



DEPARTAMENTO DE MATEMÁTICAS Y FÍSICA

Materia: **CÁLCULO INTEGRAL**

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Examen Parcial 1

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Carrera: ICI 2-A

EXAMEN PARCIAL 1

(1) Calcule el área bajo la curva de $f(x) = (x + (-7))^2 + 3$ entre los ejes x y y los rectos $x=3$ y $x=5$. Utilice el límite superior del subintervalo. Muestre el valor de Δx y de x_i^* y el valor del área.

$$f(x) = (x - 7)^2$$

$$q = 3$$

$$b = 5$$

$$\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \frac{2}{n}$$

$$X_i^* = i\Delta x + q = i\left(\frac{2}{n}\right) + 3$$

$$x_i^* = \frac{2}{n} i + 3 = \frac{2i + 3n}{n}$$

$$x_i^* = \left(\left(\frac{2}{n}i + 3 \right) - 7 \right)^2 + 3$$

$$= \left(\left(\frac{2}{n}i + 3 \right)^2 - 2\left(\frac{2}{n}i + 3 \right)(7) + 7^2 \right) + 3$$

$$= \left(\left(\frac{2}{n} i + 3 \right)^2 + 2 \left(\frac{2}{n} i + 3 \right) (3) + (3)^2 - \frac{28}{n} \right) (-42 + 49) + 3$$

$$= \frac{4}{n^2} i^2 + \frac{18}{n} i + 9 - \frac{28i}{n} - 42 + 49 + 3$$

$$= \frac{4i^2 - 10i + 10}{n^2}$$

$$A_i = f(x_i^*) \Delta x$$

$$A_i = \left(\frac{4}{n^2} i^2 - \frac{10i}{n} + 10 \right) \left(\frac{2}{n} \right)$$

$$A_i = \left(\frac{8}{n^3} i^2 - \frac{20i}{n^2} + \frac{20}{n} \right)$$

$$\sum_{i=1}^n A_i = \sum_{i=1}^n \left(\frac{8}{n^3} i^2 - \frac{20i}{n^2} + \frac{20}{n} \right)$$

$$\sum_{i=1}^n A_i = \frac{8}{n^3} \sum_{i=1}^n i^2 - \frac{90}{n^2} \sum_{i=1}^n i + \frac{20}{n} n$$

$$= \frac{8}{n^2} \left(\frac{4(n+1)(2n+1)}{6} \right) - \frac{90}{n^2} \left(\frac{n(n+1)}{2} \right) + \frac{20}{n} (n)$$

$$= \frac{8}{n^2} \left(\frac{2n^2 + 3n + 1}{2} \right) - \frac{45}{n} - \left(\frac{45}{n} + 20 \right)$$

$$= \frac{8}{n^2} \left(\frac{2n^2}{2} \right) + \frac{9}{n^2} \left(\frac{3n}{2} \right) + \frac{9}{n^2} \left(\frac{1}{2} \right) - 45 - \frac{45}{n} + 20$$

$$= 8 + \frac{27}{2n} + \frac{9}{2n^2} - 45 - \frac{45}{n} + 20$$

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n A_i = \lim_{n \rightarrow \infty} \left(\frac{27}{2n} + \frac{9}{n^2} - \frac{45}{n} + \frac{45}{n} \right) = 45v^2$$

② Usando sumas especiales, calcula el valor de la suma dada

$$\sum_{i=1}^{74} (-s_i + 4)^3$$

$$\sum_{n=m}^n = m - \sum_{n=1}^m = -1$$

$$\sum_{i=1}^{74} (-s_i + 4)^3 = \sum_{i=1}^{70} (-s_i + 4)^3$$

$$= (-s_i)^3 + 3(-s_i)^2 \cdot (4 + 3(-s_i)) - 4^2 + 4^3$$

$$= -125s_i^3 + 300s_i^2 + 240s_i + 64$$

$$\sum_{i=1}^{74} -125s_i^3 + 2 \sum_{i=1}^{74} 300s_i^2 - \sum_{i=1}^{74} 240s_i + \sum_{i=1}^{74} 64$$

①

$$\sum_{n=1}^n n^3 = \frac{1}{n} n^2 (n+1)^3 = \frac{1}{4} 74^2 (74+1)^2 = 7700525 \cdot 125 \\ = -962578125$$

② $\sum a_n = C \sum g_n = 300 \sum_{i=1}^{74} i^2 = \frac{1}{6} n(n+1)(2n+1)$

$$= \frac{1}{6} (74)(74+1)(2 \cdot 74 + 1) = 137825 \cdot 300 \\ = 41347500$$

③ $240 \sum_{i=1}^{74} i = \frac{1}{2} 74(74+1) = 2775 \cdot 240 \\ = 666000$

④ $\sum_{i=1}^{74} 64 = 64 \cdot 74 = 4736 \quad -962578125 + 41347500 - 6660000 \\ + 4736 = -921891889$

$$13) - \sum_{i=1}^{10} 125i^3 + \sum_{i=1}^{10} 300i^2 + \sum_{i=1}^{10} 240i + \sum_{i=1}^{10} 64$$

$$① \frac{1}{4} 10^2 (10+1)^2 = 3025 \cdot 125 = 378125$$

$$② \frac{1}{6} 10(10+1)(2(10+1)) = 385 \cdot 300 = 115500$$

$$③ \frac{1}{2} 10(10+1) = 55 \cdot 240 = 13200$$

$$④ 10(64) = 640$$

$$= -378125 + 115500 - 13200 + 640 = -271581$$

$$\sum_{i=11}^{94} (-5i+4)^3 = -921891889 - (-271581)$$

$$= -921616704 \quad \cancel{t}$$

④ Calcular la integral indefinida que se muestra

$$\int (\operatorname{sen}(2x)^5 \cos(2x)^4 dx)$$

$$u = 2x$$

$$du = 2 dx$$

$$dx = \frac{1}{2} du$$

$$\operatorname{sen}^2 u = 1 - \cos^2 u$$

$$= \int \operatorname{sen}(u)^5 \cos(u)^4 \left(\frac{1}{2} du\right)$$

$$= \frac{1}{2} \int \operatorname{sen}^5(u) \cos^4(u) du$$

$$= \int \cos^4(u)((\cos^2 u - 1)^2) \sin(u) du$$

$$w = \cos u$$

$$dw = -\sin u du$$

$$du = -\frac{1}{\sin u}$$

$$= \int w^4 (w^2 - 1)^2 \sin(u) \frac{1}{\sin u} du$$

$$= \int w^4 (w^4 - 2w^2 + 1)$$

$$= \int w^8 - 2w^6 + w^4$$

$$= \int w^8 - 2 \int w^6 + \int w^4 = \frac{w^9}{9} - 2 \frac{w^7}{7} + \frac{w^5}{5}$$

$$= \frac{1}{2} \left[\frac{\cos^9 u}{9} - \frac{2 \cos^7 u}{7} + \frac{\cos^5 u}{5} \right] = \frac{1}{18} \cos^9 u - \frac{2}{14} \cos^7 u + \frac{1}{10} \cos^5 u$$

$$= \frac{1}{18} \cos^9(2x) - \frac{2}{14} \cos^7(2x) + \frac{1}{10} \cos^5(2x)$$

$$= -\frac{1}{18} \cos^9(2x) - \frac{1}{7} \cos^7(2x) + \frac{1}{10} \cos^5(2x) + C$$

⑤ Resuelva la integral indefinida indicada:

$$\int \left(\frac{7x^9 + 3x^5 + 4x^2}{9x^7} \right) dx$$

$$\begin{aligned} & \frac{1}{9} \int \frac{7x^9}{x^7} dx + \int \frac{3x^5}{x^7} dx + \int \frac{4x^2}{x^7} dx = \frac{1}{9} \int 7x^2 dx + \int \frac{3}{x^2} dx + \int \frac{4}{x^5} dx \\ &= \frac{1}{9} \left[7 \int x^2 dx + 3 \int \frac{1}{x^2} dx + 4 \int \frac{1}{x^5} dx \right] \\ &= \frac{1}{9} \left[7 \left(\frac{x^3}{3} \right) + 3 \left(-\frac{1}{x} \right) + 4 \left(-\frac{1}{4x^4} \right) \right] = \frac{1}{9} \left[\frac{7x^3}{3} - \frac{3}{x} - \frac{1}{x^4} \right] \\ &= \frac{1}{27} x^3 - \frac{1}{3x} - \frac{1}{9x^4} + C \end{aligned}$$

⑥ Resuelva la integral indicada

$$\int \frac{8x+5}{1+x^2} dx$$

$$= \int \frac{8x}{1+x^2} dx + \int \frac{5}{1+x^2} dx = 8 \int \frac{x}{1+x^2} dx + 5 \int \frac{1}{1+x^2} dx$$

$$\int \frac{x}{1+x^2} dx = \int \frac{x}{u} \left(\frac{1}{2}du\right) = \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} [\ln|u|] = \frac{\ln|u|}{2}$$

$$u = 1+x^2 \quad = \frac{\ln|1+x^2|}{2} \quad \int \frac{1}{1+x^2} dx = \tan^{-1}(x)$$
$$du = 2x dx \quad \quad \quad$$
$$dx = \frac{1}{2x} du$$

$$\int \frac{8x+5}{1+x^2} dx = 8 \left[\frac{\ln|1+x^2|}{2} \right] + 5 \tan^{-1}(x)$$

$$= 4 \ln|1+x^2| + 5 \tan^{-1}(x) + C$$

⑦ Usando la parte uno del teorema fundamental del cálculo
encontrar la derivada que se indica

$$\frac{d}{dx} \left(\int_{1}^{2x^3+9x} (\sin(5+3t))^8 dt \right)$$

$$= \int \sin(5+3t)^8 \cdot 3 \cdot 2x^2 dt = \int \frac{\sin(5+3t)}{9} du$$

$$\int a \cdot f(x) dx = a \int f(x) dx$$

$$= \frac{1}{9} \int_{(2x^3+9x)^9} \sin(5+3t) dt$$

$$\int f(g(x)) \cdot g'(x) dx = \int f(u) du, u = g(x)$$

$$v = 5+3t$$

$$\int \sin(v) \frac{1}{3} dv$$

$$w = 1 \rightarrow v = 8$$

$$dv = 3$$

$$w = (2x^3 + 9x)^9$$

$$dv = \frac{1}{3}$$

$$v = 5+3(2x^3 + 9x)^9$$

$$\frac{1}{9} \int_8^{5+3(2x^3+9x)^9} \sin(v) \frac{1}{3} dv = \frac{1}{9} \cdot \frac{1}{3} \int_8^{5+3(2x^3+9x)^9} \sin(v) dv$$

$$\int \sin v dv = -\cos v$$

$$= -\frac{1}{27} [-\cos v] \Big|_8^{5+3(2x^3+9x)^9} \int_a^b f(t) dx = f(b) - f(a)$$

$$\lim_{x \rightarrow b} f(t) - \lim_{x \rightarrow a} f(t)$$

$$\lim_{x \rightarrow 8} \cos(5+3(2x^3+9x)^9) - \cos(5+3(2x^3+9x)^9)$$

$$= 1/27 (-\cos(5+3(2x^3+9x)^9) + \cos(8))$$

$$= \sin(3(2x^3+9x)^9 + 5)(6x^2 + 9)(2x^3 + 9x)^8$$

⑧ Calcule el valor promedio de la función

$$f(t) = 16t^2 + 56t + 49 \text{ en } [4, 12]$$

$$\bar{f}(t) = \frac{\int_a^b f(t) dt}{b-a}$$

$$= \frac{1}{12-4} \left[\frac{16t^3}{3} + \frac{56t^2}{2} + 49t \right] \Big|_4^{12}$$

$$= \frac{1}{8} \left[\frac{16}{3} t^3 + 28t^2 + 49t \right] \Big|_4^{12}$$

$$= \frac{1}{8} \left(\frac{16}{3} (12)^3 + 28(12)^2 + 49(12) \right) - \left(\frac{16}{3} (4)^3 + 28(4)^2 + 49(4) \right)$$

$$= \frac{1}{8} \left[\frac{16}{3} (1728) + 28^2(144) + 588 \right] - \left[\frac{16}{3} (64) + 28(16) + 196 \right]$$

$$= \frac{1}{8} \left[\frac{27648}{3} + 4032 + 588 \right] - \left[\frac{1024}{3} + 448 + 196 \right]$$

$$= \frac{1}{8} \left[\frac{27648}{3} - \frac{1024}{3} + 4032 + 588 - 448 - 196 \right]$$

$$= \frac{1}{8} \left[\frac{26624}{3} + 4003 \right] = \frac{1}{8} \left[\frac{26624}{3} + \frac{12009}{3} \right]$$

$$= \frac{1}{8} \left(\frac{38633}{3} \right) = \frac{38633}{24}$$