Datos de Identificación de tareas

UNIVERSIDAD AUTONOMA DE AGUASCALIENTES

Centro de Ciencias Básicas

Materia: **Ecuaciones Diferenciales**

Tarea II

ED'S SEPARABLES

Ingeniería en Computación Inteligente Semestre 5° A

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TAREA II

ECUACIONES DIFERENCIALES SEPARABLES

1. Encuentre la solución general de las siguientes ecuaciones diferenciales

a) $\frac{dy}{dx} + y^{-2} = xy^{-2}$

$\frac{dy}{dx} = xy^{-2} - y^{-2} \rightarrow \overbrace{\underbrace{y^{-2}}_{h(y)}\underbrace{(x-1)}_{g(x)}}^{\text{ED es separable}} = \frac{dy}{y^{-2}} = x-1dx \rightarrow \int y^2 dy = \int x-1dx$

$\rightarrow \frac{y^3}{3} + C_1 = \frac{x^2}{2} - x + C_2 \rightarrow \frac{y^3}{3} = \frac{x^2}{2} - x + \overbrace{C_2 - C_1}^{C} \rightarrow y^3 = \left(\frac{x^2}{2} - x + C\right)(3)$

$\rightarrow y^3 = 3\frac{x^2}{2} - 3x + C \rightarrow y = \sqrt[(+)3]{\frac{3}{2}x^2 - 3x + C}$ Sol. General

b) $x^2\frac{dy}{dx} + y = 0$

$x^2\frac{dy}{dx} = -y \rightarrow \frac{dy}{dx} = \frac{\overset{h(y)}{-y}}{\underset{g(x)}{x^2}}$ ED es separable $\rightarrow dy = \frac{-y}{x^2}dx \rightarrow \frac{dy}{-y} = \frac{1}{x^2}dx$

$\rightarrow \int \frac{dy}{y} = \int -\frac{1}{x^2}dx \rightarrow \ln|y| = -\int x^{-2}dx \rightarrow \underbrace{\ln|y| = -x^{-1} + C}_{\text{Solución general Implícita}}$

$\rightarrow e^{\ln|y|} = e^{\frac{1}{x} + C} \rightarrow y = e^{\frac{1}{x}} \cdot e^{C} \rightarrow C \rightarrow y \overset{(x)}{=} Ce^{\frac{1}{x}}$ Sol. General

c) $\frac{dy}{dx} = 3x^2 + 3x^2y^2$

$\frac{dy}{dx} = 3x^2 + 3x^2y^2 \rightarrow \frac{dy}{dx} = \underbrace{3x^2}_{g(x)}\underbrace{(1+y^2)}_{h(y)} \rightarrow dy = 3x^2(1+y^2)dx$

$\frac{dy}{(1+y^2)} = 3x^2dx \rightarrow \int \frac{dy}{1+y^2} = 3\int x^2dx \rightarrow \tan^{-1}(y) = 3\frac{x^3}{3} + C$

$\rightarrow \underbrace{\tan^{-1}(y) = x^3 + C}_{\text{Sol. Implícita}} \rightarrow \cancel{\tan}(\cancel{\tan^{-1}}(y)) = \tan(x^3 + C)$

$\rightarrow y(x) = \tan(x^3 + C)$ Sol. General

d) $\frac{dy}{dx} = xy + x - 3y - 3$

$\rightarrow \frac{dy}{dx} = xy + x - 3y - 3 \longrightarrow \frac{dy}{dx} = x(y-1) - 3(y+1) \rightarrow \frac{dy}{dx} = (y+1)(x-3)$

$\rightarrow \frac{dy}{y+1} = x - 3dx \rightarrow \int \frac{dy}{y+1} = \int x - 3\,dx = \underbrace{\ln|1+y| = \frac{x^2}{2} - 3x + c}_{\text{Sol. implicita}}$

$\rightarrow e^{\ln|1+y|} = e^{\frac{x^2}{2} - 3x + c} \longrightarrow 1 + y = e^{\frac{x^2}{2}} \cdot e^{-3x} \cdot e^{c} \longrightarrow y = e^{\frac{x^2}{2}} \cdot e^{-3x} \cdot \underbrace{e^{c}}_{c} - 1$

$\rightarrow \underbrace{y(x) = ce^{\frac{x^2}{2} - 3x} - 1}_{\text{Sol. general}}$

2. Resuelva los siguientes problemas de valor inicial:

a) $\frac{dy}{dx} = y - 1$ sujeto a $y(0) = 0$

$\frac{dy}{dx} = y - 1 \rightarrow \frac{dy}{y-1} = dx \rightarrow \int \frac{dy}{y-1} = \int dx = \overbrace{\ln|y-1| = x + c}^{\text{Sol. Implicita}}$

$\rightarrow e^{\ln|y-1|} = e^{x+c} \longrightarrow y - 1 = e^{x} \cdot e^{c} \longrightarrow \underbrace{y(x) = ce^{x} + 1}_{\text{Sol. general}}$

• Se busca solución que cumpla $y(0) = 0$

$ce^{0} + 1 = 0 \rightarrow c = -1$ $\longrightarrow \underline{y(0) = -e^{0} + 1 = -1 + 1 = 0}$ comprobación

$\rightarrow \underbrace{y(x) = -e^{x} + 1}_{\text{Solución del PVI}}$

b) $\frac{dy}{dx} = \frac{y-1}{x+3}$ sujeto a $y(0) = 0$

Sol. Implicita

$\frac{dy}{dx} = \frac{y-1}{x+3} \longrightarrow \frac{dy}{y-1} = \frac{dx}{x+3} \longrightarrow \int \frac{dy}{y-1} = \int \frac{dx}{x+3} \rightarrow \ln|y-1| = \ln|x+3| + c$

$\rightarrow e^{\ln|y-1|} = e^{\ln|x+3| + c} \rightarrow \ln|y-1| = e^{c} \cdot (x+3) = y - 1 = \pm e^{c}(x+3)$

$\rightarrow \underbrace{y = c(x+3) + 1}_{\text{Sol. general}}$

$\rightarrow y - 1 = c(x+3) \longrightarrow 0 - 1 = c(0+3)$

Sol. general

PVI $\rightarrow$
$x = 0$
$y = 0$

$\rightarrow -1 = c(3)$

$\rightarrow c = -\frac{1}{3}$

• Se busca solución que cumpla $y(0)=0$

$Y(x) = \frac{-1}{3}(x+3)+1$ Solución del PVI

$y(0) = \frac{-1}{3}(0+3)+1 = -1+1=0$ comprobación

c) $(1+x)dy - ydx = 0$, $y(1)=2$

$\rightarrow \frac{(1+x)dy - ydx}{dx} = \frac{0}{dx} \rightarrow (1+x)\frac{dy}{dx} - y\frac{dx}{dx} = 0$

$\rightarrow (1+x)\frac{dy}{dx} - y = 0 \rightarrow (1+x)\frac{dy}{dx} = y \rightarrow \frac{dy}{dx} = \frac{y}{1+x}$

$\rightarrow \frac{dy}{y} = \frac{dx}{1+x} \rightarrow \int \frac{dy}{y} = \int \frac{dx}{1+x} \rightarrow \underbrace{\ln|y| = \ln|1+x| + C}_{\text{Sol. Implícita}}$

$\rightarrow e^{\ln|y|} = e^{\ln|1+x|+C} \rightarrow |y| = 1+x \cdot e^{C} \rightarrow y = \pm(1+x)e^{C}$

$y(x) = e^{c} + e^{c}x = y(x) = C + Cx \rightarrow \underbrace{y(x) = C(1+x)}_{\text{Sol general}}$

• Se busca solución que cumpla $y(1)=2$

$C(1+1) = 2$ $\quad e = 1$

$C(2) = 2$

$C = \frac{2}{2} = 1$ $\rightarrow$ $y(x) = 1(1+x)$ Sol. del PVI

$y(1) = 1(1+1) = 1(2) = 2$ Comprobación

a)

$\frac{dy}{dx} = 4y - y^2$, sujeto a $y(0) = 0$ $\quad 4 - y^2 + 4y - 4$

$\frac{dy}{dx} = 4y - y^2 \rightarrow \frac{dy}{4y - y^2} = dx \rightarrow \int \frac{dy}{4y - y^2} = \int dx = \int \frac{dy}{4 - (y-2)^2} = x + C$

$\int \frac{du}{a^2 - u^2} = x + C = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| = x + C \rightarrow \frac{1}{2(2)} \ln \left| \frac{2 + (y-2)}{2 - y + 2} \right| = x + C$

$u = y - 2$
$du = dy$
$a = 2$

$\rightarrow \frac{1}{4} \ln \left| \frac{y}{4-y} \right| = x + C \rightarrow \ln \left| \frac{y}{4-y} \right| = 4(x + c) \rightarrow \ln \left| \frac{y}{4-y} \right| = 4x + \overset{c}{4c}$

$\rightarrow e^{\ln \left| \frac{y}{4-y} \right|} = e^{4x + C} \rightarrow \frac{y}{4-y} = ce^{4x} \rightarrow 4 - y \left(\frac{y}{4-y} \right) = ce^{4x} (4 - y)$

$\rightarrow y = 4ce^{4x} - ce^{4x} y \rightarrow y + ce^{4x} y = 4ce^{4x} \rightarrow y(1 + ce^{4x}) = 4ce^{4x}$

$\rightarrow y(x) = \frac{4ce^{4x}}{1 + ce^{4x}}$

Sol. general

- Se busca solución que cumpla $y(0) = 0$

$\frac{4ce^{4(0)}}{1 + ce^{4(0)}} = 0 \rightarrow \frac{4c}{1+c} = 0 \rightarrow 4c = 0(1 + c) \rightarrow 4c = 0 \rightarrow c = \frac{0}{4} = 0$

$\therefore y(x) = \frac{4(0)e^{4x}}{1 + 0e^{4x}} = \frac{0}{1+0} = 0 \rightarrow y(x) = 0$

Sol. del PVI

③ Encuentra la solución general de la ecuación diferencial $\frac{dy}{dx}+xy=2x$

a) $\frac{dy}{dx}+xy=2x \longrightarrow \frac{dy}{dx}=2x-xy=\frac{dy}{dx}=x(2-y) \rightarrow dy=x(2-y)dx$

$\rightarrow \frac{dy}{2-y}=xdx \longrightarrow \int\frac{dy}{2-y}=\int xdx = \ln|2-y|=\frac{x^2}{2}+c$

$\rightarrow e^{\ln|2-y|}=e^{\frac{x^2}{2}+c} \longrightarrow |2-y|=e^{\frac{1}{2}x^2}\cdot e^c \longrightarrow 2-y=\pm ce^{\frac{1}{2}x^2}$

$\rightarrow y(x)=-(ce^{\frac{1}{2}x^2}-2)) \longrightarrow \underbrace{y(x)=+ce^{\frac{1}{2}x^2}+2}$ Sol. general

b) $y(0)=4$

• Se busca solución que cumpla $y(0)=4$

$ce^{\frac{1}{2}(0)}+2=4$

$ce^0+2=4$

$c(1)+2=4$

$c+2=4$

$\rightarrow c=4-2$

$c=2$

$\longrightarrow y(x)=2e^{\frac{1}{2}x^2}+2$

Sol del PVI

④ Encuentre la solución general de las ED's separables con parametros

a) $\frac{dy}{dx}+xy=Ax$, donde A es una constante positiva

$\frac{dy}{dx}+xy=Ax \longrightarrow \frac{dy}{dx}=Ax-xy \longrightarrow \frac{dy}{dx}=x(A-y) \rightarrow dy=x(A-y)dx$

$\rightarrow \frac{dy}{(A-y)}=xdx \longrightarrow \int\frac{dy}{A-y}=\int xdx \rightarrow \ln|A-y|=\frac{x^2}{2}dx$

$\rightarrow e^{\ln|A-y|}=e^{\frac{1}{2}x^2+c} \rightarrow |A-y|=ce^{\frac{1}{2}x^2} \longrightarrow A-y=\pm ce^{\frac{1}{2}x^2}$

$\rightarrow y(x)=A+ce^{\frac{1}{2}x^2}$

b) $m\frac{dv}{dt} = -kv - mg$, donde m, k y g son constantes positivas

$$m\frac{dv}{dt} = -kv - mg \rightarrow \frac{dv}{dt} = \frac{-kv-mg}{m} \rightarrow \frac{dv}{dt} = \frac{-kv}{m} - \frac{\cancel{m}g}{\cancel{m}}$$

$$\rightarrow \frac{dv}{dt} = \frac{-k}{m}v - g \rightarrow \frac{dv}{dt} = \underbrace{-1}_{g(t)}\underbrace{\left(g + \frac{k}{m}v\right)}_{h(v)} \rightarrow \frac{dv}{\left(g+\frac{k}{m}v\right)} = -1\,dt$$

$$\int \frac{dv}{\left(g+\frac{k}{m}v\right)} = \int -1\,dt \rightarrow \frac{m}{k}\int \frac{dv}{\left(g+\frac{k}{m}v\right)} = \int dt$$

$$u = g + \frac{k}{m}v \quad dv = \frac{m\,du}{k}$$

$$du = \frac{k}{m}dv$$

$$\rightarrow \underbrace{\frac{m}{k}\ln\left|g + \frac{k}{m}v\right| = -t + c}_{\text{Sol. gral implícita}} \rightarrow e^{\ln\left|g+\frac{k}{m}v\right|} = \frac{-t+c/1}{\frac{m}{k}} \rightarrow e^{\ln\left|g+\frac{k}{m}v\right|} = e^{\frac{-k}{t}+\frac{kc}{m}}$$

$$\rightarrow g + \frac{k}{m}v = \pm e^{\frac{-kt}{m}} \cdot e^{\frac{kc}{m}} \rightarrow c \qquad \rightarrow g + \frac{k}{m}v = ce^{\frac{-kt}{m}} \rightarrow \frac{k}{m} = ce^{-\frac{kt}{m}} - g \qquad \frac{m}{k}c = C$$

$$\rightarrow v = \frac{ce^{\frac{-kt}{m}} - g}{\frac{k}{m}} \rightarrow v(t) = \frac{m(ce^{\frac{-kt}{m}} - g)}{k} = v(t) = \frac{m}{k}Ce^{\frac{-kt}{m}} - \frac{mg}{k} \rightarrow$$

$$\rightarrow v(t) = Ce^{\frac{-kt}{m}} - \frac{m}{k}g \quad \text{Sol. general}$$

ED separable

c) $a\frac{dy}{dx} + by = m$

$$\rightarrow a\frac{dy}{dx} = m - bx \rightarrow \frac{dy}{dx} = \frac{m-by}{a} \rightarrow \frac{dy}{dx} = \underbrace{\frac{1}{a}}_{g(x)}\underbrace{(m-by)}_{h(y)} \rightarrow \frac{dy}{m-by} = \frac{1}{a}dx$$

$$\rightarrow \frac{dy}{by-m} = -\frac{dx}{a} \rightarrow \int \frac{dy}{by-m} = -\int \frac{dx}{a} \quad \left| \begin{array}{l} u = by-m \\ du = b\,dy \\ dy = \frac{du}{b} \end{array} \right. \rightarrow \frac{1}{b}\int \frac{dy}{by-m} = \frac{-1}{a}\int dx$$

$$\rightarrow \underbrace{\frac{1}{b}\ln|by-m| = \frac{-1}{a}x + c}_{\text{Sol. Implícita}} \rightarrow \ln|by-m| = \frac{\frac{-1}{a}x + c}{\frac{1}{b}} \rightarrow \ln|by-m| = \frac{b\left(\frac{-1}{a}x + c\right)}{1}$$

$$\rightarrow \ln|by-m| = \frac{-b}{a}x + bc \rightarrow e^{\ln|by-m|} = e^{-\frac{b}{a}x + bc} \rightarrow |by-m| = e^{-\frac{b}{a}x} \cdot e^{c} \rightarrow c$$

$$\rightarrow by - m = \pm ce^{-\frac{b}{a}x} \rightarrow by = ce^{-\frac{b}{a}x} + m \rightarrow y(x) = \frac{ce^{-\frac{b}{a}x} + m}{b}$$

$$\rightarrow y(x) = \underbrace{\frac{c}{b}}_{c}e^{-\frac{b}{a}x} + \frac{m}{b} \rightarrow \underbrace{y(x) = ce^{-\frac{b}{a}x} + \frac{m}{b}}_{\text{Sol. general}}$$