DEPARTAMENTO DE MATEMÁTICAS Y FÍSICA

Materia: **CÁLCULO INTEGRAL**

Profesor: M. C. DAVID REFUGIO ARELLANO BÁEZ

Periodo: ENERO-JUNIO / 2022

Tarea Parcial 2

fecha: 3/MAYO/2022

Carrera: ICI 2-A

TAREA 12 INTEGRACION POR PARTES

① $\int x^2 \ln x \, dx = \frac{1}{3} x^3 \ln x - \int \frac{x^3}{3} \left({}^1/_x \right)$

$u = \ln x \quad , \; du = {}^1/_x$
$dv = x^2 \quad , \; v = \frac{x^3}{3}$

$= \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^2 \, dx$

$= \frac{1}{3} x^3 \ln x - \frac{1}{3} \frac{x^3}{3}$

$= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^4 + C$

② $\int \overset{u}{t} \, \overset{dv}{e^{-3t} \, dt} = t \cdot -\frac{1}{3} e^{-3t} - \int -\frac{1}{3} e^{-3t}$

$u = t$
$dv = -3t \, dt$
$du = dt$
$v = -\frac{1}{3} e^{-3t}$

$= -\frac{1}{3} t e^{-3t} + \frac{1}{3} \int e^{-3t}$

$= -\frac{1}{3} t e^{-3t} + \frac{1}{3} \cdot \left(-\frac{1}{3} e^{-3t} \right)$

$= -\frac{1}{3} t e^{-3t} - \frac{1}{3} e^{-3t} + C$

(3) $\int \ln \sqrt[3]{x}\, dx = \ln \sqrt[3]{x}\,(x) - \int x\,(1/3x)$

$= \ln \sqrt[3]{x}\,(x) - \frac{1}{3}\int dx$

$= x \ln \sqrt[3]{x} - \frac{1}{3} x + c$

$u = \ln \sqrt[3]{x}$ dx

$du = \frac{dx}{x} \ln \sqrt[3]{x}$

$\frac{du}{dx} = \ln(x)^{1/3}$

$= \frac{1/3\, x^{1/3 - 3/3}}{x^{1/3}}$

$= \frac{1}{3} \frac{x^{-2/3}}{x^{1/3}}$

$= \frac{1}{3x^{1/3+2/3}} = \frac{1}{3}x$

$dv = dx$

$v = x$

$du = 1/3\, x$

(4) $\int t \sec^2 2t = t(\tan(2t)) - \int \tan(2t)(dt)$

$u = t$

$du = dt$

$dv = \sec^2(2t)$

$v = \tan(2t)$

=

U	dt
t	$\sec^2 2t$
1	$1/2 \tan(2t)$
0	$\frac{1}{4}\ln|\sec(2t)|$

$\int \frac{\tan(2t)}{2} = \frac{\tan(u)}{2}(1/2)$

$= t\, 1/2 \tan(2t) - \frac{1}{4}\ln|\sec(2t)| + c$

$= 2t$ $= \frac{1}{4}\int \tan(u) = \frac{1}{4}\int\frac{\sin(u)}{\cos(u)} = -\frac{1}{u} = -\ln(u) = \frac{1}{4}\ln|\cos(u)| = \frac{1}{4}\ln|\sec(2t)|$

$u = \cos(u)$

⑤ $\int (\ln x)^2 \, dx = \ln^2 x \cdot x - \int \cancel{x} \left(2\ln x \frac{1}{\cancel{x}} \, dx\right)$

$u = \ln^2 x$ $\quad = \ln^2 x \cdot x - \int 2\ln x \, dx$

$du = 2\ln x \frac{1}{x} \, dx$

$dv = dx$ $\quad = \ln^2 x \cdot x - 2\int \ln x \, dx$

$v = x$

$= \ln^2 x \cdot x - 2(x \ln x - x)$

$= x\ln^2 x - 2x\ln x + 2x + c$

⑥ $\int z^3 e^z$

U	dv
z^3	$+\ e^z$
$3z^2$	$-\ e^z$
$6z$	$+\ e^z$
6	$-\ e^z$
0	e^z

$= z^3 e^z - 3z^2 e^z + 6ze^z - 6e^z + c$

⑦ $\int_0^{1/2} x \cos \pi x$

$U = x$ $\quad = \frac{1}{\pi} x \, \text{sen}(\pi x) - \int \frac{1}{\pi} \text{sen}(\pi x)$

$du = dx$

$dv = \cos \pi x$ $\quad = \frac{1}{\pi} x \, \text{sen}(\pi x) - \frac{1}{\pi}\left[\frac{1}{\pi} \cos(\pi x)\right]$

$v = \int \cos \pi x \, dx$

$u = \pi x \quad v = \frac{1}{\pi} \text{Sen}(\pi x)$ $\quad = \frac{1}{\pi} x \, \text{sen}(\pi x) + \frac{1}{\pi^2} \cos(\pi x) \Big|_0^{1/2}$

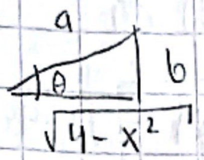
$du = \pi$

$dx = \frac{1}{\pi}$

TAREA 13

SUSTITUCION TRIGONOMETRICA

(1) $\int \frac{dx}{x^2\sqrt{4-x^2}} = \frac{1}{4}\int \frac{d\theta}{\sin^2\theta} = \frac{1}{4}\int \csc^2\theta\, d\theta = \frac{-1}{8}\cot\theta$



$\cot = \frac{ca}{co} = \frac{\sqrt{4-x^2}}{x} = \frac{-\sqrt{4-x^2}}{4} + c$

$a^2 = 4,\ a = 2$

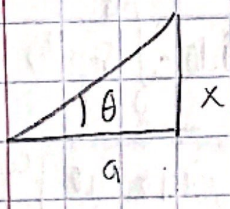
$b^2 = x^2,\ b = x$

$\sin\theta = b/a = \frac{x}{2}$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

(2) $\int_0^a \frac{dx}{(a^2+x^2)^{3/2}}$ $a>0$ $= \int_0^a \frac{dx}{(a^2+x^2)\sqrt{a^2+x^2}} = \int_0^a \frac{a\sec^2\theta\, d\theta}{(a^2+a^2\tan^2\theta)^{3/2}}$



$= \frac{1}{a^2}\int_0^a \cos\theta\, d\theta = \frac{1}{a^2}\sin\theta\Big|_0^a = \frac{1}{a^2}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$

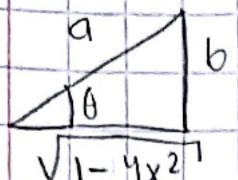
$x = a\tan\theta$

$dx = a\sec^2\theta$

$= \frac{x}{a^2\sqrt{a^2+x^2}} + C$

$= \frac{a}{a^2\sqrt{a^2+a^2}} - \frac{0}{a^2\sqrt{a^2}} = \frac{1}{a\sqrt{2a^2}} = \frac{1}{a^2\sqrt{2}}$

③ $\int \sqrt{1-4x^2}\,dx = 1/2\int \sqrt{1-4\left(\frac{\sin\theta}{2}\right)^2}\cos\theta\,d\theta = 1/2\int\sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta$



$= 1/2\int\sqrt{\cos^2\theta}\,\cos\theta\,d\theta = 1/2\int\cos^2\theta\,d\theta$

$= \frac{1}{4}\theta + 1/8\sin(2\theta) + C = \frac{1}{4}\sin^{-1}(2x) + \frac{x\sqrt{1-4x^2}}{2} + C$

$a^2 = 1,\ a = 1$

$b^2 = 4x^2,\ b = 2x$

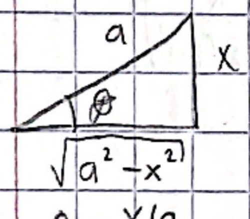
$\sin\theta = 2x$

$x = \frac{\sin\theta}{2}$ $\quad \theta = \sin^{-1}(2x)$

$dx = \frac{\cos\theta}{2}$

④ $\int_0^a x^2\sqrt{a^2-x^2}\,dx = \int_0^a a^2\sin^{2}\theta\sqrt{a^2-a^2\sin^2\theta}\,a\cos\theta\,d\theta$

$= a^4\int_0^a \sin^2\theta(1-\sin^2\theta)\,d\theta = a^4\left[\int_0^a \sin^2\theta\,d\theta - \int_0^a \sin^4\theta\,d\theta\right]$



$= a^4\left[\frac{1}{2}\theta - \frac{\cos\theta\sin\theta}{2} + \frac{\cos\theta\sin^3\theta}{4} + \frac{3}{8}\theta - \frac{3\cos\theta\sin\theta}{8}\right]_0^a$

$\sin\theta = x/a$

$a\sin\theta = x$ $\quad = a^4\left[\frac{1}{2}\sin^{-1}\left(\frac{x}{a}\right) - \frac{x\sqrt{a^2-x^2}}{2a^2} + \frac{x^3\sqrt{a^2-x^2}}{4a^4} + \frac{3}{8}\sin^{-1}\left(\frac{x}{a}\right) - \frac{3x\sqrt{a^2-x^2}}{8a^2}\right.$

$dx = a\cos\theta\,d\theta$

$\int_0^a x^2\sqrt{a^2-x^2}\,dx = a^4\left[\frac{\pi}{4} - 0 + 0 - \frac{3\pi}{16} + 0 - 0 + \frac{a\sqrt{a^2}}{2a^2} - \frac{a^3\sqrt{a^2}}{4a^4} + C\right.$

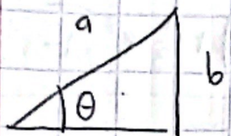
$\left. + \frac{3a\sqrt{a^2}}{8a^2}\right] = a^4\left[\frac{4\pi - 3\pi}{16} + 1/2 + \frac{1}{4} + \frac{3}{8} = \frac{\pi - 16}{16}\right.$

⑤ $\int_0^{0.6} \frac{x^2\,dx}{\sqrt{9-25x^2}} = 3/5 \int_0^{0.6} \frac{(3/5\sin\theta)^2\cos\theta\,d\theta}{\sqrt{9-25(3/5\sin\theta)^2}} = \frac{27}{125}\int_0^{0.6} \frac{\sin^2\theta\cos\theta\,d\theta}{3\sqrt{1-\sin^2\theta}}$

$a^2 = 9,\ a = 3$

$b^2 = 25x^2,\ b = 5x$

$= \frac{27}{125}\int_0^{0.6} \frac{\sin^2\theta\,\cancel{\cos\theta}\,d\theta}{\cancel{\cos\theta}}$



$= \frac{27}{125}\int_0^{0.6} \sin^2\theta\,d\theta = \frac{27}{125}\left[\frac{1}{2}\theta - \frac{1}{4}\sin(2\theta)\right]_0^{0.6}$

$\sin\theta = \frac{b}{a} = \frac{5x}{3}$

$x = \frac{3\sin\theta}{5}$

$= \frac{27}{250}\theta - \frac{54}{500}\cos\theta\sin\theta\Big]_0^{0.6}$

$\theta = \sin^{-1}\left(\frac{5x}{3}\right)$

$\cos\theta = \frac{ca}{h} = \frac{\sqrt{9-25x^2}}{3}$

$= \frac{1}{2}\sin^{-1}\left(\frac{5x}{3}\right) - \frac{\overset{6}{\cancel{54}}(5x)\sqrt{9-25x^2}}{\underset{100}{\cancel{500}}\cdot 3\cdot 5}\Bigg]_0^{0.6}$

$= \frac{1}{2}\sin^{-1}\left(\frac{5x}{3}\right) - \frac{6x\sqrt{9-25x^2}}{100} = \frac{11}{4}\,u^2$

⑥ $\int \frac{x}{\sqrt{x^2+x+1}}\,dx = \int\left(\frac{2x+1}{\sqrt{x^2+x+1}} - \frac{1}{\sqrt{x^2+x+1}}\right)dx = \int \frac{du}{\sqrt{u}} - \frac{dx}{\sqrt{(x+1/2)^2+1-1/4}}$

$= \int u^{-1/2}\,du - \int \frac{dx}{(x+1/2)^2+3/4} = \frac{u^{-1/2+1}}{-1/2+1} - \int \frac{dx}{\frac{\sqrt{3}}{2}\sqrt{\left(\frac{2x+1}{\sqrt{3}}\right)+1}}$

$u = x^2+x+1$

$du = 2x+1$

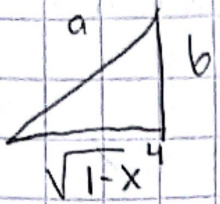
$= \frac{u^{1/2}}{1/2} - \frac{2\cdot\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}\cdot 2}\int \frac{dv}{\sqrt{v^2+1}} = 2\sqrt{x^2+x+1} - \ln\left|\sqrt{\frac{(2x+1)^2}{3}+1}\right.$

$\left. + \frac{2x+1}{\sqrt{3}}\right| + C$

$v = 2x+1$

$dv = \frac{2}{\sqrt{3}}\,dx$

$$⑦\ \int x\sqrt{1-x^4}\,dx = \frac{1}{2}\int \cancel{\sqrt{\sin\theta}}\left(\sqrt{1-\sin^2\theta}\right)\cos\theta\, d\theta \Big/ \cancel{\sqrt{\sin\theta}} = \frac{1}{2}\int (\cos)^2\theta\, d\theta$$



$$= \frac{1}{2}\left[\frac{1}{2}\theta + \frac{1}{4}\sin(2\theta)\right] = \frac{1}{4}\theta + \frac{2}{16}\theta\cos\theta\sin\theta$$

$a^2 = 1,\ a = 1$

$b^2 = x,\ b = x^2$

$\sin\theta = x^2$

$x = \sqrt{\sin\theta}$

$\theta = \sin^{-1}(x^2)$

$dx = \frac{1}{2}\frac{1}{\sqrt{\sin\theta}}\cos\theta$

$$= \frac{1}{4}\sin^{-1}(x^2) + \frac{1}{8}\left(\sqrt{1-x^4}\right)(x^2) = \frac{1}{4}\sin^{-1}(x^2) + \frac{1}{8}x^2\sqrt{1-x^4} + C$$

TAREA 14
FRACCIONES PARCIALES

① $\displaystyle\int \frac{5x+1}{(2x+1)(x-1)}\,dx = \int \frac{A}{2x+1} + \frac{B}{x-1}\,dx = \int\left(\frac{1}{2x+1} + \frac{2}{x+1}\right)dx$

$A(x-1) + B(2x+1) = Ax - A + 2Bx + B = (A+2B)x - A + B$

$A + 2B = 5$ $\quad 2 - A = 1$
$B - A = 1$ $\quad A = 2 - 1$
$3B = 6$ $\quad A = 1$
$B = 2$

$\displaystyle = \int \frac{1}{2x+1}\,dx + 2\int \frac{1}{x-1}\,dx$

$= \ln|2x+1| + 2\ln|x-1| + c$

② $\displaystyle\int \frac{ax}{x^2-6x}\,dx = \int \frac{ax}{x(x-6)}\,dx = \int\left(\frac{A}{x} + \frac{B}{x-6}\right)dx = \int \frac{0}{x} + \frac{a}{x-6}\,dx$

$\displaystyle A(x-6) + B(x) = (A+B)x - 6A = \int \frac{a}{x-6}\,dx = a\int\frac{dx}{x-6}$

$A + B = a$ $\quad A = 0$
$-A = 0$ $\quad B = a$

$= a\ln(x-6) + c$

③ $\displaystyle\int_3^4 \frac{x^3-2x^2-4}{x^3-2x^2}\,dx = \int_3^4\left(\frac{(x^3-2x^2)}{x^3-2x^2} - \frac{4}{x^3-2x^2}\right)dx$

$\displaystyle = \int_3^4 dx - 4\int_3^4 \frac{4}{x^3-2x^2}\,dx = x\Big]_3^4 - \int_3^4 \frac{4\,dx}{x^2(x-2)} = \frac{Ax+B}{x^2}\,dx + \int_3^4 \frac{1}{x-2}\,dx$

$Cx^2 + (Ax+Bx)(x-2) = C\cancel{x^2} + \cancel{Ax^2} - 2Ax + Bx - 2B$
$(A+C)x^2 - (2A-B)x - 2B$
$A + C = 0 \Leftrightarrow C = 1$
$2A - B = 0 \longleftrightarrow 2A + 2 = 0 \Leftrightarrow A = -1$
$-2B = 4 \longleftrightarrow B = -2$

$\displaystyle = -\int_3^4 \frac{1}{x}\,dx + 2\int_3^4 x^{-2}\,dx + \ln|x-2|\Big|_3^4$

$\displaystyle = x\Big|_3^4 - \ln|x|\Big|_3^4 + \frac{2}{x}\Big]_3^4 + \ln|x-2|\Big|_3^4$

$= 4 - 3 - \ln|4| + \ln|3| + \frac{1}{2} + \frac{2}{3} + \ln|2| - \ln|1| \qquad = 1.2387$

④ $\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy = \int_1^2 \left(\frac{A}{y}+\frac{B}{(y+2)}+\frac{C}{(y-3)}\right) = \int_1^2 \left(\frac{2}{y}+\frac{9}{5y+10}+\frac{1}{5y-15}\right)dy$

$A(y+2)(y-3)+B(y)(y-3)+C(y)(y+2)$

$= Ay^2-Ay-6A+By^2-3By+Cy^2+2Cy^2$

$= (A+B+C)y^2+(2C-A-3B)y-6A$

$A+B+C=4 \qquad -2(B+C=2)$

$2C-A-3B=-7 \qquad 2C-3B=-5$

$-6A=-12 \qquad -2C-2B=-4$

$A=2 \qquad -5B=-9$

$B=9/5$

$= 2\int_1^2 \frac{dy}{y}+\frac{9}{5}\int_1^2 \frac{1}{5y+2}dy+\frac{1}{5}\int_1^2 \frac{1}{y-3}dy$

$2\ln|y|\Big|_1^2+\frac{9}{5}\ln|y+10|\Big|_1^2+\frac{1}{5}\ln|y-3|\Big|_1^2$

$= 2[\ln 2-\ln 1]+9/5[\ln 12-\ln 11]$

$= \underline{1,5429\ u^2}$

⑤ $\int \frac{x^3+4}{x^2+4}dx = \int\left(x+\frac{-4x+4}{x^2+4}\right)dx = \int x\,dx-4\int\frac{x-1}{x^2+4}dx$

$= \frac{x^2}{2}-4\int\frac{x}{x^2+4}dx-4\int\frac{1}{x^2+4}$

$= \frac{x^2}{2}-2\ln|x^2+4|-\frac{4}{4}\int\frac{1}{(\frac{x}{2})^2+1} = \underline{\frac{x^2}{2}-2\ln|x^2+4|-2\tan^{-1}\left(\frac{x}{2}\right)+C}$

⑥ $\int\frac{4x\,dx}{x^3+x^2+x+1} = \int\left(\frac{A}{x+1}+\frac{Bx+C}{x^2+1}\right)dx = \int\left(\frac{-2}{x+1}+\frac{2x-2}{x^2+1}\right)dx$

$= -2\int\frac{dx}{x+1}+2\int\frac{x\,dx}{x^2+1}-2 \qquad = -2\ln|x+1|+\ln|x^2+1|-2\tan^{-1}(x)$

$= \underline{-2\ln|x+1|+\ln|x^2+1|-2\tan^{-1}(x)+C}$

(7) $\int \frac{x+4}{x^2+2x\,15}\,dx = \int \left((x-2) + 13/(x+4) \right) dx = \int x\,dx - 2\int dx + 13\int \frac{1}{x+4}$

$U = x+4$
$du = dx$

$= \left[\frac{x^{1+1}}{1+1} \right] - 2x + 13\ln|u| = \frac{x^2}{2} - 2x + 13\ln|x+4| + c$

(8) $\int_0^1 \frac{x^3+2x}{x^4+4x^2+3}\,dx = \frac{1}{4}\int_0^1 \frac{du}{u} = \frac{1}{4}\ln|x^4+4x^2+3| \Big|_0^1$

$= \frac{1}{4}\left[\ln|1+4+3| - \ln|3| \right] = 0.2452\ldots$

$U = x^4+4x^2+3$
$du = 4x^3+8x\,dx$
$\frac{du}{4(x^3+2x)} = dx$

(9) $\frac{dx}{x(x^2+4)^2}\,dx = \int \left(\frac{1}{16x} - \frac{x}{16(x^2+4)} - \frac{x}{4(x^2+4)^2} \right) dx = \frac{1}{16}\int \frac{1}{x}\,dx - \frac{1}{16}\int \frac{1}{x^2+4}\,dx$

$- \frac{1}{4}\int \frac{1}{(x^2+4)^2}\,dx$

$= \ln|x| - 1/32\ln|x^2+4| - 1/8\int u^{-2}\,du$

$= \ln|x| - 1/32\ln|x^2+4| + 1/(8x^2+32) + c$

(10) $\int \frac{\sec^2 t}{\tan^2 t + 3\tan t + 2}\,dt = \int \frac{du}{(u^2+3u+2)} = \int \frac{du}{u^2+2} + \int \frac{du}{u+1}$

$= \int \frac{1}{v}\,dv + \int \frac{1}{w}\,dw = \ln|u+2| + \ln|u+1| + c$

$= \ln|\tan t+2| + \ln|\tan t+1| + c$