

## Homework 9 - Math 300 - Discrete Mathematics

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In order to receive full credit problems and solutions must be clearly written and presented neatly. In writing up your solution, you should state the problem as explaining the solution. Merely giving answers to exercises will is not enough to receive credit. Your write up must show work and convince me that you understand the material. Late homework will not be graded. Starred (★) proof problems are eligible for inclusion in the proof portfolio assignment.

- Choose, complete, and write-up **3 exercises** and **3 proofs**.
- Exercises are worth up to 3 points, and proofs are worth up to 5 points.
- Put your name on each page you submit, and somewhere indicate who you worked with on this assignment.

**Exercise 9.1:** From the text, Exercise 5.2.

**Exercise 9.2:** From the text, Exercise 5.4.

**Exercise 9.3:** Let  $p(n)$  be the number of partition types of an  $n$  element set. Find  $p(1)$ ,  $p(2)$ ,  $p(3)$ , and  $p(4)$ .

**Exercise 9.4:** Find the coefficient of  $x$ ,  $x^2$ ,  $x^3$ , and  $x^4$  in the Taylor expansion of

$$\frac{1}{(1-x)} \frac{1}{(1-x^2)} \frac{1}{(1-x^3)} \cdots$$

and compare to  $p(1)$ ,  $p(2)$ ,  $p(3)$ ,  $p(4)$ . Do you want to make a conjecture? (Hint: find the taylor expansion, by multiplying together each of the fractions Taylor expansion).

**Exercise 9.5:** Find the definition of Stirling numbers of the first kind. What recursion relationship do these numbers satisfy? Give examples of the permutations counted by  $s(5, 2)$  and  $s(6, 3)$ , where  $s(n, k)$  is a Stirling number of the first kind.

**Exercise 9.6:** Find the definition of a equivalence relation. What is the relationship between equivalence relations and  $S(n, k)$ , Stirling numbers of the second kind.

**Proof 9.1:** Let  $s(n, k)$  denote the signless Stirling numbers of the first kind. Prove that

$$s(n, n-1) = \binom{n}{2}$$

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**Proof 9.2:** Let  $s(n, k)$  denote the signless Stirling numbers of the first kind. Prove that

$$s(n, 2) = (n-1)! \left( 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1} \right).$$

**Proof 9.3:** Let  $s(n, k)$  denote the signless Stirling numbers of the first kind. Find a formula for  $s(n, n-2)$  and prove it.

**Proof 9.4:** From the text, Exercise 5.5.

**Proof 9.5:** From the text, Exercise 5.6.

**Proof 9.6:** ★ From the text, Exercise 5.7.