In order to receive full credit problems and solutions must be clearly written and presented neatly. In writing up your solution, you should state the problem as explaining the solution. Merely giving answers to exercises will is not enough to receive credit. Your write up must show work and convince me that you understand the material. Late homework will not be graded. Starred (★) proof problems are eligible for inclusion in the proof portfolio assignment.

- Choose, complete, and write-up **3 exercises** and **3 proofs**.
- Exercises are worth up to 3 points, and proofs are worth up to 5 points.
- Put your name on each page you submit, and somewhere indicate who you worked with on this assignment.

Exercise 2.1: Many ways are there for 8 men and 5 women to stand in a line so that no two women stand next to each other? What is the answer if there are 10 men and 6 women? What is the answer if there are m men and w women with $m \geq w$?

Exercise 2.2: A **diagonal** of a polygon is a line joining two nonadjacent vertices. How may diagonals does a hexagon have? What about a octogon? What about a n-gon?

Exercise 2.3: Suppose you have 10 points in the plane with no three collinear. How many different line segments are formed by joining pairs of of these points? How many different triangles are formed by these line segments? If A is one of the 10 points, how many of the triangles formed by the line segments have A as a vertex?

Exercise 2.4: Find the definition of a multinomial coefficient. How is it related to binomial coefficients? What can a multinomial coefficient be used to count? Give examples of counting with multinomial coefficients.

Exercise 2.5: How many different strings can be made from the letters in the work PEPPERCORN when all the letters are used? How may of the strings start and end with the letter P? How many strings have 3 consecutive Ps?

Proof 2.1: Use mathematical induction to show that 21 divides $4^{n+1} + 5^{2n-1}$ for any positive integer n

Proof 2.2: (Sum of a geometric sequence) Prove that for any real numbers a and r,

$$\sum_{i=0}^{n} a \cdot r^i = \frac{a(1 - r^{n+1})}{1 - r}.$$

Proof 2.3: (Sum of an arithmetic sequence) Prove that for any real numbers a and r,

$$\sum_{i=0}^{n}(a + i \cdot r) = \frac{n+1}{2}(2a + n \cdot r).$$

Proof 2.4: ★ Prove by induction that

$$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} \leq \frac{1}{\sqrt{n+1}}$$

for n a positive integer.

Proof 2.5: Prove by induction that $3^n + 7^n - 2$ is divisible by 8 for all positive integers n

Proof 2.6: By experimenting with small values of n, guess a formula for the given sum,

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)};$$

then use induction to prove your formula.

Proof 2.7: ★ Bernoulli's inequality: Prove that if $h > -1$, then $1 + nh \leq (h+1)^n$ for all non-negative integers n.

Proof 2.8: Prove that for $n \geq 0$, $2^n > n$.

Proof 2.9: Prove that for $n \geq 4$, $n! > 2^n$.

Proof 2.10: Prove that for $n \geq 10$, $2^n > n^3$.

Proof 2.11: ★ Prove that for $n \geq 2$, the maximum number of points of intersection of n distinct lines in the plane in $n(n-1)/2$