EXAMEN PARCIAL 1

① Calcule el área bajo la curva de $f(x) = (x + (-7))^2 + 3$ el eje x y las rectas $x = 3$ y $x = 5$. Utilice el límite superior del subintervalo. Muestre el valor de Δx y de x_i^* y el valor del area

$f(x) = (x-7)^2$

$a = 3$

$b = 5$

$$\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \boxed{\frac{2}{n}}$$

$$x_i^* = i\Delta x + a = i\left(\frac{2}{n}\right) + 3$$

$$x_i^* = \frac{2}{n} i + 3 = \boxed{\frac{2i + 3n}{n}}$$

$$x_i^* = \left(\left(\frac{2}{n} i + 3\right) - 7\right)^2 + 3$$

$$= \left(\left(\frac{2}{n} i + 3\right)^2 - 2\left(\frac{2}{n} i + 3\right)(7) + 7^2\right) + 3$$

$$= \left(\left(\frac{2}{n} i + 3\right)^2 + 2\left(\frac{2}{n} i + 3\right)(3) + (3)^2 - \frac{28}{n} i - 42 + 49 + 3\right.$$

$$= \frac{4}{n^2} i^2 + \frac{18}{n} i + 9 - \frac{28i}{n} - 42 + 49 + 3$$

$$= \frac{4}{n^2} i^2 - \frac{10i}{n} + 10$$

$A_i = f(x_i^*)\Delta x$

$A_i = \left(\frac{4}{n^2} i^2 - \frac{10i}{n} + 10\right)\left(\frac{2}{n}\right)$

$A_i = \left(\frac{8}{n^3} i^2 - \frac{20i}{n^2} + \frac{20}{n}\right)$

$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \left(\frac{8}{n^3} i^2 - \frac{20i}{n^2} + \frac{20}{n}\right)$

$\sum_{i=1}^{n} A_i = \frac{8}{n^3} \sum_{i=1}^{n} i^2 - \frac{90}{n^2} \sum_{i=1}^{n} i + \sum_{i=1}^{n} \frac{81}{n}$

$= \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{90}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{81}{n}(n)$

$= \frac{8}{n^2}\left(\frac{2n^2+3n+1}{2}\right) - \frac{45}{n} - \left(\frac{45}{n}\right) + 81$

$= \frac{8}{n^2}\left(\frac{2n^2}{2}\right) + \frac{9}{n^2}\left(\frac{3n}{2}\right) + \frac{9}{n^2}\left(\frac{1}{2}\right) - 45 - \frac{45}{n} + 81$

$= 8 + \frac{27}{2n} + \frac{9}{2n^2} - 45 - \frac{45}{n} + 81$

$A = \lim_{n\to\infty} \sum_{i=1}^{n} A = \lim_{n\to\infty} \left(\frac{27}{2n} + \frac{9}{n^2} - \frac{45}{n} + 45\right) = 45u^2$

② Usando sumas especiales, calcule el valor de la suma dada

$$\sum_{i=11}^{74} (-5i+(4))^3$$

$$\sum_{n}^{n} = m = \sum_{n-1}^{n} - \sum_{n}^{m} = i^{-1}$$

$$\sum_{i=11}^{74} = (-5i+(4))^3 - \sum_{i=}^{10} (-5i+4)^3$$

$$= (-5i)^3 + 3(-5i)^2 \cdot (4+3(-5i) - 4^2 + 4^3$$

$$= -125i^3 + 300i^2 + 240i + 64$$

$$\sum_{i=1}^{74} -125i^3 + \sum_{i=1}^{74} 300i^2 - \sum_{i=1}^{74} 240i + \sum_{i=1}^{74} 64$$

①

$$\sum_{n=1}^{n} n^3 = \frac{1}{n} n^2(n+1)^3 = \frac{1}{4} 74^2(74+1)^2 = 7700625 \cdot 125$$

$$= -962578125$$

② $$\sum_c a_n = c \sum a_n = 300 \sum_{i=1}^{74} i^2 = \frac{1}{6} n(n+1)(2n+1)$$

$$= \frac{1}{6}(74)(74+1)(2 \cdot 74+1) = 137825 \cdot 300$$

$$= 41347500$$

③ $$240 \sum_{i=1}^{74} i = \frac{1}{2} 74(74+1) = 2775 \cdot 240$$

$$= 666000$$

④ $$\sum_{i=1}^{74} 64 = 64 \cdot 74 = 4736$$

$$-962578125 + 41347500 - 666000 + 4736 = -921891889$$

(3) $-\sum_{i=1}^{10} 125i^3 + \sum_{i=1}^{10} 300i^2 + \sum_{i=1}^{10} 240i + \sum_{i=1}^{10} 64$

① $\frac{1}{4} 10^2(10+1)^2 = 3025 \cdot 125 = 378125$

② $\frac{1}{6} 10(10+1)(2(10+1)) = 385 \cdot 300 = 115500$

③ $\frac{1}{2} 10(10+1) = 55 \cdot 240 = 13200$

④ $10(64) = 640$

$= -378125 + 115500 - 13200 + 640 = -271581$

$\sum_{i=11}^{94} (-5i+4)^3 = -921891889 - (-271585)$

$= \underline{-921616704}$

④ Calcular la integral indefinida que se muestra

$$\int (\operatorname{sen}(2x))^5 (\cos(2x))^4 \, dx$$

$u = 2x$

$du = 2\,dx$

$dx = \frac{1}{2} du$

$$= \int \operatorname{Sen}(u)^5 \cos(u)^4 \left(\tfrac{1}{2} du\right)$$

$$= 1/2 \int \operatorname{sen}^5(u) (\cos^4(u)) \, du$$

$\operatorname{sen}^2 u = 1 - \cos^2 u$

$$= \int \cos^4(u) ((\cos^2 u - 1)^2) \sin(u) \, du$$

$w = \cos u$

$dw = -\sin u \, du$

$du = -\frac{1}{\sin u}$

$$= \int w^4 (w^2 - 1)^2 \, \cancel{\sin(u)} \, \frac{1}{\cancel{\sin u}}$$

$$= \int w^4 (w^2 - 1)^2$$

$$= \int w^4 (w^4 - 2w^2 + 1)$$

$$= \int w^8 - 2w^6 + w^4$$

$$= \int w^8 - 2\int w^6 + \int w^4 = \frac{w^9}{9} - \frac{2(w^7)}{7} + \frac{w^5}{5}$$

$$= \frac{1}{2}\left[\frac{\cos^9 u}{9} - \frac{2\cos^7 u}{7} + \frac{\cos^5 u}{5}\right] = \frac{1}{18}\cos^9 u - \frac{2}{14}\cos^7 u + \frac{1}{10}\cos^5 u$$

$$= \frac{1}{18}\cos^9(2x) - \frac{2}{14}\cos^7(2x) + \frac{1}{10}\cos^5(2x)$$

$$= -\frac{1}{18}\cos^9(2x) - \frac{1}{7}\cos^7(2x) + \frac{1}{10}\cos^5(2x) + C$$

⑤ Resuelva la integral indefinida indicada:

$$\int \left(\frac{7x^9+3x^5+4x^2}{9x^7}\right) dx$$

$$\frac{1}{9}\int \frac{7x^9}{x^7}dx + \int\frac{3x^5}{x^7}dx + \int\frac{4x^2}{x^7}dx = \frac{1}{9}\int 7x^2\,dx + \int\frac{3}{x^2}dx + \int\frac{4}{x^5}dx$$

$$= \frac{1}{9}\left[7\int x^2\,dx + 3\int\frac{1}{x^2} + 4dx\int\frac{1}{x^5}dx\right]$$

$$= \frac{1}{9}\left[7\left(\frac{x^3}{3}\right) + 3\left(-\frac{1}{x}\right) + 4\left(-\frac{1}{4x^4}\right)\right] = \frac{1}{9}\left(\frac{7x^3}{3} - \frac{3}{x} - \frac{1}{x^4}\right)$$

$$= \frac{1}{27}x^3 - \frac{1}{3x} - \frac{1}{9x^4} + c$$

⑥ Resuelva la integral indicada

$$\int \frac{8x+5}{1+x^2}\,dx$$

$$= \int \frac{8x}{1+x^2}\,dx + \int \frac{5}{1+x^2}\,dx = 8\int \frac{x}{1+x^2}\,dx + 5\int \frac{1}{1+x^2}\,dx$$

$$\int \frac{x}{1+x^2} = \int \frac{x}{u}\left(\frac{1}{2x}\right) = \frac{1}{2}\int \frac{1}{u} = \frac{1}{2}\left[\ln|u|\right] = \frac{\ln|u|}{2}$$

$u = 1+x^2$

$du = 2x\,dx$

$dx = \frac{1}{2x}\,du$

$$= \frac{\ln|1+x^2|}{2} \qquad \int \frac{1}{1+x^2} = \tan^{-1}(x)$$

$$\int \frac{8x+5}{1+x^2}\,dx = 8\left[\frac{\ln|1+x^2|}{2}\right] + 5\tan^{-1}(x)$$

$$= 4\ln|1+x^2| + 5\tan^{-1}(x) + c$$

⑦ Usando la parte uno del teorema fundamental del calculo encontrar la derivada que se indica

$$\frac{d}{dx}\left(\int_1^{2x^3+9x} \left(t^8 \operatorname{sen}(3t^9+5)\right) dt\right)$$

$$= \int \sin(5+3t)\, t^8 \frac{1}{9t^8}\, du = \int \frac{\operatorname{Sen}(5t+3t)}{9}\, du$$

$$\int a \cdot f(x)\, dx = a \int f(x)\, dx$$

$$= \frac{1}{9} \int_1^{(2x^3+9x)^9} \operatorname{Sen}(5+3t)\, dt$$

$$\int f(g(x)) \cdot g(x)\, dx = \int f(u)\, du \quad , \quad v = g(x)$$

$v = 5+3t$ $\qquad \int \sin(v)\left(\frac{1}{3}\, dv\right)$ $\qquad w = 1 \rightarrow v = 8$

$du = 3$ $\qquad w = (2x^3+9x)^9$

$dv = \frac{1}{3}$ $\qquad \hookrightarrow v = 5+3(2x^3+9x)^9$

$$\frac{1}{9} \int_8^{5+3(2x^3+9x)^9} \sin(v) \frac{1}{3}\, du = \frac{1}{9} \cdot \frac{1}{3} \int_8^{5+3(2x^2+9x)^9} \operatorname{Sen}(v)$$

$$\int \operatorname{Sen} v\, dv = -\cos v$$

$$= -\frac{1}{27}\left[-\cos v\right]\Big|_8^{5+3(2x^3+9x)^9} \qquad \int_a^b f(x)\, dx = f(b) - f(a)$$

$$\lim_{x \to b^-} f(x) - \lim_{x \to a} f(t)$$

$\lim_{x \to 5+3(2x^3+9x)^9} - (-\cos v) = -\cos(5+3(2x^3+9x)$

$= 1/27\left(-\cos(3+3(2x^3+9x)^9) + \cos(8)\right)$

$= \operatorname{Sin}\left(3(2x^5+9x)^9+5\right)(6x^2+9)(2x^3+9x)^8$

⑧ Calcule el valor promedio de la función

$$f(t) = 16t^2 + 56t + 49 \quad en \quad [4, 12]$$

$$\bar{f}(t) = \frac{\int_a^b f(t)\,dt}{b-a}$$

$$= \frac{1}{12-4}\left[16\frac{t^3}{3} + 56\frac{t^2}{2} + 49t\right]\Bigg|_4^{12}$$

$$= \frac{1}{8}\left[\frac{16}{3}t^3 + 28t^2 + 49t\right|_4^8$$

$$= \frac{1}{8}\left(\left[\frac{16}{3}(12)^3 + 28(12)^2 + 49(12)\right] - \left[\frac{16}{3}(4)^3 + 28(4)^2 + 49(4)\right]\right)$$

$$= \frac{1}{8}\left[\frac{16}{3}(1728) + 28(144) + 588\right] - \left[\frac{16}{3}(64) + 28(16) + 196\right]$$

$$= \frac{1}{8}\left[\frac{27648}{3} + 4032 + 588\right] - \left[\frac{1024}{3} + 448 + 196\right]$$

$$= \frac{1}{8}\left[\frac{27648}{3} - \frac{1024}{3} + 4032 + 588 - 448 - 196\right]$$

$$= \frac{1}{8}\left[\frac{26,624}{3} + 4003\right] = \frac{1}{8}\left[\frac{26,624}{3} + \frac{12,009}{3}\right]$$

$$= \frac{1}{8}\left(\frac{38633}{3}\right) = \frac{38633}{24}$$