Circuit Theory and Electronics Fundamentals

Álvaro Dias Patrício, 95768

Inês de Sousa Afonso, 95797

José Severino Medeiro, 95811

Integrated Master in Aerospace Engineering, Técnico, University of Lisbon

Laboratory 1 Report

Group 8

February 25, 2021

Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit with four meshes containing a capacitor, a sinusoidal voltage source v_s and two linearly dependent sources: a voltage controlled current source I_b and a current controlled voltage source V_d. The circuit also contains seven resistors from R_1 to R_7 as it is shown in Figure 1. The values for the caracteristics of this components, apart from I_d and V_b and including v_s, C, K_b and K_d, are given by Python and are:

Name	Value [A, V, S, Ω or F]
#R_1	1.032580e+03
#R_2	2.058543e+03
#R_3	3.056589e+03
#R_4	4.120838e+03
#R_5	3.102237e+03
#R_6	2.099094e+03
#R_7	1.015699e+03
V_s	5.198324e+00
&C	1.047395e-06
§K_b	7.074481e-03
#K_d	8.223457e+03

Table 1: Variables in the Nodal Method. A variable preceded by @ is of type *current* and expressed in Ampere; variables preceded by # is of type *resistance* and expressed in Ohm; variables preceded by § is of type *conductance* and expressed in Seimens; ariables preceded by & is of type *capacitance* and expressed in Farad; other variables are of type *voltage* and expressed in Volt.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical ones obtained in Section 2. The conclusions of this study are outlined in Section 4.

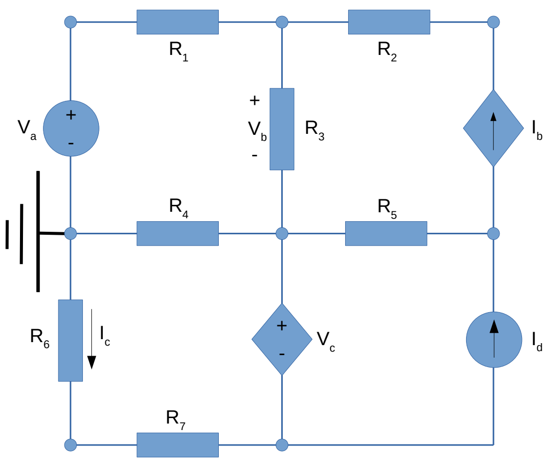

Figure 1: Circuit analysed.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analyzed theoretically with the Nodal Method and General Solution Method in order to caracterize it in the interval $[-5, 20[$.

2.1 Step 1: Solution for $t < 0$

When $t < 0$ we consider that the circuit is in equilibrium and, because of that, no current passes through the capacitor, since the relation between current and voltage in this component is:

$$\frac{dV}{dt} = \frac{I}{C} \Rightarrow I = 0. \tag{1}$$

Hence, we can simplify the circuit:

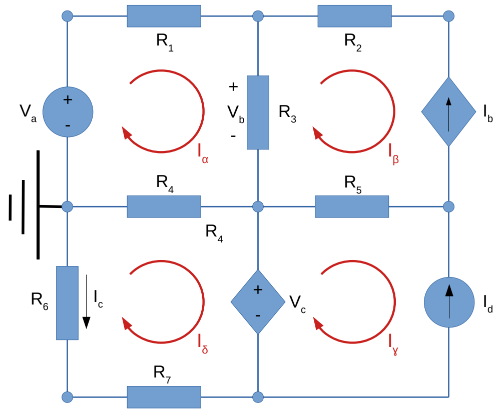

Figure 2: Circuit analysed with mesh currents.

Applying the Nodal Method to this circuit leads to the following equations:

$$\begin{cases} v_1 - v_4 & = V_s; \\ \frac{1}{R_1}v_1 - (\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_3})v_2 + (\frac{1}{R_2})v_3 + \frac{1}{R_3}v_5 & = 0; \\ \frac{1}{R_2}v_2 - \frac{1}{R_2}v_3 + I_b & = 0; \\ -\frac{1}{R_1}v_1 + \frac{1}{R_1}v_2 - (\frac{1}{R_4} + \frac{1}{R_6})v_4 + \frac{1}{R_4}v_5 + \frac{1}{R_6}v_7 & = 0; \\ v_5 - v_8 - V_d & = 0; \\ \frac{1}{R_5}v_5 - \frac{1}{R_5}v_6 - I_b & = 0; \\ -(\frac{1}{R_7} + \frac{1}{R_6})v_7 + \frac{1}{R_7}v_8 + \frac{1}{R_6}v_4 & = 0; \\ v_2 - v_3 - V_b & = 0; \\ \frac{1}{R_6}v_4 - \frac{1}{R_6}v_7 - I_d & = 0; \\ v_4 & = 0; \\ -K_bV_b + I_b & = 0; \\ V_d - K_cI_c & = 0. \end{cases} \tag{2}$$

Where the 2^{nd}, 3^{rd}, 6^{th} and 7^{th} are the equations in the respective nodes; the 1^{st} and 5^{th} are the relations imposed by the voltages soureces in those nodes; the 4^{th} is the supernode that bypasses V_s; the 8^{th} to 10^{th} are the relations between the circuit and, in order, V_b, I_d and v_4; the final two equations are the ones that describe I_b and V_d, respectivly. The solution to this linear system of equations is determined by Octave, plus the currents flowing in the circuit using Ohm's Law to the resistor's branches and Kirchhoff Current Law (KCL) to the branches with voltage sources:

Name	Value [A or V]
$@I_c$	0.000000e+00
$@I_D$	9.770070e-04
$@I_1$	2.342031e-04
$@I_2$	2.455590e-04
$@I_3$	-1.135597e-05
$@I_4$	-1.211210e-03
$@I_5$	-2.455590e-04
$@I_6$	9.770070e-04
$@I_7$	9.770070e-04
v_1	5.198324e+00
v_2	4.956490e+00
v_3	4.450997e+00
v_4	0.000000e+00
v_5	4.991201e+00
v_6	5.752983e+00
v_7	-2.050829e+00
v_8	-3.043174e+00
V_b	-3.471054e-02
V_d	8.034375e+00
$@I_b$	-2.455590e-04
$@I_d$	9.770070e-04
V_c	8.796158e+00

Table 2: Variables in the Mesh Method. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

All the currents were measured from the lower numbered node to the higher one; so, the direction of the current that passes through R_6 is, if the result is positive, from v_4 to v_7.

2.2 Step 2: Solution for $t = 0$ and R_{eq}

Once found the solution for the nodes in $t < 0$, the next step is to calculate the boundary conditions for the knots in the circuit, because the voltage in the nodes doesn't necessarily need to be continuos, only the difference of potencial in the capacitor, V_c. Therefor, a non continuos change in a power source, in this case in the voltage source V_s, might lead to a diferent voltage in the nodes. Since the solution to this circuit will also be divided in natural and forced, we can use this step to evaluate the equivelent resistance of the circuit, R_{eq}.

To evaluate the circuit at $t = 0$, we must have in mind that V_c is continuos, so $V_c(0) = V_c(0^-)$. Another point to take into account is that, for the natural solution, we can ignore the sinusoidal part of V_s, which leaves us with $V_s = 0$. In this conditions, we can say that the circuit 1 is identical to this one:

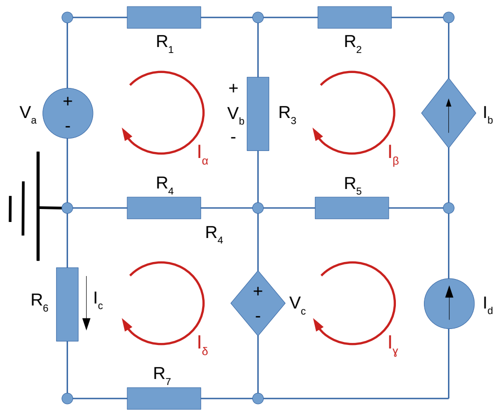

Figure 3: Circuit analysed with mesh currents.

The set of equations that describe it are, using the Nodal Method:

$$\begin{cases} v_1 - v_4 & = 0; \\ \frac{1}{R_1}v_1 - (\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_3})v_2 + (\frac{1}{R_2})v_3 + \frac{1}{R_3}v_5 & = 0; \\ \frac{1}{R_2}v_2 - \frac{1}{R_2}v_3 + I_b & = 0; \\ -\frac{1}{R_1}v_1 + \frac{1}{R_1}v_2 - (\frac{1}{R_4} + \frac{1}{R_6})v_4 + \frac{1}{R_4}v_5 + \frac{1}{R_6}v_7 & = 0; \\ v_5 - v_8 - V_d & = 0; \\ v_6 - v_8 - V_c & = 0; \\ -(\frac{1}{R_7} + \frac{1}{R_6})v_7 + \frac{1}{R_7}v_8 + \frac{1}{R_6}v_4 & = 0; \\ v_2 - v_3 - V_b & = 0; \\ \frac{1}{R_6}v_4 - \frac{1}{R_6}v_7 - I_d & = 0; \\ v_4 & = 0; \\ -K_bV_b + I_b & = 0; \\ V_d - K_cI_c & = 0. \end{cases} \tag{3}$$

Where the 2^{nd}, 3^{rd} and 7^{th} are the equations in the respective nodes; the 1^{st}, 5^{th} and 6^{th} are the relations imposed by the voltages soureces in those nodes; the 4^{th} is the supernode that bypasses V_s; the 8^{th} to 10^{th} are the relations between the circuit and, in order, V_b, I_d and v_4; the final two equations are the ones that describe I_b and V_d, respectivly. We note that

$$I_c = I_b + \frac{v_6 - v_5}{R_5}, \tag{4}$$

$$R_{eq} = \frac{V_c}{I_c}, \text{ and} \tag{5}$$

$$\tau = R_{eq}C. \tag{6}$$

The solution to this linear system of equations is determined by Octave, plus the currents flowing in the circuit using Ohm's Law to the resistor's branches and Kirchhoff Current Law (KCL) to the branches with voltage sources:

Name	Value [A, V or FΩ]
@I_D	4.336809e-19
@I_1	-8.399090e-22
@I_2	-4.027896e-19
@I_3	1.862716e-20
@I_4	1.360604e-20
@I_5	-2.835424e-03
@I_6	-1.165351e-20
@I_7	-1.276613e-20
v_1	0.000000e+00
v_2	8.672734e-19
v_3	8.300268e-16
v_4	0.000000e+00
v_5	-5.606829e-17
v_6	8.796158e+00
v_7	2.679731e-17
v_8	3.976385e-17
V_c	8.796158e+00
@I_c	2.835424e-03
#R_{eq}	3.102237e+03
*τ	3.249269e-03

Table 3: Variables in the Mesh Method. A variable preceded by @ is of type *current* and expressed in Ampere; variables preceded by * are expressed in Farad Ohm;other variables are of type *voltage* and expressed in Volt.

2.3 Step 3: Natural Solution for $t > 0$

Using R_{eq} and v_6 from the previous step we can compute the natural solution of $v_6(t)$:

$$\begin{aligned} v_6(t) &= v_6(+\infty) - (v_6(+\infty) - v_6(0))e^{-\frac{1}{\tau}t} \\ &= v_6(0)e^{-\frac{1}{\tau}t}, \text{ dado que } v_6(+\infty) = 0. \end{aligned} \tag{7}$$

Using Octave to plot this equation gives us:

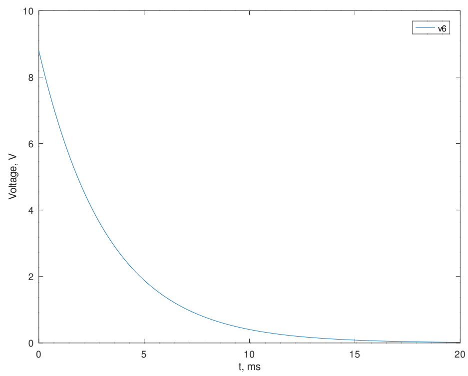

Figure 4: Natural solution of $v_6(t)$.

2.4 Step 4: Forced Solution for $t > 0$

Since V_s is a sinusoidal source in $t > 0$, we can perform a complex analysis, replacing all the componets for their impedances. In this case, only the capacitor changes (since all resistors have the same impedance):

$$Z_c = \frac{1}{j\omega C}, \tag{8}$$

where ω is the frequency ($1k\ rads^{-1}$). Saying that $V_s = 1$ to calculate the phasors relative to V_s and using the Nodal Method we arive to a set of equations:

$$\begin{cases} v_1 - v_4 & = 1; \\ \frac{1}{R_1}v_1 - (\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_3})v_2 + (\frac{1}{R_2})v_3 + \frac{1}{R_3}v_5 & = 0; \\ \frac{1}{R_2}v_2 - \frac{1}{R_2}v_3 + I_b & = 0; \\ -\frac{1}{R_1}v_1 + \frac{1}{R_1}v_2 - (\frac{1}{R_4} + \frac{1}{R_6})v_4 + \frac{1}{R_4}v_5 + \frac{1}{R_6}v_7 & = 0; \\ v_5 - v_8 - V_d & = 0; \\ \frac{1}{R_5}v_5 - (\frac{1}{R_5} + j\omega C)v_6 + (j\omega C)v_8 - I_b & = 0; \\ -(\frac{1}{R_7} + \frac{1}{R_6})v_7 + \frac{1}{R_7}v_8 + \frac{1}{R_6}v_4 & = 0; \\ v_2 - v_3 - V_b & = 0; \\ \frac{1}{R_6}v_4 - \frac{1}{R_6}v_7 - I_d & = 0; \\ v_4 & = 0; \\ -K_bV_b + I_b & = 0; \\ V_d - K_cI_c & = 0. \end{cases} \tag{9}$$

Where the 2^{nd}, 3^{rd}, 6^{th} and 7^{th} are the equations in the respective nodes; the 1^{st} and 5^{th} are the relations imposed by the voltages soureces in those nodes; the 4^{th} is the supernode that bypasses V_s; the 8^{th} to 10^{th} are the relations between the circuit and, in order, V_b, I_d and v_4; the final two equations are the ones that describe I_b and V_d, respectivly.

The solution to this linear system of equations is determined by Octave:

Name	**Value**
Pv_1	1.000000e+00 0.000000e+00 j
Pv_2	9.534786e-01 0.000000e+00 j
Pv_3	8.562369e-01 0.000000e+00 j
Pv_4	0.000000e+00 0.000000e+00 j
Pv_5	9.601558e-01 0.000000e+00 j
Pv_6	-5.813645e-01 -8.268436e-02 j
Pv_7	-3.945174e-01 0.000000e+00 j
Pv_8	-5.854145e-01 0.000000e+00 j

Table 4: Variables in the Nodal Method. Variables are adimentional.

The forced solution of $v_6(t)$ is given by $v_6(t) = V_s(t)Pv_6$.

2.5 Step 5: Total Solution for $t \in [-5, 20]ms$

The total solution of v_6 will be the solution from step 1 for $t \in [-5, 0[ms$ and the sum of the natural solution and forced solution for $t \in [0, 20]ms$ The plot given by Octave is:

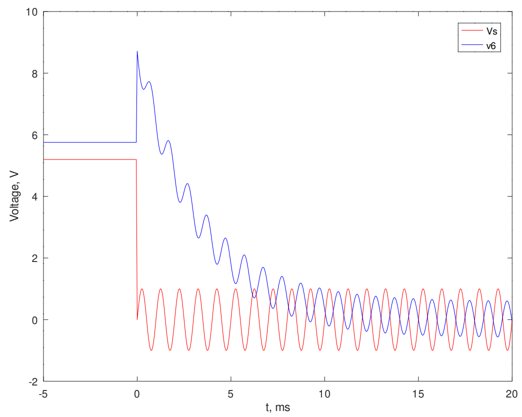

Figure 5: Total solution of $v_6(t)$ and $V_s(t)$.

2.6 Step 6: Frequency Response of v_6 and V_c

In this subsection is made an analysis of the frequency response of v_6 and V_c in order to V_s. for that we need to solve the same system as in step 4 (equation 9), with ω as a parameter. The magnitude (absolute value of the complex vectors) and phase (angle of the complex vectors is ploted using Octave:

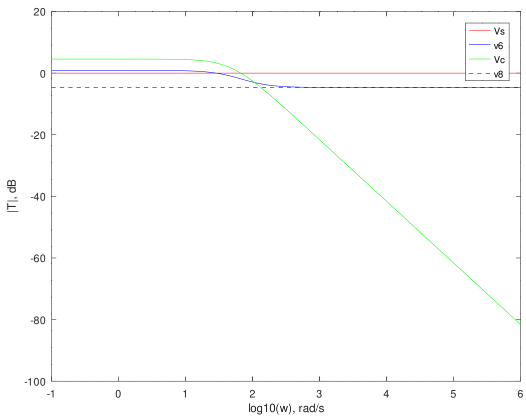

Figure 6: Magnitude response of V_s, v_6, V_c and v_8.

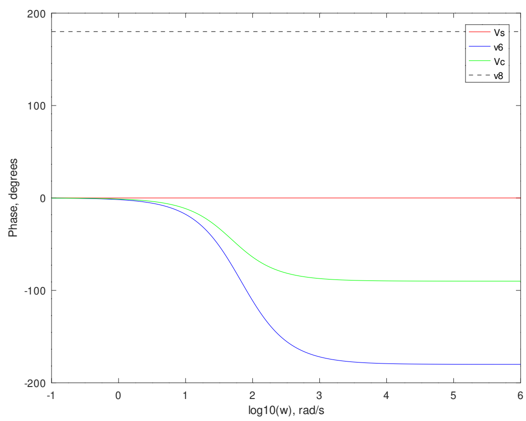

Figure 7: Angle response of V_s, v_6, V_c and v_8.

The results of V_s are to be expected, since $\frac{V_s}{V_s} = 1$. The results of V_c are also expected, since a capacitor is a low pass filter and this plots match to the ones of the classes. The results of v_6 are, at first glance, unexpected, since we are used to suppose that $v_6 \rightarrow 0$; but in fact what happens is that $V_c \rightarrow 0 \Rightarrow v_6 - v_8 \rightarrow 0$. Because the phase response of v_8 is constant, $v_6 - v_8 \rightarrow 0 \Rightarrow v_6 \rightarrow v_8$, so the results of v_6 are, indeed, expected.

3 Simulation Analysis

3.1 Operating Point Analysis for $t < 0$

Table 5 shows the simulated operating point results for the circuit under analysis for $t < 0$.

Name	Value [A or V]
@c1[i]	0.000000e+00
@gb[i]	-2.45559e-04
@r1[i]	2.342031e-04
@r2[i]	2.455590e-04
@r3[i]	-1.13560e-05
@r4[i]	-1.21121e-03
@r5[i]	-2.45559e-04
@r6[i]	9.770070e-04
@r7[i]	9.770070e-04
v(1)	5.198324e+00
v(2)	4.956490e+00
v(3)	4.450997e+00
v(4)	0.000000e+00
v(5)	4.991201e+00
v(6)	5.752983e+00
v(7)	-2.05083e+00
v(8)	-3.04317e+00

Table 5: Operating point for $t < 0$. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

It is important to notice that it has been necessary to implement an extra test voltage sorce between the node 7 and R_6 providing 0V so that is does not interfere with the rest of the circuit and enable us to measure the voltage V_d flowing in the dependent source. Therefore, this creation does not change any results and it is only based on the Ngspice requirements to define a current controlled voltage source.

Compared to the theoretical analysis the simulation showed practically identical results, except for a small divergence in the last decimal place that probably occurs when Ngspice rounds the numbers. Thus, the maximum relative error is 10^{-5}. It is worth mentioning that Ngspice software also uses the same mathematical methods as octave to find results, hence, this result was already expected.

3.2 Operating Point Analysis for $t = 0$

The circuit was simulated using an operating point analysis with $V_s(0) = 0$ and replacing que capacitor with a voltage source $V_x = V(6) - V(8)$ as these were obtained in the previous step. This step in required to compute the boundary conditions that ensure the continuity in the capacitor. Table 6 shows the results of the simulation. Notice that:

$$R_{eq} = \frac{V_x}{I_x} \tag{10}$$

Name	Value [A or V]
@gb[i]	-2.10600e-18
@r1[i]	2.008605e-18
@r2[i]	2.105998e-18
@r3[i]	-9.73927e-20
@r4[i]	4.310669e-19
@r5[i]	-2.83542e-03
@r6[i]	-2.85149e-18
@r7[i]	-2.85149e-18
v(1)	0.000000e+00
v(2)	-2.07405e-15
v(3)	-6.40933e-15
v(4)	0.000000e+00
v(5)	-1.77636e-15
v(6)	8.796153e+00
v(7)	5.985534e-15
v(8)	8.881784e-15

Table 6: Operating point for $t < 0$. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3.3 Natural Solution for V_6

In order to study the natural response of the circuit in the interval [0,20]ms, a transient analysis was made, using the results for V_6 and V_8 computed in the previous step. Figure 8 shows the results for the natural solution for V_6.

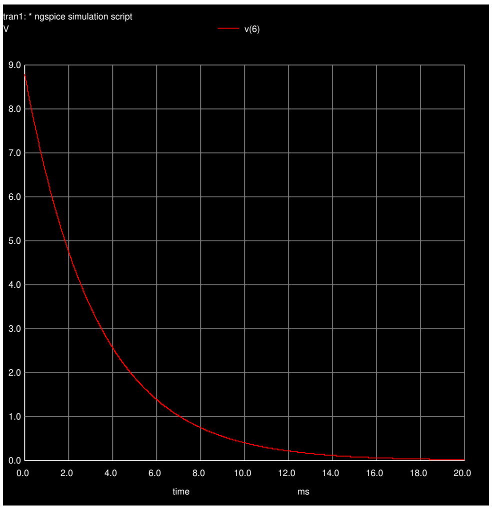

Figure 8: Natural solution of $v_6(t)$.

3.4 Total Solution for V_6

The results obtained in the last section were used to make this analysis,but considering V_s a sinusoidal voltage source with f=1kHz. The total response of the circuit is the result of summing the natural analysis with the forced one (natural + forced). Figure 9 shows the transient analysis results for $V_s(t)$.

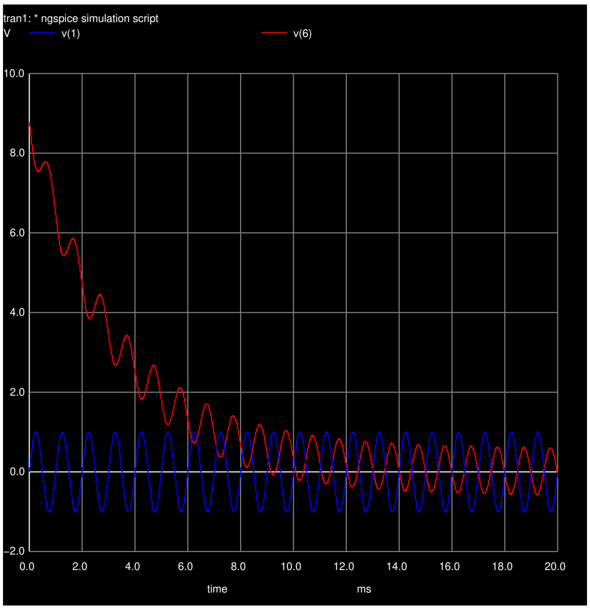

Figure 9: Natural solution of $v_6(t)$ and V_s.

3.5 Frequency Response

In this section, the frequency range considered to analyze the frequency response in node 6 is from 0.1 Hz to 1 MHz. The figures show the magnitude and phase of the frequency response for V_s, V_c and V_6.

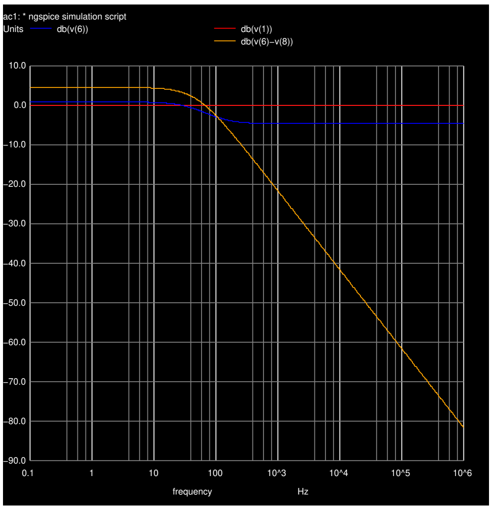

Figure 10: Magnitude response of V_s, $v_6(t)$ and $V_c(t)$.

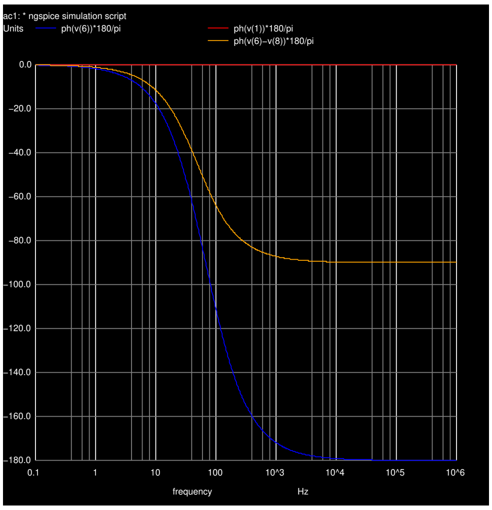

Figure 11: Angle response of V_s, $v_6(t)$ and $V_c(t)$.

When comparing the graphics obtained in Ngspice and Octave, the similarity between the results is noticeable and small differences can be explained by approximation errors.

4 Conclusion

To conclude, in this laboratory assigment the objective of analysing a circuit containing a sinusoidal voltage source and a capacitor has been achieved. Static, time and frequency analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool.

Name	Value [A or V]
$@I_c$	0.000000e+00
$@I_D$	9.770070e-04
$@I_1$	2.342031e-04
$@I_2$	2.455590e-04
$@I_3$	-1.135597e-05
$@I_4$	-1.211210e-03
$@I_5$	-2.455590e-04
$@I_6$	9.770070e-04
$@I_7$	9.770070e-04
v_1	5.198324e+00
v_2	4.956490e+00
v_3	4.450997e+00
v_4	0.000000e+00
v_5	4.991201e+00
v_6	5.752983e+00
v_7	-2.050829e+00
v_8	-3.043174e+00
V_b	-3.471054e-02
V_d	8.034375e+00
$@I_b$	-2.455590e-04
$@I_d$	9.770070e-04
V_c	8.796158e+00

Name	Value [A or V]
@c1[i]	0.000000e+00
@gb[i]	-2.45559e-04
@r1[i]	2.342031e-04
@r2[i]	2.455590e-04
@r3[i]	-1.13560e-05
@r4[i]	-1.21121e-03
@r5[i]	-2.45559e-04
@r6[i]	9.770070e-04
@r7[i]	9.770070e-04
v(1)	5.198324e+00
v(2)	4.956490e+00
v(3)	4.450997e+00
v(4)	0.000000e+00
v(5)	4.991201e+00
v(6)	5.752983e+00
v(7)	-2.05083e+00
v(8)	-3.04317e+00

Table 7: Operating point for $t < 0$. in Octave and NGSpice, respectivily A variable preceded by @ is of type *current* are expressed in Ampere; other variables are of type *voltage* and expressed in Volt. (As shown in Tables 2 and 5)

Name	Value [A or V]
@I_D	4.336809e-19
@I_1	-8.399090e-22
@I_2	-4.027896e-19
@I_3	1.862716e-20
@I_4	1.360604e-20
@I_5	-2.835424e-03
@I_6	-1.165351e-20
@I_7	-1.276613e-20
v_1	0.000000e+00
v_2	8.672734e-19
v_3	8.300268e-16
v_4	0.000000e+00
v_5	-5.606829e-17
v_6	8.796158e+00
v_7	2.679731e-17
v_8	3.976385e-17
V_c	8.796158e+00
@I_c	2.835424e-03
#R_{eq}	3.102237e+03
*τ	3.249269e-03

Name	Value [A or V]
@gb[i]	-2.10600e-18
@r1[i]	2.008605e-18
@r2[i]	2.105998e-18
@r3[i]	-9.73927e-20
@r4[i]	4.310669e-19
@r5[i]	-2.83542e-03
@r6[i]	-2.85149e-18
@r7[i]	-2.85149e-18
v(1)	0.000000e+00
v(2)	-2.07405e-15
v(3)	-6.40933e-15
v(4)	0.000000e+00
v(5)	-1.77636e-15
v(6)	8.796153e+00
v(7)	5.985534e-15
v(8)	8.881784e-15

Table 8: Variables in the Mesh Method. A variable preceded by @ is of type *current* and expressed in Ampere; variables preceded by * are expressed in Farad Ohm;other variables are of type *voltage* and expressed in Volt. (As shown in Tables 3 and 6)

The results match considerably, as expected, since this is a relatively straightforward circuit containing only one capacitor apart from linear components, therefore the theoretical and simulation models shouldn't differ. Values of the order of 10^{-14} and lower should be taken as 0.