

Materia: Calculo Diferencial

Tarea

DERIVADAS DE ORDEN SUPERIOR

Ingeniería en Computación Inteligente

Semestre 1° A

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TAREA DERIVADAS ORDEN SUPERIOR

① Obtenga la primera y segunda derivada de los siguientes funciones.

$$a) f(x) = x^2 - 4x^3$$

$$\begin{aligned} f'(x) &= 2x - 12x^2 \quad (1) \\ f''(x) &= \boxed{2 - 24x} \quad (2) \end{aligned}$$

$$b) f(x) = x^3 - 4x$$

$$\begin{aligned} f'(x) &= 3x^2 - 4 \quad (1) \\ f''(x) &= \boxed{6x} \quad (2) \end{aligned}$$

$$c) f(x) = x^2 \cos(2x)$$

$$\begin{aligned} f'(x) &= (x^2)'(\cos(2x))' + (x^2)(\cos(2x))' \\ &= (x^2)[- \sin(2x) \cdot 2] + 2x(\cos(2x)) \\ &= \boxed{-2x^2 \sin(2x) + 2x \cos(2x)} \quad (1) \end{aligned}$$

$$\begin{aligned} f''(x) &= (-2x^2)[(\sin(2x))' \cdot (2x)'] + (-2x^2)[2 \sin(2x) + (2x)(\cos(2x))] \\ &\quad \cdot (2x) + (2x)^2 \cos(2x) \\ &= (-2x^2)[(\cos(2x) \cdot 2)] + (-4x)\sin(2x) + [2x(-\sin(2x) \cdot 2)] + \\ &\quad (2)\cos(2x) \\ &= -4x^2 \cos(2x) - 4x \sin(2x) - 4x \sin(2x) + 2 \cos(2x) \\ &= \boxed{(2 - 4x^2)\cos(2x) - 8x \sin(2x)} \quad (2) \end{aligned}$$

$$d) f(x) = 2x^3 e^{6x}$$

$$\begin{aligned} f'(x) &= (2x^3)(e^{6x})' + (2x^3)'(e^{6x}) \\ &= 2x^3(6e^{6x}) + (6x^2)(e^{6x}) \\ &= \boxed{12x^3 e^{6x} + 6x^2 e^{6x}} \quad (1) \end{aligned}$$

$$\begin{aligned} f''(x) &= (12x^3)(e^{6x})' + (12x^3)'(e^{6x}) + (6x^2)(e^{6x})' + (6x^2)'(e^{6x}) \\ &= 12x^3(6e^{6x}) + (36x^2)(e^{6x}) + 6x^2(6e^{6x}) + (12x)(e^{6x}) \\ &= 72x^3 e^{6x} + 36x^2 e^{6x} + 36x^2 e^{6x} + 12x e^{6x} \\ &= \boxed{72x^3 e^{6x} + 72x^2 e^{6x} + 12x e^{6x}} \quad (2) \end{aligned}$$

$$e) f(x) = e^{6x} \sin(3x)$$

$$\begin{aligned}f'(x) &= (e^{6x}) [(\sin(3x))' - (3x)] + (e^{6x})' (\sin(3x)) \\&= (e^{6x}) [\cos(3x) \cdot 3] + (6e^{6x}) (\sin(3x)) \\&= \underline{3e^{6x} (\cos(3x)) + 6e^{6x} \sin(3x)} \quad ①\end{aligned}$$

$$\begin{aligned}f''(x) &= (3e^{6x}) [(\cos(3x))' - (3x)] + (3e^{6x})' + ((\cos(3x))'') \\&\quad + (6e^{6x}) [\sin(3x)' - (3x)] + (6e^{6x})' (\sin(3x))\end{aligned}$$

$$\begin{aligned}&= 3e^{6x} [-\sin(3x) \cdot 3] + 18e^{6x} (\cos(3x)) + (6e^{6x})' \\&\quad [\cos(3x) \cdot 3] + (36e^{6x}) \sin(3x) \\&= 9e^{6x} \sin(3x) + 18e^{6x} \cos(3x) + 18e^{6x} \cos(3x) + 36e^{6x} \sin(3x) \\&= \underline{36e^{6x} (\cos(3x)) + 27e^{6x} \sin(3x)} \quad ②\end{aligned}$$

$$f) f(x) = e^{6x} (\cos(3x) + \sin(3x))$$

$$\begin{aligned}f'(x) &= (e^{6x}) [((\cos(3x) + \sin(3x))')] + (e^{6x})' (\cos(3x) + \sin(3x)) \\&= (e^{6x}) [(-\sin(3x) \cdot 3) + (\cos(3x) \cdot 3)] + 6e^{6x} (\cos(3x) + \sin(3x)) \\&= -3e^{6x} \sin(3x) + 3e^{6x} \cos(3x) + 6e^{6x} \cos(3x) + 6e^{6x} \sin(3x) \\&= \underline{3e^{6x} \sin(3x) + 9e^{6x} \cos(3x)} \quad ①\end{aligned}$$

$$\begin{aligned}f''(x) &= 3e^{6x} [(\sin(3x) + 3x)'] + (3e^{6x})' (\sin(3x)) + 9e^{6x} \\&\quad [(\cos(3x))' - (3x)'] + (9e^{6x})' (\cos(3x)) \\&= 3e^{6x} [\cos(3x) \cdot 3] + (18e^{6x}) (\sin(3x)) + 9e^{6x} [-\sin(3x) \cdot 3] \\&\quad + (54e^{6x}) \cos(3x) \\&= 9e^{6x} (\cos(3x)) + 18e^{6x} \sin(3x) - 27e^{6x} (\sin(3x)) + \\&\quad 54e^{6x} \cos(3x) \\&= \underline{63e^{6x} (\cos(3x)) - 9e^{6x} \sin(3x)} \quad ③\end{aligned}$$

$$g) f(x) = \frac{1}{x-1} = (x-1)^{-1}$$

$$f'(x) = [(x-1)^{-1}]' = -1(x-1)^{-2} = \frac{-1}{(x-1)^2} \quad ①$$

$$f''(x) = -[(x-1)^{-2}] = -[-2(x-1)^{-3}] = \frac{2}{(x-1)^3} \quad ②$$

$$h) f(x) = x^2 \cos(2x)$$

$$\begin{aligned} f'(x) &= x^2 [(\cos(2x))' \cdot (2x)' + (x^2)' \cos(2x)] \\ &= x^2 [(-\sin(2x) \cdot 2) + (2x)(\cos(2x))] \\ &= -2x^2 \sin(2x) + 2x \cos(2x), \quad ① \end{aligned}$$

$$\begin{aligned} f''(x) &= (-2x^2)[(\sin(2x))' \cdot (2x)'] + (-2x^2)' \sin(2x) + (2x)[(\cos(2x))' \cdot (2x)'] \\ &\quad + (2x)' \cos(2x) \\ &= (-2x^2)[(\cos(2x)) \cdot 2] + (-4x) \sin(2x) + 2x[-\sin(2x) \cdot 2] \\ &\quad + (2)(\cos(2x)) \\ &= -4x^2 \cos(2x) - 4x \sin(2x) - 4x \sin(2x) + 2 \cos(2x) \\ &= (-4x^2 + 2) \cos(2x) - 8x \sin(2x), \quad ② \end{aligned}$$

$$i) f(x) = (e^{x^2} + e^{-x^2})$$

$$f'(x) = \underline{2x e^{x^2} - 2x e^{-x^2}} \quad ①$$

$$\begin{aligned} f''(x) &= (2x)(e^{x^2})' + (2x)'(e^{x^2}) + (-2x)(e^{-x^2})' + (-2x)'(e^{-x^2}) \\ &= (2x)(2x e^{x^2}) + (2)(e^{x^2}) + (-2x)(e^{-x^2}) + (-2)(e^{-x^2}) \\ &= 4x^2 e^{x^2} + 2e^{x^2} + 4x^2 e^{-x^2} - 2e^{-x^2} \\ &= \underline{4x^2 e^{x^2} + 4x^2 e^{-x^2} + 2e^{x^2} - 2e^{-x^2}}, \quad ② \end{aligned}$$

Propiedades de la función exponencial

② Utiliza las propiedades de la función exponencial para realizar los productos.

$$\begin{aligned} a) & (e^{2x} - 4)(e^{2x} + 4) \\ & = e^{2x+2x} + 4e^{2x} - 4e^{2x} - 16 \\ & = \boxed{e^{4x} - 16} \end{aligned}$$

$$\begin{aligned} b) & (e^{2x} - 4)(e^{2x} - 4) \\ & = e^{2x+2x} - 4e^{2x} - 4e^{2x} + 16 \\ & = \boxed{e^{4x} - 8e^{2x} + 16} \end{aligned}$$

$$\begin{aligned} c) & (e^{3x} + 6)(e^{3x} - 4) \\ & = e^{3x+3x} - 4e^{3x} + 6e^{3x} - 24 \\ & = \boxed{e^{6x} + 2e^{3x} - 24} \end{aligned}$$

$$\begin{aligned} d) & (e^{2x} - e^{3x})(e^{2x} + e^{3x}) \\ & = e^{2x+2x} + e^{2x+3x} - e^{3x+2x} - e^{3x+3x} \\ & = e^{4x} + e^{5x} - e^{5x} - e^{6x} \\ & = \boxed{e^{4x} - e^{6x}} \end{aligned}$$

$$\begin{aligned} e) & (e^{2x} - e^{3x})(e^{2x} - e^{3x}) \\ & = e^{2x+2x} - e^{3x+2x} - e^{3x+2x} + e^{3x+3x} \\ & = e^{4x} - e^{5x} - e^{5x} + e^{6x} \\ & = \boxed{e^{4x} - 2e^{5x} + e^{6x}} \end{aligned}$$

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$$\begin{aligned}
 f) & (e^{3x} + 6)(e^{-3x} - 4) \\
 & = e^{3x-3x} - 4e^{3x} + 6e^{-3x} - 24 \\
 & = e^0 - 4e^{3x} + 6e^{-3x} - 24 \\
 & = 1 - 4e^{3x} + 6e^{-3x} - 24
 \end{aligned}$$

Derivadas de funciones con parámetros $\text{Examen } 1 \text{ A} = (x) \text{ } 1$

③ Encuentre la primera derivada de las siguientes funciones.

a) $f(x) = Ae^{ax}$

$$\begin{aligned}
 f'(x) &= (A)(e^{ax})' + (A)'(e^{ax}) \\
 &= A(ae^{ax}) + (0)(e^{ax}) \\
 &= Ae^{ax}
 \end{aligned}$$

b) $f(x) = e^{ax} \operatorname{Sen} bx$

$$\begin{aligned}
 f'(x) &= (e^{ax})[(\operatorname{Sen} bx)' \cdot (bx)'] + (e^{ax})'(\operatorname{Sen} bx) \\
 &= (e^{ax})[(\operatorname{Cos} bx) \cdot (b)] + (ae^{ax})(\operatorname{Sen} bx) \\
 &= be^{ax}(\operatorname{Cos} bx) + ae^{ax}(\operatorname{Sen} bx)
 \end{aligned}$$

c) $f(x) = e^{ax} \operatorname{Cos} bx$

$$\begin{aligned}
 f'(x) &= (e^{ax})[(\operatorname{Cos} bx)' \cdot (bx)'] + (e^{ax})'(\operatorname{Cos} bx) \\
 &= (e^{ax})[-\operatorname{Sin} (bx) \cdot (b)] + (ae^{ax})(\operatorname{Cos} (bx)) \\
 &= -be^{ax}\operatorname{Sin} (bx) + ae^{ax}(\operatorname{Cos} (bx))
 \end{aligned}$$

d) $f(x) = x^2 e^{-ax}$

$$\begin{aligned}
 f'(x) &= (x^2)(e^{-ax})^2 + (x^2)'(e^{-ax}) \\
 &= (x^2(-ae^{-ax})) + (2x)(e^{-ax}) \\
 &= -ax^2 e^{-ax} + 2x e^{-ax}
 \end{aligned}$$

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$$e) f(x) = Ax e^{3x}$$

$$\begin{aligned}f'(x) &= (Ax)(e^{3x})' + (Ax)'(e^{3x}) \\&= (Ax)(3e^{3x}) + (A)(e^{3x}) \\&= \underline{3Ax e^{3x} + Ae^{3x}}\end{aligned}$$

$$f) f(x) = A \operatorname{sen}(bx) + B \cos(bx)$$

$$\begin{aligned}f'(x) &= (A)[(\operatorname{sen}(bx))' \cdot (bx)] + (A)'(\operatorname{sen}(bx)) + (B)[(\cos(bx))' \cdot (bx)] \\&\quad + (B)'(\cos(bx)) \\&= (A)[(b \operatorname{sen}(bx) \cdot b)] + (\emptyset) \operatorname{sen}(bx) + (B)[- \operatorname{sen}(bx) \cdot b] + \\&\quad (\emptyset)(\cos(bx)) \\&= Ab \operatorname{sen}(bx) + \emptyset - Bb \operatorname{sen}(bx) + \emptyset \\&= \underline{Ab \operatorname{sen}(bx) - Bb \operatorname{sen}(bx)}\end{aligned}$$

9) Encuentre la segunda derivada de las siguientes funciones

$$a) f(x) = Ax^3 e^{ax}$$

$$\begin{aligned}f'(x) &= (Ax^3)(e^{ax})' + (Ax^3)'(e^{ax}) \\&= (Ax^3)(ae^{ax}) + (3Ax^2)(e^{ax}) \\&= \underline{Aa x^3 e^{ax} + 3Ax^2 e^{ax}}, \quad (1)\end{aligned}$$

$$\begin{aligned}f''(x) &= (Aa x^3)(e^{ax})' + (Aa x^3)'(e^{ax}) + (3Ax^2)(e^{ax})' + (3Ax^2)(e^{ax})' \\&= (Aa x^3)(ae^{ax}) + (3Aa x^2)(e^{ax}) + (3Ax^2)(ae^{ax}) + (6Ax)(e^{ax}) \\&= Aa^2 x^3 e^{ax} + 3Aa x^2 e^{ax} + 3Aa x^2 e^{ax} + 6Ax e^{ax} \\&= \underline{Aa^2 x^3 e^{ax} + 6Aa^2 e^{ax} + 6Ax e^{ax}}, \quad (2)\end{aligned}$$

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$$b) f(x) = e^{ax} + e^{-ax}$$

$$\begin{aligned}f'(x) &= ae^{ax} - ae^{-ax} \quad (1) \\f''(x) &= a^2e^{ax} + a^2e^{-ax} \quad (2)\end{aligned}$$

$$c) f(x) = 2 \operatorname{sen} bx + 3 \cos bx$$

$$\begin{aligned}f'(x) &= 2[(\operatorname{sen}(bx))' + (bx)'] + 3[(\cos(bx))' + (bx)'] \\&= 2[(\cos(bx) \cdot b)] + 3[-\operatorname{sen}(bx) \cdot b], \\&= 2b\cos(bx) - 3b\operatorname{sen}(bx), \quad (1)\end{aligned}$$

$$\begin{aligned}f''(x) &= (2b)[(\cos(bx))' + (bx)'] + (2b)'(0\operatorname{sen}(bx) + (-3b)[(\operatorname{sen}(bx))' + (bx)']) \\&\quad + (-3b)'(\operatorname{sen}(bx)) \\&= (2b)(-\operatorname{sen}(bx) \cdot b) + (0)(0\operatorname{sen}(bx)) + (-3b)[(0\operatorname{sen}(bx)) - b] \\&\quad + (0)(\operatorname{sen}(bx)) \\&= -2b^2\operatorname{sen}(bx) + 0 - 3b^2(0\operatorname{sen}(bx)) + 0 \\&= -2b^2\operatorname{sen}(bx) - 3b^2(0\operatorname{sen}(bx)), \quad (2)\end{aligned}$$

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