

## **Datos de Identificación de tareas**



**Centro de Ciencias Básicas**

**Materia: Ecuaciones Diferenciales**

**Tarea II**

**ED'S LINEALES**

**Ingeniería en Computación Inteligente Semestre 5° A**

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# TAREA III

## ED'S LINEALES 1 Orden

Instrucciones: Todas las ecuaciones diferenciales de esta tarea deben ser resueltas por el método de ED lineales. Aunque sean separables,

① Encuentra la solución general de cada ecuación

$$a) \frac{dy}{dx} = x e^{-4x} - 4y$$

① Encontramos forma  $\frac{dy}{dx} + P(x)y = h(x)$

$$\rightarrow \frac{dy}{dx} + 4y = x e^{-4x}$$

② Encontrar FI

$$M(x) = e^{\int 4 dx} = e^{4x}$$

③ Multiplicando ED por FI

$$e^{4x} \left( \frac{dy}{dx} + 4y \right) = x e^{-4x} (e^{4x})$$

④ Integramos

$$e^{4x} \frac{dy}{dx} + 4e^{4x}y = x \quad \rightarrow \int \frac{d(e^{4x}y)}{dx} - \int x dx$$

$$\frac{d(e^{4x}y)}{dx} = x$$

b)  $x \frac{dy}{dx} + y = 0 \rightarrow \frac{d(xy)}{dx} = \int 0 dx$

derivada  
de un producto

① Integramos

$$\int \frac{xy}{dx} = \int 0 dx$$

$$xy = x + C$$

② Despejamos y sacando Sol. general

$$xy = x + C \rightarrow y = \frac{x + C}{x} = Cx^{-1}$$

Sol. general

c)  $x \frac{dy}{dx} - y = 2x^2 - x$

① Dividimos entre x para encontrar forma  $\frac{dy}{dx} + P(x)y = h(x)$

$$\frac{x \frac{dy}{dx} - y}{x} = \frac{2x^2 - x}{x}$$

$$\frac{dy}{dx} - \frac{1}{x}y = \frac{2x^2}{x} - \frac{x}{x} = 2x - 1$$

$$\frac{dy}{dx} - \frac{1}{x}y = 2x - 1 \quad \left( \frac{dy}{dx} + P(x)y = h(x) \right)$$

② Encontramos F1

$$M(x) = e^{\int p(x) dx} = e^{\int -\frac{1}{x} dx} = e^{-\ln|x|} = e^{\ln x^{-1}} = x^{-1} = \frac{1}{x} \rightarrow x \in (0, \infty)$$

③ Multiplicamos F1 por ED

$$\frac{1}{x} \left( \frac{dy}{dx} - \frac{1}{x} y \right) = 2x - 1 \left( \frac{1}{x} \right)$$

$$\rightarrow \underbrace{\frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2} y}_{\frac{d(\frac{1}{x} \cdot y)}{dx}} = 2x - \frac{1}{x}$$

$$\rightarrow \frac{d(\frac{1}{x} \cdot y)}{dx} = 2x - \frac{1}{x}$$

④ Integrando

$$\int \frac{d(\frac{1}{x} \cdot y)}{dx} dx = \int 2x - \frac{1}{x} dx$$

$$\rightarrow \frac{1}{x} y = 2 \int dx - \int \frac{1}{x} dx$$

$$\rightarrow \frac{1}{x} y = 2x - \ln|x| + C$$

⑤ Encuentramos solución despejando y(x)

$$y(x) = \frac{x^2 + 1 + C}{\frac{1}{x}} = \frac{2x}{\frac{1}{x}} - \frac{\ln|x|}{\frac{1}{x}} + \frac{C}{\frac{1}{x}} \rightarrow y(x) = 2x^2 - x \ln|x| + Cx$$

Sol. general

② Encuentra el PVI

$$a) \underbrace{\frac{dy}{dx} + xy = 2x}_{\frac{dy}{dx} + p(x)y = h(x)} \quad y(0) = 4$$

$$\frac{dy}{dx} + p(x)y = h(x)$$

① Encontramos F1

$$M(x) = e^{\int p(x) dx} = e^{\int x dx} = e^{\frac{x^2}{2}}$$

② Multiplicamos F1 por ED

$$e^{\frac{x^2}{2}} \left( \frac{dy}{dx} + xy \right) = 2x \left( e^{\frac{x^2}{2}} \right)$$

$$e^{\frac{x^2}{2}} \frac{dy}{dx} + xy e^{\frac{x^2}{2}} = 2x e^{\frac{x^2}{2}}$$

③ Integrando

$$\int \frac{d(e^{\frac{x^2}{2}} y)}{dx} dx = \int 2x e^{\frac{x^2}{2}} dx \quad \begin{cases} v = \frac{x^2}{2} \\ dv = x dx \\ dx = \frac{dv}{x} \end{cases}$$

$$e^{\frac{x^2}{2}} y = 2 \int x e^{\frac{x^2}{2}} dx$$

$$e^{\frac{x^2}{2}} y = 2 \int x e^v \frac{dv}{x} = 2 \int x e^v \frac{1}{x} dv$$

$$= 2 e^v = 2 e^{\frac{x^2}{2}} + C$$

④ Se busca sol que satisfaga  
y(0) = 4

$$\frac{2+C}{e^{\frac{x^2}{2}}} = 4$$

$$\rightarrow \frac{C}{e^{\frac{x^2}{2}}} = 4 - 2$$

$$\rightarrow C = 2e^{\frac{x^2}{2}}$$

④ Encontramos sol despejando y(x)

$$y(x) = \frac{2e^{\frac{x^2}{2}} + C}{e^{\frac{x^2}{2}}} = 2 + \frac{C}{e^{\frac{x^2}{2}}}$$

sol general

$$\rightarrow 2 + \frac{2e^{\frac{x^2}{2}}}{e^{\frac{x^2}{2}}} = y(x) \quad \text{Sol del PVI}$$

$$\rightarrow 2 + \frac{2e^{\frac{x^2}{2}}}{e^{\frac{x^2}{2}}} = 2 + 2 = 4 \quad \text{Comprobación}$$

③ Encuentra las soluciones de los PVI

$$a) x \frac{dy}{dx} + 3y + 2x = 3x^2, \quad y(1) = 1$$

① Encontramos forma  $\frac{dy}{dx} + p(x)y = h(x)$

$$x \frac{dy}{dx} + 3y + 2x = 3x^2 \quad \rightarrow \frac{x \frac{dy}{dx} + 3y}{x} = \frac{3x^2}{x} - \frac{2x}{x}$$

$$\rightarrow x \frac{dy}{dx} + 3y = 3x^2 - 2x$$

$$\rightarrow \underbrace{\frac{dy}{dx} + \frac{3}{x}y}_{\frac{dy}{dx} = p(x) \quad y} = \underbrace{3x^2 - 2x}_{n(x)}$$

④ Integrando

$$\int \frac{d(x^3 \cdot y)}{dx} = \int 3x^4 - 2x^3 dx$$

$$\rightarrow x^3 \cdot y = 3 \int x^4 dx - 2 \int x^3 dx$$

② Encontramos F1

$$M(x) = e^{\int \frac{3}{x} dx} = e^{3 \int \frac{1}{x} dx} = e^{3 \ln|x|} = e^{\ln|x^3|} = x^3 \rightarrow x^3 \cdot y = 3 \frac{x^5}{5} + C_1 - 2 \frac{x^4}{4} + C_2 \quad C_1 + C_2 = C$$

③ Multiplicando F1 por ED

$$(x^3)(\frac{dy}{dx} + \frac{3}{x}y) = (3x^2 - 2x^3)(x^3)$$

$$\rightarrow x^3 \cdot y = \frac{3}{5}x^5 - \frac{1}{2}x^4 + C$$

$$\rightarrow y = \frac{\frac{3x^5}{5}}{x^3} - \frac{\frac{1x^4}{2}}{x^3} + \frac{C}{x^3}$$

$$\rightarrow y = \frac{3x^5}{5x^3} - \frac{x^4}{2x^3} + \frac{C}{x^3}$$

$$\rightarrow y = \frac{3}{5}x^2 - \frac{1}{2}x + \frac{C}{x^3}$$

$$\underbrace{d(x^3 \cdot y)}_{dx}$$

Sol. general

⑤ Se busca solución que satisface  $y(1)=1$

$$1 = \frac{3}{5}x^2 - \frac{1}{2}x + \frac{C}{x^3}$$

$$\rightarrow 1 = \frac{3(1)^2}{5} - \frac{1}{2}(1) + \frac{C}{1^3}$$

$$\rightarrow 1 = \frac{3}{5} \cdot \frac{1}{2} + \frac{C}{1^3}$$

$$\rightarrow 1 = \frac{3}{5} - \frac{1}{2} + \frac{C}{1}$$

$$\rightarrow 1 = \frac{6}{10} - \frac{5}{10} + C$$

$$\rightarrow 1 = \frac{1}{10} + C$$

$$\rightarrow 1 - \frac{1}{10} = C$$

$$\frac{10}{10} - \frac{1}{10} = C$$

$$C = \frac{9}{10}$$

⑥ Sustituir  $C$  en sol general

$$y(x) = \frac{3}{5}x^2 - \frac{1}{2}x + \frac{\frac{9}{10}}{x^3} = \underbrace{\frac{3}{5}x^2 - \frac{1}{2}x}_{\text{Sol del PVI}} + \frac{9}{10x^3}$$

$$b) x \frac{dy}{dx} - y = x^3 \cos(x), \quad y(\pi) = \pi$$

① Encontrar forma  $\frac{dy}{dx} + p(x)y = h(x)$

$$\rightarrow x \frac{dy}{dx} - y = x^3 \cos(x)$$

$$\rightarrow \frac{x \frac{dy}{dx} - y}{x} = \frac{x^3 \cos(x)}{x}$$

$$\rightarrow \frac{dy}{dx} - \frac{1}{x}y = x^2 \cos(x)$$

② Encontramos FI

$$y(x) = e^{\int -\frac{1}{x} dx} = e^{-\int \frac{1}{x} dx} = e^{-\ln|x|} = e^{\ln|x^{-1}|} = x^{-1} = \frac{1}{x}$$

③ Multiplicamos FI por ED

$$\left(\frac{1}{x}\right)\left(\frac{dy}{dx} - \frac{1}{x}y\right) = (x^2 \cos(x))\left(\frac{1}{x}\right) \rightarrow \underbrace{\frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2}y}_{\frac{d(\frac{1}{x} \cdot y)}{dx}} = x \cos(x)$$

④ Integraremos

$$\int \frac{d(\frac{1}{x} \cdot y)}{dx} = \int x \cos(x) dx$$

$$\frac{1}{x} \cdot y = \int x \cos(x) dx$$

$$\rightarrow \frac{1}{x} \cdot y = x \sin(x) + \cos(x) + C$$

$$\rightarrow y = \frac{x \sin(x)}{\frac{1}{x}} + \frac{\cos(x)}{\frac{1}{x}} + \frac{C}{\frac{1}{x}}$$

$$\rightarrow y(x) = x^2 \sin(x) + x \cos(x) + Cx$$

sol. general

$$\int x \cos(x) dx \longrightarrow uv - \int v du$$

$$u = x \quad dv = \cos(x) \\ du = dx \quad v = \sin(x)$$

$$\begin{aligned} \int x \cos(x) dx &= x \sin(x) - \int \sin(x) dx \\ &= x \sin(x) - (-\cos(x)) \\ &= x \sin(x) + \cos(x) + C \end{aligned}$$

⑤ Se busca solución que satisface  $y(\pi) = \pi$

$$\pi = \pi^2 \sin(\pi) + \pi \cos(\pi) + C\pi$$

$$\pi = -\pi + C\pi$$

$$\pi + \pi = C\pi$$

$$2\pi = C\pi$$

$$C = \frac{2\pi}{\pi} \rightarrow C = 2$$

⑥ Sustituir C en sol. general

$$y(x) = x^2 \sin(x) + x \cos(x) + 2x$$

sol. del PVI

④ Resolver la ED lineal con parámetros positivos a, b

$$a \frac{dy}{dx} + by = 0$$

① Encontramos forma  $\frac{dy}{dx} + p(x)y = h(x)$

$$a \frac{dy}{dx} + by = 0$$

$$\rightarrow \frac{a \frac{dy}{dx} + by}{a} = 0$$

$$\rightarrow \frac{dy}{dx} + \frac{b}{a}y = 0$$

$\underbrace{\frac{dy}{dx}}_{p(x) y} \quad \underbrace{0}_{h(x)}$

② encontramos F1

$$M(x) = e^{\int \frac{b}{a} dx} = e^{\frac{b}{a} x} = e^{\frac{b}{a} x} =$$

③ Multiplicar ED por F1

$$e^{\frac{b}{a}x} \left( \frac{dy}{dx} + \frac{b}{a}y \right) = 0(e^{\frac{b}{a}x})$$

$$\rightarrow e^{\frac{b}{a}x} \frac{dy}{dx} + e^{\frac{b}{a}x} \frac{b}{a}y = 0$$

$\underbrace{d(e^{\frac{b}{a}x} \cdot y)}_{dx} = 0$

④ Integrando

$$\int d(e^{\frac{b}{a}x} \cdot y) = \int 0 dx$$

$$e^{\frac{b}{a}x} \cdot y = 0 \int dx \rightarrow e^{\frac{b}{a}x} \cdot y = 0(x) + C_1$$

$$y(x) = \frac{C}{e^{\frac{b}{a}x}}$$

sol. general

⑤ Resolver la ED lineal con parámetros positiva k, Tm

$$\frac{dT}{dt} = -k(T - T_m)$$

② Encuentra F1

① Encontrar forma  $\frac{dy}{dx} + p(x)y = h(x)$

$$M(t) = e^{\int -k dt} = e^{-kt} = \bar{e}^{-kt}$$

$$\frac{dT}{dt} = -k(T - T_m)$$

③ Multiplicar F1 por ED

$$\bar{e}^{-kt} \left( \frac{dT}{dt} - kT \right) = \bar{e}^{-kt} (kT_m)$$

$$\bar{e}^{-kt} \frac{dT}{dt} - \bar{e}^{-kt} kT = kT_m \bar{e}^{-kt}$$

$\underbrace{\frac{d(\bar{e}^{-kt} \cdot T)}{dt}}_{=}$

$\underbrace{\frac{dy}{dx}}_{p(x) y} \quad \underbrace{h(x)}_{=}$

#### ④ Integración

$$\int \frac{dt(e^{-kt} \cdot T)}{dt} = \int kTm e^{-kt}$$

$$\int e^u = e^0$$

$$u = -kt$$

$$du = -k dt$$

$$dt = \frac{du}{-k}$$

$$\rightarrow e^{-kt} \cdot T = kTm \int e^{-kt} dt$$

$$\rightarrow e^{-kt} \cdot T = kTm \frac{1}{k} e^{-kt} + C$$

$$\rightarrow e^{-kt} \cdot T = \frac{kTm e^{-kt}}{k} = Tm e^{-kt} + C$$

$$\rightarrow e^{-kt} \cdot T = Tm e^{-kt} + C$$

$$\rightarrow T(t) = \frac{Tm e^{-kt} + C}{e^{-kt}} = \frac{C}{e^{-kt}} + Tm$$

$$\rightarrow T(t) = Tm + \frac{C}{e^{-kt}}$$

Sol. general de la ED

#### ⑤ Resolver la ED lineal con parámetros positivos $a, b$

$$ax \frac{dy}{dx} + by = 1$$

② Encontrar FI

$$P(x) = e^{\int \frac{b}{a} \frac{dx}{dx}} = e^{\frac{b}{a} \int \frac{1}{x} dx} = e^{b \ln|x|} = x^{\frac{b}{a}}$$

① Encontramos forma  $\frac{dy}{dx} + p(x)y = h(x)$

$$ax \frac{dy}{dx} + by = 1$$

③ Multiplicar FI por ED

$$x^{\frac{b}{a}} \left( \frac{dy}{dx} + \frac{b}{a} y \right) = \frac{1}{a x} (x^{\frac{b}{a}})$$

$$x^{\frac{b}{a}} \frac{dy}{dx} + \frac{b}{a} x^{\frac{b}{a}} y = \frac{x^{\frac{b-a}{a}}}{a}$$

$$x^{\frac{b}{a}} \frac{dy}{dx} + \frac{b x^{\frac{b-a}{a}}}{a} y = \frac{x^{\frac{b-a}{a}}}{a}$$

$$\frac{d(x^{\frac{b}{a}} \cdot y)}{dx}$$

$$\rightarrow \frac{dy}{dx} + \frac{b}{a x} y = \frac{1}{a x}$$

#### ④ Integral

$$\int \frac{dx(x^{\frac{b}{a}} \cdot y)}{dx} = \int \frac{x^{\frac{b-a}{a}}}{a} dx$$

$$x^{\frac{b}{a}} \cdot y = \frac{1}{a} \int x^{\frac{b-a}{a}} dx$$

$$x^{\frac{b}{a}} \cdot y = \frac{1}{a} \frac{x^{\frac{b-a}{a}+1}}{\frac{b-a}{a}+1} = \frac{1}{a} x^{\frac{b-a}{a} + \frac{a}{a} = \frac{b}{a}}$$

$$x^{\frac{b}{a}} \cdot y = \frac{1}{a} \frac{x^{\frac{b}{a}}}{\frac{1}{a}} + C$$

$$x^{\frac{b}{a}} \cdot y = \frac{1}{a} x^{\frac{b}{a}} + C$$

$$\begin{aligned} x \cdot y &= \frac{x^{\frac{b}{a}}}{x^{\frac{b}{a}}} + C \\ y(x) &= \frac{x^{\frac{b}{a}}}{x^{\frac{b}{a}}} + \frac{C}{x^{\frac{b}{a}}} \end{aligned}$$

$$\rightarrow y(x) = \frac{x^{\frac{b}{a}}}{b x^{\frac{b}{a}}} + \frac{C}{x^{\frac{b}{a}}}$$

$$\rightarrow y(x) = \frac{1}{b} + \frac{C}{x^{\frac{b}{a}}}$$

Sol. general

⑦ Resolver la ED lineal con parámetros positivos a, b

$$\frac{dy}{dx} + ay = be^{ax}$$

① Encontrar forma  $\frac{dy}{dx} + p(x)y = h(x)$

$$\frac{dy}{dx} + ay = be^{ax}$$

$\downarrow \quad \downarrow \quad \downarrow$

$\frac{dy}{dx}$     $p(x) \quad y$     $h(x)$

② Encontrar FI

$$y(x) = e^{\int adx} = e^{\frac{aydx}{a}} = e^{ax}$$

③ Multiplicar ED por FI

$$e^{ax}\left(\frac{dy}{dx} + ay\right) = be^{ax}(e^{ax})$$

$$\rightarrow e^{ax}\frac{dy}{dx} + aye^{ax} = be^{2ax}$$

$$\frac{d(e^{ax} \cdot y)}{dx}$$

④ Integrando

$$\int \frac{d(e^{ax} \cdot y)}{dx} = \int be^{2ax} dx$$

$$e^{ax} \cdot y = b \int e^{2ax} dx \rightarrow \int e^u du = e^u$$
$$u = 2ax \quad u = 2ax$$
$$du = 2a dx \quad dx = \frac{du}{2a}$$

$$y(x) = \frac{be^{2ax}}{2a} + \frac{C}{e^{ax}} \rightarrow y(x) = \frac{be^{2ax}}{2a e^{ax}} + \frac{C}{e^{ax}}$$
$$= \frac{be^{ax}}{2a} + \frac{C}{e^{ax}}$$

$\underbrace{\qquad \qquad \qquad}_{\text{sol. general}}$

(8) Resuelve el PVI

$$x \frac{dy}{dx} - 2y = x^3 \cos(x)$$

① Encuentra forma  $\frac{dy}{dx} + p(x)y = h(x)$

$$\rightarrow x \frac{dy}{dx} - 2y = x^3 \cos(x)$$

$$\rightarrow \frac{dy}{dx} - \frac{2}{x}y = x^2 \cos(x)$$

② Encuentra F1

$$M(x) = e^{\int \frac{2}{x} dx} = e^{2 \int \frac{1}{x} dx} = e^{-2 \ln(x)} = e^{\ln(\bar{x}^2)} = \bar{x}^2 = \frac{1}{x^2}$$

③ Multiplicar F1 por ED

$$\frac{1}{x^2} \left( \frac{dy}{dx} - \frac{2}{x}y \right) = (x^2 \cos(x)) \left( \frac{1}{x^2} \right)$$

$$\rightarrow \frac{1}{x^2} \frac{dy}{dx} - \frac{2}{x^3}y = \frac{x^2 \cos(x)}{x^2}$$

$$\rightarrow \underbrace{\frac{1}{x^2} \frac{dy}{dx} - \frac{2}{x^3}y}_{\frac{d(\frac{1}{x^2} \cdot y)}{dx}} = \cos(x)$$

(4) Integración

$$\int \frac{d(\frac{1}{x^2} \cdot y)}{dx} dx = \int \cos(x) dx$$

$$\frac{1}{x^2} \cdot y = \sin(x) + C$$

$$y(x) = \frac{\sin(x)}{x^2} + \frac{C}{x^2}$$

$$y(x) = \underbrace{x^2 \sin(x) + Cx^2}_{\text{Sol. general}}$$

④ Se busca solución que satisface

Sol. general

$$y(0) = 0$$

$$0 = (0)^2 \sin(0) + C(0)^2$$

$$-\sin(0) = C(0)^2 \rightarrow C = 0$$

$$\frac{0}{0} = 0$$

⑤ Sustituyendo C en sol. general

$$y(x) = x^2 \sin(x) + 0x^2 = \underbrace{x^2 \sin(x)}$$

Sol. del PVI