TAREA 1 — FUNCIONES

1.- Obtenga el dominio de las siguientes funciones.

a) $f(x) = 4x - x^2$

$f: A \longrightarrow B$

$f: \mathbb{R} \longrightarrow \mathbb{R}$

$f(x) = 4x - x^2 \mid x \in \mathbb{R}$

$\boxed{D_F = \mathbb{R}}$

b) $f(x) = \sqrt{x-1}$

$x - 1 \geq 0 \;;\; x \geq 1$

$f: [1 + \infty) \longrightarrow \mathbb{R}$

$f(x) = \sqrt{x-1} \quad x \in \mathbb{R}$

$\boxed{D_F = [1 + \infty)}$

c) $f(x) = \underset{(a)}{\sqrt{2-x}} + \underset{(b)}{\sqrt{x+1}}$

$2 - x \geq 0 \qquad x + 1 \geq 0$

$x \leq 2 \qquad x \geq -1$

$D_a = (-\infty, 2] + D_b = [-1, \infty) = D_F$

$D_F = D_a \cap D_b = [2, \infty) \cap [-1, \infty)$

$\boxed{D_F = [-1, 2]}$

d) $f(x) = \dfrac{2x - 3}{4x - x^2}$

$x^2 - 4x = 0$

$x = -(-4) \pm \sqrt{(-4)^2 - 4(1)(0)} \,/\, 2(1)$

$= 4 \pm \sqrt{16} \,/\, 2$

$= 4 \pm 4 / 2$

$x_1 = 4 + 4/2 = 4$

$x_2 = 4 - 4/2 = 0$

$\boxed{x_1 = 4} \quad \boxed{x_2 = 0}$

$\boxed{D_F = \mathbb{R} - \{0, 4\}}$

e) $f(x) = \dfrac{x}{x^2 + 1}$

$f: A \longrightarrow \mathbb{R}$

$f(x) = x$

$x \in \mathbb{R}$

$D_F = \mathbb{R}$

$g: A \rightarrow \mathbb{R}$

$g(x) = x^2 + 1$

$x \in \mathbb{R}$

$D_g = \mathbb{R}$

$\boxed{D_F = D_g \cap D_F = \mathbb{R}}$

José Luis Sandoval Perez

f)

$$f(x) = \sqrt{\overset{(a)}{1-x}} + \sqrt{\overset{(b)}{x-1}}$$

$1 - x \geq 0$ $\quad$ $x - 1 \geq 0$

$1 - x = 0$ $\quad$ $x - 1 = 0$

$\lfloor x = 1 \rfloor$ $\quad$ $\lfloor x = 1 \rfloor$

$D_a = [1, \infty)$ $\quad$ $D_b = [1, \infty)$

$D_F = D_a \cap D_b = [1, \infty) \cap [1, \infty)$

$D_F = 1$

② Sean $f(x) = (x^2+1)$, $g(x) \dfrac{1}{2x-1}$ y $h(x) = (3x-2)$

Determine el dominio y la regla de correspondencia para las siguientes composiciones.

a) $f \circ g$

$f : \mathbb{R} \longrightarrow \mathbb{R}$ $\qquad$ $g : \mathbb{R} \longrightarrow \mathbb{R} - \{1/2\}$

$f(x) = (x^2+1)$ $\qquad$ $g(x) - 1/2x - 1$

$$(f \circ g)(x) = f(g(x)) = f(1/2x-1) = \left(\frac{1}{2x-1}\right)^2 + 1$$

$$= \frac{1}{(2x-1)^2} + 1 = \frac{1}{4x^2-4x+1} + 1 = f(x)$$

$$f(x) = \underset{(a)}{\frac{1}{4x^2-4x+1}} + \underset{(b)}{1}$$

$D_f = D_a \cap D_b$

$D_a = \mathbb{R} - \{+1/2\}$ $\qquad$ $D_b = \mathbb{R}$

$4x^2 - 4x + 1 = (2x-1)^2 = (2x-1)(2x-1)$

$2x - 1 = 0$ $\qquad$ $2x - 1 = 0$

$x = 1/2$ $\qquad$ $x = 1/2$

$D_F = \mathbb{R} \cap \mathbb{R} - \{1/2\}$ Rango $= (-\infty, 1/2) \cup (1/2, \infty)$

$\boxed{D_{f \circ g} = \mathbb{R} - \{1/2\}, \quad (f \circ g)(x) = \mathbb{R} - \{1/2\}}$

b) f o h

$f: \mathbb{R} \longrightarrow \mathbb{R}$ $h: \mathbb{R} \longrightarrow \mathbb{R}$

$f(x) = (x^2+1$ $h(x) = (3x-2)$

$(f \circ h)(x) = f(h(x)) = f(3x-2) = (3x-2)^2 + 1 = f(x)$

$b(x) = (3x-2)^2$ $z(x) = 1$ $D_b = \mathbb{R}$ $D_z = \mathbb{R}$

$D_{f \circ h} = D_b \cap D_z = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$ $\boxed{D_{f \circ h} = \mathbb{R}}$

$$\boxed{\begin{array}{ll} b: \mathbb{R} \longrightarrow \mathbb{R} & z: \mathbb{R} \to \mathbb{R} \\ b(x) = (3x-2)^2 & z(x) = 1 \end{array}}$$

c) g o f

$g: \mathbb{R} \longrightarrow \mathbb{R} - \{1/2\}$ $f: \mathbb{R} \longrightarrow \mathbb{R}$

$g(x) = 1/2x-1$ $f(x) = x^2+1$

$$(g \circ f)(x) = g(f(x)) = g(x^2+1) = \frac{1}{2(x^2+1)-1} = f(2)$$

$$f(2) = \frac{1}{2(x^2+1)-1} = \frac{1}{2x^2+2-1} = \frac{1}{2x^2+1} =$$

$\boxed{f(2) = \mathbb{R}}$ $\boxed{(f \circ g)(x) = \mathbb{R} \longrightarrow \mathbb{R}}$

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d) h o f

$h: \mathbb{R} \longrightarrow \mathbb{R}$ $\quad f: \mathbb{R} \longrightarrow \mathbb{R}$

$(3x - 2)$ $\quad (x^2 + 1)$

$(h \circ f)(x) = h(f(x)) = h(x^2+1) = 3(x^2+1) - 2 = z(x)$

$z(x) = \underset{(a)}{3(x^2+1)} - \underset{(b)}{2}$ $\quad D_{h \circ f} = D_a \cap D_b$

$D_z = \mathbb{R} \cap \mathbb{R}$

$D_a = \mathbb{R} \to \mathbb{R}$

$D_b = \mathbb{R} - R$

$\boxed{D_{h \circ f} = \mathbb{R}}$ $\quad \boxed{(h \circ f)(x) = \mathbb{R} \to \mathbb{R}}$

e) g o h

$g: \mathbb{R} \to \mathbb{R} - \{1/2\}$ $\quad h: \mathbb{R} \to \mathbb{R}$

$g(x) = 1/2x - 1$ $\quad h: (3x - 2)$

$(g \circ h)(x) = g(h(x)) = g(3x-2) = \dfrac{1}{2(3x-2) - 1}$

$D_{g \circ h} = \dfrac{1}{2(3x-2)-1} = \dfrac{1}{6x-4-1} = \dfrac{1}{6x-5}$

$6x - 5 = 0$

$6x = 5$

$x = 5/6$

$\boxed{D_{g \circ h} = \mathbb{R} - \{5/6\}}$

$(g \circ h)(x) = \mathbb{R} \to \mathbb{R} - \{5/6\}$

③ Sean $f(x) = 2x - 4$, $g(x) = \frac{1}{2}x + 2$ muestre que ambas funciones son inversas.

$f: \mathbb{R} \rightarrow \mathbb{R}$ $\quad$ $g: \mathbb{R} \rightarrow \mathbb{R}$

$f(x) = 2x - 4$ $\quad$ $g(x) = 1/2x + 2$

$$(f \circ g)(x) = f(g(x)) = f(1/2x + 2) = 2(1/2x + 2) - 4$$

$$f \circ g = 2(1/2x + 2) - 4 = x + 4 - 4 = x \qquad \boxed{f \circ g = x}$$

$$(g \circ f)(x) = g(f(x)) = g(2x - 4) = 1/2(2x - 4) + 2$$

$$g \circ f = 1/2(2x - 4) + 2 = x - 2 + 2 = x$$

$$\boxed{g \circ f = x}$$

$$\boxed{f \circ g = x = g \circ f}$$

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