EXAMEN 3PARCIAL

①

a) $F(x) = 3x^2 + 2x - x^0$

$f'(x) = (3x^2 + 2x - x)'$

$f'(x) = 6x + 2 - 1$

$\underbrace{f'(x) = 6x - 1}$

b) $F(x) = \frac{2}{x} + \frac{3}{x^2} + \sqrt{x}$

c) $3e^x \cos x$

$f'(x) = 3e^x((\cos(x))' + (3e^x)'(\cos(x))$

$= 3e^x(-\sin(x)) + 3e^x(\cos(x))$

$\underbrace{= -3e^x \sin(x) + 3e^x \cos(x)}$

d) $F(x) = 2x\ln(x)$

$f'(x) = 2x(\ln(x))' + (2x)'(\ln(x))$

$\underbrace{f'(x) = 2x \frac{1}{x} + 2\ln(x)}$

②

i)

a) $f(x) = \frac{\sqrt{x}}{x^2 - 8}$

$f'(x) = \frac{(\sqrt{x})' \cdot (x^2-8) - \sqrt{x} \cdot (x^2-8)'}{(x^2-8)^2}$

$f'(x) = \frac{1/2\, x^{-1/2} \cdot (x^2-8) - \sqrt{x}(2x)}{(x^2-8)^2}$

$f'(x) = \frac{\frac{x^2-8}{2\sqrt{x}} - \sqrt{x}(2x)}{(x^2-8)^2}$

$\underbrace{= \frac{\frac{x^2-8}{2\sqrt{x}} - 2x^{3/2}}{(x^2-8)^2}}$

$\underbrace{x = (0, \infty)}$

b) $f(x) = \cot(x)$

$f'(x) = \frac{\cos(x)}{\sin(x)} = f'(x) = \left(\frac{\cos(x)}{\sin(x)}\right)'$

$f'(x) = \frac{(\cos(x))' \cdot \sin x - \cos x \cdot (\sin x)'}{\sin^2 x}$

$f'(x) = \frac{(-\sin(x)) \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$

$f'(x) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

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$$f(x) = -\frac{1}{\sin^2(x)}$$

(3)

a) $F(x) = \sin^4(x)$

$f'(x) = [\sin^4(x)]'$

$f'(x) = [4(\sin(x))^3 \cdot (\sin(x)')]$

$f'(x) = 4(\sin(x))^3 \cdot (\cos(x))$

$f'(x) = 4\cos(x)\sin^3(x)$

b) $f(x) = \cos(x^4)$

$f'(x) = -\sin(x^4) \cdot (x^4)'$

$f'(x) = -\sin(x^4) \cdot (4x^3)$

$f'(x) = -4x^3 \sin(x^4)$

c) $f(x) = e^{3x} + e^{-3x}$

$f'(x) = e^{3x} \cdot (3x)' + e^{-3x} \cdot (-3x)'$

$f'(x) = e^{3x} \cdot 3 + e^{-3x} \cdot -3$

$f'(x) = 3e^{3x} + -3e^{-3x}$

(4)

a) $f(x) = 4\sin(\pi - x^2) + 3\cos(x^2 - \pi)$

$f'(x) = 4\cos(\pi - x^2)(\pi - x^2)' - 3\sin(x^2 - \pi)(x^2 - \pi)'$

$f'(x) = 4\cos(\pi - x^2)(-2x) - 3\sin(x^2 - \pi)(2x)$

$f'(x) = -8\cos x(\pi - x^2) - 6x\sin(x^2 - \pi)$

b) $f(x) = 2\ln(1+x^2) - 3\ln(1-x^3)$

$$f'(x) = \frac{2}{(1+x^2)} \cdot (1+x^2)' - \frac{3}{(1-x^3)} \cdot (1-x^3)$$

$$f'(x) = \frac{2}{(1+x^2)} \cdot (2x) - \frac{3}{(1-x^3)} \cdot (-3x^2) = \frac{4x}{(1+x^2)} + \frac{9x^2}{(1-x^3)}$$

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5

a) $f(x) = x^2\sqrt{1-x}$

$f'(x) = x^2((1-x)^{1/2}) + (x^2)'\sqrt{1-x}$

$f'(x) = x^2(1/2(1-x)^{1/2-1} \cdot (1-x)') + (2x)\sqrt{1-x}$

$f'(x) = x^2(1/2(1-x)^{-1/2} \cdot (-1) + (2x)\sqrt{1-x}$

$f'(x) = \underline{x^2\left(-\frac{1}{2\sqrt{1-x}}\right) + 2x\sqrt{1-x}}$

b) $f(x) = \sin^2(x)\cos(x)$

$f'(x) = \sin^2(x)((\cos(x))' + (\sin^2(x))'\cos(x)$

$f'(x) = \sin^2(x)(-\operatorname{sen}(x)) + [2\sin(x)\cos(x)]\cos(x)$

$f'(x) = \underline{-\sin^3(x) + 2\cos^2(x)\sin(x)}$

c) $f(x) = \tan(\pi - x^2)$

$f'(x) = (\tan(\pi - x^2)')$

$f'(x) = \sec^2(\pi - x^2) \cdot (\pi - x^2)'$

$f'(x) = \sec^2(\pi - x^2) \cdot (-2x)$

$f'(x) = \underline{-2x\sec^2(\pi - x^2)}$

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⑥

a) $F(x) = x^3 e^{-x^3}$

$F'(x) = (x^3)' e^{-x^3} + x^3 (e^{-x^3})'$

$F'(x) = (3x^2) e^{-x^3} + x^3 e^{-x^3} \cdot (-x^3)$

$f'(x) = (3x^2) e^{-x^3} + x^3 e^{-x^3} \cdot (-3x^2)$

$f'(x) = (3x^2) e^{-x^3} + x^3 e^{-x^2} \cdot 3x^2$

$f'(x) = 3x^2 e^{-x^3} - x^3 e^{-x^2} \cdot 3x^2$

$f'(x) = 3x^2 e^{-x^3} - 3x^5 e^{-x^2}$

$f''(x) = [(3x^2)' e^{-x^3} + (3x^2)(e^{-x^3})'] - [(3x^5)' e^{-x^2} + (3x^5)(e^{-x^2})']$

$f''(x) = [6x e^{-x^3} + 3x^2 e^{-x^3} \cdot (-x^3)'] - [15x^4 e^{-x^2} + 3x^5 e^{-x^2} \cdot (-x^2)]$

$f''(x) = [6x e^{-x^3} + 3x^2 e^{-x^3} \cdot (-3x^2)] - [15x^4 e^{-x^2} + 3x^5 e^{-x^2} \cdot (-2x)]$

$f''(x) = [6x e^{-x^3} + 3x^2 e^{-x^3}] - [15x^4 e^{-x^2} - 6x^2 e^{x^2}]$

$f''(x) = [6x e^{-x^3} - 9x^4 e^{-x^3}] - [15x^4 e^{-x^2} - 6x^2 e^{x^2}]$

b) $F(x) = x^2 \ln(x)$

$f'(x) = x^2 (\ln(x))' + (x^2)' + \ln(x)$

$f'(x) = x^2 \left(\frac{1}{x}\right) + (2x) + \ln(x)$

$f'(x) = \frac{x^2}{x} + 2x \ln(x)$

$f''(x) = \frac{(x^2)'(x) - (x^2)(x)'}{(x)^2} + [2x (\ln(x))' + (2x)' \ln(x)]$

$f''(x) = \frac{2x(x) - (x^2)}{(x)^2} + [2x \left(\frac{1}{x}\right) + (2) \ln(x)]$

$f''(x) = \frac{2x^2 - x^2}{x^2} + [\frac{2x}{x} + 2\ln(x)]$

$f''(x) = \frac{\cancel{x^2}}{\cancel{x^2}} + [\frac{2\cancel{x}}{\cancel{x}} + 2\ln(x)] = 4\ln(x)$

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(7)

a) $f(x) = e^{-3x} \sin(2x) + e^{-3x} \cos(2x)$

$f'(x) = [e^{-3x} \sin(2x)]' + [e^{-3x} \cos(2x)]$

$f'(x) = [(e^{-3x})' \sin(2x) + e^{-3x} (\sin(2x))'] + [(e^{-3x})' \cos(2x) + e^{-3x} (\cos(2x))'$

$f'(x) = [(e^{-3x}) \cdot (-3x)' \sin(2x) + e^{-3x} (\sin(2x)'] + [e^{-3x} . (-3x)' \cos(2x) + e^{-3x} (\cos(2x))']$

$f'(x) = [-3e^{-3x} \sin(2x) + e^{-3x} + e^{-3x} \cos(2x)] + [-3e^{-3x} \cos(2x) + e^{-3x} (-\sin(2x))]$

$f'(x) = [-3e^{-3x} \sin(2x) + e^{-3x} + e^{-3x} \cos(2x)] + [-6xe^{-3x} \cos(x) - 2xe^{-3x} \sin(x)]$

$f'(x) = [-6xe^{-3x} \sin(x) + 2xe^{-3x} \cos(x)] + [-6xe^{-3x} \cos(x) - 2xe^{-3x} \sin(x)]$

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8

a) $f(x) = Ax^3e^{-x^2}$

$f'(x) = A[x^3e^{-x^2}]'$

$f'(x) = A[(x^3)'e^{-x^2} + x^3(e^{-x^2})']$

$f'(x) = A[3x^2e^{-x^2} + x^3(e^{-x^2} \cdot (-x^2)')]$

$f'(x) = A[3x^2e^{-x^2} + x^3e^{-x^2} \cdot -(2x)]$

$f'(x) = A[3x^2e^{-x^2} - x^3e^{-x^2} \cdot (2x)]$

$f'(x) = A[3x^2e^{-x^2} - 2x^4e^{-x^2}$

b) $f(x) = x^n \ln x$

$f'(x) = (x^n)' \ln(x) + x^n (\ln(x))'$

$f'(x) = nx^{n-1} \ln(x) + x^n \cdot \frac{1}{x}$

$f'(x) = nx^{n-1} \ln(x) + x^{n-1}$

c) $f(x) = \sin^n(x) + \sin(x^n)$

$f'(x) = [\sin^n(x)]' + [\sin(x^n)]$

$f'(x) = [n\sin^{n-1}(x) \cdot (\sin(x))' + \cos(x^n) \cdot (x^n)'$

$f'(x) = n\sin^{n-1}(x)\cos(x) + \cos(x^n) \cdot (nx^{n-1})$

$f'(x) = n\cos(x)\sin^{n-1} + nx^{n-1}\cos(x^n)$

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