Circuit Theory and Electronics Fundamentals

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Laboratory 5 Report

Group 8

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Contents

1 Introduction

The aim of the laboratory assignment was to create a BandPass Filter (BPF, using an OP-AMP, shown in Figure 1. It should have a central frequency of 1 kHz a gain of 40 dB, or 100 times, at this frequency. In order to implement the BPF, the following components were available:

- One 741 OP-AMP;
- At most three $1k\Omega$ resistors;
- At most three $10k\Omega$ resistors;
- At most three $100k\Omega$ resistors;
- At most three $220nF$ capacitors;
- At most three 10μ capacitors.

In order to have a good voltage amplifier, a high gain A_v, a hight input impedance Z_i and a low output impedance are required, so that the input and output voltages do not get degraded. There are two stages in the audio amplifier circuit: the gain and output stages.

The quality of the BPF, comparing to others, is determined by a merit classification system. It takes into account the cost of the components used, the voltage gain, the cut off frequency and the bandwidth. The merit is calculated according the following equation:

$$MERIT = \frac{1}{Cost * (Gaindeviation + CentralFrequencydeviation + 10{-}6)} \tag{1}$$

And the cost of the components is:

- cost of resistors = 1 monetary unit (MU) per $k\Omega$;
- cost of capacitors = 1 MU/ μF;
- cost of transistors = 0.1 MU per transistor.

Figure 1 shows the layout of the implemented circuit.

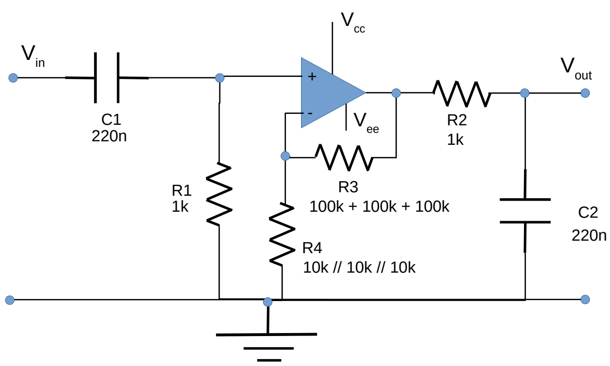

Figure 1: Circuit analysed.

Table 1 shows the resistors, capacitors and number of transistors, as well as the total cost of the circuit.

Name	**Value**
AMPOP	1
#R_1	1.000000e+03
#R_2	5.000000e+02
#R_3	3.000000e+05
#R_4	3.333333e+03
&C_1	2.200000e-07
&C_2	2.200000e-07
£Cost	1.362527e+04

Table 1: Variables preceded by # is of type *resistance* and expressed in Ohm; variables preceded by & is of type *capacitance* and expressed in Farad; A variable preceded by £ is of type *cost* and expressed in Unit of Cost; the numbers of OP-AMPs are in units.

In Section 2, the circuit is analyzed by simulation in Ngspice, using the OPAMP model provided. It allowed us to obtain the output voltage gain in the passband, the central frequency and the input and output impedances.

Then, in Section 3, the circuit is analyzed theoretically using Octave tools, where the frequency response $V_o(f)/V_i(f)$, the gain and the input and output impedances at the central frequency are determined.

In the conclusion, Section 4 the theoretical and the simulation results were compared.

2 Simulation Analysis

In this section we evaluated the BPF proposed. In order to analise the circuit after the transiant period, we have chosen $t \in [1.5ms; 11.5ms[$.

The graphs for NGSpice are displayed here, alongside the table with the required elements:

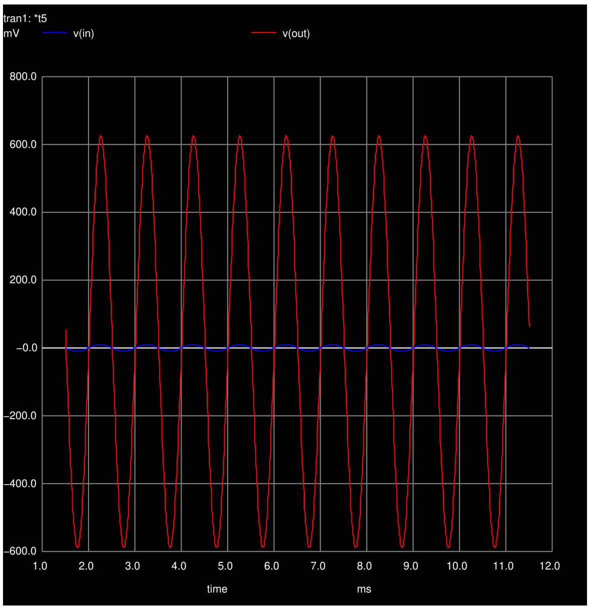

Figure 2: v_{in} and v_{out}.

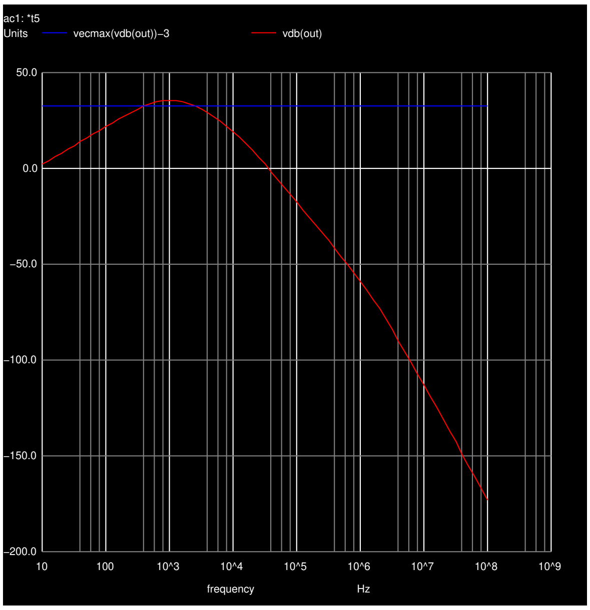

Figure 3: $db(v_{out})$ and $max(db(v_{out})) - 3$.

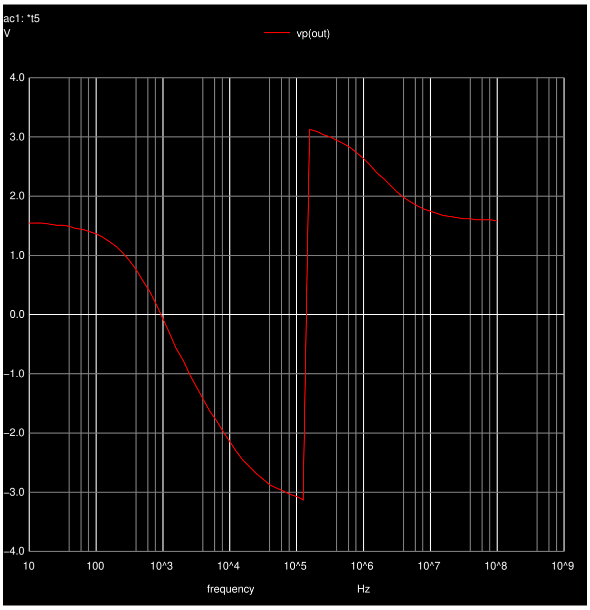

Figure 4: Phasor of v_{out}, rad

Table 2 shows the simulated frequency response results for the circuit.

Name	Value
f1	4.083079e+02
f2	2.500013e+03
f0	1.010334e+03
f0 - 1000	1.033413e+01
amplitude db	3.565074e+01
amplitude	6.713499e+01
amplitude - 100	-3.28650e+01
cost	1.365644e+04
merit	3.250008e-06

Table 2: Merit is in *per voltage per cost* and expressed in Volt^{-1}UC^{-1}; $f1$, $f2$, $f0$, $f0 - 1000$ are of the type *frequency* and expressed in Hz; amplitude ande amplitude - 100 are of the type *voltage* in Volt; other variables are adimentional.

Table 3 and table 4 show the input and output impedances, respectivly.

Name	Value
zin	9.999944e+02 + j*-7.23526e+02

Table 3: Variables are of type *impedance* and expressed in Ohm.

Name	Value [A or V]
zout	3.413639e+02 + j*-2.33092e+02

Table 4: Variables are of type *impedance* and expressed in Ohm.

As will be discused in the next chapter, the output impedance is high, meaning that this particular circuit wuold face chalanges connecting to low impedance loads.

3 Theoretical Analysis

In this section, a theoretical analysis of the previously shown circuit was conducted. An OP-AMP BandPass filter blocks the undesired frequencies, allowing only a certain bandwidth of frequencies to pass through. Besides that, it also amplifies the signal for these frequencies. As it is an active filter, transistors are used, apart from resistors and capacitors, with the purpose of amplifying the signal. In this laboratory, the 741 OP-AMP is used in an non inverting manner and the input signal is filtered.

The transfer function for the circuit is:

$$T(s) = \frac{R_1C_1s}{R_1C_1s+1}\frac{R_3+R_4}{R_4}\frac{1}{R_2C_2s+1}, \tag{2}$$

where the first part is the result of the high pass subcircuit, the second is due to the OP-AMP (assumig that $A_v = \infty$) and the third and last due to the low pass subcircuit after the OP-AMP.

The lower and upper cut-off frequencies, respectively, are given by the following expressions:

$$\omega_L = \frac{1}{R_1C_1}; \tag{3}$$

$$\omega_H = \frac{1}{R_2C_2}. \tag{4}$$

Therefore, the central angular frequency, which corresponds to the geometric centre of there two frequencies above, is given by

$$\omega_O = \sqrt{\omega_L\omega_H} = \sqrt{\frac{1}{R_1C_1R_2C_2}}. \tag{5}$$

The gain will be given when $s = j\omega_O$.

Finally, the input and output impedances, respectively, are given by:

$$Z_I = Z_{R1} + Z_{C1} = R_1 + \frac{1}{j\omega_O C_1}; \tag{6}$$

$$Z_O = Z_{R2} \parallel Z_{C2} = \frac{1}{\frac{1}{R_2} + j\omega_O C_2}. \tag{7}$$

The following table shows the theoretical results obtained using the previous formulas to optimise the merit:

Name	**Value**
#Z_{in}	1.000000e+03 +j*-7.234316e+02
#Z_{out}	3.383662e+02 +j*-2.338619e+02

Table 5: The variables of type *impedance* and expressed in Ohm.

We can observe that the output impedance of the circuit is non trivial, posing problems when considering connecting this amplifier to a low impedance load; this is the primary reason we have tested this circuit without a load connected ($R_L = \infty$).

Name	**Value**
$Gain$	6.066649e+01

Table 6: Variables are adimentional.

The gain we see here is significantly lower than the requested 100. The problem arises with the limitations in the options of the resistors, since the gain comes from $\frac{R_3+R_4}{R_4}$, as seen in equation 2. With a low R_4, we could increase this gain significantly; unfortunatly, due to the necessary use of the $1k\Omega$ resistors in the filters, we are only left with $10k\Omega$ and $100k\Omega$ resistors, from each we take full advantage, pairing in series the $100k\Omega$ resistors (to create the biggest resistor possible) and in parallel the $10k\Omega$ resistor (to create the smallest possible resistor), which is not perfect. A simple solution to this problem would be to replace the $10k\Omega$ resistors with $2k\Omega$ ones. In that case, the pairing of $R_3 = 100k\Omega$ and $R_4 = 2k \parallel 2k \parallel 2k\Omega$ would be a almost perfect solution (given the limitations of discreet components).

The next figures show the frequency response obtained in this analysis.

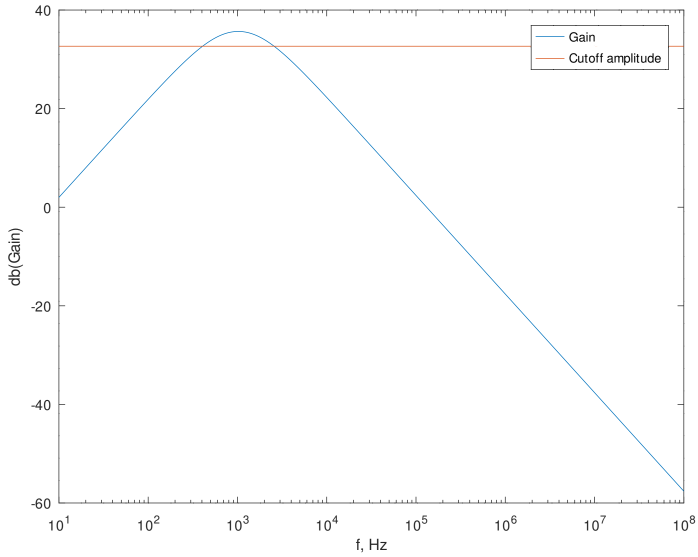

Figure 5: $db(v_{out})$ and $max(db(v_{out})) - 3$.

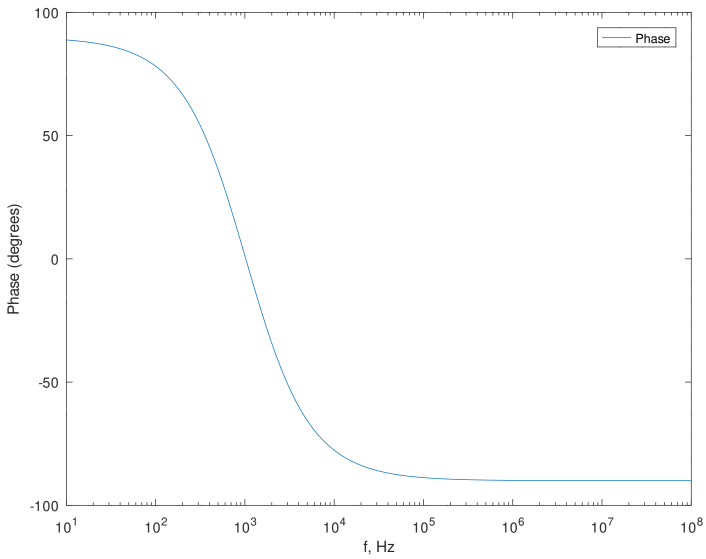

Figure 6: Phasor of v_{out}, degrees.

We can see the two poles of the transfer function acting on the phase, subtracting 180 degrees from the initial phase and on the magnitude, only alowing a narrow set of frequencies to have a significant gain.

4 Conclusion

In this fifth TCFE lab, we had to design a bandpass filter with an OP-AMP. Through mathematical tools such as OCTAVE, we were able to produce a theoretical analysis of the circuit and through NGSPICE we created a simulation of the circuit.

For the most part, there was not a significant difference between the theoretical results and the simulation results, so we have decided not to use a more acurate theoretical model of the OP-AMP, having instead chosen to maintain the ideal model.

However, the same cannot be said in regard to the phase response (Figure 4 or Figure 10). A possible explanation is the presence of two extra poles in the transfer function that are hidden in a more realistic OP-AMP, since there is a diference of -180 degrees to what would be expected if the transfer function 2. Upon a closer inspection, the plot of the amplitude response (Figure 3 or Figure 8) does indicate the same two poles, one around 10^4 Hz and 10^6 Hz, that change de slope of the plot. With this knowladge, we could, in theory, discart the low pass filter after the OP-AMP, and adjust the high pass filter accordingly. The problem with this approach is that we are not sure about the origin of additional low pass filter, or if it changes with input amplitude or other unknown paramaters.

The gain obtained by the two models was also different between simulation and theory, and the output impedance values were,as hinted in the beggining of this chapter, not very diferent from each other.

Name	**Value**
$\#Z_{in}$	1.000000e+03 +j*-7.234316e+02
$\#Z_{out}$	3.383662e+02 +j*-2.338619e+02

Name	**Value**
zin	9.999944e+02 + j*-7.23526e+02
Name	**Value [A or V]**
zout	3.413639e+02 + j*-2.33092e+02

Table 7: Results in Octave and NGSpice, respectivily. NGSpice values are of of type *impedance* and expressed in Ohm; variables preceded by # are of type *impedance* and expressed in Ohm; other variables are adimentional. (As shown in Tables 5, 3 and 4)

Name	**Value**
$Gain$	6.066649e+01

Name	**Value**
f1	4.083079e+02
f2	2.500013e+03
f0	1.010334e+03
f0 - 1000	1.033413e+01
amplitude db	3.565074e+01
amplitude	6.713499e+01
amplitude - 100	-3.28650e+01
cost	1.365644e+04
merit	3.250008e-06

Table 8: Results in Octave and NGSpice, respectivily. Merit is in *per voltage per cost* and expressed in Volt^{-1}UC^{-1}; $f1 - f2$ is of the type *frequency* and expressed in Hz; amplitude ande amplitude - 100 are of the type *voltage* in Volt; other variables are adimentional. (As shown in Tables 6 and 2)

The Merit result was $3.25 * 10^{-6} V^{-1} uc^{-1}$ by Ngspice. It was agreed by the members of the group that the main goal of task was completed, given the circunstances. We note that the main drives for the low figure of merit are two: the elevated cost of the OP-AMP and the lack of low resistors to use, as said in section 3.

The obtained plot with NGSpice and Octave are the following:

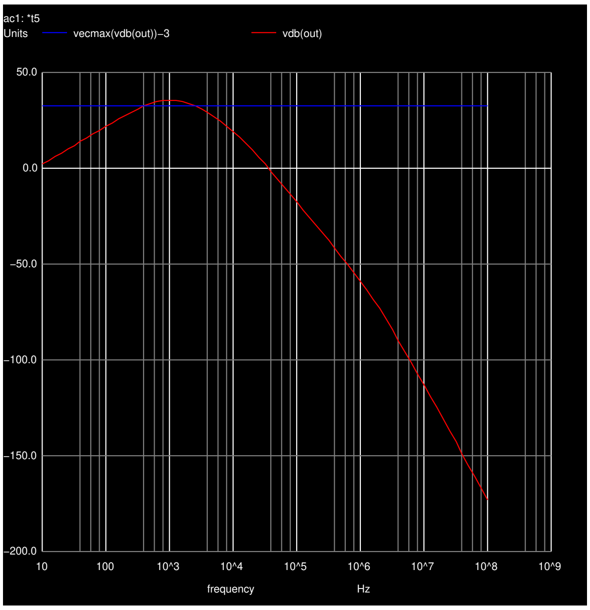

Figure 7: NGSpice plot:$db(v_{out})$ and $max(db(v_{out})) - 3$.

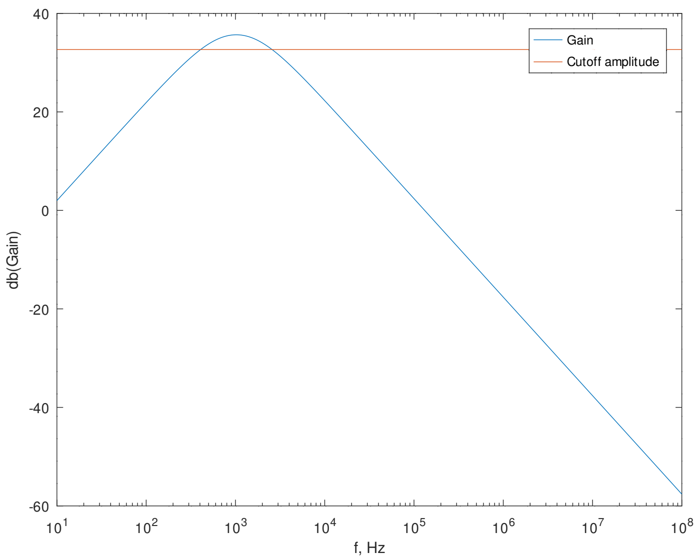

Figure 8: Octave plot:$db(v_{out})$ and $max(db(v_{out})) - 3$.

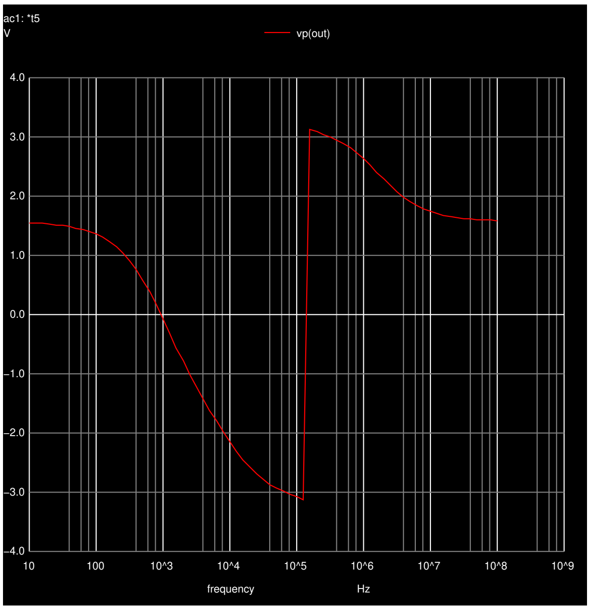

Figure 9: NGSpice plot: Phasor of v_{out}, rad

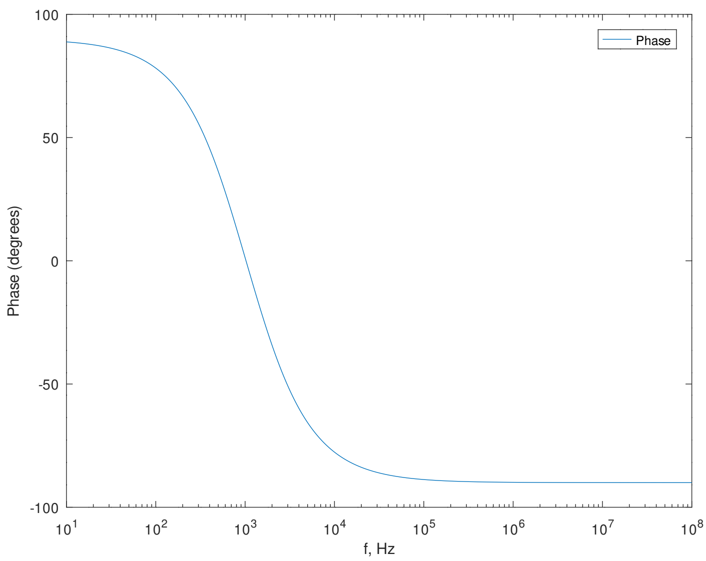

Figure 10: Octave plot: Phasor of v_{out}, rad

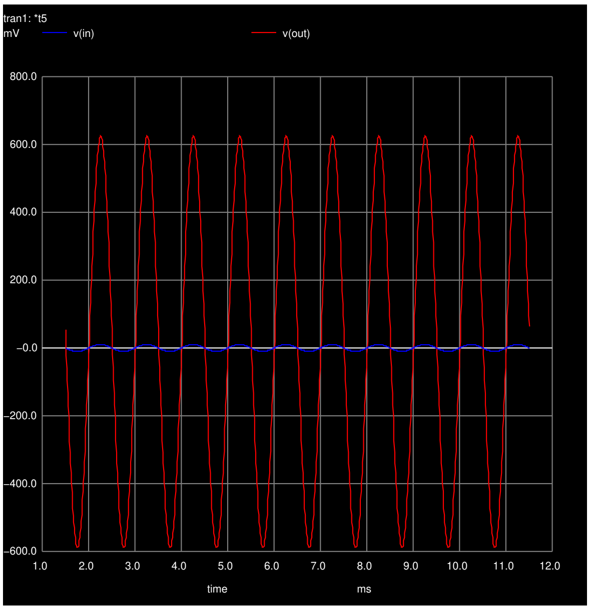

Figure 11: NGSpice plot: v_{in} and v_{out}.