DEPARTAMENTO DE MATEMÁTICAS Y FÍSICA

Materia: **CÁLCULO INTEGRAL**

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TAREA 10 REGLA L'HOPITAL

① $\lim_{x \to 1} \frac{\ln(2x^2-1)}{\tan(x-1)} = \frac{\ln(2(1)^2-1)}{\tan(1-1)} = \frac{\ln(2-1)}{\tan(0)} = \frac{0}{0}$ IND

$$= \frac{\frac{d}{dx}\ln(2x^2-1)}{\frac{d}{dx}\tan(x-1)} = \frac{\frac{4x}{2x^2-1}}{\sec^2(x-1)\cdot(1)} = \frac{\frac{4x}{2x^2-1}}{\sec^2(x-1)}$$

$$= \frac{\frac{4(1)}{2(1)^2-1}}{\sec(1-1)} = \frac{\frac{4}{2-1}}{\sec(0)} = \frac{\frac{4}{1}}{\sec(0)} = \frac{4}{\sec(0)} = \frac{4}{1} = 4$$

② $\lim_{x \to 1} \frac{1-\cos(x-1)}{(\ln x)^2} = \frac{1-(\cos(1-1))}{(\ln 1)^2} = \frac{0}{0}$ IND

$$\lim_{x \to 1} \frac{(1-\cos(x-1))'}{((\ln x)^2)'} = \frac{\operatorname{Sen}(x-1)}{\frac{2}{x}\ln(x)} = \frac{\operatorname{Sen}(1-1)}{\frac{2}{1}\ln 1} = \frac{0}{0}$$ IND

$$\lim_{x \to 1} \frac{(\operatorname{Sen}(x-1))'}{(\frac{2}{x}\ln x)'} = \frac{\cos(x-1)}{-2/x^2\ln(x) + 2/x^2} = \frac{\cos(1-1)}{-\frac{2}{1^2}\ln(1) + \frac{2}{1^2}} = \frac{\cos(0)}{-2(0)+2} = \frac{1}{2}$$

③ $\lim_{x \to 0} (\cot x - 1/x) = \lim_{x \to 0} \left(\frac{\cos x}{\operatorname{Sen} x} - \frac{1}{x}\right) = \lim_{x \to 0} \left(\frac{x\cos x - \operatorname{Sen} x}{x \operatorname{sen}(x)}\right)$

$$= \lim_{x \to 0} \left(\frac{0(\cos(0)) - \operatorname{sen}(0)}{0(\operatorname{sen}(0))}\right) = \frac{0}{0}$$ IND.

$$\lim_{x \to 0} \left(\frac{x\cos(x) - sen(x)}{x\ sen\ x} \right)' = \cos x - x sen x - \cos x = -x\ sen\ x$$

$$= x' sen x + sen(x)' x = sen(x) + x\cos(x)$$

$$= \frac{-x\ sen x}{sen(x) + x\cos(x)} = \frac{0}{0}$$

$$\lim_{x \to 0} \left(\frac{-x\ sen(x)}{sen(x) + x\cos(x)} \right)' = \lim_{x \to 0} \frac{-sen x - x\cos x}{2\cos x - x\ sen x} = \frac{-sen(0) - (0)\cos(0)}{2\cos(0) - 0(sen(0))}$$

$$= \frac{-0 - (0)(1)}{2(1) - (0)(0)} = \frac{-0-0}{2-0} = \frac{0}{2} = 0$$

④ $\lim_{x \to 0^+} (2x)^{1/x}$

$$= \lim_{x \to 0} \left(\frac{\ln(2x)}{x} \right)' = \lim_{x \to 0} \left(\frac{1}{x} \right)$$

$y = 2x^{1/x}$

$$\ln y = \frac{1}{x} \ln(2x) = \frac{\ln(2x)}{x} \qquad = \lim_{x \to 0} \left(\frac{0}{1} \right) = 0$$

$$\lim_{x \to 0^+} (2x)^{1/x} = 0$$

⑤ $\lim\limits_{x\to 0} (\cot x)^{\operatorname{sen} x}$

$y = (\cot x)^{\operatorname{sen} x}$

$\ln(y) = \ln(\cot x)^{\operatorname{sen} x} =$ $\lim\limits_{x\to 0} \ln\left(\cot x^{\operatorname{sen} x}\right) = \lim\limits_{x\to 0} \left(\operatorname{sen} x \ln(\cot(x))\right)$

$= \lim\limits_{x\to 0} \dfrac{\ln \cot(x)}{\frac{1}{\operatorname{sen}}} = \dfrac{\infty}{\infty}$

$= \lim\limits_{x\to 0} \dfrac{\ln(\cot x)}{\csc(x)} = \lim\limits_{x\to 0} \dfrac{\frac{1}{\cot x}(-\csc^2 x)}{-\csc(x)\cot x}$

$= \lim\limits_{x\to 0} \dfrac{\csc^{\cancel{2}}(x)}{\cancel{\csc}(x)\cot^2(x)} = \lim\limits_{x\to 0} \left(\dfrac{\csc(x)}{\cot^2(x)}\right)' = \dfrac{\infty}{\infty}$

$= \lim\limits_{x\to 0} \dfrac{-\csc(x)\cot(x)}{2\cot(x)(-\csc^2(x))} = \lim\limits_{x\to 0} \dfrac{\cancel{\csc}(x)\cancel{\cot}(x)}{2\cancel{\cot}(x)\csc^{\cancel{2}}(x)}$

$= \lim\limits_{x\to 0} \dfrac{1}{2\csc(x)} = \dfrac{1}{2}\lim\limits_{x\to 0} \operatorname{sen}(x)$

$\lim\limits_{x\to 0}(\ln y) = 0 = \ln\left(\lim\limits_{x\to 0} y\right) = \emptyset = \lim\limits_{x\to 0} y = e^0 = 1$

TAREA 11 INTEGRALES IMPROPIAS

① $\int_3^{\infty} \frac{1}{x^4}\, dx = \int_3^{b} \frac{1}{x^4}\, dx = \lim_{b\to\infty} \int_2^{b} \frac{1}{x^4}$

$= \int \frac{1}{x^4}\, dx = \int x^{-4}\, dx = x^{-4+1} = \frac{x^{-3}}{-3} = -\frac{1}{3x^3}$

$= \lim_{b\to\infty} \left[-\frac{1}{3x^3} \Big|_2^b \right] = \lim_{b\to\infty} \left[-\frac{1}{3(b)^3} - \left(-\frac{1}{3(3)^3} \right) \right]$

$= -\cancel{\frac{1}{3(\infty)^3}} + \cancel{\frac{1}{3(3)^3}} = \cancel{\frac{1}{\infty 3}}^{0} + \frac{1}{81} = \frac{1}{81}$

② $\int_{-\infty}^{3} e^{2x} = \int_a^3 e^{2x}\, dx = \lim_{a\to-\infty} \int_a^3 e^{2x}$

$= \lim_{a\to-\infty} \left[\frac{1}{2} e^{2x} \Big|_a^3 \right] = \lim_{a\to-\infty} \left[\frac{1}{2} e^{2(3)} - e^{2(a)} \right]$

$= \lim_{a\to-\infty} \left[\frac{1}{2} e^{2(3)} - \cancel{e^{2(-\infty)}}^{0} \right] = \frac{1}{2} e^{6}$

(3) $\int_e^{\infty} \frac{1}{x(\ln x)^3} dx = \lim_{b \to \infty} \int_e^b \frac{1}{x(\ln x)^3} dx$

$= \int \frac{1}{x(\ln x)^3} dx = \int \frac{1}{\cancel{x}(u)^3} (\cancel{x}\, du) = \int u^{-3} du = -\frac{1}{2} u^{-2}$

$u = \ln x$
$du = 1/x \, dx$
$dx = x\, du$

$= -\frac{1}{2(\ln x)^2} =$

$= \lim_{b \to \infty} \left[-\frac{1}{2(\ln x)^2} \Big|_e^a \right] = \lim_{b \to \infty} \left[-\frac{1}{2(\ln a)^2} - \left(-\frac{1}{2(\ln e)^2} \right) \right]$

$= -\frac{1}{2(\ln \infty)^2} + \frac{1}{2(\ln 1)^2} = -\cancel{\frac{1}{2 \cdot \infty}}^{0} + \frac{1}{2} = \frac{1}{2}$ //

(4) $\int_{-\infty}^{0} \frac{x}{(x^2+9)^2} dx = \lim_{a \to -\infty} \int_a^0 \frac{x}{(x^2+9)^2} dx$

$u = x^2+9$
$du = 2x\, dx$
$dx = \frac{1}{2x} du$

$= \int \frac{x}{u^2} dx = \int \frac{\cancel{x}}{u^2} \left(\frac{1}{2\cancel{x}} \right) = \frac{1}{2} \int u^{-2} = \frac{1}{2} \frac{u^{-1}}{1}$

$= -\frac{1}{2} \frac{1}{x^2+9} = -\frac{1}{2x^2+18}$

$= \lim_{a \to -\infty} \left[-\frac{1}{2x^2+18} \Big|_a^0 \right] = \lim_{a \to -\infty} \left[-\frac{1}{2(0)^2+18} + \frac{1}{2a^2+18} \right] =$

$= -\frac{1}{18} + \frac{1}{2(-\infty)+18} = -\frac{1}{18} + \cancel{\frac{1}{\infty}}^{0} = -\frac{1}{18}$ //

⑤ $\int_{\frac{2}{\pi}}^{\infty} \frac{sen(1/x)}{x^2} dx = \lim_{b\to\infty} \int_{\frac{2}{\pi}}^{b} \frac{sen(1/x)}{x^2} dx$

$= \int_{\frac{\pi}{x}}^{b} \frac{sen(1/x)}{x^2} dx = \int \frac{sen(u)}{\cancel{x^2}}\left(-\cancel{x^2}\right) = -\int sen(u)\, du$

$u = 1/x$

$du = -x^{-2} dx$ $\quad = -\left(-cos(u)\right) = cos(1/x)$

$dx = -x^2 du$

$= \lim_{b\to\infty} \left[cos(1/x) \Big|_{\frac{2}{\pi}}^{b} \right] = \lim_{b\to\infty} \left[cos\left(\frac{1}{b}\right) - cos\left(\frac{1}{\frac{2}{\pi}}\right) \right]$

$= cos\left(\cancel{\frac{1}{\infty}}^{0}\right) - \cancel{cos\left(\frac{\pi}{2}\right)}^{0} = cos(0) - 0 = 1 - 0 = \underline{1}$

⑥ $\int_{0}^{\infty} e^{-x} senx\, dx = \lim_{b\to\infty} \int_{0}^{b} e^{x} sen x\, dx$

$\int e^{-x} sen\, x\, dx$ $\quad = sen\, x\left(-e^{-x}\right) - \int -e^{-x}\left(cos\, x\, dx\right)$

$u = sen\, x \to du = cos\, x\, dx$ $\quad u = cos\, x \to du = -sen\, x$

$dv = e^{-x}$ $\quad v = \int e^{-x} = -e^{-x}$ $\quad dv = -e^{-x} \to v = -\int e^{-x} = +e^{-x}$

$= -e^{-x} senx - \left[cos\, x\left(e^{-x}\right) - \int e^{-x}\left(-sen\, x\right)\right.$

$= -e^{-x} sen\, x - e^{-x} cos\, x + \int e^{-x} sen\, x$

$2\int e^{-x} sen\, x\, dx = -e^{-x} sen\, x - e^{-x} cos\, x = -1/2\, e^{-x} sen\, x - 1/2\, e^{-x} cos\, x$

$= \lim_{b\to\infty} \left[-1/2\, e^{-x} sen\, x - 1/2\, e^{-x} cos\, x \Big|_{0}^{b} \right]$

$$= \lim_{b\to\infty} \left[\left(-\tfrac{1}{2} e^{-b} \operatorname{sen} b - \tfrac{1}{2} e^{-b} \cos b \right) - \left(-\tfrac{1}{2} \cancel{e^{0}}^{1} \operatorname{sen}(0)^{0} - \tfrac{1}{2} \cancel{e^{0}}^{1} \cos(0)^{1} \right) \right]$$

$$= -\tfrac{1}{2} e^{-\infty} \operatorname{sen}(\infty) - \tfrac{1}{2} e^{-\infty} \cos(\infty) + \tfrac{1}{2}(1)(0) + \tfrac{1}{2}(1)(1)$$

$$= \underline{1/2} \;/\!\!/$$

⑦ $\displaystyle\int_1^{\infty} \left[1/x - 1/x+1 \right] dx = \lim_{b\to\infty} \int_1^{b} \left[\frac{1}{x} - \frac{1}{x+1} \right] dx$

$$= \int 1/x - \frac{1}{x+1} \, dx = \int \frac{1}{x} \, dx - \int \frac{1}{x+1} \, dx \qquad \begin{aligned} u &= x+1 \\ du &= dx \end{aligned}$$

$$= \ln|x| - \ln|x+1|$$

$$= \lim_{b\to\infty} \left[\ln|x| - \ln|x+1| \Big|_1^b \right] = \lim_{b\to\infty} \left[(\ln(b) - \ln(b+1)) - (\ln(1) - (\ln(1+1)) \right]$$

$$= \lim_{b\to\infty} \ln(b) - \ln(b+1) + \lim_{b\to\infty} \ln(2) = \lim_{b\to\infty} \ln\left(\frac{b}{b+1}\right) + \ln(2)$$

$$= \ln\left(\lim_{b\to\infty} \left(\frac{b}{b+1} \right) \right) + \ln(2)$$

$$= \ln(\lim(1/1)) + \ln(2)$$

$$= \ln(\lim(1)) + \ln(2)$$

$$= \ln(1) + \ln(2) = \underline{\ln(2)} \;/\!\!/$$

⑧ $\int_{-\infty}^{-2} \frac{x^2}{(x^3-1)^2} dx = \lim_{a \to -\infty} \int_a^{-2} \frac{x^2}{(x^3-1)^2} dx$

$u = x^3 - 1$
$du = 3x^2$
$dx = 1/3x^2 \, du$

$= \int \frac{x^2}{(x^3-1)^2} dx = \int \frac{\cancel{x^2}}{u^2} \left(\frac{1}{3\cancel{x^2}} du \right) = \frac{1}{3} \int u^{-2} du$

$= \frac{1}{3} \frac{u^{-1}}{-1} = -\frac{1}{3x^3-3}$

$= \lim_{a \to \infty} \left[-\frac{1}{3x^3-3} \Big|_a^{-2} \right] = \lim_{a \to -\infty} \left[-\frac{1}{3(-2)^3-3} - \left(-\frac{1}{3a^3-3} \right) \right]$

$= -\frac{1}{-27} + \frac{1}{3(\infty)^3-3}^{\,0} = -\frac{1}{27}$

⑨ $\int_0^1 \frac{1}{x^{0.99}} dx = \lim_{a \to 0^+} \int_a^1 \frac{1}{x^{0.99}} dx$

$= \lim_{a \to 0^+} \int_a^1 \frac{1}{x^{0.99}} dx = \int \frac{1}{x^{0.99}} dx = \int x^{-.99} dx = \frac{x^{0.01}}{0.01}$

$= \lim_{a \to 0^-} \left(\frac{1^{0.01}}{0.01} - \frac{a^{0.01}}{0.01} \right) = \frac{1}{0.01} - \frac{\cancel{0^{0.01}}}{\cancel{0.01}} = 100$

⑩ $\int_0^2 \frac{1}{\sqrt{2-x}} dx = \lim_{a \to 0^+} \int_a^2 \frac{1}{\sqrt{2-x}} dx$

$u = 2-x$
$du = -1$

$\int \frac{1}{\sqrt{2-x}} dx = -\int u^{-1/2} du = -\frac{u^{1/2}}{1/2} = -\frac{2u^{1/2}}{1} = -2\sqrt{2-x}$

$= \lim_{a \to 0^+} \left[-2\sqrt{2-x} \Big|_a^2 \right] = \lim_{a \to 0^+} \left[-2\sqrt{\cancel{2-2}}^{\,0} - \left(-2\sqrt{2-a} \right) \right] = 2\sqrt{2} - 0 = \boxed{2\sqrt{2}}$

(11) $\int_{-1}^{1} \frac{1}{x^{5/3}}\,dx = \underbrace{\lim_{b\to 0^-} \int_{-1}^{b} \frac{1}{x^{5/3}}\,dx}_{I_1} + \underbrace{\lim_{b\to 0^+} \int_{a}^{1} \frac{1}{x^{5/3}}\,dx}_{I_2}$

$I_1 = \lim_{b\to 0^-} \int_{-1}^{b} \frac{1}{x^{5/3}}$

$\int \frac{1}{x^{5/3}}\,dx = \int u^{-5/3} = \frac{u}{-2/3}) = -\frac{3u^{-2/3}}{2} = -\frac{3}{2x^{2/3}}$

$I_1 = \lim_{b\to 0^-} \left[-\frac{3}{2x^{2/3}} \Big|_{-1}^{b} \right] = \lim_{b\to 0^-} \left[\frac{3}{2b^{2/3}} + \frac{3}{2(-1)^{2/3}} \right]$

$= \left(-\frac{3}{2(0)^{2/3}} + \frac{3}{-2^{2/3}} = \infty + \frac{3}{-2^{2/3}} = \infty \right.$

$\hookrightarrow$ DIVERGE

(12) $\int_{0}^{\frac{\pi}{2}} \tan t\,dt = \lim_{b\to \pi/2^+} \int_{0}^{b} \tan t\,dt$

$= \int \tan\,dt = \ln|\sec t| \Big|_{0}^{b}$

$= \lim_{b\to \frac{\pi}{2}} \left[\ln|\sec(b)| + \ln|\sec(0)| \right]$

$= \ln|\sec(\tfrac{\pi}{2})| - \cancel{\ln(1)}^{0}$

$= \infty \longrightarrow$ DIVERGE

(13) $\int_{-1}^{0} \frac{x}{\sqrt{1+x}}\,dx = \lim_{a\to -1} \int_{a}^{0} \frac{x}{\sqrt{1+x}}\,dx$

$\int \frac{x}{\sqrt{1+x}}\,dx = \int \frac{u-1}{\sqrt{u}}\,du = \int \frac{u}{\sqrt{u}} - \int \frac{1}{\sqrt{u}}$

$u = 1+x$
$du = dx$
$u = x-1$

$= \int u\,u^{-1/2} - \int u^{-1/2} = \int u^{1/2}\,du - \int u^{1/2}\,du$

$= \frac{u^{3/2}}{\frac{3}{2}} - \frac{u^{1/2}}{\frac{1}{2}} = \frac{2}{3}(1+x)^{3/2} - 2\sqrt{1+x}\,\Big|_{a}^{0}$

$\lim_{a\to -1}\left[(2/3)(1+0)^{3/2} - 2\sqrt{1+0} - \left(\frac{2}{3}(1+a)^{3/2} - 2\sqrt{1+a}\right)\right]$

$= \frac{2}{3} - 2 - \frac{2}{3}(1+(-1))^{3/2} + 2\sqrt{1-1}$

$= -\frac{4}{3} - \cancel{\frac{2}{3}(0)^{0}} + \cancel{2\sqrt{0}}^{0}$

$= -\frac{4}{3}$

(14) $\int_{1}^{3} \frac{1}{\sqrt{3+2x-x^2}}\,dx$

COMPLETAMOS TCP
$-x^2+2x+3$
$-(x^2-2x-3)$
$-(x^2-2x-3+1-1) = -(x-1)^2-4$
$= 4-(x-1)^2$

$\lim_{b\to 3^+} \int_{1}^{b} \frac{1}{\sqrt{4-(x-1)^2}}\,dx = \lim_{b\to 3^+} \left[\operatorname{sen}^{-1}\left(\frac{x-1}{2}\right)\Big|_{1}^{b} \right]$

$$= \lim_{b \to 3^+} \left[\operatorname{sen}^{-1}\left(\frac{b-1}{2}\right) - \operatorname{sen}^{-1}\left(\frac{1-1}{2}\right) \right)$$

$$= \operatorname{Sen}^{-1}\left(\frac{3-1}{2}\right) - \operatorname{sen}^{-1}(0) = \operatorname{sen}^{-1}\left(\frac{2}{2}\right) - 0 = \operatorname{Sen}^{-1}(1) = 90$$

$$V = \frac{90 - \pi}{180} = \frac{\pi}{2} = \underline{\frac{\pi}{2}}$$

Encuentra el área bajo la grafica de la función dada sobre el intervalo indicado

(15) $\int_1^{\infty} \frac{1}{(2x+1)^2}, (1,\infty) = \lim_{b \to \infty} \int_1^{\infty} \frac{1}{(2x+1)^2}$

$u = 2x+1$

$du = 2\ dx$

$dx = 1/2$

$$\int \frac{1}{(2x+1)^2} = \int u^{-2}\left(\frac{1}{2} du\right) = \frac{1}{2}\int u^{-2} du = \frac{1}{2}\int u^{-2} du$$

$$= \frac{1}{2} \frac{u^{-1}}{-1} = \left(-\frac{1}{4x+2}\right.$$

$$= \lim_{b \to \infty} \left[-\frac{1}{4x+2} \Big|_1^b \right) = \lim_{b \to \infty} \left(-\frac{1}{4b+2} + \frac{1}{4(1)+2} \right] = \frac{1}{4(\infty)+2} + \frac{1}{6} = \underline{\frac{1}{6}}$$

(16) $f(x) = e^x$

$$L\left[e^t\right] = \int_0^{\infty} e^t e^{-st}\, dt = \frac{1}{1-s} e^{(1-s)t} \Big|_0^{\infty}$$

$$= \frac{1}{1-s} \lim_{t \to \infty} e^{(1-s)t} - \frac{1}{1-s} = \frac{1}{1-s} = \frac{1}{s-1} \quad \text{si} \quad (1-s < 0)$$

$(1 > s)$

$= \underline{(s > 1)}$

17) $f(x) = \text{sen}\, x$

$$L[\text{sen}\, t] = \int_0^{\infty} e^{-st}\, \text{sen}(t)\, dt$$

$$\int e^{-st}\, \text{sen}(t) = \text{sen}(t)\left(-\frac{1}{se^{-st}}\right) - \int -\frac{1}{se^{-st}}\left(\cos(t)\, dt\right)$$

$u = \text{sen}(t)$

$du = \cos(t)$

$dv = e^{-st}$

$v = -\frac{1}{s} e^{-st}$

$$= -\frac{1}{s} e^{-st}\, \text{sen}(t) - \frac{1}{s^2} e^{-st} \cos t + \frac{1}{s}\left(-\frac{1}{s} \int e^{-st}\, \text{sen}\, t\right.$$

$$\int e^{-st}\, \text{sen}(t) + \frac{1}{s^2} \int e^{-st}\, \text{sen}\, t = -\frac{1}{s} e^{-st}\, \text{sen}(t) - \frac{1}{s^2} e^{-st} \cos t$$

$$= \frac{s^2+1}{s^2} = \frac{s^2}{s^2+1}$$

$$= \frac{s^2}{s^2+1}\left(-\frac{1}{s^2} e^{-st}\, \text{sen}(t) - \frac{1}{s^2} e^{-st} \cos(t)\right)$$

$$= \frac{s^2}{s^2+1(s)} e^{-st}\, \text{sen}(t) - \frac{s^2}{s^2+1(s^2)} e^{-st} \cos(t)$$

$$= \frac{-s}{s^2+1} e^{-st}\, \text{sen}(t) - \frac{1}{s^2+1} e^{-st} \cos(t) \Big|_0^{\infty}$$

$$\lim_{t \to \infty} \left[-\frac{s}{s^2+1} \cancel{e^{-st}}\, \text{sen}(t) - \frac{1}{s^2+1} \cancel{e^{-st}} \cos(t)\right] - \left(\frac{s}{s^2+1} \cancel{e^{-s(0)}}\, \text{sen}(0) - \cancel{\frac{1}{s^2+1}} \cancel{e^{-s(0)}} \cancel{\cos(0)}\right)$$

$= -s < 0 = 0 < s$

$s > 0$

$$= +\frac{1}{s^2+1}$$