DEPARTAMENTO DE MATEMÁTICAS Y FÍSICA

Materia: **CALCULO INTEGRAL**

Profesor: M. C. DAVID REFUGIO ARELLANO BÁEZ

Periodo: ENERO-JUNIO / 2022

Tarea Mini Examen 2 Periodo Parcial 1

fecha: 23/FEB/2022

Carrera: ICI 2-A

Tarea Mini Examen 2
Parcial 2

① $\int sen^2 x\, cos^3 x\, dx = \int Sen^2 x\, Cos^2 x\, Cos\, x\, dx = \int sen^2 x\, cos\, x\, (Sin^2 x - 1)$

$U = Sin\, x$
$du = cos\, x\, dx$
$dx = \frac{1}{cos\, x}$

$= \int U^2 \cancel{cos\, x} (U^2 - 1) \frac{1}{\cancel{cos\, x}} = \int U^2 (U^2 - 1) = \int U^4 - U^2$

$= \frac{U^5}{5} - \frac{U^3}{3} = \frac{Sin^5(x)}{5} - \frac{Sin^3(x)}{3} + c$

② $\int sen^3\theta\, cos^4\theta\, d\theta = \int cos^4\theta\, sen^2\theta\, sen\,\theta\, d\theta = \int cos^4 (1 - cos^2\theta)\, sen\,\theta\, d\theta$

$U = cos\,\theta$
$du = -Sin\,\theta\, d\theta$
$d\theta = \frac{1}{-sin\,\theta}$

$= \int U^4 (1 - U^2\theta)\, sen\,\cancel{\theta} \left(\frac{1}{\cancel{sen\,\theta}}\right) = \int U^4 (1 - U^2)$

$= \int U^4 - U^6 = \int U^4 - \int U^6 = \frac{U^5}{5} - \frac{U^7}{7}$

$= \frac{Cos^5}{5} - \frac{Cos^7}{7} + c$

③ $\int sen^2(\pi x)\, cos^5(\pi x)\, dx = \frac{1}{\pi} \int sen^2(u)\, cos^5(u)\, du$

$U = \pi x$
$du = \pi\, dx$
$dx = \frac{1}{\pi}$

$w = sen\, u$
$dw = cos\, u$
$dx = \frac{1}{cos\, u}$

$= \int Sen^2 u\, Co\cancel{s}\, u\, (1 - sin^2 u)^2 \left(\frac{1}{\cancel{cos\, u}}\right) = \int w^2 (1 - w^2)^2$

$= \int (w^2 - 1)^2 w^2 = \int (w^4 - 2w^2 + 1) w^2 = \int w^6 - 2w^4 + w^2$

$= \int w^6 - 2\int w^4 + \int w^2 = \frac{w^7}{7} - 2\left[\frac{w^5}{5}\right] + \frac{w^3}{3}$

$= \frac{w^7}{7} - \frac{2w^5}{5} + \frac{w^3}{3} = \frac{Sen^7(u)}{7\pi} - \frac{2\,sen^5(u)}{5\pi} + \frac{Sen^3(u)}{3\pi}$

$= \frac{Sin^7(\pi x)}{7\pi} - \frac{2 sen^5(\pi x}{5\pi} + \frac{sen^3(\pi x)}{3\pi} + C$

④ $\int \frac{\text{sen}^3(\sqrt{x})}{\sqrt{x}} = \int \frac{\text{sen}^3(u)}{\sqrt{x}}(2\sqrt{x}$

$w = \cos u \qquad dx = \frac{1}{-\sin x}$

$dw = -\sin u \, dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx$

$dx = 2\sqrt{x}\, du$

$= 2\int \text{sen}^3 x = \int (1-\cos^2(u))(\sin(u))^{du} = \int (1-w^2)\sin(u)\left(\frac{1}{-\sin(u)}\right)$

$= -2\int 1-w^2 \, du = \frac{w^3}{3} - w = 2\left[\frac{\cos^3(u)}{3} - \cos u\right]$

$= \frac{2\cos^3\sqrt{x}}{3} - \cos\sqrt{x} + c$

⑤ $t\,\text{sen}^2 t \, dt = \int t\left(\frac{1}{2} - \frac{\cos(2t)}{2}\right)dt = \int \frac{t}{2} - \frac{t\cos(2t)}{2}\, dt$

$= \frac{1}{2}\int t\, dt - \frac{1}{2}\int t\cos(2t)\, dt = \frac{t^2}{4}\left[\frac{t\sin(2t)}{2} - \int \frac{\sin(2t)}{2} dt\right]$ $\quad u = 2t \quad du = 2 \quad dx = \frac{1}{2} du$

$= \frac{t^2}{4}\left[\frac{t\sin(u)}{2} - \int \frac{\sin(u)}{2}\left(\frac{1}{2}\right)\right] = \frac{t^2}{4}\left[\frac{t\sin(u)}{2} - \frac{1}{4}\int \sin\, du\right]$

$= \frac{1}{2}\left(\frac{t^2}{2}\left[\frac{t\sin(u)}{2} - \frac{\cos u}{4}\right] = \frac{t^2}{2}\left[\frac{t\sin(2t)}{2} - \frac{\cos(2t)}{4}\right]\right)$

$= \frac{t^2}{4} - \frac{t\sin(2t)}{4} - \frac{\cos(2t)}{8} + c$

⑥ $\int \cos\theta \cos^5(\text{sen}\,\theta)\, d\theta = \int \cancel{\cos\theta}\cos^5\theta(u)\frac{1}{\cancel{\cos\theta}} = \int \cos^5 u\, du$

$u = \text{sen}\,\theta$

$du = \cos\theta$

$dx = \frac{1}{\cos} d\theta$

$w = \text{sen}\, u$

$dw = \cos u\, dx$

$dx = \frac{1}{\cos u}$

$= \int \cos u(1-\text{sen}^2 u)^2 = \int \cancel{\cos u}(1-u^2)^2\frac{1}{\cancel{\cos u}}$

$= \int (1-u^2)^2 = \int (u^4 - 2u^2 + 1)\, du$

$= \frac{u^5}{5} - 2\left(\frac{u^3}{3}\right) + u = \frac{\text{sen}^5(\text{sen}\,\theta)}{5} - \frac{2\,\text{sen}^3(\text{sen}\,\theta)}{3} + \text{sen}(\text{sen}\,\theta)$

$= \frac{\text{sen}^5(\text{sen}\,\theta)}{5} - \frac{2}{3}\text{sen}^3(\text{sen}\,\theta) + \text{sen}(\text{sen}\,\theta) + c$

⑦ $\int \frac{\cos^5\alpha}{\sqrt{\operatorname{sen}\alpha}}\,d\alpha = \int \cos\alpha \left(\frac{\sin^2(\alpha)-1}{\sqrt{\sin\alpha}}\right)^2 d\alpha = \int \cancel{\cos\alpha} \left(\frac{u^2-1}{\sqrt{u}}\right)\frac{1}{\cancel{\cos\alpha}}$

$u = \sin\alpha$

$du = \cos\alpha$

$dx = \frac{1}{\cos\alpha}$

$= \int \frac{u^2-1}{\sqrt{u}}\,du = \int \left(u^{7/2} - 2u^{3/2} + \frac{1}{\sqrt{u}}\right) du$

$= \int u^{7/2} - 2\int u^{3/2} + \int \frac{1}{\sqrt{u}}\,du$

$= \frac{2u^{9/2}}{9} - \frac{4u^{5/2}}{5} + 2\sqrt{u} = \frac{2\sin^{9/2}\alpha}{9} - \frac{4\sin^{5/2}\alpha}{5} + 2\sqrt{\sin(\alpha)}$

$= \frac{2\sin^{9/2}\alpha}{9} - \frac{4\sin^{5/2}\alpha}{5} + 2\sqrt{\sin(\alpha)} + c$

⑧ $\int x\,\operatorname{sen}^3 x = \int -\left(\frac{x\sin(3x) - 3\sin(x)}{4}\right) dx = \int \frac{3x\sin(x)}{4} - \frac{x\sin(3x)}{4}$

$= \frac{3}{4}\underbrace{\int x\sin x\,dx}_{A} - \frac{1}{4}\underbrace{\int x\sin(3x)\,dx}_{B}$

$A = \sin x - x\cos x$

$B = \frac{\sin(3x)}{9} - \frac{x\cos 3x}{3}$

$= \frac{3}{4}(\sin x - x\cos x) - \frac{1}{4}\left(\frac{\sin 3x}{9} - \frac{x\cos 3x}{3}\right)$

$= \frac{3}{4}\sin(x) - \frac{3}{4}x\cos(x) - \frac{1}{36}\sin(3x) + \frac{1}{12}x\cos(3x) + c$

⑨ $\int \cos^2 x \tan^3 x\,dx = \int \sin^2 x \tan x\,dx = \int (1-\cos^2 x)\tan x\,dx$

$u = \sin x$

$du = \cos x$

$dx = \frac{1}{\cos x}$

$= \int \tan x - \cos x \sin x\,dx = \int \tan x\,dx - \int \cos x \sin x\,dx$

$= -\ln|\cos x| - \int \cancel{\cos x}\sin x\left(\frac{1}{\cancel{\cos x}}\right) = -\ln|\cos x| - \frac{u^2}{2}$

$= -\ln|\cos x| - \frac{\sin^2 x}{2} + c$

(10) $\int \cot^4_6 \text{sen}^4\theta \, d\theta = \int \cos^4\theta \, d\theta = \frac{\cos^3\theta \sin\theta}{4} + \frac{3}{4}\int \cos^2\theta \, d\theta$

$= \frac{\cos^3\theta \sin\theta}{4} + \frac{3}{4}\left[\frac{\cos x \sin x}{2} + \frac{x}{2}\right]$ $\quad \int \cos^2\theta = \frac{\cos x \sin x}{2} + \frac{x}{2}$

$= \frac{\cos^3\theta \sin\theta}{4} + \frac{3}{8}\cos x \sin x + \frac{3x}{8} + c$

(11) $\int \frac{\cos x + \text{sen } 2x}{\text{sen } x} dx = \int \frac{2\cos x \sin x + \cos x}{\sin x} = \int \frac{2\cancel{\cos x}\, u + \cancel{\cos x}}{u} \frac{1}{\cancel{\cos} x}$

$u = \sin x$
$du = \cos x$
$dx = \frac{1}{\cos x}$

$= \int \frac{2u+1}{u} = \int \frac{1}{u} + 2 = \int \frac{1}{u} + 2\int 1 \, du$

$= \ln u + 2(u) = \ln|\sin x| + 2\sin(x) + c$

(12) $\int \cos^2 x \text{ sen } 2x \, dx = \int -(-2\cos^3(x)\sin(x)) dx = \int -(-2u^3 \cancel{\sin x}) \cdot \frac{1}{\cancel{\sin x}}$

$u = \cos x$
$du = -\sin x$
$dx = \frac{1}{-\sin x}$

$= -2\int u^3 \, du = 2\left[\frac{u^4}{4}\right] = 2\left[\frac{\cos^4 x}{4}\right] = \frac{2}{4}\cos^4 x$

$= \frac{1}{2}\cos^4 x + c$

(13) $\int \tan x \sec^3 x \, dx = \int \cancel{\tan} x \, \cancel{\sec^3} x \left(\frac{1}{3\cancel{\sec x}\cancel{\tan x}}\right) = \frac{1}{3}\int 1 \, du$

$u = \sec^3 x$
$du = 3\sec^3 x \tan x \, dx$
$dx = \frac{1}{3\sec x \tan x}$

$= \frac{1}{3}u = \frac{1}{3}\sec^3 x + c$

(14) $\int \tan^2\theta \sec^4\theta \, d\theta = \int \sec^2\theta \tan^2\theta(\tan^2\theta + 1) d\theta = \int \cancel{\sec^2\theta}\, u^2(u^2+1) \frac{1}{\cancel{\sec^2 x}}$

$u = \tan\theta$
$d\theta = \sec^2\theta$
$dx = \frac{1}{\sec^2\theta}$

$= \int (u^2+1)u^2 = \int u^4 + \int u^2 = \frac{u^5}{5} + \frac{u^3}{3}$

$= \frac{\tan^5\theta}{5} + \frac{\tan^3\theta}{3} + c$

(15) $\int \tan^2 x \, dx = \int \sec^2 x - 1 \, dx = \int \sec^2 x \, dx - \int 1 dx$

$= \tan x - x + c$ //

(16) $\int (\tan^2 x + \tan^4 x) \, dx = \int \sec^2(x) \tan^2(x) = \int \cancel{\sec^2(x)} \tan^2(x) \frac{1}{\cancel{\sec^2 x}}$

$u = \tan x$
$du = \sec^2 x$
$dx = \frac{1}{\sec^2 x}$

$= \int u^2 = \frac{u^3}{3} = \frac{\tan^3 x}{3} + c$ //

(17) $\int \tan^4 x \sec^6 x \, dx = \int \sec^2 \tan^4 x (\tan^2 x + 1)^2 \, dx = \int \cancel{\sec^2 x} \, u^4 (u^2+1)^2 \left(\frac{1}{\cancel{\sec^2 x}}\right)$

$u = \tan x$
$du = \sec^2 x$
$dx = \frac{1}{\sec^2 x}$

$= \int u^4 (u^2+1)^2 \, dx = \int u^8 + 2u^6 + u^4 \, dx$

$= \frac{u^9}{9} + 2\left[\frac{u^7}{7}\right] + \frac{u^5}{5}$

$= \frac{\tan^9 x}{9} + \frac{2}{7} \tan^7 x + \frac{\tan^5}{5} + c$ //

(18) $\int \tan^5 x \sec^3 x \, dx = \int \sec^2 x (\sec^2 x - 1) \cdot (\sec x \tan x) = \int \sec^2 x (\sec^2 x - 1)^2 \cdot (\cancel{\sec x \tan x}) \frac{1}{\cancel{\sec x \tan x}}$

$u = \sec x$
$du = \sec x \tan x$
$dx = \frac{1}{\sec x \tan x}$

$= \int u^2 (u^2+1)^2 = \int u^6 - 2u^4 + u^2$

$= \frac{u^7}{7} - \frac{2}{5} u^5 + \frac{u^3}{3}$

$= \frac{\sec^7 x}{7} - \frac{2}{5} \sec^5 x + \frac{\sec^3 x}{3} + c$ //

(19) $\int \tan^3 x \sec x \, dx$

$\int (\sec^2 x - 1) \sec x \tan \, dx = \int (u^2-1) \cancel{\sec} x \cancel{\tan} x \left(\frac{1}{\cancel{\sec x} \cancel{\tan x}}\right)$

$u = \sec x$

$du = \sec x \tan x \, dx$

$dx = \frac{1}{\sec x \tan x}$

$= \int u^2 - 1 = \int u^2 \, du - \int 1 \, du$

$= \frac{u^3}{3} - u = \frac{\sec^3}{3} - \sec x$

$= \frac{\sec^3}{3} - \sec x + c$

(20) $\int \tan^5 x \, dx$

$= \int (\tan^2 x)^2 \tan x \, dx = \int (\sec^2 x - 1)^2 \tan x$

$u = \sec x$

$du = \sec x \tan x \, dx$

$dx = \frac{1}{\sec x \tan x}$

$= \int (u^2-1)^2 \cancel{\tan} \left(\frac{1}{u \cancel{\tan x}}\right)$

$= \frac{\int (u^2-1)^2}{u} = \int \left(u^3 - 2u + \frac{1}{u}\right)$

$= \int u^3 \, du - 2 \int u \, du + \int \frac{1}{u} \, du$

$= \frac{u^4}{4} - 2\left(\frac{u^2}{2}\right) + \ln|u| + c$

$= \ln|u| + \frac{u^4}{4} - u^2 + c = \ln|\sec x| + \frac{\sec^4 x}{4} - \sec^2 x$

(21) $\int \tan^2 x \sec x \, dx$

$= \int (\sec^2 x - 1) \sec x \, dx = \int \sec^3 x - \sec x \, dx$

$= \int \sec^3 x \, dx - \int \sec x \, dx$

$= \frac{1}{2} \sec x \tan x \quad \frac{1}{2} \int \sec x \, dx$

$= \frac{1}{2} \ln|\sec x + \tan x| + \frac{\sec x \tan x}{2} = \int \sec^3 x \, dx$

$\int \sec x \, dx = \ln|\tan x + \sec x|$

$= \frac{\sec x \tan x}{2} - \frac{\ln|\tan x + \sec x|}{2} + c$

$$\int f g' = fg - \int f g'$$

(22) $\int x \sec x \tan x \, dx = \int x \, dx - \int \sec x \, dx$

$= x \sec x - \ln|\tan x + \sec x|$

$= x \sec x - \ln(|\tan x + \sec x|) + c$ //

(23) $\int \frac{\operatorname{sen} \phi}{\cos^3 \phi} \, d\phi = \int \frac{\cancel{\operatorname{sen} \phi}}{u^3 \phi} \left(\frac{1}{-\cancel{\sin \phi}}\right) = -\int \frac{1}{u^3 \phi}$

$u = \cos \phi$

$du = -\sin \phi$

$d\phi = \frac{1}{-\sin \phi}$

$= \int u^{-3} = \frac{u^{-2}}{2} = \frac{1}{2u^2} = -\frac{1}{2u^2}$

$= -\frac{1}{2(\cos x)^2} + c$ //

(24) $\int \csc^4 x \cot^6 x \, dx = \int \cot^6 x (-\cot^2 + 1) \csc^2 x \, dx$

$= \int -(-\cot^6 x (\cot^2 x + 1) \cancel{\csc^2} x \, dx \, (1/-\cancel{\csc^2} x)$

$u = \cot x$

$du = -\csc^2 dx$

$dx = \frac{1}{-\csc^2 x}$

$= -\int u^6 (u^2 + 1) \, du$

$= -\int u^8 + u^6 du = \int u^8 du + \int u^6 du$

$= \frac{u^9}{9} + \frac{u^7}{7} = -\left[\frac{u^9}{9} + \frac{u^7}{7}\right]$

$= -\frac{\cot^9 x}{9} - \frac{\cot^7 x}{7} + c$ //

(25) $\int \csc x \, dx = \ln|\csc x - \cot u| + c$

$= -\ln|\csc x + \cot u| + c$ //

(26) $\int \operatorname{sen} 8x \cos 5x = \frac{1}{2}\int \left[\operatorname{sen}(8x+5x) + \operatorname{sen}(8x-5x)\right] dx$

$= \frac{1}{2}\int \left[\operatorname{sen}(13x) + \operatorname{sen}(3x)\right] dx$

$u = 13x \quad w = 3x$
$du = 13 \quad dw = 3$

$= \frac{1}{2}\int \operatorname{sen}(13x)\, dx + \frac{1}{2}\int \operatorname{sen}(3x)\, dx$

$= \frac{1}{26}\int \operatorname{sen}(u)\, dx + \frac{1}{6}\int \operatorname{sen}(w)\, dx$

$= \frac{1}{26}(-\cos(u)) + \frac{1}{6}(-\cos(w)) = -\frac{1}{26}\cos(13x) - \frac{1}{6}\cos(3x) + c$

(27) $\int \cos \pi x \cos 4\pi x\, dx = \int \cos \pi \cos(4\pi)\, x^2\, dx$

$= \int -x^2 = -\int x^2 = \frac{-x^3}{3}$

$= \frac{-x^3}{3} + c$

(28) $\int \operatorname{sen} 5\theta \operatorname{sen} \theta\, d\theta = \frac{1}{2}\int \left[\cos(4\theta) - \cos(6\theta)\right] d\theta$

$u = 4\theta\, d\theta \quad w = 6\theta$
$du = 4\, d\theta \quad dw = 6\, d\theta$
$d\theta = \frac{1}{4} \quad d\theta = 1/6$

$= \frac{1}{2}\int [\cos(u)^{du} - \frac{1}{2}\int \cos(w)\, dw$

$= \frac{1}{8}\int \cos(u)\, du - \frac{1}{12}\int \cos(w)\, dw$

$= \frac{1}{8} + \operatorname{sen}(u) - \frac{1}{12} + \operatorname{sen}(w)$

$= \frac{1}{8}\operatorname{sen}(4\theta) - \frac{1}{12}\operatorname{sen}(6\theta)$

(29) $\int \frac{\cos x + \operatorname{sen} x}{\operatorname{sen} 2x} = \int \frac{\cos x + \operatorname{sen} x}{2\cos x \operatorname{sen} x}\, dx = \int \frac{\cancel{\cos x}}{2\operatorname{sen} \cancel{\cos x}}\, dx + \int \frac{\cancel{\operatorname{sen} x}}{2\cancel{\operatorname{sen} x} \cos x}$

$\operatorname{sen} 2x = 2\cos x \operatorname{sen} x$

$= \frac{1}{2}\int \frac{1}{\operatorname{sen} x}\, dx + \frac{1}{2}\int \frac{1}{\cos x}\, dx$

$= \frac{1}{2}\int \csc x\, dx + \frac{1}{2}\int \sec x\, dx$

$= \frac{1}{2}\ln|\csc x - \cot x| + \frac{1}{2}\ln|\sec x + \tan x| + c$

(30) $\int \frac{1-\tan^2 x}{\sec^2 x} = \int \frac{1-\left(\frac{\operatorname{sen} x}{\cos x}\right)^2}{\sec^2 x}\,dx = \int \frac{1-\frac{\operatorname{sen}^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}\,dx$

$\tan x = \frac{\operatorname{sen} x}{\cos x}$

$\sec x = \frac{1}{\cos x}$

$= \int \frac{\frac{\cos^2 x - \operatorname{sen}^2 x}{\cancel{\cos^2 x}}}{\frac{1}{\cancel{\cos^2 x}}}\,dx = \int \cos^2 x - \operatorname{sen}^2 x\, dx$

$\cos^2 - \operatorname{sen}^2 x = \cos(2x)$

$u = 2x$
$du = 2\,dx$
$dx = \frac{1}{2}\,du$

$= \int \cos(2x)\,dx$

$= \frac{1}{2}\int \cos u\,du = \frac{1}{2}\operatorname{sen}(2x) + c$

(31) $\int \frac{dx}{\cos x - 1} = \int \frac{1}{\cos x - 1}\,dx = \int \frac{1}{\cos x - 1}\left(\frac{\cos x + 1}{\cos x + 1}\right)dx = \int \frac{\cos x - 1}{\cos^2 x - 1}\,dx$

$1 - \cos^2 x = \sin^2 x$

$= \int \frac{\cos x - 1}{-\sin^2 x} = \int \frac{\cos x\,dx}{-\sin^2 x}\ \frac{1}{\sin^2 x}\,dx = \int -\cot x \csc x - \csc^2 x\,dx$

$= -\int \cot x \csc x\,dx - \int \csc^2\,dx$

$= -(-\csc x) - (-\cot x) + c$

$= \csc x + \cot x + c$

(32) $\int x\tan^2 x\,dx = x(-x + \tan x) - \int -x + \tan x\,dx$

$\int fg' = fg - \int f'g$

$= x(-x + \tan x) - \left[\int \tan x - \int x\right]$

$= x(-x + \tan x) - \left[-\ln|\cos x| - \frac{x^2}{2}\right]$

$= \ln|\cos x| + x(-x + \tan x) + \frac{x^2}{2} + c$

(33) $\int x\,sen^2(x^2)\,dx = \int x\,sen^2 u\left(\frac{1}{2x}\right) d$

$U = x^2$

$du = 2x\,dx$

$dx = \frac{1}{2x}\,du$

$= \frac{1}{2}\int sen^2 u$

$\frac{1}{2}\left[\frac{1}{2}u - \frac{1}{4}\,sen\,2u\right]$

$\frac{1}{2}\left[\frac{1}{2}(x^2) - \frac{1}{4}\,sen\,2(x^2)\right]$

$= \frac{1}{4}\left(x^2 - \frac{1}{2}\sin(2x^2)\right) + C$

(34) $\int sen^5 x\,cos^3 x\,dx = \int sen^5 x\,cos^2 x\,cos\,x\,dx$

$U = \sin x$

$du = +\cos x$

$dx = \frac{1}{\cos}x$

$= \int sen^5 x(1-\sin^2 x)\cancel{\cos x}\left(\frac{1}{\cancel{\cos x}}\right)$

$= \int u^5(1-u^2)$

$= \int u^7 - u^5 = \int u^7 - \int u^5$

$= \frac{u^8}{8} - \frac{u^6}{6} = \frac{\sin^8 x}{8} - \frac{\sin^6 x}{6}$

$= \frac{sen^8 x}{8} - \frac{sen^6 x}{6} + C$

(35) $\int sen\,3x\,sen\,6x = \frac{1}{2}\left[\int \cos(3x-6x) - \cos(3x+6x)\,dx\right.$

$u = 3x$ $\quad w = 9x$

$du = 3$ $\quad dw = 9$

$dx = \frac{1}{3}$ $\quad dx = \frac{1}{9}$

$= \frac{1}{2}\left[\int \cos(-3x) - \cos(9x)\,dx\right.$

$= \frac{1}{2}\left(\int \cos(u) - \cos(w)\,dx\right.$

$= \frac{1}{2}\frac{1}{3}\int \cos(u) - \frac{1}{2}\frac{1}{9}\int \cos(w)$

$= \frac{1}{6}\,Sen(u) - \frac{1}{18}\,sin(u)$

$= \frac{1}{6}\,sen(3x) - \frac{1}{18}\,sen(9x) + C$

(36) $\int \sec^4 \frac{x}{2}\, dx = 2\int \sec^4 (u)$

$u = \left(\frac{x}{2}\right)$

$du = \frac{1}{2}\, dx$

$dx = 2\, du$

$u = \tan$

$du = \sec^2 x$

$dx\ \frac{1}{\sec^2} x$

$= 2\int \sec^2 u (\tan^2 u + 1)\, du$

$= 2\int \cancel{\sec^2} u\, (u^2+1) \frac{1}{\cancel{\sec^2} u}$

$= 2\int (u^2+1)\, du$

$= 2\left[\frac{u^3}{3} + u\right]$

$= 2\left[\frac{\tan^3(u)}{3} + \tan(u)\right]$

$= \frac{2\tan^3(u)}{3} + 2\tan(u)$

$= \frac{2\tan^3\left(\frac{x}{2}\right)}{3} + 2\tan\left(\frac{x}{2}\right) + C$