

Homework 5 - Math 300 - Discrete Mathematics

In order to receive full credit problems and solutions must be clearly written and presented neatly. In writing up your solution, you should state the problem as explaining the solution. Merely giving answers to exercises will not be enough to receive credit. Your write up must show work and convince me that you understand the material. Late homework will not be graded. Starred (★) proof problems are eligible for inclusion in the proof portfolio assignment.

- Choose, complete, and write-up **3 exercises** and **3 proofs**.
- Exercises are worth up to 3 points, and proofs are worth up to 5 points.
- Put your name on each page you submit, and somewhere indicate who you worked with on this assignment.

Exercise 5.1: What is Reve's Puzzle? Describe it, and describe the Frame-Stewart Algorithm in general and using an example.

Exercise 5.2: From the text, Exercise 2.1, parts b and c.

Exercise 5.3: From the text, Exercise 2.2.

Exercise 5.4: From the text, Exercise 2.7.

Exercise 5.5: From the text, Exercise 2.10.

Exercise 5.6: Solve the recurrence relation

$$a_n = \sqrt{\frac{a_{n-2}}{a_{n-1}}}$$

with initial conditions $a_0 = 8$ and $a_1 = 1/(2\sqrt{2})$ by taking the logarithm of both sides and making the substitution $b_n = \ln a_n$.

Exercise 5.7: Let $a_0 = 1$, $a_1 = 2$, and $a_n = 4a_{n-1} - a_{n-2}$ for $n \geq 2$. Find an odd factor of a_{2015} .

Exercise 5.8: Solve the following recurrence relations:

$$a_n = 6a_{n-1} - 8a_{n-2}; a_0 = 1, a_1 = 0$$

$$a_n = 2a_{n-1} + 8a_{n-2}; a_0 = 4, a_1 = 10$$

Exercise 5.9: Find the first 5 terms of the Taylor expansion of $\sqrt{1-4x}$. By examining the pattern of coefficients find a_n where $\sqrt{1-4x} = \sum_{n=0}^{\infty} a_n x^n$.

Proof 5.1: From the text, Exercise 2.3.

Proof 5.2: From the text, Exercise 2.12.

Proof 5.3: From the text, Exercise 2.13.

Proof 5.4: Prove

$$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n(n+1)!} 4^n = \frac{1}{n+1} \binom{2n}{n}.$$

Proof 5.5: ★ Find a definition for $\binom{n}{k}$ when $n < 0$. Use this to prove

$$\binom{-1/2}{n} = \frac{\binom{2n}{n}}{(-4)^n}$$

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