Circuit Theory and Electronics Fundamentals

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Laboratory 1 Report

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Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit with four meshes containing a voltage source V_a, a current source I_d, and two linearly dependent sources: a voltage controlled current source I_b and a current controlled voltage source V_c. The circuit also contains seven Resistors from R_1 to R_7 as it is shown in Figure 1. The values for the caracteristics of this components, apart from I_b and V_c and including K_b and K_c, are given by Python and are:

Name	**Value [A, V, S or Ω]**
#R_1	1.03258022265e+03
#R_2	2.05854281116e+03
#R_3	3.05658918951e+03
#R_4	4.12083818633e+03
#R_5	3.10223748153e+03
#R_6	2.09909352125e+03
#R_7	1.01569886691e+03
V_a	5.19832384287e+00
@I_d	1.04739543259e-03
§K_b	7.07448059081e-03
#K_c	8.22345657857e+03

Table 1: Variables in the Nodal Method. A variable preceded by @ is of type *current* and expressed in Ampere; variables preceded by # is of type *resistance* and expressed in Ohm; variables preceded by § is of type *conductance* and expressed in Seimens; other variables are of type *voltage* and expressed in Volt.

In Section 2, two different theoretical analysis of the circuit are presented using the mesh method and the nodal method. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical ones obtained in Section 2. The conclusions of this study are outlined in Section 4.

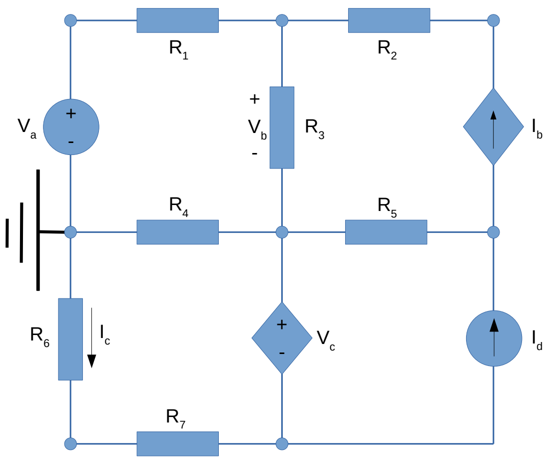

Figure 1: Circuit analysed.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analyzed theoretically with the Mesh Method and with the Nodal Method, in order to complement each other results.

2.1 Mesh Method

The Mesh Method consists in introducing currents that circulate in the meshes of the circuit, which are the loops that do not contain other loops, as shown in Figure 2, and then evaluate the circuit based on the new currents.

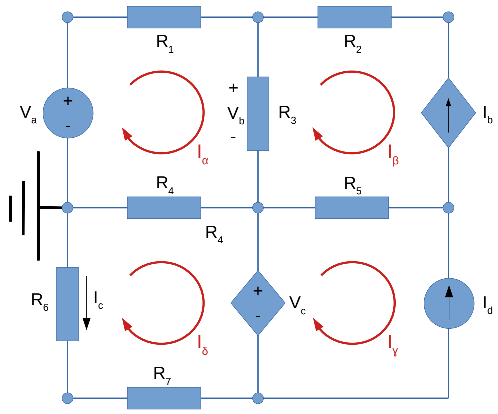

Figure 2: Circuit analysed with mesh currents.

After identifying the mesh currents, the next step in this method is to use the Kirchhoff Voltage Law (KVL) in the meshes that do not contain current sources (mesh α (1) and δ (4)) and to relate the mesh currents to the currents imposed by the sources (mesh β (2) and γ (3)):

$$R_1 I_\alpha + R_3(I_\alpha - I_\beta) + R_4(I_\alpha - I_\delta) - V_a = 0; \tag{1}$$

$$I_\beta = -I_b; \tag{2}$$

$$I_\gamma = -I_d. \tag{3}$$

$$R_6 I_\delta + R_4(I_\delta - I_\alpha) + V_c + R_7 I_\delta = 0; \tag{4}$$

Since there are 8 variables in the circuit, I_α, I_β, I_γ, I_δ, V_b, V_c, I_b, I_c, there must be more four independent equations: two of them are already given,

$$I_b = K_b V_b; \tag{5}$$

$$V_c = K_c I_c. \tag{6}$$

The other two are found by examining the circuit and with Ohm's Law:

$$I_c = -I_\delta; \tag{7}$$

$$V_b = R_3(I_\alpha - I_\beta) \tag{8}$$

In resume, the linear system of equations is:

$$\begin{cases} (R_1+R_3+R_4)I_\alpha - R_3 I_\beta - R_4 I_\delta &= V_a; \\ I_\beta + I_b &= 0; \\ I_\gamma &= -I_d. \\ (R_6+R_4+R_7)I_\delta - R_4 I_\alpha + V_c &= 0; \\ I_\delta + I_c &= 0; \\ R_3 I_\alpha - R_3 I_\beta - V_b &= 0. \\ -K_b V_b + I_b &= 0; \\ V_c - K_c I_c &= 0. \end{cases} \tag{9}$$

The solution to this linear system of equations is determined by Octave:

Name	Value [A or V]
@I_α	2.342031e-04
@I_β	2.455590e-04
@I_γ	-1.047395e-03
@I_δ	-9.770070e-04
V_b	-3.471054e-02
V_c	8.034375e+00
@I_b	-2.455590e-04
@I_c	9.770070e-04

Table 2: Variables in the Mesh Method. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

2.2 Nodal Method

Firstly, computing the values of current and voltage using the nodal method requires finding all the knots in the circuit, as it is presented in Figure 3.

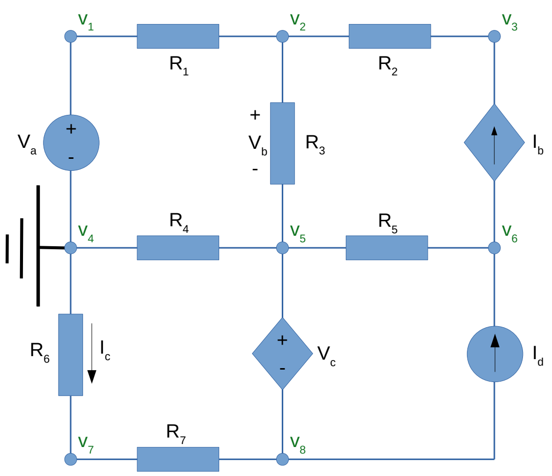

Figure 3: Circuit analysed with nodal voltages.

Then, the Kirchhoff Current Law (KCL) is used in the nodes not connected to voltage sources (Equations 10 to 13) and two additional equations relating the knots voltages with the voltage sources connected to them are presented (Equations 14 and 15):

$$\frac{v_3 - v_2}{R_2} + \frac{v_2 - v_5}{R_3} + \frac{v_1 - v_2}{R_1} = 0; \tag{10}$$

$$I_b + \frac{v_2 - v_3}{R_2} = 0; \tag{11}$$

$$\frac{v_5 - v_6}{R_5} - I_b + I_d = 0; \tag{12}$$

$$\frac{v_4 - v_7}{R_6} + \frac{v_8 - v_7}{R_7} = 0; \tag{13}$$

$$v_1 - v_4 = V_a; \tag{14}$$

$$v_5 - v_8 = V_c. \tag{15}$$

Since there are 8+4 variables (v_1 to v_8, V_b, V_c, I_b, I_c), the system is defined by twelve independent equations. Two are provided in the circuit and are the same as in the Mesh Method, (5) and (6). By observing the circuit,

$$v_2 - v_5 = V_b. \tag{16}$$

Using Ohm's Law, the following relation is found:

$$\frac{v_4 - v_7}{R_6} = I_c. \tag{17}$$

Because there needs to be a knot with a defined voltage, we chose v_4 to be connected to the ground:

$$v_4 = 0. \tag{18}$$

For the last equation, the continuity of current in the circuit can be used to create a "super-knot", bypassing the voltage sources V_a and V_c, from which the equations are (19) and (20), respectively:

$$\frac{v_5 - v_4}{R_4} + \frac{v_2 - v_1}{R_1} - \frac{v_4 - v_7}{R_6} = 0; \tag{19}$$

$$\frac{v_7 - v_8}{R_7} - Id + \frac{v_6 - v_5}{R_5} + \frac{v_2 - v_5}{R_3} + \frac{v_4 - v_5}{R_4} = 0. \tag{20}$$

Given that only one more equation is needed, we chose to use the simpler one (19).

$$\begin{cases} v_1 - v_4 & = V_a; \\ \frac{1}{R_1}v_1 - (\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_3})v_2 + (\frac{1}{R_2})v_3 + \frac{1}{R_3}v_5 & = 0; \\ \frac{1}{R_2}v_2 - \frac{1}{R_2}v_3 + I_b & = 0; \\ -\frac{1}{R_1}v_1 + \frac{1}{R_1}v_2 - (\frac{1}{R_4} + \frac{1}{R_6})v_4 + \frac{1}{R_4}v_5 + \frac{1}{R_6}v_7 & = 0; \\ v_5 - v_8 - V_c & = 0; \\ \frac{1}{R_5}v_5 - \frac{1}{R_5}v_6 - I_b & = -I_d; \\ -(\frac{1}{R_7} + \frac{1}{R_6})v_7 + \frac{1}{R_7}v_8 + \frac{1}{R_6}v_4 & = 0; \\ v_2 - v_3 - V_b & = 0; \\ \frac{1}{R_6}v_4 - \frac{1}{R_6}v_7 - I_c & = 0; \\ v_4 & = 0; \\ -K_bV_b + I_b & = 0; \\ V_c - K_cI_c & = 0. \end{cases} \tag{21}$$

The solution to this linear system of equations is determined by Octave:

Name	Value [A or V]
v_1	5.198324e+00
v_2	4.956490e+00
v_3	4.450997e+00
v_4	0.000000e+00
v_5	4.991201e+00
v_6	9.002253e+00
v_7	-2.050829e+00
v_8	-3.043174e+00
V_b	-3.471054e-02
V_c	8.034375e+00
@I_b	-2.455590e-04
@I_c	9.770070e-04

Table 3: Variables in the Nodal Method. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3 Simulation Analysis

3.1 Operating Point Analysis

Table 4 shows the simulated operating point results for the circuit under analysis. All the currents were measured from the lower numbered node to the higher one; so, the direction of the current that passes through R_6 is, if the result is positive, from v_4 to v_7.

Name	**Value [A or V]**
@gb[i]	-2.45559e-04
@id[current]	1.047395e-03
@r1[i]	2.342031e-04
@r2[i]	2.455590e-04
@r3[i]	-1.13560e-05
@r4[i]	-1.21121e-03
@r5[i]	-1.29295e-03
@r6[i]	9.770070e-04
@r7[i]	9.770070e-04
v(1)	5.198324e+00
v(2)	4.956490e+00
v(3)	4.450997e+00
v(4)	0.000000e+00
v(5)	4.991201e+00
v(6)	9.002253e+00
v(7)	-2.05083e+00
v(8)	-3.04317e+00
v(2,5)	-3.47105e-02
v(5,8)	8.034375e+00

Table 4: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

It is important to notice that it has been necessary to implement an extra test voltage sorce V_3 connecting v_4 to R_6 providing 0V so that is does not interfere with the rest of the circuit and enable us to measure the current I_c flowing in the above mentioned resistor. Therefore, this creation does not change any results and it is only based on the Ngspice requirements to define a current controlled voltage source.

Compared to the theoretical analysis the simulation showed practically identical results, except for a small divergence in the last decimal place that probably occurs when NGSpice rounds the numbers. Thus, the maximum relative error is 10^{-5}. It is worth mentioning that ngspice software also uses the same mathematical methods as octave to find results, hence, this result was already expected.

4 Conclusion

To conclude, in this laboratory assigment the aim of analysing the circuit has been achieved. The values of the currents and voltages have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The results match precisely, as expected, since this is a straightforward circuit containing only linear components, therefore the theoretical and simulation models shouldn't differ.