Tarea III
ED's Lineales 1 Orden

Instrucciones: Todas las ecuaciones diferenciales de esta tarea deben ser resueltas por el método de ED lineales. Aunque sean separables.

① Encuentra la solución general de cada ecuación

a) $\frac{dy}{dx} = xe^{-4x} - 4y$

① Encontramos forma $\frac{dy}{dx} + P(x)y = h(x)$

$\rightarrow \frac{dy}{dx} + 4y = xe^{-4x}$

② Encontrar FI

$\mu(x) = e^{\int 4dx} = e^{4x}$

③ Multiplicamos ED por FI

$e^{4x}\left(\frac{dy}{dx} + 4y\right) = xe^{-4x}(e^{4x})$

$\underbrace{e^{4x}\frac{dy}{dx} + 4e^{4x}y}_{\frac{d\, e^{4x} y}{dx}} = x$

$\frac{d\, e^{4x} y}{dx} = x$

④ Integramos

$\int \frac{d e^{4x} \cdot y}{\cancel{dx}} = \int x\,dx$

⑤ Resolvemos y sacamos sol. general

$e^{4x} \cdot y = \frac{x^2}{2} + C \longrightarrow y = \frac{\frac{x^2}{2}}{\frac{e^{4x}}{1}} + \frac{C}{e^{4x}} = \frac{x^2}{2e^{4x}} + \frac{C}{e^{4x}} = \frac{1}{2}x^2e^{-4x} + Ce^{-4x}$

★ Solución general

b) $\underbrace{x\frac{dy}{dx} + y}_{\text{derivada de un producto}} = 0 \rightarrow \frac{d\,xy}{dx} = \int 0\,dx$

① Integramos

$\int \cancel{d}\frac{xy}{\cancel{dx}} = \int 0\,dx$

$xy = x + c$

② Despejamos y sacamos sol. general

$xy = 0x + c$

$\rightarrow y = \frac{\cancel{0x}^{\,0}}{\cancel{x}} + \frac{C}{x} = Cx^{-1}$

Sol. general

c) $x\frac{dy}{dx} - y = 2x^2 - x$

① Dividimos entre x para encontrar forma $\frac{dy}{dx} + P(x)y = h(x)$

$\frac{x\frac{dy}{dx} - y}{x} = \frac{2x^2 - x}{x} = \frac{dy}{dx} - \frac{1}{x}y = \frac{2x^2}{x} - \frac{x}{x} = 2x - 1$

$= \frac{dy}{dx} - \frac{1}{x}y = 2x - 1 \quad \left(\frac{dy}{dx} + P(x)y = h(x)\right)$

② Encontramos FI

$$\mu(x) = e^{\int p(x)} = e^{\int -\frac{1}{x}dx} = e^{-\ln|x|} = e^{\ln x^{-1}} = x^{-1} = \frac{1}{x} \rightarrow x \in (0,\infty)$$

③ Multiplicamos FI por ED

$$\frac{1}{x}\left(\frac{dy}{dx} - \frac{1}{x}y\right) = 2x - 1\left(\frac{1}{x}\right)$$

$$\rightarrow \underbrace{\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y}_{\frac{d\left(\frac{1}{x}\cdot y\right)}{dx}} = 2 - \frac{1}{x}$$

④ Integramos

$$\int \frac{d\left(\frac{1}{x}\cdot y\right)}{\cancel{dx}} = \int 2 - \frac{1}{x}\,dx$$

$$\rightarrow \frac{1}{x}y = 2\int dx - \int \frac{1}{x}dx$$

$$\rightarrow \frac{1}{x}y = 2x - \ln|x| + C$$

⑤ Encontramos solución despejando $y(x)$

$$y(x) = \frac{x^2+1+c}{\frac{1}{x}} = \frac{\frac{2x}{1}}{\frac{1}{x}} - \frac{\frac{\ln|x|}{1}}{\frac{1}{x}} + \frac{\frac{c}{1}}{\frac{1}{x}} \rightarrow y(x) = 2x^2 - x\ln|x| + Cx$$

Sol. general

② Encuentra el PVI

a) $\underbrace{\frac{dy}{dx} + xy = 2x}_{\frac{dy}{dx} + p(x)y = h(x)}$ $\quad y(0) = 4$

① Encontramos FI

$$\mu(x) = e^{\int p(x)dx} = e^{\int x dx} = e^{\frac{x^2}{2}}$$

② Multiplicamos FI por ED

$$e^{\frac{x^2}{2}}\left(\frac{dy}{dx} + xy\right) = 2x\left(e^{\frac{x^2}{2}}\right)$$

$$\underbrace{e^{\frac{x^2}{2}}\frac{dy}{dx} + xye^{\frac{x^2}{2}}}_{\frac{d\,e^{\frac{x^2}{2}}y}{dx}} = 2xe^{\frac{x^2}{2}}$$

③ Integramos

$$\int \frac{d\left(e^{\frac{x^2}{2}}y\right)}{\cancel{dx}} = \int 2xe^{\frac{x^2}{2}}dx \qquad u = \frac{x^2}{2}, \quad du = x\,dx, \quad dx = \frac{du}{x}$$

$$e^{\frac{x^2}{2}}y = 2\int xe^{\frac{x^2}{2}}dx$$

$$e^{\frac{x^2}{2}}y = 2\int xe^{u}du$$

$$= 2\int \cancel{x}e^{u}\frac{1}{\cancel{x}}$$

$$= 2e^{u}$$

$$= 2e^{\frac{x^2}{2}} + C$$

④ Encontramos sol despejando $y(x)$

$$y(x) = \frac{2e^{\cancel{\frac{x^2}{2}}} + C}{e^{\cancel{\frac{x^2}{2}}}} = \underbrace{2 + \frac{C}{e^{\frac{x^2}{2}}}}_{\text{sol general}}$$

⑤ Se busca sol que satisfaga $y(0) = 4$

$$2 + \frac{C}{e^{\frac{x^2}{2}}} = 4$$

$$\rightarrow \frac{C}{e^{\frac{x^2}{2}}} = 4 - 2$$

$$\rightarrow C = 2e^{\frac{x^2}{2}}$$

$\rightarrow 2 + \dfrac{2e^{\frac{x^2}{2}}}{e^{\frac{x^2}{2}}} = Y(x)$ Sol del PVI

$\rightarrow 2 + \dfrac{2e^{\frac{0^2}{2}}}{e^{\frac{0^2}{2}}} = 2+2 = 4$ Comprobación

③ Encuentra las soluciones de los PVI

a) $x\dfrac{dy}{dx} + 3y + 2x = 3x^2 \quad , \quad y(1) = 1$

① Encontramos forma $\frac{dy}{dx} + P(x)y = h(x)$

$x\dfrac{dy}{dx} + 3y + 2x = 3x^2 \longrightarrow \dfrac{x\frac{dy}{dx} + 3y}{x} = \dfrac{3x^2}{x} - \dfrac{2x}{x}$

$\rightarrow x\dfrac{dy}{dx} + 3y = 3x^2 - 2x$

$\rightarrow \underbrace{\dfrac{dy}{dx}}_{\frac{dx}{dy}} + \underbrace{\dfrac{3}{x}}_{p(x)}\underbrace{y}_{y} = \underbrace{3x-2}_{h(x)}$

② Encontramos FI

$M(x) = e^{\int \frac{3}{x}dx} = e^{3\int \frac{1}{x}dx} = e^{3\ln|x|} = e^{\ln|x^3|} = x^3$

③ Multiplicamos FI por ED

$(x^3)\left(\dfrac{dy}{dx} + \dfrac{3}{x}y\right) = (3x-2)(x^3)$

$\rightarrow \underbrace{x^3\dfrac{dy}{dx} + 3x^2y}_{\frac{d(x^3\cdot y)}{dx}} = 3x^4 - 2x^3$

④ Integramos

$\int \dfrac{d(x^3\cdot y)}{dx} = \int 3x^4 - 2x^3\, dx$

$\rightarrow x^3\cdot y = 3\int x^4 dx - 2\int x^3 dx$

$\rightarrow x^3\cdot y = 3\dfrac{x^5}{5} + C_1 - 2\dfrac{x^4}{4} + C_2$

$C_1 + C_2 = C$

$\rightarrow x^3\cdot y = \dfrac{3}{5}x^5 - \dfrac{1}{2}x^4 + c$

$\rightarrow y = \dfrac{\frac{3x^5}{5}}{\frac{x^3}{1}} - \dfrac{\frac{1x^4}{2}}{\frac{x^3}{1}} + \dfrac{c}{x^3}$

$\rightarrow y = \dfrac{3x^5}{5x^3} - \dfrac{x^4}{2x^3} + \dfrac{c}{x^3}$

$\rightarrow \underbrace{y = \dfrac{3}{5}x^2 - \dfrac{1}{2}x + \dfrac{c}{x^3}}$

Sol. general

⑤ Se busca solución que satisfaga $y(1)=1$

$1 = \frac{3}{5}x^2 - \frac{1}{2}x + \frac{c}{x^3}$

$\rightarrow 1 = \frac{3(1)^2}{5} - \frac{1}{2}(1) + \frac{c}{1^{(3)}}$

$\rightarrow 1 = \frac{3}{5} \cdot \frac{1}{2} + \frac{c}{1^3}$

$\rightarrow 1 = \frac{3}{5} - \frac{1}{2} + \frac{c}{1}$

$\rightarrow 1 = \frac{6}{10} - \frac{5}{10} + c$

$\rightarrow 1 = \frac{1}{10} + c$

$\rightarrow 1 - \frac{1}{10} = c$

$\frac{10}{10} - \frac{1}{10} = c$

$c = \frac{9}{10}$

⑥ Sustituir c en sol general

$y(x) = \frac{3}{5}x^2 - \frac{1}{2}x + \frac{\frac{\frac{9}{10}}{x3}}{1} = \underbrace{\frac{3}{5}x^2 - \frac{1}{2} + \frac{9}{10x^3}}_{\text{Sol del PVI}}$

b) $x\frac{dy}{dx} - y = x^3\cos(x)$, $y(\pi) = \pi$

① Encontrar forma $\frac{dy}{dx} + p(x)y = h(x)$

$\rightarrow x\frac{dy}{dx} - y = x^3\cos(x)$

$\rightarrow \frac{x\frac{dy}{dx} - y}{x} = \frac{x^3\cos(x)}{x}$

$\rightarrow \frac{dy}{dx} - \frac{1}{x}y = x^2\cos(x)$

② Encontramos FI

$\mu(x) = e^{\int -\frac{1}{x}dx} = e^{-\int\frac{1}{x}dx} = e^{-\ln|x|} = e^{\ln|x^{-1}|} = x^{-1} = \frac{1}{x}$

③ Multiplicamos FI por ED

$\left(\frac{1}{x}\right)\left(\frac{dy}{dx} - \frac{1}{x}y\right) = \left(x^2\cos(x)\right)\left(\frac{1}{x}\right) \rightarrow \underbrace{\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y}_{\frac{d\left(\frac{1}{x}\cdot y\right)}{dx}} = x\cos(x)$

④ Integramos

$$\int \frac{d(\frac{1}{x}\cdot y)}{dx} = \int x\cos(x)\,dx$$

$$\frac{1}{x}\cdot y = \int x\cos(x)$$

$$\rightarrow \frac{1}{x}\cdot y = x\,sen(x) + \cos(x) + c$$

$$\rightarrow y = \frac{x\,sen(x)}{\frac{1}{\frac{1}{x}}} + \frac{\cos(x)}{\frac{1}{\frac{1}{x}}} + \frac{c}{\frac{1}{\frac{1}{x}}}$$

$$\rightarrow \underbrace{y(x) = x^2 sen(x) + x\cos(x) + Cx}_{\text{Sol. general}}$$

$$\int x\cos(x)\,dx \longrightarrow uv - \int v\,du$$

$u = x \qquad dv = \cos(x)$

$du = dx \qquad v = sen(x)$

$$\int x\cos(x)\,dx = x\,sen(x) - \int sen(x)\,dx$$

$$= x\,sen(x) - (-\cos(x))$$

$$= x\,sen(x) + \cos(x) + C$$

⑤ Se busca solución que satisfaga $y(\pi) = \pi$

$$\pi = \pi^2 \cancel{sen(\pi)} + \pi\cos(\pi) + C\pi$$

$$\pi = -\pi + C\pi$$

$$\pi + \pi = C\pi$$

$$2\pi = C\pi$$

$$C = \frac{2\pi}{\pi} \rightarrow C = 2$$

⑥ Sustituir C en sol. general

$$\underbrace{y(x) = x^2 sen(x) + x\cos(x) + 2x}_{\text{Sol. del PVI}}$$

④ Resolver la ED lineal con parametros positivos a, b

$$a\frac{dy}{dx}+by=0$$

① Encontramos forma $\frac{dy}{dx}+p(x)y=h(x)$

$$a\frac{dy}{dx}+by=0$$

$$\rightarrow \frac{a\frac{dy}{dx}+by}{a}=\frac{0}{a}$$

$$\rightarrow \underbrace{\frac{dy}{dx}}_{\frac{dy}{dx}}+\underbrace{\frac{b}{a}}_{p(x)}\underbrace{y}_{y}=\underbrace{0}_{h(x)}$$

② encontramos FI

$$\mu(x)=e^{\int\frac{b}{a}dx}=e^{\frac{b}{a}\int dx}=e^{\frac{b}{a}x}=$$

③ Multiplicar ED por FI

$$e^{\frac{b}{a}x}\left(\frac{dy}{dx}+\frac{b}{a}y\right)=0\left(e^{\frac{b}{a}x}\right)$$

$$\rightarrow \underbrace{e^{\frac{b}{a}x}\frac{dy}{dx}+e^{\frac{b}{a}}\frac{b}{a}y}_{\frac{d\left(e^{\frac{b}{a}x}\cdot y\right)}{dx}}=0$$

④ Integramos

$$\int\frac{d\left(e^{\frac{b}{a}x}\cdot y\right)}{dx}=\int 0dx$$

$$e^{\frac{b}{a}x}\cdot y=0\int dx\rightarrow e^{\frac{b}{a}x}\cdot y=0(x)+c$$

$$\underbrace{y(x)=\frac{c}{e^{\frac{b}{a}x}}}_{\text{sol. general}}$$

⑤ Resolver la ED lineal con parametros positiva k, Tm

$$\frac{dT}{dt}=-k(T-Tm)$$

① Encontrar forma $\frac{dy}{dx}+p(x)y=h(x)$

$$\frac{dT}{dt}=-k(T-Tm)$$

$$\frac{dT}{dt}=-kT+kTm$$

$$\underbrace{\frac{dT}{dt}}_{\frac{dy}{dx}}\underbrace{-k}_{p(x)}\underbrace{T}_{y}=\underbrace{kTm}_{h(x)}$$

② Encontra FI

$$\mu(t)=e^{\int-kdt}=e^{-k\int dt}=e^{-kt}=e^{-kt}$$

③ Multiplicar FI por ED

$$e^{-kt}\left(\frac{dT}{dt}-kT\right)=e^{-kt}(kTm)$$

$$\underbrace{e^{-kt}\frac{dT}{dt}-e^{-kt}kT}_{\frac{d\left(e^{-kt}\cdot T\right)}{dt}}=kTme^{-kt}$$

④ Integrar

$$\int \frac{d(e^{-kt}\cdot T)}{\cancel{dt}} = \int kT_m e^{-kt}$$

$$\int e^{u} = e^{u}$$

$u = -kt$

$du = -k\,dt$

$dt = \frac{du}{k}$

$\rightarrow e^{-kt}\cdot T = kT_m \int e^{-kt}\,dt$

$\rightarrow e^{-kt}\cdot T = kT_m \frac{1}{k} e^{-kt} + C$

$\rightarrow e^{-kt}\cdot T = \frac{\cancel{k}T_m e^{-kt}}{\cancel{k}} = T_m e^{-kt} + C$

$\rightarrow e^{-kt}\cdot T = T_m e^{-kt} + C$

$\rightarrow T(t) = \frac{T_m \cancel{e^{-kt}} + C}{\cancel{e^{-kt}}} = \frac{C}{e^{-kt}} + T_m$

$\rightarrow \underbrace{T(t) = T_m + \frac{C}{e^{-kt}}}_{\text{Sol. general de la ED}}$

⑥ Resolver la ED lineal con parametros positivos a, b

$$ax\frac{dy}{dx} + by = 1$$

① Encontramos forma $\frac{dy}{dx} + p(x)y = h(x)$

$ax\frac{dy}{dx} + by = 1$

$\rightarrow \frac{ax\frac{dy}{dx} + by}{ax} = \frac{1}{ax}$

$\rightarrow \underbrace{\frac{dy}{dx}}_{\frac{dy}{dx}} + \underbrace{\frac{b}{ax}y}_{p(x)\ y} = \underbrace{\frac{1}{ax}}_{h(x)}$

② Encontrar FI

$\mu(x) = e^{\int \frac{b}{ax}dx} = e^{\frac{b}{a}\int \frac{1}{x}dx} = e^{\frac{b}{a}\ln|x|}$

$= e^{\ln|x^{\frac{b}{a}}|} = x^{\frac{b}{a}}$

③ Multiplicar FI por ED

$x^{\frac{a}{b}}\left(\frac{dy}{dx} + \frac{b}{ax}y\right) = \frac{1}{ax}\left(x^{\frac{b}{a}}\right)$

$x^{\frac{a}{b}}\frac{dy}{dx} + \frac{b}{ax}yx^{\frac{a}{b}} = \frac{x^{\frac{b}{a}}}{ax}$

$\underbrace{x^{\frac{a}{b}}\frac{dy}{dx} + \frac{bx^{\frac{b-a}{a}}}{a}}_{\frac{d(x^{\frac{a}{b}}\cdot y)}{dx}} = \frac{x^{\frac{b-a}{a}}}{a}$

④ Integrar

$$\int \frac{d\left(x^{\frac{b}{a}} \cdot y\right)}{d\cancel{x}} = \int \frac{x^{\frac{b-a}{a}}}{a}\,dx$$

$$x^{\frac{b}{a}} \cdot y = \frac{1}{a}\int x^{\frac{b-a}{a}}\,dx$$

$$x^{\frac{b}{a}} \cdot y = \frac{1}{a}\,\frac{x^{\frac{b-a}{a}+1}}{\frac{b-a}{a}+1} = \frac{1}{a}\,x^{\frac{b-a}{a}+\frac{a}{a}=\frac{b}{a}}$$

$$x^{\frac{b}{a}} \cdot y = \frac{\frac{1}{a}\,x^{\frac{b}{a}}}{\frac{1}{\frac{b}{a}}} + c$$

$$x^{\frac{b}{a}} \cdot y = \frac{\frac{1}{\cancel{a}}\cancel{a}\,x^{\frac{b}{a}}}{b} + c$$

$$x^{\frac{b}{a}} \cdot y = \frac{x^{\frac{b}{a}}}{b} + c$$

$$y(x) = \frac{\frac{x^{\frac{b}{a}}}{b}}{\frac{x^{\frac{b}{a}}}{1}} + \frac{c}{x^{b/a}}$$

$$\rightarrow y(x) = \frac{\cancel{x^{\frac{b}{a}}}}{b\,\cancel{x^{b/a}}} + \frac{c}{x^{b/a}}$$

$$\rightarrow \underbrace{y(x) = \frac{1}{b} + \frac{c}{x^{\frac{b}{a}}}}_{\text{Sol. general}}$$

7. Resolver la ED lineal con parametros positivos a, b

$$\frac{dy}{dx} + ay = be^{ax}$$

1. Encontrar forma $\frac{dy}{dx} + p(x)y = h(x)$

$$\underbrace{\frac{dy}{dx}}_{\frac{dy}{dx}} + \underbrace{a}_{p(x)}\underbrace{y}_{y} = \underbrace{be^{ax}}_{h(x)}$$

2. Encontrar FI

$$\mu(x) = e^{\int a\,dx} = e^{a\int dx} = e^{ax}$$

3. Multiplicar ED por FI

$$e^{ax}\left(\frac{dy}{dx} + ay\right) = be^{ax}(e^{ax})$$

$$\rightarrow \underbrace{e^{ax}\frac{dy}{dx} + aye^{ax}}_{\frac{d(e^{ax}\cdot y)}{dx}} = be^{2ax}$$

4. Integramos

$$\int \frac{d(e^{ax}\cdot y)}{\cancel{dx}} = \int be^{2ax}\,dx$$

$$e^{ax}\cdot y = b\int e^{2ax}\,dx \qquad \rightarrow \int e^{u}du = e^{u}$$

$$= b\frac{1}{2a}e^{2ax} + C \qquad u = 2ax,\quad du = 2a\,dx,\quad dx = \frac{du}{2a}$$

$$y(x) = \frac{\frac{be^{2ax}}{2a}}{\frac{e^{ax}}{1}} + \frac{C}{e^{ax}} \longrightarrow y(x) = \frac{be^{2ax}}{2a\,e^{ax}} + \frac{C}{e^{ax}}$$

$$= \underbrace{\frac{be^{ax}}{2a} + \frac{C}{e^{ax}}}_{\text{Sol. general}}$$

⑧ Resuelve el PVI

$$x\frac{dy}{dx} - 2y = x^3\cos(x)$$

① Encontrar forma $\frac{dy}{dx} + p(x)y = h(x)$

$$\rightarrow \frac{x\frac{dy}{dx} - 2y = x^3\cos(x)}{x}$$

$$\rightarrow \frac{dy}{dx} - \frac{2}{x}y = x^2\cos(x)$$

② Encontrar FI

$$\mu(x) = e^{\int \frac{2}{x}dx} = e^{-2\int \frac{1}{x}dx} = e^{-2\ln|x|} = e^{\ln|x^{-2}|} = x^{-2} = \frac{1}{x^2}$$

③ Multiplicar FI por ED

$$\frac{1}{x^2}\left(\frac{dy}{dx} - \frac{2}{x}y\right) = (x^2\cos(x))\left(\frac{1}{x^2}\right)$$

$$\rightarrow \frac{1}{x^2}\frac{dy}{dx} - \frac{2}{x^3}y = \frac{\cancel{x^2}\cos(x)}{\cancel{x^2}}$$

$$\rightarrow \underbrace{\frac{1}{x^2}\frac{dy}{dx} - \frac{2}{x^3}y}_{\frac{d\left(\frac{1}{x^2}\cdot y\right)}{dx}} = \cos(x)$$

④ Integramos

$$\int \frac{d\left(\frac{1}{x^2}\cdot y\right)}{\cancel{dx}} = \int \cos(x)$$

$$\frac{1}{x^2}\cdot y = \operatorname{sen}(x) + c$$

$$y(x) = \frac{\operatorname{sen}(x)}{\frac{1}{x^2}} + \frac{c}{\frac{1}{x^2}}$$

$$\underbrace{y(x) = x^2\operatorname{sen}(x) + cx^2}_{\text{Sol. general}}$$

⑤ Se busca solución que satisfaga $y(0) = 0$

$$0 = (0)^2\operatorname{sen}(0) + c(0)^2$$

$$-\operatorname{sen}(0) = c(0)^2 \rightarrow c = 0$$

$$\frac{0}{0} = c$$

⑥ Sustituimos c en sol. general

$$y(x) = x^2\operatorname{sen}(x) + 0x^2 = \underbrace{x^2\operatorname{sen}(x)}_{\text{Sol del PVI}}$$