In order to receive full credit problems and solutions must be clearly written and presented neatly. In writing up your solution, you should state the problem as explaining the solution. Merely giving answers to exercises will is not enough to receive credit. Your write up must show work and convince me that you understand the material. Late homework will not be graded. Starred (★) proof problems are eligible for inclusion in the proof portfolio assignment.

- Choose, complete, and write-up **3 exercises** and **3 proofs**.
- Exercises are worth up to 3 points, and proofs are worth up to 5 points.
- Put your name on each page you submit, and somewhere indicate who you worked with on this assignment.

Exercise 3.1: From the text, Exercise 1.2.

Exercise 3.2: How many strings of 6 letters of the english alphabet contain one vowel? exactly two vowels? at least one vowel? at least two vowels? Generalize this to the case when the alphabet has c consonants and v vowels.

Exercise 3.3: A string that contains only 0s,1s, and 2s is called a **ternary string**. How many strings of 10 ternary digits (0, 1, or 2) are there that contain exactly two 0s, three 1s, and five 2s? Explain your answer.

Exercise 3.4: A **palindrome** is a string whose reverse is identical to the original string. Give examples of bit strings of length 5 and 6 that are palindromes. How many bit string palindromes of length n are there? How many ternary string palindromes of length n are there?

Exercise 3.5: Evaluate the sum

$$\sum_{k=1}^{n}(-1)^k k/(4k^2-1)$$

Proof 3.1: From the text, Exercise 1.9

Proof 3.2: From the text, Exercise 1.10

Proof 3.3: Prove $\binom{n}{k}\binom{n-k}{l} = \binom{n}{l}\binom{n-l}{k}$ with $k + l \geq n$.

Proof 3.4: Let n be a positive integer. Show that $\binom{2n}{n+1} + \binom{2n}{n} = \frac{1}{2}\binom{2n+2}{n+1}$.

Proof 3.5: Prove the hexagon property:

$$\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = \binom{n-1}{k}\binom{n+1}{k+1}\binom{n}{k-1}$$

Proof 3.6: ★ Find and prove a formula the for value of $\sum_{k=0}^{n} \binom{n}{k}^3(-1^k)$.

Proof 3.7: ★ Find and prove a formula the for value of $\sum_{k=0}^{n} \binom{n+k}{n}2^{-k}$.

Proof 3.8: ★ Find and prove a formula the for value of $\sum_{k=0}^{n} \binom{2k}{k}4^{-k}$.

Proof 3.9: ★ Find and prove a formula the for value of $\sum_{k=0}^{n} \binom{\binom{k}{2}}{2}\binom{2n-k}{n}$.

Proof 3.10: ★ For every postivie integer n, prove the identity

$$\sum_{k=1}^{n-1} \binom{n}{k} \frac{kn^{n-k}}{k+1} = \frac{n(n^n-1)}{n+1}.$$

Proof 3.11: ★ Find and prove a formula for the value of $\sum_{k=0}^{n} \binom{n+k}{2k}(-4)^k$.

Proof 3.12: ★ Prove Lagrange's identity:

$$\sum_{1\le j<k\le n} (a_j b_k - a_k b_j)^2 = \left(\sum_{k=1}^{n} a_k^2\right)\left(\sum_{k=1}^{n} b_k^2\right) - \left(\sum_{k=1}^{n} a_k b_k\right)^2$$

Proof 3.13: ★ Riemann's zeta function $\zeta(k)$ is defined to be

$$\zeta(k) = \sum_{j\ge 1} 1/j^k = 1 + 1/2^k + 1/3^k + 1/4^k + \dots$$

Prove $\sum_{k\ge 2}(\zeta(k)-1) = 1$.