Ejercicio Conocimientos Previos

0. Resolver las siguientes ecuaciones lineales

a) $2x+18=0$
$2x=-18$
$x=\frac{-18}{2}$
$\boxed{x=-9}$

b) $3x-81=0$
$3x=81$
$x=\frac{81}{3}$
$\boxed{x=27}$

c) $12x-36=20$
$12x=20+36$
$x=\frac{56}{12}$
$\boxed{x=4.666}$

d) $5x-3=2x$
$5x-2x=3$
$3x=3$
$\boxed{x=1}$

e) $16x+24=4-3x$
$16x+3x=4-24$
$19x=-20$
$x=-20/19$
$\boxed{x=-1.05}$

f) $\frac{4}{2x-3}=6$
$4=6(2x-3)$
$4=12x-18$
$4+18=12x$
$12x=22$
$x=\frac{22}{12}$
$\boxed{x=1.833}$

g) $\frac{x}{2-x}=6$
$x=6(2-x)$
$x=12-6x$
$7x=12$
$x=\frac{12}{7}$
$\boxed{x=1.71}$

h) $\frac{4}{x-3}=0$
No hay solución
$x\neq 3$

1.) Efectúe los siguientes productos notables:

a) $(x+7)(x-13)=$
$x^2-13x+7x-91$
$=x^2-6x-91$

b) $(2x-7)(2x-7)=$
$4x^2-14x-14x+49$
$=4x^2-28x+49$

c) $(5x-2)(5x+2)=$
$=25x^2+\cancel{10x}-\cancel{10x}-4$
$=25x^2-4$

d) $(s^2-6)(s^2-4)=$
$=s^4-4s^2-6s^2+24$
$=s^4-10s^2+24$

e) $(2x-3)^3=$
$=(2x)^3-3(2x)^2(3)+3(2x)(3)^2-(3)^3$
$=8x^3-36x^2+54x-27$

f) $(m+2)(m+2)=$
$=m^2+2m+2m+4$
$=m^2+4m+4$

g) $(x-\sqrt{2})(x+\sqrt{2})=$
$=(x)^2-(\sqrt{2})^2$
$=x^2-2$

h) $(x-\sqrt{2})^2=$
$=(x)^2-2(x)(\sqrt{2})+(\sqrt{2})^2$
$=x^2-2x\sqrt{2}+2$
$=x^2-2\sqrt{2}x+2$

i) $(x-3)(x-3)(x^2+9)=$
$=(x-3)^2(x^2+9)$
$=x^2+9-2(3x)(x^2+9)$
$=(x^2-6x+9)(x^2+9)$

$=x^4-6x^3+9x^2+9x^2-54x+81$
$=x^4-6x^3+18x^2-54x+81$

2.) Factorice las siguientes expresiones algebraicas

a) $x^2+64x =$
$= x(x+36)$

b) $x^2-81 =$
$= (x-9)(x+9)$

c) $3x^2-6x^2y =$
$= x(3x-6xy)$

d) $4s-s^2 =$
$= s(4-s)$

e) $y^2-64 =$
$= (y-8)(y+8)$

f) $s^2-16s+64 =$
$= (s-8)(s-8)$

g) $y^4-64 =$
$= (y^2-8)(y^2+8)$

h) $m^2-m-20 =$
$= (m-5)(m+4)$

i) $s^3+9s^2 =$
$s(s^2+9s)$

j) $x^2-12x+35$
$= (x-7)(x-5)$

k) $x^2+12x+36 =$
$= (x-6)(x-6)$

i) $x^2-13x+36 =$
$= (x-9)(x-4)$

3) Resolver las siguientes ecuaciones cuadraticas usando factorización

a) $x^2-11x+18=0$
$(x-2)(x-9)$
$x-2=0 \quad x-9=0$
$\boxed{x=2} \quad \boxed{x=9}$

b) $y^2-81=0$
$(y-9)(y+9)$
$y-9=0 \; ; \; y+9=0$
$\boxed{y=9} \quad \boxed{y=-9}$

c) $m^2+12m+36=0$
$(m+6)(m+6)$
$m+6=0 \; , \; m+6=0$
$\boxed{m=-6} \; ; \; \boxed{m=-6}$

d) x^2-2x
$\frac{\cancel{x^2}}{\cancel{x}} = 2$
$\boxed{x=2}$

e) $x^2-16x+64=0$
$(x-8)(x-8)$
$x-8=0 \; ; \; \boxed{x=8}$

f) $3m^2-9m=0$
$m(3m-9)=0$
$m=0 \; , \; 3m-9=0$
$\boxed{m=0} \; , \; 3m=9$
$\boxed{m=3}$

g) $m^2-10m+24=0$
$(m-6)(m-4)$
$m-6=0 \; , \; m-4=0$
$\boxed{m=6} \; , \; \boxed{m=4}$

h) $3x^2=6x$
$3x^2-6x$
$x(3x-6)$
$\boxed{x=0} \; , \; 3x-6=0$
$3x=6$
$x=6/3$
$\boxed{x=2}$

4) Resolver las siguientes ecuaciones cuadráticas usando formula general

a) $x^2 = 4x - 9$

$= x^2 - 4x + 9$

$= -(-4) \pm \sqrt{(-4)^2 - 4(1)(9)} / 2(1)$

$= \dfrac{4 \pm \sqrt{16 - 36}}{2}$

$= \dfrac{4 \pm \sqrt{-20}}{2}$

$X_1 = 4 + 2\sqrt{5i} / 2$

$= 2 + \sqrt{5i}$

$X_2 = 4 - 2\sqrt{5i} / 2$

$= 2 - \sqrt{5i}$

b) $\underset{a}{x^2} \underset{b}{-10x} + \underset{c}{24} = 0$

$x = -(-10) \pm \sqrt{(-10)^2 - 4(1)(24)} / 2(1)$

$x = 10 \pm \sqrt{100 - 96} / 2$

$x = 10 \pm \sqrt{4} / 2$

$\boxed{X_1 = 10 + 2/2 = 6}$

$X_2 = \dfrac{10 - 2}{2} \rightarrow \boxed{X_2 = 4}$

c) $\underset{a}{x^2} + \underset{b}{12x} + \underset{c}{36} = 0$

$x = -(12) \pm \sqrt{12^2 - 4(1)(36)} / 2(1) = -12 \pm \sqrt{144 - 144} / 2 = -12 \pm 0 / 2$

$\boxed{X_1 = \dfrac{-12 + 0}{2} = -6}$

$\boxed{X_2 = \dfrac{-12 - 0}{2} = -6}$

d) $\underset{a}{x^2} - \underset{b}{2x} = 0$

$x = -(-2) \pm \sqrt{(-2)^2 - 4(1)(0)} / 2(1) = 2 \pm \sqrt{4} / 2 = \boxed{X_1 = \dfrac{2+2}{2} = 2}$ $\boxed{X_2 = \dfrac{2-2}{2} = 0}$

e) $\underset{a}{m^2} + \underset{c}{81} = 0$

$m = -(0) \pm \sqrt{(0)^2 - 4(1)(81)} / 2(1) = \pm \sqrt{-324} / 2 = \boxed{X_1 = +81\sqrt{4i} / 2}$ $\boxed{X_2 = -81\sqrt{4i} / 2}$

f) $m^2 - 4m + 8 = 0$

$m = -(-4) \pm \sqrt{(-4)^2 - 4(1)(8)} / 2(1) = 4 \pm \sqrt{16 - 32} / 2 = 4 \pm \sqrt{-16} / 2$

$\boxed{X_1 = 4 + 4\sqrt{4i} / 2}$

$\boxed{X_2 = 4 - 4(4i) / 2}$

g) $2x^2 + 7x = 15$

$2x^2 + 7x - 15 = 0$

$x = -(7) \pm \sqrt{(7)^2 - 4(2)(-15)} / 2(2) = -7 \pm \sqrt{49 + 120} / 4$

$\boxed{X_1 = -7 + 13/4 = +\dfrac{6}{4}}$

$\boxed{X_2 = -7 - 13/4 = -5}$

h) $3r^2 = 9r$

$\underset{a}{3r^2} - \underset{b}{9r} = 0$

$r = -(-9) \pm \sqrt{(-9)^2 - 4(3)(0)} / 2(3)$

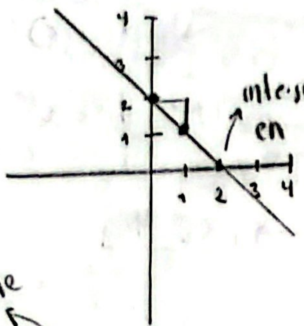
$= +9 \pm \sqrt{81} / 6$

$\boxed{r_1 = \dfrac{+9+9}{6} = 3}$

$\boxed{r_2 = \dfrac{+9-9}{6} = 0}$

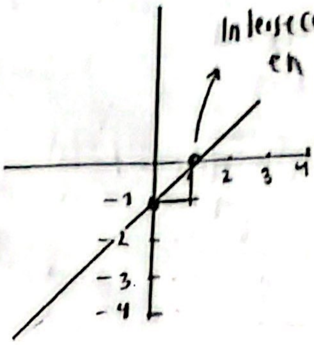
5) Encuentre la gráfica de las siguientes funciones lineales. Ordenada al origen, pendiente e intersección en x

a) $F(x) = \overset{\text{pendiente}}{-x} + \underset{\text{ordenada al origen}}{2}$



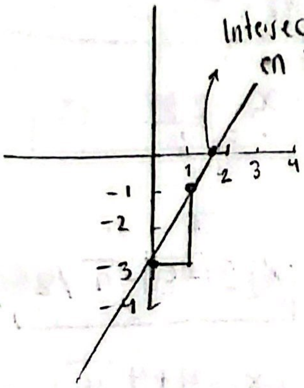
b) $F(x) = x - 1$ ordenada al origen

pendiente = 1



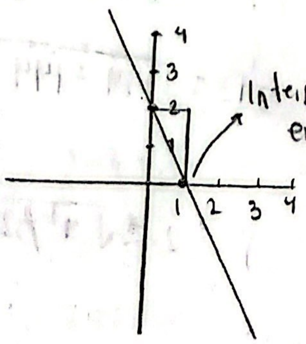
c) $g(x) = 2x - 3$ → ordenada al origen

pendiente



d) $g(x) = -2x + 2$ → ordenada al origen

pendiente



6) Resuelve cada sistema de ecuaciones lineales por los tres métodos

a) Sumas y restas

1) ① $2x - y = 3$
② $-x + y = 1$

$$\begin{array}{r} 2x - \cancel{y} = 3 \\ -x + \cancel{y} = 1 \\ \hline x \quad = 4 \end{array}$$

Sustituimos en ① para encontrar y

$2x - 3 = y$
$2(4) - 3 = y$

$\boxed{x = 4 \quad y = 5}$

2) ① $2x - 3y = 0$
② $2x + 3y = 12$

$$\begin{array}{r} 2x - \cancel{3y} = 0 \\ 2x + \cancel{3y} = 12 \\ \hline 4x \quad = 12 \end{array}$$

$x = \frac{12}{4} = 3$

Sustituimos en ① para encontrar y

$2x = 3y$
$\frac{2x}{3} = y$

$\frac{2(3)}{3} = y$

$\boxed{x = 3 \quad y = 2}$

3) ① $4x + 3y = 18$
② $7x - 5y = 11$

$5(4x + 3y = 18)$
$3(7x - 5y = 11)$

$$\begin{array}{r} 20x + \cancel{15y} = 90 \\ 21x - \cancel{15y} = 33 \\ \hline 41x \quad = 123 \end{array}$$

$x = \frac{123}{41}$

$x = 3$

Sustituimos en ① para encontrar y

$4x - 18 = -3y$
$\frac{4x - 18}{-3} = y$

$\boxed{x = 3 \quad y = +2}$

5) $2x - 4y = 6$ ①
$3x - 7y = 6$ ②

$\longrightarrow 7(2x - 4y = 6)$
$-4(3x - 7y = 6)$

$\longrightarrow 14x - \cancel{28y} = 56$
$-12x + \cancel{28y} = -24$
$2x = 32$
$x = 32/2$
$= 16$

→ Sustituimos en 1 para obtener y

$2x - 6 = 4y$
$= \frac{2x + 6}{4} = y$

→ $\boxed{\begin{array}{l} x = 16 \\ y = 6.5 \end{array}}$

6) Determinantes

1) ① $2x - y = 3$
② $-x + y = 1$

$\Delta = \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} = 2 - 1 = 1$

$\Delta x = \begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix} = 3 + 1 = 4$

$\Delta y = \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} = 2 + 3 = 5$

$x = \frac{\Delta x}{\Delta} = \frac{4}{1} = 4$

$y = \frac{\Delta y}{\Delta} = \frac{5}{1} = 5$

2) ① $2x - 3y = 0$
$2x + 3y = 12$

$\Delta = \begin{vmatrix} 2 & -3 \\ 2 & 3 \end{vmatrix} = 6 + 6 = 12$

$\Delta x = \begin{vmatrix} 0 & -3 \\ 12 & 3 \end{vmatrix} = 0 + 36 = 36$

$\Delta y = \begin{vmatrix} 2 & 0 \\ 2 & 12 \end{vmatrix} = 24$

$\boxed{\begin{array}{l} x = \frac{36}{12} = 3 \\ y = \frac{24}{12} = 2 \end{array}}$

3) ① $4x + 3y = 18$
② $7x - 5y = 11$

$\Delta = \begin{vmatrix} 4 & 3 \\ 7 & -5 \end{vmatrix} = -20 - 21 = -41$

$\Delta x = \begin{vmatrix} 18 & 3 \\ 11 & -5 \end{vmatrix} = -90 - 33 = -123$

$\Delta y = \begin{vmatrix} 4 & 18 \\ 7 & 11 \end{vmatrix} = 44 - 126 = -82$

$\boxed{\begin{array}{l} x = \frac{-123}{-41} = +3 \\ y = \frac{-82}{-41} = 2 \end{array}}$

4) ① $2x - 4y = 6$
② $3x - 7y = 6$

$\Delta = \begin{vmatrix} 2 & -4 \\ 3 & -7 \end{vmatrix} = -14 + 12 = -2$

$\Delta x = \begin{vmatrix} 6 & -4 \\ 6 & -7 \end{vmatrix} = -42 + 24 = -18$

$\Delta y = \begin{vmatrix} 2 & 6 \\ 3 & 6 \end{vmatrix} = 12 - 18 = -6$

$\boxed{\begin{array}{l} x = \frac{-18}{-2} = 9 \\ y = \frac{-6}{-2} = 3 \end{array}}$

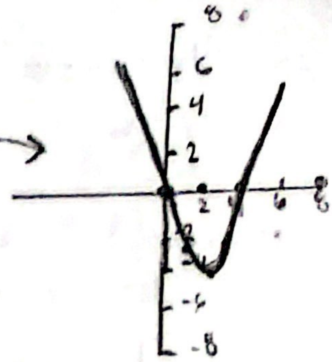
7) Oblenga la gráfica de las siguientes funciones no lineales

a) $f(x) = 4x - x^2$
$= x^2 - 4x$

x	0	1	2	3	4
f(x)	0	-3	-4	-3	0

$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$

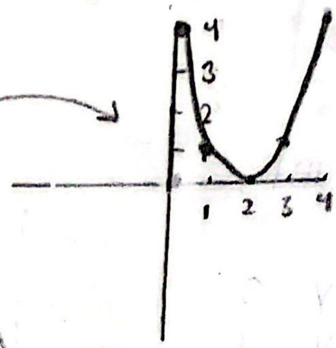
$f(2) = 2^2 + 4(2) = 4 - 8 = -4$



b) $f(x) = x^2 - 4x + 4$

x	0	1	2	3	4
f(x)	4	1	0	1	4

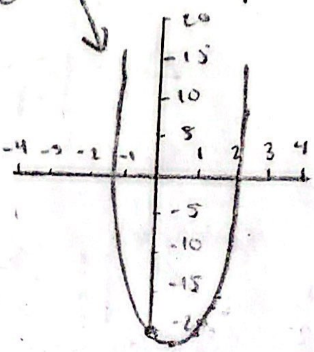
$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$



c) $f(y) = y^2 - y - 20$

y	0	0.5	1	1.5	2
f(y)	-20	-20.25	-20	-19.25	-18

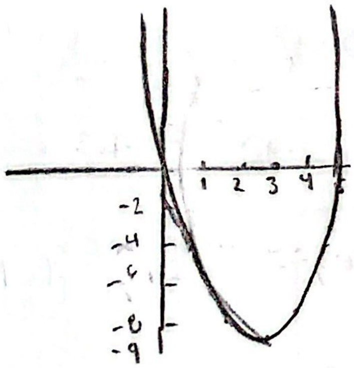
$y = \frac{-b}{2a} = \frac{-(-1)}{2a} = \frac{+1}{2} = \frac{1}{2}$



d) $f(y) = y^2 - 6y$

y	1	2	3	4	5
f(y)	-6	-8	-9	-8	-5

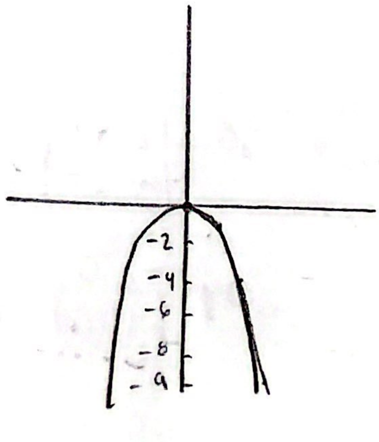
$y = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = \frac{+6}{2} = 3$



f) $f(y) = -y^2$

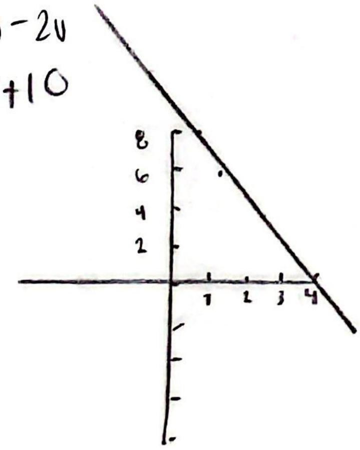
$y = \frac{-b}{2(1)} = \frac{0}{2} = 0$

y	0	1	2	3
f(y)	0	-1	-4	-9



e) $f(v) = 10 - 2v$
$= -2v + 10$

v	1	2	3
f(v)	+8	+6	+4



11) Resuelve las siguientes ecuaciones

8) Descomponer en fracciones parciales

a) $\frac{18}{x^2+2x-8} = \frac{18}{(x+4)(x-2)} = \frac{A}{(x+4)} + \frac{B}{(x-2)} = \frac{A(x-2)+B(x+4)}{(x+4)(x-2)}$

$\frac{18}{\cancel{(x+4)(x-2)}} = \frac{A(x-2)+B(x+4)}{\cancel{(x+4)(x-2)}} \rightarrow 18 = A(x-2)+B(x+4)$

$x=-4 \longrightarrow 18 = A(-4-2)+B(\cancel{-4+4})^{0}$

$= 18 = -6A$

$= \frac{18}{-6} = \boxed{A=-3}$

$x = 2 \longrightarrow 18 = A(\cancel{2-2})^{0}+B(2+4)$

$18 = 6B$

$\frac{18}{6} = \boxed{B = 3}$

$\rightarrow \frac{18}{(x+4)(x-2)} = \frac{-3}{(x+4)} + \frac{3}{(x-2)}$

b) $\frac{12}{s^2-16} = \frac{12}{\cancel{(s+4)(s-4)}} = \frac{A}{(s+4)} + \frac{B}{(s-4)} = \frac{A(s-4)+B(s+4)}{\cancel{(s+4)(s-4)}}$

$\Rightarrow 12 = A(s-4)+B(s+4)$

$s=-4$

$= 12 = A(-4-4)+B(\cancel{-4+4})^{0}$

$12 = -8A$

$\frac{12}{-8} = A$

$s = 4$

$= 12 = A(\cancel{4-4})^{0}+B(4+4)$

$12 = 8B$

$\frac{12}{8} = B$

$\frac{12}{(s+4)(s-4)} = \frac{-\frac{3}{2}}{s+4} + \frac{\frac{3}{2}}{s-4}$

c) $\dfrac{6x^2+50}{(x+3)(x^2+4)} =$

9) Aplica propiedades de logaritmos para expresar c/u de las siguientes expresiones con un solo logaritmo

a) $\ln(x+2)+\ln(x-2)-3\ln(x)$

$= \ln((x+2)(x-2)) - 3\ln(x)$

$= \ln((x+2)(x-2)) - \ln(x^3)$

$= \ln\left(\dfrac{(x+2)(x-2)}{x^3}\right)$

b) $\frac{1}{2}\ln(x^2-4)^2+\ln(x^2-5x+6)-2\ln(x-2)$

$= 2\cdot\frac{1}{2}\ln(x^2-4)+\ln(x^2-5x+6)-\ln((x-2)^2)$

$= 2\cdot\frac{1}{2}\ln(x^2-4)+\ln\left(\dfrac{x^2+5x+6}{(x-2)^2}\right)$

$= 2\cdot\frac{1}{2}\ln(x^2-4)+\ln\left(\dfrac{(\cancel{x-2})(x-3)}{(x-2)^{\cancel{2}}}\right)$

$= 2\cdot\frac{1}{2}\ln(x^2-4)+\ln\left(\dfrac{x-3}{x-2}\right)$

$= \ln(x^2-4)+\ln\left(\dfrac{x-3}{x-2}\right)$

10) Utiliza propiedades de la función exponencial para realizar operaciones

a) $e^{2x}(e^{3x})$

$= e^{2x+3x}$

$= e^{5x}$

b) $(e^{3x})^2$

$= e^{3x(2)}$

$= e^{6x}$

c) $e^{2x}e^{-3x}$

$= e^{2x-3x}$

$= e^{-x}$

d) $e^{-2x}e^{-3x}$

$= e^{-2x-3x}$

$= e^{-5x}$

e) $(e^{2x}-4)(e^{+2x}-4)$

$= (e^{2x}-4)^2$

$= (e^{2x})^2-2(e^{2x})(4)-(4)^2$

$= e^{4x}-8e^{2x}+16$

$=$

f) $(e^{3x}-e^{-3x})(e^{3x}+e^{-3x})$

$= (e^{3x})(e^{3x})+(e^{3x})(e^{-3x})-(e^{-3x})(e^{3x})-(e^{-3x})(e^{-3x})$

$= e^{6x}\cancel{+e}\cancel{-e}-e^{-6x}$

$= e^{6x}-e^{-6x}$

k) $\dfrac{e^{4\ln x}}{x^6} = \dfrac{x^4}{x^6}$

g) $\dfrac{e^{3x}}{e^{4x}}$

$= e^{3x-4x}$

$= e^{-x}$

h) $\dfrac{e^{2x}e^{-4x}}{e^{-3x}}$

$= \dfrac{e^{-2x}}{e^{-3x}}$

$= e^{-2x-(-3x)}$

$= e^{x}$

j) $\dfrac{e^{4x}e^{2x}}{(e^{-2x})^2}$

$= \dfrac{e^{6x}}{e^{-4x}}$

$= e^{6x-(-4x)}$

$= e^{10x}$

i) $\dfrac{e^{4\ln x}}{x-3} = \dfrac{e^{\ln(x^4)}}{x^{-3}}$

$= \dfrac{x^4}{x^{-3}} = x^7$

7) $f(x) \quad x \quad f(x)' (x^2+1)(e^x)' - e^x(x^2+1)' = \frac{(x^2+1)e^x - e^x 2x+1}{}$

11) Resuelve las siguientes ecuaciones

a) $|x-2| = 6$

$= x - 2 = 6$

$x = 6+2$

$\boxed{x = 8}$

b) $\frac{x}{2-3x} = 2$

$= x = 2(2-3x)$

$= x = 4 - 6x$

$= x + 6x = 4$

$7x = 4$

$\boxed{x = 4/7}$

c) $\frac{2x-2}{2-x} = 4$

$= 2x - 2 = 4(2-x)$

$= 2x - 2 = 8 - 4x$

$= 2x + 4x = 8+2$

$= 6x = 10$

$x = \frac{10}{6}$

d) $e^{5-3x} = 10$

$= 5 - 3x = \ln(10)$

$-3x = \ln(10) - 5$

$x = \frac{\ln(10) - 5}{-3}$

e) $e^{2x} - 4 = 0$

$2x = \ln(4)$

$x = \frac{\ln(4)}{2}$

$x = \ln(2)$

f) $\ln(3x-10) = 2$

$3x - 10 = e^2$

$3x = e^2 + 10$

$x = \frac{e^2+10}{3}$

g) $\ln(y-1) + \ln(y+1) = 0$

$\ln((y-1)(y+1)) = 0$

$\ln(y^2 - 1) = 0$

$y^2 - 1 = e^0$

$y^2 = 1 + 1$

$y = \sqrt{2}$

h) $\ln(x) + \ln(x-1) = 1$

$\ln(x(x-1)) = 1$

$\ln(x^2 - x) = 1$

$x^2 - x = e^1$

$x^2 - x - e^1 = 0$

$x = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2}$

$x = \frac{1 \pm \sqrt{1+4e}}{2}$

$x_1 = \frac{1 + \sqrt{1+4e}}{2}$

$x_2 = \frac{1 - \sqrt{1+4e}}{2}$ X

$x < 0 \therefore$

no solución

i) $e^{1-x} = x$

$= e^x x = e$

$= e^{1-x} - x = 0$

$= x = 1$

12) Obtenga la primera y segunda derivada de las siguientes funciones

a) $f(x) = x^2 - 4x^3$

$f'(x) = 2x - 12x^2$

$f''(x) = 2 - 24x$

b) $f(y) = y^3 - 4y$

$f'(y) = 3y^2 - 4$

$f''(y) = 6y$

c) $f(x) = x^2 \cos(2x)$

$f'(x) = (x^{2})' \cos 2x + x^2 (\cos 2x)'$

$= 2x \cos 2x + x^2 [-\sin 2x (2)$

$= 2x \cos 2x - 2x^2 \sin(2x)$

$f''(x) = (2x)' \cos 2x + 2x (\cos 2x)' - [(2x^2)' \sin(2x) + 2x^2 (\sin(2x))'$

$= x \cos(2x) + 2x(-\sin(2x)(2)) - [4x \sin 2x + 2x^2 (\cos(2x)(2)]$

$= 2\cos(x) - 4x \sin(2x) - 4x \sin(2x) - 4x^2 \cos(2x)$

$= (2 - 4x^2) \cos x - 8x \sin(2x)$

d) $f(x) = 2xe^{6x}$

$f'(x) = (2x)' e^{6x} + 2x (e^{6x})'$

$= 2e^{6x} + 2x(6e^{6x})$

$= 2e^{6x} + 2x6e^{6x}$

$= 2e^{6x} + 12xe^{6x}$

$f''(x) = [\cancel{(2)'}^{\,0} e^{6x} + 2(e^{6x})'] + [(12x)' e^{6x} + 12x (e^{6x})']$

$= 12e^{6x} + 12e^{6x} + 72xe^{6x}$

$= 24e^{6x} + 72xe^{6x}$

e) $f(x) = e^{6x} \sin(3x)$

$f'(x) = (e^{6x})' \sin(3x) + e^{6x} (\sin(3x))'$

$= 6e^{6x} \sin(3x) + e^{6x} (\cos(3x)(3))$

$= 6e^{6x} \sin(3x) + 3e^{6x} \cos(3x)$

$f''(x) = [(6e^{6x})' \sin 3x + 6e^{6x} (\sin(3x))']$

$+ [(3e^{6x})' \cos(3x) + 3e^{6x} (\cos(3x))']$

$= 36e^{6x} \sin(3x) + 18e^{6x} \cos(3x)$

$+ 18e^{6x} \cos(3x) - 9e^{6x} \sin(3x)$

$= 27e^{6x} \sin(3x) + 36e^{6x} \cos(3x)$

f) $f(x) = e^{2x} \cos(3x)$

$f'(x) = (e^{2x})' \cos 3x + e^{2x} (\cos(3x))'$

$= 2e^{2x} \cos 3x - 3e^{2x} \sin(3x)$

$f''(x) = [(2e^{2x})' \cos 3x + 2e^{2x} (\cos(3x))'] - [(3e^{2x})' \sin(3x) + (3e^{2x})(\sin(3x))']$

$= 4e^{2x} \cos 3x - 6e^{2x} \sin(3x) - 6e^{2x} \sin(3x) - 9e^{2x} \cos(3x)$

$= -5e^{2x} \cos(3x) - 12e^{2x} \sin(3x)$

g) $f(x) = \frac{e^x}{x^2+1}$ $f(x)' = \frac{(x^2+1)(e^x)' - e^x(x^2+1)'}{(x^2+1)^2} = \frac{(x^2+1)e^x - e^x 2x}{x^4+2x^2+1}$ A

$$f(x)'' = \frac{(x^4+2x^2+1)((x^2+1)e^x - e^x 2x)' - (x^4+2x^2+1)'[(x^2+1)e^x - e^x 2x]}{(x^4+2x^2+1)^2}$$

$$f''(x) = \frac{(x^4+2x^2+1)[(x^2+1)e^x - 2e^x] - [4x^3+4x)[(x^2+1)e^x - 2xe^x]}{(x^4+2x^2+1)^2}$$ A

h) $f(x) = \ln(1-r)$

$f'(r) = \frac{1}{1-r}(1-r)' = \frac{-1}{1-r} = -(1-r)^{-1}$ A

$f''(1) - (1-r)^{-2} = \frac{1}{(1-r)^2}$ A

i) $f(t) = te^{t^2}$

$f'(t) = t(e^{t^2})' + t'(e^{t^2}) = te^{t^2}(t^2)' + e^{t^2}$

$f'(t) = te^{t^2}(2t) + e^{t^2} = 2t^2e^{t^2} + e^{t^2}$ A

$f''(t) = 2t^2(e^{t^2})' + (2t^2)'(e^{t^2}) + e^{t^2}(t^2)'$

$= 4t^3e^{t^2} + 6te^{t^2}$ A

13) Obtenga la derivada implicita de las siguientes relaciones

a) $x^2+y^2=4$

$\frac{d\, x^2+y^2}{dx} = \frac{d4}{dx}$

$\frac{dx^2}{dx} + \frac{dy^2}{dx} = 0$

$2x + 2y\frac{dx}{dx} = 0$

$= 2y\frac{dy}{dx} = -2x$

$= \frac{dy}{dx} = \frac{-2x}{2y}$

$= \frac{-x}{y}$ A

b) $x^3y - y^2 = 3$

$$\frac{dx^3y - y^2}{dx} = \frac{d3}{dx}$$

$$\frac{dx^3y}{dx} - \frac{dy^2}{dx} = 0$$

$$3x^2y + x^3 - 2y\frac{dy}{dx} = 0$$

$$x^3 - 2y\frac{dy}{dx} = -3x^2y$$

$$\frac{dy}{dx} = \frac{-3x^2y}{x^3-2y}$$

c) $xy^2 - y = 3$

$$\frac{dxy^2 - y}{dx} = \frac{d3}{dx}$$

$$\frac{dxy^2}{dx} - \frac{dy}{dx} = 0$$

$$y^2 + 2xy - 1\frac{dy}{dx} = 0$$

$$2xy - 1\frac{dy}{dx} = y^2$$

$$\frac{dy}{dx} = \frac{-y^2}{2xy-1}$$

14) Utilice integración por sustitución, por partes o por formula

a) $\int x^2 - 4x^3 dx = \int x^2 dx - 4\int x^3 dx = \frac{x^3}{3} - 4\left(\frac{x^4}{4}\right) = \frac{x^3}{3} - x^4 + c$

b) $\int 3e^{6x} dx = 3\int e^{6x} dx = 3\int e^u \left(\frac{1}{6}\right) du = \frac{3}{6}\int e^u du = \frac{1}{2}e^u + c$

c) $\int \frac{4}{2x-4} dx = 4\int \frac{dx}{2x-4} = 4\int \frac{1}{u}\left(\frac{du}{2}\right) = \frac{4}{2}\int \frac{du}{u} = 2\ln|u| + c$

$u = 2x-4$, $\frac{du}{dx} = 2$, $dx = \frac{du}{2}$ $= 2\ln|2x-4| + c$

d) $\int \cos(3x)dx = \int \cos(u)\frac{du}{3} = \frac{1}{3}\int \cos(u)du = \frac{1}{3}\operatorname{sen}(u) + c = \frac{1}{3}\operatorname{sen}(3x) + c$

$u = 3x$, $\frac{du}{dx} = 3$, $dx = \frac{du}{3}$

e) $\int \frac{2}{y-2} dy = 2\int \frac{1}{u} dy = 2\int \frac{du}{u} = 2\ln|u| + c = 2\ln(y-2) + c$

$u = y-2$, $\frac{du}{dy} = 1$, $dy = du$

f) $\int \frac{3}{4-y} dy = 3\int \frac{1}{u} dy = 3\int \frac{-du}{u} = -3\ln|u| + c = -3\ln|4-y| + c$

$u = 4-y$, $\frac{du}{dy} = -1$, $dy = -du$

g) $\int r^2\sqrt{1+r^3}\,dr = \int r^2\sqrt{u}\,\frac{du}{3r^2} = \frac{1}{3}\int r^2\sqrt{u}\,du = \frac{1}{3}\int (u)^{1/2}du$

$u = 1+r^3,\ \frac{du}{dr} = 3r^2,\ dr = \frac{du}{3r^2}$ $= \frac{1}{3}\cdot\left(\frac{u^{3/2}}{3/2}\right) = \frac{2}{9}u^{3/2}+c = \frac{2}{9}(1+r^3)^{3/2}+c$

h) $\int y^2+\frac{1}{y^2}\,dy = \int y^2dy + \int \frac{1}{y^2}dy = \frac{y^3}{3}+\frac{y^{-1}}{-1} = \frac{y^3}{3}-\frac{1}{y}+c$

i) $\int x\,sen(2x)dx = \frac{-x\cos(2x)}{2} - \int -\frac{\cos(2x)}{2}dx \rightarrow u=2x,\ \frac{du}{dx}=2 \rightarrow dx=\frac{du}{2}$

$u = x \quad dv = sen(2x)$

$du = 1 \quad v = -\frac{1}{2}\cos(2x)$ $= \frac{-x\cos(2x)}{2} - -\frac{1}{4}\int \cos(u)\,du$

$= \frac{-x\cos(2x)}{2}+\frac{sen(2x)}{4}+c$

j) $x\ln(x)dx = \frac{x^2\ln(x)}{x} - \int\frac{x^2dx}{2x} = \frac{x^2\ln(x)}{x} - \int\frac{x}{2}dx$

$u = \ln(x)\ dv = x$

$du = \frac{1}{x}\quad v = \frac{x^2}{x}$ $= \frac{x^2\ln(x)}{x} - \frac{1}{2}\int x\,dx = \frac{x^2\ln(x)}{x} - \frac{1}{2}\frac{x^2}{2}$

$= \frac{x^2\ln(x)}{x} - \frac{x^2}{4}+c$

k) $x\cos(x)dx = uv-\int v\,du = x\,sen\,x + \int sen\,x\,dx$

$u = x\quad dv = \cos(x)$ $= x\,sen\,x - (-\cos(x))$

$du = dx\quad v = sen(x)$ $= x\,sen\,x + \cos(x)+c$

l) $\int e^{6x}\cos(3x)dx = uv-\int v\,du = \cos(3x)\frac{1}{6}e^{6x} - \int\frac{1}{6}e^{6x}(-3sen(3x))dx$ $\quad u = sen(3x)\ dv = e^{6x}\quad du = 3\cos(3x)\ v = \frac{1}{6}e^{6x}$

$u = \cos(3x) \quad dv = e^{6x}$ $= \frac{1}{6}e^{6x}\cos(3x)+\frac{1}{2}\int e^{6x}sen(3x)dx$

$du = -3sen(3x)\quad v = \frac{1}{6}e^{6x}$

$= \frac{1}{6}e^{6x}\cos(3x)+\frac{1}{2}(uv-\int v\,du) = \frac{1}{6}e^{6x}\cos(3x)+\frac{1}{2}\left[sen(3x)\left(\frac{1}{6}e^{6x}\right)\right]$

$- \int\frac{1}{6}e^{6x}(3\cos(3x)dx)\Big] = \frac{1}{6}e^{6x}\cos(3x)+\frac{1}{12}e^{6x}sen(3x) - \frac{1}{4}\int e^{6x}\cos(3x)dx$

$\Rightarrow \int e^{6x}\cos(3x)dx + \frac{1}{4}\int e^{6x}\cos(3x)dx = \frac{1}{6}e^{6x}\cos(3x)+\frac{1}{12}e^{6x}sen(3x)$

$\Rightarrow \frac{5}{4}\int e^{6x}\cos(3x)dx = \frac{1}{6}e^{6x}\cos(3x)+\frac{1}{12}e^{6x}sen(3x)$

$\Rightarrow \int e^{6x}\cos(3x)dx = \frac{2}{15}e^{6x}\cos(3x)+\frac{1}{15}e^{6x}sen(3x)+c$

15) Utilice integración por fracciones parciales

a) $\int \frac{4}{x^2-x}\,dx = 4\int \frac{dx}{x^2-x} = 4\int \frac{dx}{x(x-1)} = 4\left[\frac{1}{x(x-1)}\right] = \int \frac{A}{x} + \frac{B}{(x-1)}\,dx$

$= 4\int \frac{A(x-1)+Bx}{x(x-1)}\,dx \rightarrow 1 = Ax - A + Bx = A + B(x) - A$

$\rightarrow A+B=0$ $\quad -1+B=0$

$A=1$ $\quad B=1$

$4 = \int -\frac{1}{x} + \frac{1}{x-1}\,dx = 4\left[\int \frac{dx}{x} + \int \frac{dx}{x-1}\right.$

$= 4\left[-\ln|x| + \int \frac{du}{u}\right. = -4\ln x + 4\ln|u| + c$

$u = x-1,\ \frac{du}{dx} = 1,\ dx = du$

$= -4\ln|x| + 4\ln|x-1| + c$

b) $\int \frac{4}{y^2-4}\,dy = 4\int \frac{dy}{(y+2)(y-2)} = 4\int \frac{A}{y+2} + \frac{B}{y-2}\,dy = 4\int \frac{A(y-2)+B(y+2)}{(y+2)(y-2)}\,dy$

$1 = Ay - 2A + By + 2B = A + By + (-2A+2B)$

$A+B=0$ $\quad 2(A+B=0)$ $\quad 2\cancel{A}+2B=0$ $\quad B=1/4$ $\quad A+B=0$, $A=-1/4$

$-2A+2B=1$ $\quad -2A+2B=1$ $\quad -2\cancel{A}+2B=1$

$4B=1$

$= 4\int \frac{-1/4}{y+2} + \frac{\frac{1}{4}}{y-2}\,dy = 4\left[-\frac{1}{4}\int \frac{dy}{y+2} + \frac{1}{4}\int \frac{dy}{y-2}\right. = -\int \frac{du}{u} + \frac{dv}{v}$

$u = y+2$, $du = 1$, $dy = du$ | $v = y-2$, $v = 1$, $y = dv$

$= -\ln|u| + \ln|v| + c$

$= -\ln|y+2| + \ln|y-2| + c$

16) Calcule los siguientes determinantes

a) $\begin{vmatrix} 2 & 4 \\ 2 & 5 \end{vmatrix} = 10 - 8 = 2$

b) $\begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 12 - 12 = 0$

c) $\begin{vmatrix} 2 & 4 \\ 3 & 9 \end{vmatrix} = 18 - 12 = 6$