

## **Circuit Theory and Electronics Fundamentals**

Álvaro Dias Patrício, 95768

Inês de Sousa Afonso, 95797

José Severino Medeiro, 95811

Integrated Master in Aerospace Engineering, Técnico, University of Lisbon

Laboratory 1 Report

Group 8

February 25, 2021

# Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
<b>2</b>	<b>Theoretical Analysis</b>	<b>4</b>
2.1	Mesh Method . . . . .	4
2.2	Nodal Method . . . . .	5
<b>3</b>	<b>Simulation Analysis</b>	<b>7</b>
3.1	Operating Point Analysis . . . . .	7
<b>4</b>	<b>Conclusion</b>	<b>8</b>

# 1 Introduction

The objective of this laboratory assignment is to study a circuit with four meshes containing a voltage source  $V_a$ , a current source  $I_d$ , and two linearly dependent sources: a voltage controlled current source  $I_b$  and a current controlled voltage source  $V_c$ . The circuit also contains seven Resistors from  $R_1$  to  $R_7$  as it is shown in Figure 1. The values for the characteristics of this components, apart from  $I_b$  and  $V_c$  and including  $K_b$  and  $K_c$ , are given by Python and are:

Name	Value [A, V, S or $\Omega$ ]
# $R_1$	1.03258022265e+03
# $R_2$	2.05854281116e+03
# $R_3$	3.05658918951e+03
# $R_4$	4.12083818633e+03
# $R_5$	3.10223748153e+03
# $R_6$	2.09909352125e+03
# $R_7$	1.01569886691e+03
$V_a$	5.19832384287e+00
@ $I_d$	1.04739543259e-03
§ $K_b$	7.07448059081e-03
# $K_c$	8.22345657857e+03

Table 1: Variables in the Nodal Method. A variable preceded by @ is of type *current* and expressed in Ampere; variables preceded by # is of type *resistance* and expressed in Ohm; variables preceded by § is of type *conductance* and expressed in Seimens; other variables are of type *voltage* and expressed in Volt.

In Section 2, two different theoretical analysis of the circuit are presented using the mesh method and the nodal method. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical ones obtained in Section 2. The conclusions of this study are outlined in Section 4.

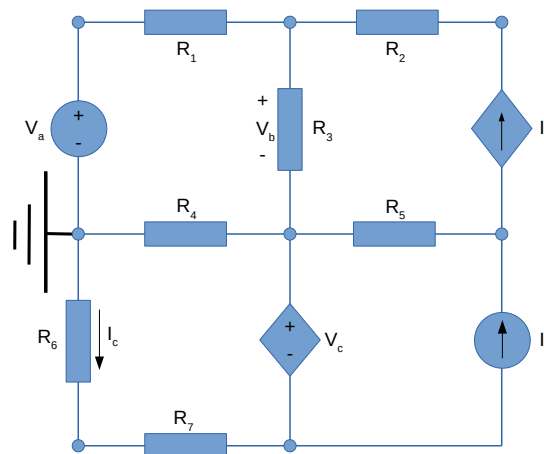


Figure 1: Circuit analysed.

## 2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analyzed theoretically with the Mesh Method and with the Nodal Method, in order to complement each other results.

### 2.1 Mesh Method

The Mesh Method consists in introducing currents that circulate in the meshes of the circuit, which are the loops that do not contain other loops, as shown in Figure 2, and then evaluate the circuit based on the new currents.

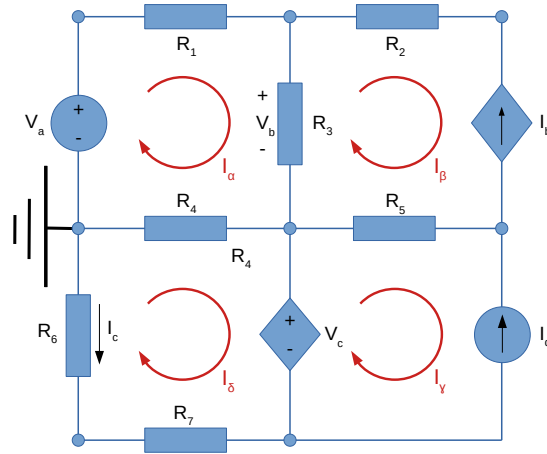


Figure 2: Circuit analysed with mesh currents.

After identifying the mesh currents, the next step in this method is to use the Kirchhoff Voltage Law (KVL) in the meshes that do not contain current sources (mesh  $\alpha$  (1) and  $\delta$  (2)) and to relate the mesh currents to the currents imposed by the sources (mesh  $\beta$  (3) and  $\gamma$  (4)):

$$R_1 I_\alpha + V_b + R_4 (I_\alpha - I_\delta) - V_a = 0; \quad (1)$$

$$R_6 I_\delta + R_4 (I_\delta - I_\alpha) + V_c + R_7 I_\delta = 0; \quad (2)$$

$$I_\beta = -I_b; \quad (3)$$

$$I_\gamma = -I_d. \quad (4)$$

Since there are 8 variables in the circuit,  $I_\alpha, I_\beta, I_\gamma, I_\delta, V_b, V_c, I_b, I_c$ , there must be more four independent equations: two of them are already given,

$$I_b = K_b V_b; \quad (5)$$

$$V_c = K_c I_c. \quad (6)$$

The other two are found by examining the circuit and with Ohm's Law:

$$I_c = -I_\delta; \quad (7)$$

$$V_b = R_3 (I_\alpha - I_\beta) \quad (8)$$

The solution to this linear system of equations is determined by Octave:

Name	Value [A or V]
@ $I_\alpha$	2.342031e-04
@ $I_\beta$	2.455590e-04
@ $I_\gamma$	-1.047395e-03
@ $I_\delta$	-9.770070e-04
$V_b$	-3.471054e-02
$V_c$	8.034375e+00
@ $I_b$	-2.455590e-04
@ $I_c$	9.770070e-04

Table 2: Variables in the Mesh Method. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

## 2.2 Nodal Method

Firstly, computing the values of current and voltage using the nodal method requires finding all the knots in the circuit, as it is presented in Figure 3.

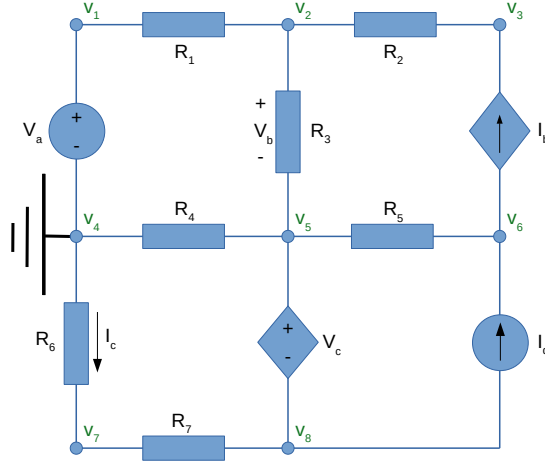


Figure 3: Circuit analysed with nodal voltages.

Then, the Kirchhoff Current Law (KCL) is used in the nodes not connected to voltage sources (Equations 9 to 12) and two additional equations relating the knots voltages with the voltage sources connected to them are presented (Equations 13 and 14):

$$\frac{v_3 - v_2}{R_2} - \frac{V_b}{R_3} + \frac{v_1 - v_2}{R_1} = 0; \quad (9)$$

$$I_b + \frac{v_2 - v_3}{R_2} = 0; \quad (10)$$

$$\frac{v_5 - v_6}{R_5} - I_b + I_d = 0; \quad (11)$$

$$I_c + \frac{v_8 - v_7}{R_7} = 0; \quad (12)$$

$$v_1 - v_4 = V_a; \quad (13)$$

$$v_5 - v_8 = V_c. \quad (14)$$

Since there are 8+4 variables ( $v_1$  to  $v_8$ ,  $V_b$ ,  $V_c$ ,  $I_b$ ,  $I_c$ ), the system is defined by twelve independent equations. Two are provided in the circuit and are the same as in the Mesh Method, (5) and (6). By observing the circuit,

$$v_5 - v_8 = V_b. \quad (15)$$

Using Ohm's Law, the following relation is found:

$$I_c = \frac{v_4 - v_7}{R_6}. \quad (16)$$

Because there needs to be a knot with a defined voltage, we chose  $v_4$  to be connected to the ground:

$$v_4 = 0. \quad (17)$$

For the last equation, the continuity of current in the circuit can be used to create a “super-knot”, bypassing the voltage sources  $V_a$  and  $V_c$ , from which the equations are (18) and (19), respectively:

$$\frac{v_5 - v_4}{R_4} + \frac{v_2 - v_1}{R_1} - I_c = 0; \quad (18)$$

$$\frac{v_7 - v_8}{R_7} - I_d + \frac{v_6 - v_5}{R_5} + \frac{V_b}{R_3} + \frac{v_4 - v_5}{R_4} = 0. \quad (19)$$

Given that only one more equation is needed, we chose to use the simpler one (18). The solution to this linear system of equations is determined by Octave:

Name	Value [A or V]
$v_1$	5.198324e+00
$v_2$	4.956490e+00
$v_3$	4.450997e+00
$v_4$	0.000000e+00
$v_5$	4.991201e+00
$v_6$	9.002253e+00
$v_7$	-2.050829e+00
$v_8$	-3.043174e+00
$V_b$	-3.471054e-02
$V_c$	8.034375e+00
@ $I_b$	-2.455590e-04
@ $I_c$	9.770070e-04

Table 3: Variables in the Nodal Method. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

### 3 Simulation Analysis

#### 3.1 Operating Point Analysis

Table 4 shows the simulated operating point results for the circuit under analysis. All the currents were measured from the lower numbered node to the higher one; so, the direction of the current that passes through  $R_6$  is, if the result is positive, from  $v_4$  to  $v_7$ .

Name	Value [A or V]
@gb[i]	-2.45559e-04
@id[current]	1.047395e-03
@r1[i]	2.342031e-04
@r2[i]	2.455590e-04
@r3[i]	-1.13560e-05
@r4[i]	-1.21121e-03
@r5[i]	-1.29295e-03
@r6[i]	9.770070e-04
@r7[i]	9.770070e-04
v(1)	5.198324e+00
v(2)	4.956490e+00
v(3)	4.450997e+00
v(4)	0.000000e+00
v(5)	4.991201e+00
v(6)	9.002253e+00
v(7)	-2.05083e+00
v(8)	-3.04317e+00
v(2,5)	-3.47105e-02
v(5,8)	8.034375e+00

Table 4: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

It is important to notice that it has been necessary to implement an extra test voltage source  $V_3$  connecting  $v_4$  to  $R_6$  providing 0V so that it does not interfere with the rest of the circuit and enable us to measure the current  $I_c$  flowing in the above mentioned resistor. Therefore, his creation does not change any results and it is only based on the Ngspice requirements to define a current controlled voltage source.

Compared to the theoretical analysis the simulation showed practically identical results, except for a small divergence in the last decimal place that probably occurs when NGSpice rounds the numbers. Thus, the maximum relative error is  $10^{-5}$ . It is worth mentioning that ngspice software also uses the same mathematical methods as octave to find results, hence, this result was already expected.

## 4 Conclusion

To conclude, in this laboratory assignment the aim of analysing the circuit has been achieved. The values of the currents and voltages have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The results match precisely, as expected, since this is a straightforward circuit containing only linear components, therefore the theoretical and simulation models shouldn't differ.