.770070e-04
@ $I_7$	9.770070e-04
$v_1$	5.198324e+00
$v_2$	4.956490e+00
$v_3$	4.450997e+00
$v_4$	0.000000e+00
$v_5$	4.991201e+00
$v_6$	5.752983e+00
$v_7$	-2.050829e+00
$v_8$	-3.043174e+00
$V_b$	-3.471054e-02
$V_d$	8.034375e+00
@ $I_b$	-2.455590e-04
@ $I_d$	9.770070e-04
@ $I_D$	0.000000e+00
$V_c$	8.796158e+00

Table 2: Variables in the Mesh Method. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

All the currents were measured from the lower numbered node to the higher one; so, the direction of the current that passes through  $R_6$  is, if the result is positive, from  $v_4$  to  $v_7$ .

## 2.2 Step 2: Solution for $t = 0$ and $R_{eq}$

Once found the solution for the nodes in  $t < 0$ , the next step is to calculate the boundary conditions for the knots in the circuit, because the voltage in the nodes doesn't necessarily need to be continuous, only the difference of potential in the capacitor,  $V_c$ . Therefore, a non continuous change in a power source, in this case in the voltage source  $V_s$ , might lead to a different voltage in the nodes. Since the solution to this circuit will also be divided in natural and forced, we can use this step to evaluate the equivalent resistance of the circuit,  $R_{eq}$ .

To evaluate the circuit at  $t = 0$ , we must have in mind that  $V_c$  is continuous, so  $V_c(0) = V_c(0^-)$ . Another point to take into account is that, for the natural solution, we can ignore the sinusoidal part of  $V_s$ , which leaves us with  $V_s = 0$ . In this conditions, we can say that the circuit 1 is identical to this one:

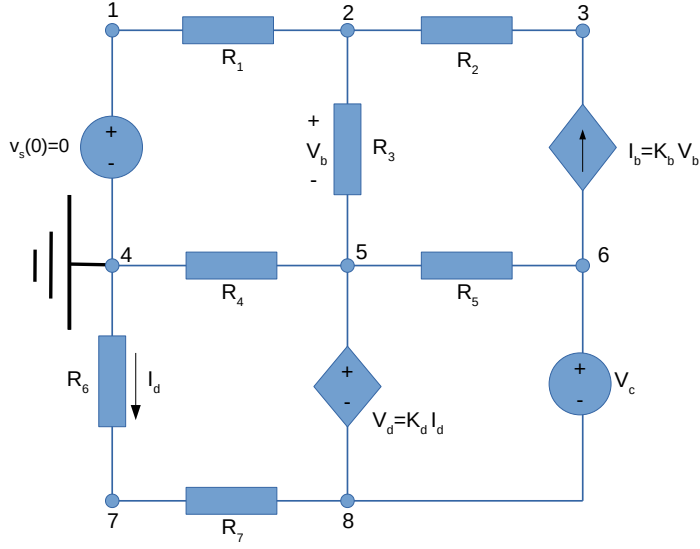


Figure 3: Equivalent circuit when  $t = 0$ .

The set of equations that describe it are, using the Nodal Method:

$$\left\{ \begin{array}{lcl} v_1 - v_4 & = & 0; \\ \frac{1}{R_1}v_1 - (\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_3})v_2 + (\frac{1}{R_2})v_3 + \frac{1}{R_3}v_5 & = & 0; \\ \frac{1}{R_2}v_2 - \frac{1}{R_2}v_3 + I_b & = & 0; \\ -\frac{1}{R_1}v_1 + \frac{1}{R_1}v_2 - (\frac{1}{R_4} + \frac{1}{R_6})v_4 + \frac{1}{R_4}v_5 + \frac{1}{R_6}v_7 & = & 0; \\ v_5 - v_8 - V_d & = & 0; \\ v_6 - v_8 & = & V_c; \\ -(\frac{1}{R_7} + \frac{1}{R_6})v_7 + \frac{1}{R_7}v_8 + \frac{1}{R_6}v_4 & = & 0; \\ v_2 - v_3 - V_b & = & 0; \\ \frac{1}{R_6}v_4 - \frac{1}{R_6}v_7 - I_d & = & 0; \\ v_4 & = & 0; \\ -K_bV_b + I_b & = & 0; \\ V_d - K_cI_c & = & 0. \end{array} \right. \quad (3)$$

Where the  $2^{nd}$ ,  $3^{rd}$  and  $7^{th}$  are the equations in the respective nodes; the  $1^{st}$ ,  $5^{th}$  and  $6^{th}$  are the relations imposed by the voltages sources in those nodes; the  $4^{th}$  is the supernode that bypasses  $V_s$ ; the  $8^{th}$  to  $10^{th}$  are the relations between the circuit and, in order,  $V_b$ ,  $I_d$  and  $v_4$ ; the final two equations are the ones that describe  $I_b$  and  $V_d$ , respectively. We note that

$$I_c = I_b + \frac{v_6 - v_5}{R_5}, \quad (4)$$

$$R_{eq} = \frac{V_c}{I_c}, \text{ and} \quad (5)$$

$$\tau = R_{eq}C. \quad (6)$$

The solution to this linear system of equations is determined by Octave, plus the currents flowing in the circuit using Ohm's Law to the resistor's branches and Kirchhoff Current Law (KCL) to the branches with voltage sources:

Name	Value [A, V or FΩ]
@ $I_b$	4.027896e-19
@ $I_1$	-8.399090e-22
@ $I_2$	-4.027896e-19
@ $I_3$	1.862716e-20
@ $I_4$	1.360604e-20
@ $I_5$	-2.835424e-03
@ $I_6$	-1.165351e-20
@ $I_7$	-1.276613e-20
$v_1$	0.000000e+00
$v_2$	8.672734e-19
$v_3$	8.300268e-16
$v_4$	0.000000e+00
$v_5$	-5.606829e-17
$v_6$	8.796158e+00
$v_7$	2.679731e-17
$v_8$	3.976385e-17
$V_c$	8.796158e+00
@ $I_c$	2.835424e-03
# $R_{eq}$	3.102237e+03
* $\tau$	3.249269e-03

Table 3: Variables in the Mesh Method. A variable preceded by @ is of type *current* and expressed in Ampere; variables preceded by # is of type *resistance* and expressed in Ohm; variables preceded by \* are expressed in Farad Ohm; other variables are of type *voltage* and expressed in Volt.

### 2.3 Step 3: Natural Solution for $t > 0$

Using  $R_{eq}$  and  $v_6$  from the previous step we can compute the natural solution of  $v_6(t)$ :

$$\begin{aligned}
 v_6(t) &= v_6(+\infty) - (v_6(+\infty) - v_6(0))e^{-\frac{1}{\tau}t} \\
 &= v_6(0)e^{-\frac{1}{\tau}t}, \text{ dado que } v_6(+\infty) = 0.
 \end{aligned} \tag{7}$$

Using Octave to plot this equation gives us:

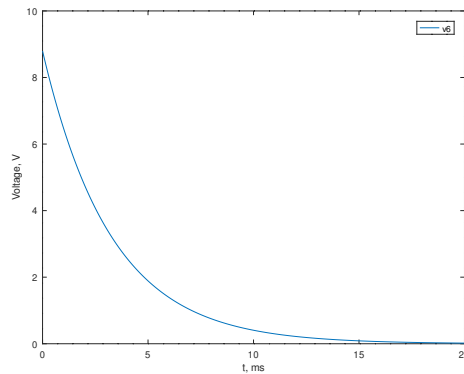


Figure 4: Natural solution of  $v_6(t)$ .

## 2.4 Step 4: Forced Solution for $t > 0$

Since  $V_s$  is a sinusoidal source in  $t > 0$ , we can perform a complex analysis, replacing all the componets for their impedances. In this case, only the capacitor changes (since all resistors have the same impedance):

$$Z_c = \frac{1}{j\omega C}, \quad (8)$$

where  $\omega$  is the frequency ( $1k \text{ rads}^{-1}$ ). Saying that  $V_s = 1$  to calculate the phasors relative to  $V_s$  and using the Nodal Method we arrive to a set of equations:

$$\begin{cases} v_1 - v_4 & = 1; \\ \frac{1}{R_1}v_1 - (\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_3})v_2 + (\frac{1}{R_2})v_3 + \frac{1}{R_3}v_5 & = 0; \\ \frac{1}{R_2}v_2 - \frac{1}{R_2}v_3 + I_b & = 0; \\ -\frac{1}{R_1}v_1 + \frac{1}{R_1}v_2 - (\frac{1}{R_4} + \frac{1}{R_6})v_4 + \frac{1}{R_4}v_5 + \frac{1}{R_6}v_7 & = 0; \\ v_5 - v_8 - V_d & = 0; \\ \frac{1}{R_5}v_5 - (\frac{1}{R_5} + j\omega C)v_6 + (j\omega C)v_8 - I_b & = 0; \\ -(\frac{1}{R_7} + \frac{1}{R_6})v_7 + \frac{1}{R_7}v_8 + \frac{1}{R_6}v_4 & = 0; \\ v_2 - v_3 - V_b & = 0; \\ \frac{1}{R_6}v_4 - \frac{1}{R_6}v_7 - I_d & = 0; \\ v_4 & = 0; \\ -K_b V_b + I_b & = 0; \\ V_d - K_c I_c & = 0. \end{cases} \quad (9)$$

Where the  $2^{nd}$ ,  $3^{rd}$ ,  $6^{th}$  and  $7^{th}$  are the equations in the respective nodes; the  $1^{st}$  and  $5^{th}$  are the relations imposed by the voltages sources in those nodes; the  $4^{th}$  is the supernode that bypasses  $V_s$ ; the  $8^{th}$  to  $10^{th}$  are the relations between the circuit and, in order,  $V_b$ ,  $I_d$  and  $v_4$ ; the final two equations are the ones that describe  $I_b$  and  $V_d$ , respectively.

The solution to this linear system of equations is determined by Octave:

Name	Value
$Pv_1$	1.000000e+00 0.000000e+00 j
$Pv_2$	9.534786e-01 0.000000e+00 j
$Pv_3$	8.562369e-01 0.000000e+00 j
$Pv_4$	0.000000e+00 0.000000e+00 j
$Pv_5$	9.601558e-01 0.000000e+00 j
$Pv_6$	-5.813645e-01 -8.268436e-02 j
$Pv_7$	-3.945174e-01 0.000000e+00 j
$Pv_8$	-5.854145e-01 0.000000e+00 j

Table 4: Variables in the Complex Nodal Method. Variables are adimentional.

The forced solution of  $v_6(t)$  is given by  $v_6(t) = V_s(t)Pv_6$ .



## 2.5 Step 5: Total Solution for $t \in [-5, 20]$ ms

The total solution of  $v_6$  will be the solution from step 1 for  $t \in [-5, 0[$  ms and the sum of the natural solution and forced solution for  $t \in [0, 20]$  ms. The plot given by Octave is:

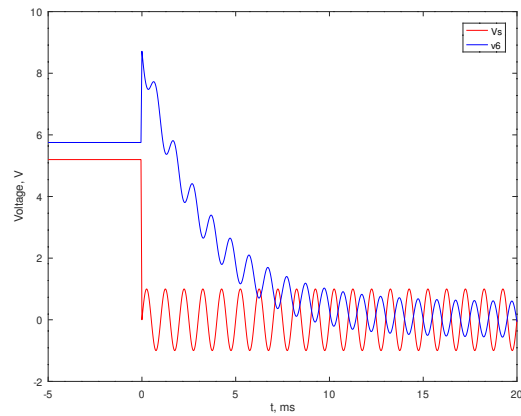


Figure 5: Total solution of  $v_6(t)$  and  $V_s(t)$ .

## 2.6 Step 6: Frequency Response of $v_6$ and $V_c$

In this subsection is made an analysis of the frequency response of  $v_6$  and  $V_c$  in order to  $V_s$ . for that we need to solve the same system as in step 4 (equation 9), with  $\omega$  as a parameter. The magnitude (absolute value of the complex vectors) and phase (angle of the complex vectors) is plotted using Octave:

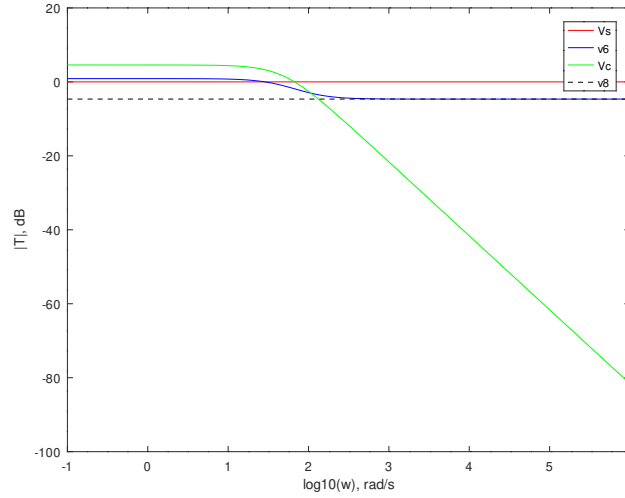


Figure 6: Magnitude response of  $V_s$ ,  $v_6$ ,  $V_c$  and  $v_8$ .

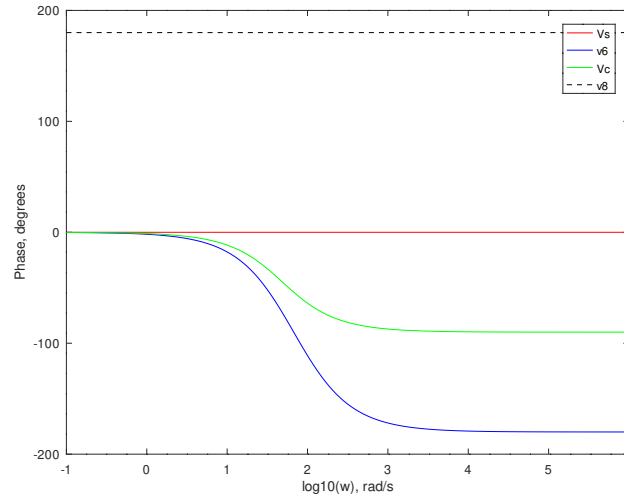


Figure 7: Angle response of  $V_s$ ,  $v_6$ ,  $V_c$  and  $v_8$ .

The results of  $V_s$  are to be expected, since  $\frac{V_s}{V_s} = 1$ . The results of  $V_c$  are also expected, since a capacitor is a low pass filter and this plots match to the ones of the classes. The results of  $v_6$  are, at first glance, unexpected, since we are used to suppose that  $v_6 \rightarrow 0$ ; but in fact what happens is that  $V_c \rightarrow 0 \Rightarrow v_6 - v_8 \rightarrow 0$ . Because the phase response of  $v_8$  is constant,  $v_6 - v_8 \rightarrow 0 \Rightarrow v_6 \rightarrow v_8$ , so the results of  $v_6$  are, indeed, expected.

### 3 Simulation Analysis

#### 3.1 Operating Point Analysis for $t < 0$

Table 5 shows the simulated operating point results for the circuit under analysis for  $t < 0$ .

Name	Value [A or V]
@c1[i]	0.000000e+00
@gb[i]	-2.45559e-04
@r1[i]	2.342031e-04
@r2[i]	2.455590e-04
@r3[i]	-1.13560e-05
@r4[i]	-1.21121e-03
@r5[i]	-2.45559e-04
@r6[i]	9.770070e-04
@r7[i]	9.770070e-04
v(1)	5.198324e+00
v(2)	4.956490e+00
v(3)	4.450997e+00
v(4)	0.000000e+00
v(5)	4.991201e+00
v(6)	5.752983e+00
v(7)	-2.05083e+00
v(8)	-3.04317e+00

Table 5: Operating point for  $t < 0$ . A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

It is important to notice that it has been necessary to implement an extra test voltage source between the node 7 and  $R_6$  providing 0V so that it does not interfere with the rest of the circuit and enable us to measure the voltage  $V_d$  flowing in the dependent source. Therefore, this creation does not change any results and it is only based on the Ngspice requirements to define a current controlled voltage source.

Compared to the theoretical analysis the simulation showed practically identical results, except for a small divergence in the last decimal place that probably occurs when Ngspice rounds the numbers. Thus, the maximum relative error is  $10^{-5}$ . It is worth mentioning that Ngspice software also uses the same mathematical methods as octave to find results, hence, this result was already expected.

#### 3.2 Operating Point Analysis for $t = 0$

The circuit was simulated using an operating point analysis with  $V_s(0) = 0$  and replacing the capacitor with a voltage source  $V_c = V(6) - V(8)$  as these were obtained in the previous step. This step is required to compute the boundary conditions that ensure the continuity in the capacitor. Table 6 shows the results of the simulation. Notice that:

$$R_{eq} = \frac{V_c}{I_c} \quad (10)$$

Name	Value [A or V]
@gb[i]	-2.10600e-18
@r1[i]	2.008605e-18
@r2[i]	2.105998e-18
@r3[i]	-9.73927e-20
@r4[i]	4.310669e-19
@r5[i]	-2.83542e-03
@r6[i]	-2.85149e-18
@r7[i]	-2.85149e-18
v(1)	0.000000e+00
v(2)	-2.07405e-15
v(3)	-6.40933e-15
v(4)	0.000000e+00
v(5)	-1.77636e-15
v(6)	8.796153e+00
v(7)	5.985534e-15
v(8)	8.881784e-15

Table 6: Operating point for  $t = 0$ . A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

### 3.3 Natural Solution for $V_6$

In order to study the natural response of the circuit in the interval  $[0,20]$ ms, a transient analysis was made, using the results for  $V_6$  and  $V_8$  computed in the previous step. Figure 8 shows the results for the natural solution for  $V_6$ .

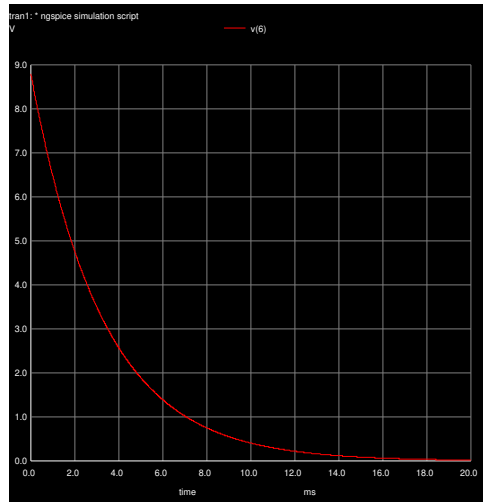


Figure 8: Natural solution of  $v_6(t)$ .

### 3.4 Total Solution for $V_6$

The results obtained in the last section were used to make this analysis, but considering  $V_s$  a sinusoidal voltage source with  $f=1\text{kHz}$ . The total response of the circuit is the result of summing the natural analysis with the forced one (natural + forced). Figure 9 shows the transient analysis results for  $V_s(t)$ .

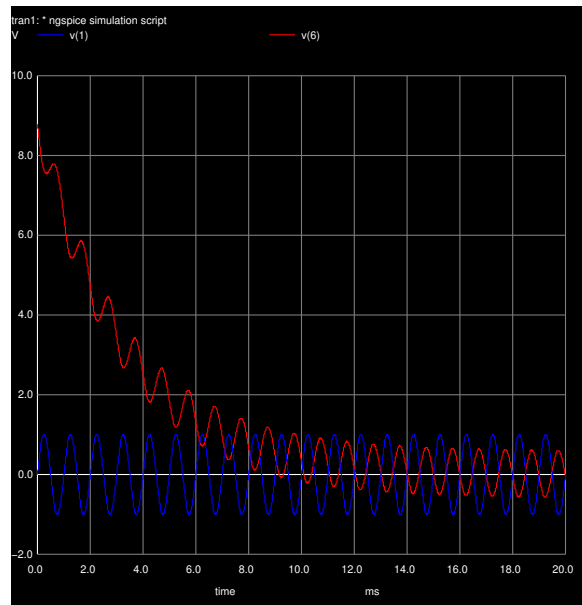


Figure 9: Natural solution of  $v_6(t)$  and  $V_s$ .

### 3.5 Frequency Response

In this section, the frequency range considered to analyze the frequency response in node 6 is from 0.1 Hz to 1 MHz. The figures show the magnitude and phase of the frequency response for  $V_s$ ,  $V_c$  and  $V_6$ .

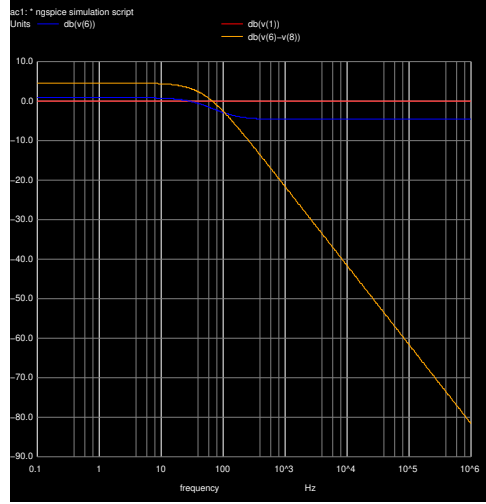


Figure 10: Magnitude response of  $V_s$ ,  $v_6(t)$  and  $V_c(t)$ .

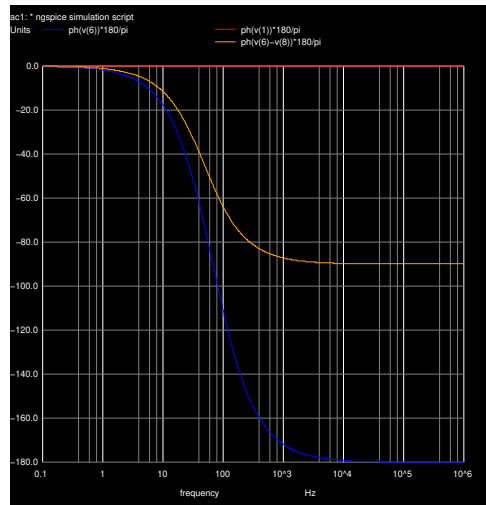


Figure 11: Angle response of  $V_s$ ,  $v_6(t)$  and  $V_c(t)$ .

When comparing the graphics obtained in Ngspice and Octave, the similarity between the results is noticeable and small differences can be explained by approximation errors.

## 4 Conclusion

To conclude, in this laboratory assignment the objective of analysing a circuit containing a sinusoidal voltage source and a capacitor has been achieved. Static, time and frequency analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool.

Name	Value [A or V]	Name	Value [A or V]
@ $I_c$	0.000000e+00	@c1[i]	0.000000e+00
@ $I_b$	-2.455590e-04	@gb[i]	-2.45559e-04
@ $I_1$	2.342031e-04	@r1[i]	2.342031e-04
@ $I_2$	2.455590e-04	@r2[i]	2.455590e-04
@ $I_3$	-1.135597e-05	@r3[i]	-1.13560e-05
@ $I_4$	-1.211210e-03	@r4[i]	-1.21121e-03
@ $I_5$	-2.455590e-04	@r5[i]	-2.45559e-04
@ $I_6$	9.770070e-04	@r6[i]	9.770070e-04
@ $I_7$	9.770070e-04	@r7[i]	9.770070e-04
$v_1$	5.198324e+00	v(1)	5.198324e+00
$v_2$	4.956490e+00	v(2)	4.956490e+00
$v_3$	4.450997e+00	v(3)	4.450997e+00
$v_4$	0.000000e+00	v(4)	0.000000e+00
$v_5$	4.991201e+00	v(5)	4.991201e+00
$v_6$	5.752983e+00	v(6)	5.752983e+00
$v_7$	-2.050829e+00	v(7)	-2.05083e+00
$v_8$	-3.043174e+00	v(8)	-3.04317e+00
$V_b$	-3.471054e-02		
$V_d$	8.034375e+00		
@ $I_b$	-2.455590e-04		
@ $I_d$	9.770070e-04		
@ $I_D$	0.000000e+00		
$V_c$	8.796158e+00		

Table 7: Operating point for  $t < 0$ . in Octave and NGSpice, respectively. A variable preceded by @ is of type *current* are expressed in Ampere; other variables are of type *voltage* and expressed in Volt. (As shown in Tables 2 and 5)

Name	Value [A or V]	Name	Value [A or V]
@ $I_b$	4.027896e-19	@gb[i]	-2.10600e-18
@ $I_1$	-8.399090e-22	@r1[i]	2.008605e-18
@ $I_2$	-4.027896e-19	@r2[i]	2.105998e-18
@ $I_3$	1.862716e-20	@r3[i]	-9.73927e-20
@ $I_4$	1.360604e-20	@r4[i]	4.310669e-19
@ $I_5$	-2.835424e-03	@r5[i]	-2.83542e-03
@ $I_6$	-1.165351e-20	@r6[i]	-2.85149e-18
@ $I_7$	-1.276613e-20	@r7[i]	-2.85149e-18
$v_1$	0.000000e+00	v(1)	0.000000e+00
$v_2$	8.672734e-19	v(2)	-2.07405e-15
$v_3$	8.300268e-16	v(3)	-6.40933e-15
$v_4$	0.000000e+00	v(4)	0.000000e+00
$v_5$	-5.606829e-17	v(5)	-1.77636e-15
$v_6$	8.796158e+00	v(6)	8.796153e+00
$v_7$	2.679731e-17	v(7)	5.985534e-15
$v_8$	3.976385e-17	v(8)	8.881784e-15
$V_c$	8.796158e+00		
@ $I_c$	2.835424e-03		
# $R_{eq}$	3.102237e+03		
* $\tau$	3.249269e-03		

Table 8: Operating point for  $t = 0$ . in Octave and NGSpice, respectively A variable preceded by @ is of type *current* and expressed in Ampere; variables preceded by \* are expressed in Farad Ohm; variables preceded by # is of type *resistance* and expressed in Ohm; other variables are of type *voltage* and expressed in Volt. (As shown in Tables 3 and 6)

The results match considerably, as expected, since this is a relatively straightforward circuit containing only one capacitor apart from linear components, therefore the theoretical and simulation models shouldn't differ. Values of the order of  $10^{-14}$  and lower should be taken as 0.