In order to receive full credit problems and solutions must be clearly written and presented neatly. In writing up your solution, you should state the problem as explaining the solution. Merely giving answers to exercises will is not enough to receive credit. Your write up must show work and convince me that you understand the material. Late homework will not be graded. Starred (★) proof problems are eligible for inclusion in the proof portfolio assignment.

- Choose, complete, and write-up **3 exercises** and **3 proofs**.
- Exercises are worth up to 3 points, and proofs are worth up to 5 points.
- Put your name on each page you submit, and somewhere indicate who you worked with on this assignment.

Exercise 7.1: From the text, Exercise 3.2.

Exercise 7.2: From the text, Exercise 3.11.

Exercise 7.3: How many non-isomorphic simple graphs are there on n vertices when n is 2? 3? 4? and 5?

Exercise 7.4: A simple graph G is **self-complimentary** if G and $\bar{G}$ are isomorphic. Find a self-complimentary graph having five vertices. Find another self complimentary graph with more than 1 vertex.

Exercise 7.5: Create a list of non-isomorphic trees on 7 vertices. How many of the these graphs satisfy the condition: the degree of each vertex is at most 2?

Exercise 7.6: A tree with n vertices is called **graceful** if it's vertices can be labeled with the integers 1, 2 , ..., n such that the absolute values of the difference of the labels of adjacent vertices are all different. Draw and label two non-isomorphic graceful trees on 6 vertices.

Exercise 7.7: A **forest** is a simple graph where each component is a tree. Find all the non-isomorphic forests with five vertices and two or more components.

Exercise 7.8: A vertex v in a connected graph G is an **articulation point** if the removal of v and all edges incident to v disconnects G. Give an example of a graph with six vertices that has exactly two articulation points. Give an example of a graph on six vertices that has no articulation points.

Proof 7.1: From the text, Exercise 3.3. (Try Contradiction)

Proof 7.2: Prove that if there are two different paths between vertices x and y in a simple graph, then there must be a cycle in the graph.

Proof 7.3: Prove the statement: In any tree there is exactly one path from any vertex to any other vertex. (Hint: Try contradiction. If there are two paths try to show there is a cycle.)

Proof 7.4: ★ Let v and w be distinct vertices in the complete graph K_n for $n \geq 2$. Prove the number of paths form v to w is

$$(n-2)! \sum_{k=0}^{n-2} \frac{1}{k!}$$

Proof 7.5: ★ Prove that a vertex v in a connected graph G is an articulation point if and only if there are vertices w and x in G having the property that every path from w to x passes through v.

Proof 7.6: ★ Prove that in any simple graph with more than one vertex, there must be two vertices with the same degree. (Hint: Try contradiction.)

Proof 7.7: A **forest** is a simple graph where each component is a tree. Find a formula for the number of edges in a forest with n vertices and c components. Prove your formula.

Proof 7.8: Prove the average degree in a tree is always less than 2. More specifically express this average as a function of the number of vertices in tree.

Proof 7.9: ★ Prove the number of vertices of degree 1 in an tree must be greater than or equal to the maximum degree in the tree.