

Determinar la magnitud y el módulo de los gradientes de pixeles de esta imagen utilizando las siguientes fórmulas.

1	2	6	4
3	7	5	3
9	5	8	6

Diferencia central

$$\delta f_x = \frac{f(x+1, y) - f(x-1, y)}{2} \quad \delta f_y = \frac{f(x, y+1) - f(x, y-1)}{2}$$

Diferencia hacia delante

$$\Delta f_x = f(x+1, y) - f(x, y) \quad \Delta f_y = f(x, y+1) - f(x, y)$$

Diferencia hacia atrás

$$\nabla f_x = f(x, y) - f(x-1, y) \quad \nabla f_y = f(x, y) - f(x, y-1)$$

$$\begin{aligned} f(x_{1,1}) &= 2 - 1 = 1 & f(x_{1,2}) &= \frac{6-1}{2} = 2.5 & f(x_{1,3}) &= \frac{4-2}{2} = 1 & f(x_{1,4}) &= 4 - 6 = -2 \\ f(x_{2,1}) &= 7 - 3 = 4 & f(x_{2,2}) &= \frac{5-3}{2} = 1 & f(x_{2,3}) &= \frac{6-1}{2} = 2.5 & f(x_{2,4}) &= 3 - 5 = -2 \\ f(x_{3,1}) &= 5 - 9 = -4 & f(x_{3,2}) &= \frac{8-9}{2} = -0.5 & f(x_{3,3}) &= \frac{6-1}{2} = 2.5 & f(x_{3,4}) &= 6 - 8 = -2 \end{aligned}$$

$$\begin{aligned} f(y_{1,1}) &= 1 - 3 = -2 & f(y_{1,2}) &= 2 - 7 = -5 & f(y_{1,3}) &= 6 - 5 = 1 & f(y_{1,4}) &= 4 - 3 = 1 \\ f(y_{2,1}) &= \frac{1-9}{2} = -4 & f(y_{2,2}) &= \frac{2-5}{2} = -1.5 & f(y_{2,3}) &= \frac{6-8}{2} = -1 & f(y_{2,4}) &= \frac{4-6}{2} = -1 \\ f(y_{3,1}) &= 3 - 9 = -6 & f(y_{3,2}) &= 7 - 5 = 2 & f(y_{3,3}) &= 5 - 8 = -3 & f(y_{3,4}) &= 3 - 6 = -3 \end{aligned}$$

$$g_x = \begin{bmatrix} 1 & 2.5 & 1 & -2 \\ 4 & 1 & -2 & -2 \\ -4 & -0.5 & 0.5 & -2 \end{bmatrix}$$

$$g_y = \begin{bmatrix} -2 & -5 & 1 & 1 \\ -4 & -1.5 & -1 & -1 \\ -6 & 2 & -3 & -3 \end{bmatrix}$$

Aplicando la siguiente fórmula, obtenemos la magnitud del vector de los gradientes de los pixeles:

$$g = \sqrt{g_x^2 + g_y^2}$$

$$\begin{aligned}
 g_{1,1} &= \sqrt{1^2 + (-2)^2} = 2.236 & g_{1,2} &= \sqrt{2.5^2 + (-5)^2} = 5.59 & g_{1,3} &= \sqrt{1^2 + (-1)^2} = 1.414 \\
 g_{2,1} &= \sqrt{4^2 + (-4)^2} = 5.657 & g_{2,2} &= \sqrt{1^2 + (-1.5)^2} = 1.803 & g_{2,3} &= \sqrt{(-2)^2 + (-1)^2} = 2.236 \\
 g_{3,1} &= \sqrt{(-4)^2 + (-6)^2} = 7.211 & g_{3,2} &= \sqrt{(-0.5)^2 + 2^2} = 2.062 & g_{3,3} &= \sqrt{(-0.5)^2 + (-3)^2} = 3.041 \\
 \\
 g_{1,4} &= \sqrt{(-2)^2 + 1^2} = 2.236 \\
 g_{2,4} &= \sqrt{(-2)^2 + (-1)^2} = 2.236 \\
 g_{3,4} &= \sqrt{(-2)^2 + (-3)^2} = 3.606
 \end{aligned}$$

$$g = \begin{bmatrix} 2.236 & 5.59 & 1.414 & 2.236 \\ 5.657 & 1.803 & 2.236 & 2.236 \\ 7.211 & 2.062 & 3.041 & 3.606 \end{bmatrix}$$

Aplicando la siguiente fórmula, obtenemos el módulo del vector de los gradientes de los pixeles, el resultado en radianes lo pasamos a grados.

$$\theta = \arctan\left(\frac{g_y}{g_x}\right)$$

$$\begin{aligned}
 \theta_{1,1} &= \arctan\left(\frac{-2}{1}\right) = -1.1071487177941 & \theta_{1,2} &= \arctan\left(\frac{-5}{2.5}\right) = -1.1071487177941 \\
 \theta_{2,1} &= \arctan\left(\frac{-4}{4}\right) = -0.78539816339745 & \theta_{2,2} &= \arctan\left(\frac{-1.5}{1}\right) = -0.98279372324733 \\
 \theta_{3,1} &= \arctan\left(\frac{-6}{-4}\right) = 0.98279372324733 & \theta_{3,2} &= \arctan\left(\frac{2}{-0.5}\right) = -1.325817663668 \\
 \\
 \theta_{1,3} &= \arctan\left(\frac{1}{1}\right) = 0.78539816339745 & \theta_{1,4} &= \arctan\left(\frac{-1}{-2}\right) = 0.46364760900081 \\
 \theta_{2,3} &= \arctan\left(\frac{-1}{-2}\right) = 0.46364760900081 & \theta_{2,4} &= \arctan\left(\frac{-1}{-2}\right) = 0.46364760900081 \\
 \theta_{3,3} &= \arctan\left(\frac{-3}{0.5}\right) = -1.4056476493803 & \theta_{3,4} &= \arctan\left(\frac{-3}{-2}\right) = 0.98279372324733
 \end{aligned}$$

Pasando a grados los valores anteriores, tenemos el siguiente resultado:

$$\theta = \begin{bmatrix} -63.435^\circ & -63.435^\circ & 45^\circ & 26.565^\circ \\ -45^\circ & -56.31^\circ & 26.565^\circ & 26.565^\circ \\ 56.31^\circ & -75.964^\circ & -80.538^\circ & 56.31^\circ \end{bmatrix}$$