Tarea Derivadas Orden Superior

① Obtenga la primer y segunda derivada de los siguientes funciones.

a) $f(x) = x^2 - 4x^3$

$f'(x) = \underbrace{2x - 12x^2}$ ①

$f''(x) = \underbrace{2 - 24x}$ ②

b) $f(x) = x^3 - 4x$

$f'(x) = \underbrace{3x^2 - 4}$ ①

$f''(x) = \underbrace{6x}$ ②

c) $f(x) = x^2 \cos(2x)$

$f'(x) = (x^2)(\cos(2x))' + (x^2)'(\cos(2x))$

$= (x^2)[-\text{Sen}(2x) \circ 2] + 2x \cos(2x)$

$= \underbrace{-2x^2 \text{Sen}(2x) + 2x\cos(2x)}$ ①

$f''(x) = (-2x^2)[(\text{sen}(2x))' \cdot (2x)'] + (-2x^2)'\text{sen}(2x) + (2x)[(\cos(2x)) \circ (2x)] + (2x)' \cos 2x$

$= (-2x^2)[\cos(2x) \circ 2] + (-4x)\text{sen}(2x) + (2x)[-\text{Sen}(2x) \cdot 2] + (2)\cos(2x)$

$= -4x^2 \cos 2x - 4x\,\text{Sen}(2x) - 4x\,\text{Sen}(2x) + 2\cos(2x)$

$= \underbrace{(2 - 4x^2)\cos(2x) - 8x\,\text{Sen}(2x)}$ ②

d) $f(x) = 2x^3 e^{6x}$

$f'(x) = (2x^3)(e^{6x})' + (2x^3)'(e^{6x})$

$= 2x^3(6e^{6x}) + (6x^2)(e^{6x})$

$= \underbrace{12x^3 e^{6x} + 6x^2 e^{6x}}$ ①

$f''(x) = (12x^3)(e^{6x})' + (12x^3)'(e^{6x}) + (6x^2)(e^{6x})' + (6x^2)'(e^{6x})$

$= 12x^3(6e^{6x}) + (36x^2)(e^{6x}) + 6x^2(6e^{6x}) + (12x)(e^{6x})$

$= 72x^3 e^{6x} + 36x^2 e^{6x} + 36x^2 e^{6x} + 12xe^{6x}$

$= \underbrace{72x^3 e^{6x} + 72x^2 e^{6x} + 12xe^{6x}}$ ②

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e) $f(x) = e^{6x}\sin(3x)$

$f'(x) = (e^{6x})[(\sin(3x)' \cdot (3x))] + (e^{6x})'(\sin(3x))$

$= (e^{6x})[\cos(3x)\cdot 3] + (6e^{6x})(\sin(3x))$

$= \underbrace{3e^{6x}\cos(3x) + 6e^{6x}\sin(3x)}$ ①

$f''(x) = (3e^{6x})[(\cos(3x)' \cdot (3x)')] + (3e^{6x})' + (\cos(3x))$
$+ (6e^{6x})[\sin(3x)' \cdot (3x)'] + (6e^{6x})'(\sin(3x))$

$= 3e^{6x}[-\sin(3x)\cdot 3] + 18e^{6x}(\cos(3x)) + (6e^{6x})\cdot$
$[\cos(3x)\cdot 3] + (36e^{6x})\sin(3x)$

$= 9e^{6x}\sin(3x) + 18e^{6x}\cos(3x) + 18e^{6x}\cos(3x) + 36e^{6x}\sin(3x)$

$= \underbrace{36e^{6x}\cos(3x) + 27e^{6x}\sin(3x)}$ ②

f) $f(x) = e^{6x}(\cos(3x) + \sin(3x))$

$f'(x) = (e^{6x})[(\cos(3x) + \sin(3x))'] + (e^{6x})'(\cos(3x) + \sin(3x))$

$= (e^{6x})[(-\text{sen}(3x)\cdot 3) + (\cos(3x\cdot 3))] + 6e^{6x}(\cos(3x) + \sin(3x))$

$= -3e^{6x}\,\text{sen}(3x) + 3e^{6x}\cos(3x) + 6e^{6x}\cos(3x) + 6e^{6x}\sin(3x)$

$= \underbrace{3e^{6x}\sin(3x) + 9e^{6x}\cos(3x)}$ ①

$f''(x) = 3e^{6x}[(\sin(3x)' + (3x)')] + (3e^{6x})'(\text{sen}(3x)) + 9e^{6x}$
$[(\cos(3x)' \cdot (3x)')] + (9e^{6x})'(\cos(3x))$

$= 3e^{6x}[\cos(3x)\cdot 3] + (18e^{6x})(\text{sen}(3x)) + 9e^{6x}[-\sin(3x)\cdot 3]$
$+ (54e^{6x})\cos(3x)$

$= 9e^{6x}(\cos 3x) + 18e^{6x}\,\text{sen}(3x) - 27e^{6x}(\text{sen}(3x) +$
$54e^{6x}\cos(3x)$

$= \underbrace{63e^{6x}\cos(3x) - 9e^{6x}\,\text{sen}(3x)}$ ②

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g) $f(x) = \frac{1}{x-1} = (x-1)^{-1}$

$f'(x) = [(x-1)^{-1}]' = -1(x-1)^{-2} = \frac{-1}{(x-1)^2}$ ①

$f''(x) = -[(x-1)^{-2}] = -[-2(x-1)^{-3}] = \frac{2}{(x-1)^3}$ ②

h) $f(x) = x^2 \cos(2x)$

$f'(x) = x^2 ((\cos(2x)' \cdot (2x)') + (x^2)' \cos(2x)$
$= x^2 [(-\text{sen}(2x) \cdot (2)) + (2x)(\cos(2x)$
$= -2x^2 \text{sen}(2x) + 2x \cos(2x)$ ①

$f''(x) = (-2x^2)[(\text{sen}(2x)' \cdot (2x)'] + (-2x^2)' \text{sen}(2x) + (2x)[((\cos(2x)' \cdot (2x)']$
$+ (2x)' \cos(2x)$
$= (-2x^2)[\cos(2x) \cdot (2)] + (-4x)\text{sen}(2x) + 2x[-\text{sen}(2x) \cdot (2)]$
$+ (2)(\cos 2x)$
$= -4x^2 \cos(2x) - 4x\,\text{sen}(2x) - 4x\,\text{sen}(2x) + 2\cos(2x)$
$= (-4x^2 + 2)\cos(2x) - 8x\,\text{sen}(2x)$ ②

i) $f(x) = (e^{x^2} + e^{-x^2})$

$f'(x) = 2xe^{x^2} - 2xe^{-x^2}$ ①

$f''(x) = (2x)(e^{x^2})' + (2x)'(e^{x^2}) + (-2x)(e^{-x^2})' + (-2x)'(e^{-x^2})$
$= (2x)(2xe^{x^2}) + (2)(e^{x^2}) + (-2x)(e^{-x^2}) + (-2)(e^{-x^2})$
$= 4x^2e^{x^2} + 2e^{x^2} + 4x^2e^{-x^2} - 2e^{-x^2}$
$= 4x^2e^{x^2} + 4x^2e^{-x^2} + 2e^{x^2} - 2e^{-x^2}$ ②

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Propiedades de la función exponencial

② Utiliza las propiedades de la función exponencial para realizar los productos.

a) $(e^{2x}-4)(e^{2x}+4)$

$= e^{2x+2x} + 4e^{2x} - 4e^{2x} - 16$

$= e^{4x} - 16$

b) $(e^{2x}-4)(e^{2x}-4)$

$= e^{2x+2x} - 4e^{2x} - 4e^{2x} + 16$

$= e^{4x} - 8e^{2x} + 16$

c) $(e^{3x}+6)(e^{3x}-4)$

$= e^{3x+3x} - 4e^{3x} + 6e^{3x} - 24$

$= e^{6x} + 2e^{3x} - 24$

d) $(e^{2x}-e^{3x})(e^{2x}+e^{3x})$

$= e^{2x+2x} + e^{2x+3x} - e^{3x+2x} - e^{3x+3x}$

$= e^{4x} + e^{5x} - e^{5x} - e^{6x}$

$= e^{4x} - e^{6x}$

e) $(e^{2x}-e^{3x})(e^{2x}-e^{3x})$

$= e^{2x+2x} - e^{3x+2x} - e^{3x+2x} + e^{3x+3x}$

$= e^{4x} - e^{5x} - e^{5x} + e^{6x}$

$= e^{4x} - 2e^{5x} + e^{6x}$

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F) $(e^{3x}+6)(e^{-3x}-4)$

$= e^{3x-3x} - 4e^{3x} + 6e^{-3x} - 24$

$= e^{0} - 4e^{3x} + 6e^{-3x} - 24$

$= \underbrace{1 - 4e^{3x} + 6e^{-3x} - 24}$

Derivadas de funciones con parámetros

③ Encuentre la primera derivada de las siguientes funciones.

a) $f(x) = Ae^{ax}$

$f'(x) = (A)(e^{ax})' + (A)'(e^{ax})$

$= A(ae^{ax}) + (0)(e^{ax})$

$= \underbrace{Aae^{ax}}$

b) $f(x) = e^{ax}\, \text{Sen}\, bx$

$f'(x) = (e^{ax})[(\text{sen}\, bx)' \cdot (bx)'] + (e^{ax})'(\text{sen}\, bx)$

$= (e^{ax})[(\cos bx) \cdot (b)] + (ae^{ax})(\text{Sen}\, bx)$

$= \underbrace{be^{ax} \cos bx + ae^{ax}\, \text{Sen}(bx)}$

c) $f(x) = e^{ax} \cos bx$

$f'(x) = (e^{ax})[(\cos bx)' \cdot (bx)'] + (e^{ax})'(\cos(bx))$

$= (e^{ax})[-\sin(bx) \cdot (b)'] + (ae^{ax})(\cos(bx))$

$= \underbrace{-be^{ax} \sin(bx) + ae^{ax} \cos(bx)}$

d) $f(x) = x^{2}e^{-ax}$

$f'(x) = (x^{2})(e^{-ax})^{2} + (x^{2})'(e^{-ax})$

$= (x^{2}(-ae^{-ax}) + (2x)(e^{-ax})$

$= \underbrace{-ax^{2}e^{-ax} + 2xe^{-ax}}$

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e) $f(x) = Axe^{3x}$

$f'(x) = (Ax)(e^{3x})' + (Ax)'(e^{3x})$

$= (Ax)(3e^{3x}) + (A)(e^{3x})$

$= 3Axe^{3x} + Ae^{3x}$

f) $f(x) = A\,\text{sen}\,bx + B\cos bx$

$f'(x) = (A)[(\text{sen}\,bx)' \cdot (bx)'] + (A)'(\text{sen}\,(bx)) + (B)[(\cos(bx))' \cdot (bx)'] + (B)'(\cos(bx))$

$= (A)[\cos bx \cdot b] + (\emptyset)\,\text{Sen}\,(bx) + (B)[-\text{Sen}\,(bx) \cdot b] + (\emptyset)\cos bx$

$= Ab\cos bx + \emptyset - Bb\,\text{Sen}(bx) + \emptyset$

$= Ab\cos(bx) - Bb\,\text{sen}(bx)$

(4) Encuentre la segunda derivada de las siguientes funciones

a) $f(x) = Ax^3e^{ax}$

$f'(x) = (Ax^3)(e^{ax})' + (Ax^3)'(e^{ax})$

$= (Ax^3)(ae^{ax}) + (3Ax^2)(e^{ax})$

$= Aax^3e^{ax} + 3Ax^2e^{ax}$ (1)

$f''(x) = (Aax^3)(e^{ax})' + (Aax^3)'(e^{ax}) + (3Ax^2)(e^{ax})' + (3Ax^2)'(e^{ax})$

$= (Aax^3)(ae^{ax}) + (3Aax^2)(e^{ax}) + (3Ax^2)(ae^{ax}) + (6Ax)(e^{ax})$

$= Aa^2x^3e^{ax} + 3Aax^2e^{ax} + 3Aax^2e^{ax} + 6Axe^{ax}$

$= Aa^2x^3e^{ax} + 6Aa^2e^{ax} + 6Axe^{ax}$ (2)

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b) $f(x) = e^{ax} + e^{-ax}$

$f'(x) = \underbrace{ae^{ax} - ae^{-ax}}$ ①

$f''(x) = \underbrace{a^2e^{ax} + a^2e^{-ax}}$ ②

c) $f(x) = 2\,\text{sen}\,bx + 3\cos bx$

$f'(x) = 2[(\text{sen}\,bx)' \cdot (bx)'] + 3[(\cos(bx))' \cdot (bx)']$

$= 2[(\cos bx \cdot b)] + 3[-\text{sen}\,bx \cdot b]$

$= \underbrace{2b\cos(bx) - 3b\,\text{sen}(bx)}$ ①

$f''(x) = (2b)[(\cos(bx))' \cdot (bx)')] + (2b)'\cos(bx) + (-3b)[(\text{sen}(bx))' \cdot (bx)')]$
$+ (-3b)'(\text{sen}(bx))$

$= (2b)(-\text{sen}(bx \cdot b)) + (\emptyset)(\cos(bx)) + (-3b)[\cos(bx) \cdot b]$
$+ (\emptyset)(\text{sen}(bx))$

$= -2b^2\,\text{sen}(bx) + \emptyset - 3b^2\cos(bx) + \emptyset$

$= \underbrace{-2b^2\,\text{Sen}(bx) - 3b^2\cos(bx)}$ ②

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