

## Homework 10 - Math 300 - Discrete Mathematics

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In order to receive full credit problems and solutions must be clearly written and presented neatly. In writing up your solution, you should state the problem as explaining the solution. Merely giving answers to exercises will not be enough to receive credit. Your write up must show work and convince me that you understand the material. Late homework will not be graded. Starred (★) proof problems are eligible for inclusion in the proof portfolio assignment.

- Choose, complete, and write-up **3 exercises** and **3 proofs**.
- Exercises are worth up to 3 points, and proofs are worth up to 5 points.
- Put your name on each page you submit, and somewhere indicate who you worked with on this assignment.

**Exercise 10.1:** From the text, Exercise 6.6.

**Exercise 10.2:** From the text, Exercise 6.7.

**Exercise 10.3:** From the text, Exercise 6.8.

**Exercise 10.4:** From the text, Exercise 6.9.

**Exercise 10.5:** From the text, Exercise 6.10.

**Exercise 10.6:** From the text, Exercise 6.13.

**Proof 10.1:** Consider two sets  $X$  and  $Y$  with  $|X| = m$  and  $|Y| = n$ . A function  $f : X \rightarrow Y$  is called **almost-onto** if  $f$  misses at most one element of  $Y$ . Find and prove a formula for the number of almost onto functions from  $X$  to  $Y$ .

**Proof 10.2:** ★ Show that if  $n$  is a positive integer, then

$$n! = \binom{n}{0}d_n + \binom{n}{1}d_{n-1} + \binom{n}{2}d_{n-2} + \cdots + \binom{n}{n-1}d_1 + \binom{n}{n}d_0$$

where  $d_n = (n-1)(d_{n-1} + d_{n-2})$  with  $d_1 = 0$  and  $d_2 = 1$  is the number of derangements of  $n$  objects.

**Proof 10.3:** ★ Let  $A$  be a subset of  $B$  with  $|A| = m$  and  $|B| = n$ , and let  $r$  be an integer  $m \leq r \leq n$ . Show the number of  $r$  element subsets of  $B$  which contain  $A$  as a subset is  $\binom{n-m}{r-m}$ .

**Proof 10.4:** ★ Use the Inclusion-Exclusion Principle to show that for any non-negative integers  $m, r, n$  such that  $m \leq r \leq n$ ,

$$\binom{n-m}{r-m} = \sum_{i=0}^m (-1)^i \binom{m}{i} \binom{n-i}{r}.$$