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HASLab/INESC TEC & UM & INL

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1 Linear relations — 20200920

Linear relations R are such that relational types

$$R \xleftarrow{(+)} R \times R \tag{1}$$

$$R \xleftarrow{s*} R \tag{2}$$

$$R \xleftarrow{\underline{0}} 1 \tag{3}$$

hold. We abbreviate "copy" by $\delta = \langle id, id \rangle$. Then Axiom 3 is the free theorem of δ:

$$\delta \cdot R \;\subseteq\; (R \times R) \cdot \delta$$

Its color-dual is:

$$(+) \cdot (R \times R) \;\subseteq\; R \cdot (+) \tag{4}$$

which is a re-statement of (1). In the opposite direction, take (3), that is $\underline{0} \;\subseteq\; R \cdot \underline{0}$ saying that $(0, 0) \in R$, and color-dualize it:

$$! \cdot R \;\subseteq\; !$$

This is the free theorem of !.

Now something more elaborate, Axiom 4 ('Wrong Way') of the paper:

$$\langle R, id \rangle \;\subseteq\; \langle id, R^\circ \rangle \cdot R \tag{5}$$

From

$$\begin{aligned} & \langle R, id \rangle \;\subseteq\; \langle id, R^\circ \rangle \cdot R \\ \equiv \quad & \{\ \} \\ & (R \times id) \cdot \delta \;\subseteq\; (id \times R^\circ) \cdot \delta \cdot R \end{aligned}$$

its color-dual is:

$$R \cdot (+) \cdot (id \times R^\circ) \;\subseteq\; (+) \cdot (R \times id) \tag{6}$$

Let us check this: first think of special case $R := f$, shunt immediately

$$\begin{aligned} & f \cdot (+) \;\subseteq\; (+) \cdot (f \times f) \\ = \quad & \{\ \} \\ & (+) \cdot (f \times f)^\circ \;\subseteq\; f^\circ \cdot (+) \\ \equiv \quad & \{\ \} \end{aligned}$$

Ok - f° linear...

from (4) we get $(+) \cdot (R \times R) \cdot (id \times R^\circ) \;\subseteq\; R \cdot (+) \cdot (id \times R^\circ)$

$$R \cdot (+) \cdot (id \times R^\circ) \;\subseteq\; (+) \cdot (R \times id)$$

$\Box$

Pointwise: $y\ R\ x \Rightarrow \langle \exists\ y' \ ::\ y'\ R\ x \wedge y = y' \wedge \ldots \rangle$

$$\begin{aligned} & \langle id, R^\circ \rangle \cdot R = \langle id \cdot R, R^\circ \cdot R \rangle \\ & \quad \Leftarrow\ id \cdot (\mathrm{img}\ R) \subseteq id \vee R^\circ \cdot (\mathrm{img}\ R) \subseteq R^\circ \end{aligned}$$

$$\begin{aligned} & \langle R, S \rangle \cdot T = \langle R \cdot T, S \cdot T \rangle \\ & \quad \Leftarrow\ R \cdot (\mathrm{img}\ T) \subseteq R \vee S \cdot (\mathrm{img}\ T) \subseteq S \end{aligned} \tag{7}$$