UNIVERSIDAD AUTONOMA
DE AGUASCALIENTES

**DEPARTAMENTO DE MATEMÁTICAS Y FÍSICA**

Materia: **CALCULO INTEGRAL**

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**Periodo: ENERO-JUNIO / 2022**

**TAREA PARCIAL 3**

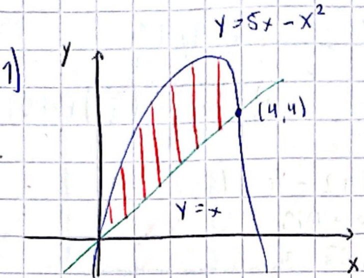
**fecha: 05/06/2022**

**Carrera: ICI 2-A**

# TAREA CALCULO TERCER PARCIAL

## AREA ENTRE CURVAS

1)




$$A=\int_a^b f(x)\,dx-\int_a^b g(x)\,dx$$

$$A=\int_a^b f(x)-g(x)\,dx$$

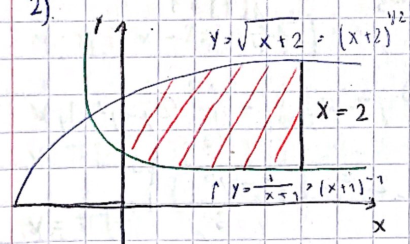
$$A=\int_0^4 (5x-x^2)-(x)\,dx$$

$$A=\int_0^4 (4x-x^2)\,dx$$

$$A=\left(4\frac{x^2}{2}-\frac{x^3}{3}\right)\Big|_0^4=2(4)^2-\frac{(4)^3}{3}-0=32-\frac{64}{3}=\frac{96-64}{3}$$

$A=32/3\ u^2$

2)




$$A=\int_a^b f(x)\,dx-\int_a^b g(x)\,dx$$

$$A=\int_a^b f(x)-g(x)\,dx$$

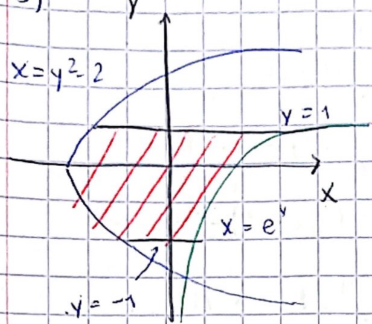
$$A=\int_0^2 (x+2)^{1/2}-\left(\frac{1}{x+1}\right)$$

$$A=\frac{(x+2)^{3/2}}{3/2}-\ln(x+1)\Big|_0^2$$

$$A=\left[\frac{(2+2)^{3/2}}{3/2}-\ln(2+1)\right]-\left[\frac{(0+2)^{3/2}}{3/2}-\ln(0+1)\right]$$

$A=5.333-1.0986-1.8856-0=2.3488\ u^2$

3)




$$A = \int_{-1}^{1} \left( \left( e^{y} - (y^{2}-2) \right) \right) dy.$$

$$= e^{y} - \frac{y^{3}}{3} - 2y\ dy \Big|_{-1}^{1}$$

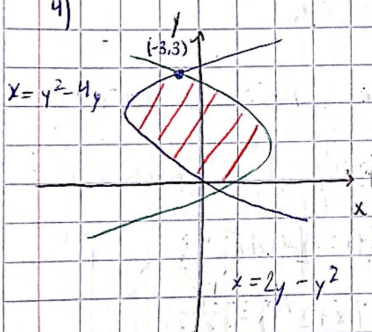
$$\left( e^{1} - \frac{1^{3}}{3} - 2(1) \right) - \left( e^{-1} - \left( \frac{-1}{3} \right) + 2(-1) \right)$$

$$= e - 1/3 + 2 - 1/e - 1/3 + 2$$

$$= e - 1/e + 10/3$$

$$= 5.6837\ u^{2}$$

4)




$F(y) = y^{2} - 4y \qquad g(1) = 2y - y^{2}$

$f(1) = 1^{2} - 4(1) \qquad g(1) = 2(1) - 1(1)$

$F(1) = -3 \qquad g(1) = 1$

$$A = \int_{0}^{3} y^{2} - 4y\ dy - \int_{0}^{3} 2y - y^{2}\ dy$$

$$A = \frac{y^{3}}{3} - 2y^{2} - \left[ y^{2} - \frac{y^{3}}{3} \right] \Big|_{0}^{3}$$

$x = y^{2} - 4y$

$x = 2y - y^{2}$

$[0, 3]$

$$A = \frac{3^{3}}{3} - 2(3)^{2} - 3^{2} + \frac{3^{3}}{3} - \left[ \frac{0^{3}}{3} - 2(0)^{2} - 0^{2} - \frac{0^{3}}{3} \right]$$

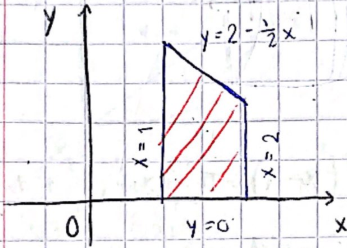
$$A = \frac{27}{3} - 2(9) - 9 + \frac{27}{3} - \left[ \frac{0}{3} - 2(0) - 0 + \frac{3}{0} \right]$$

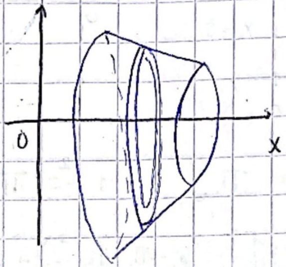
$$A = 9 - 18 - 9 + 9 = \quad A = -9\,u^{2}$$

giro en $x = x_1, x_2$ y $y = f(x)$
giro en $y = y_1, y_2$ — $x = f(y)$

# Volumenes (solidos de revolución)

a) $y = 2 - \frac{1}{2}x$, $y = 0$, $x = 1$, $x = 2$ alrededor del eje $x$




$$V = \int_1^2 \pi r^2\, dx = \pi \int_1^2 \left(2 - \tfrac{1}{2}x\right)^2 = \pi \int_1^2 \left(4 - 2(x) + \tfrac{1}{4}x^2\right) dx$$

$$V = \pi \int_1^2 \left(4 - 2x + \tfrac{1}{4}x^2\right) dx = V = \pi \int_1^2 4\, dx - 2\int x\, dx - \tfrac{1}{4}\int_1^2 x^2\, dx$$

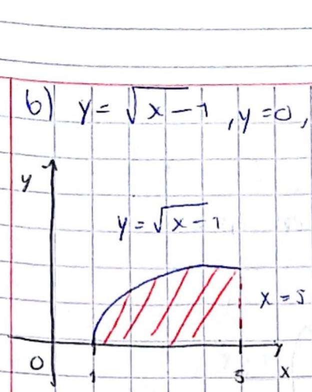
$$V = \pi \left[4x - 2\frac{x^2}{2} + \frac{1}{4}\frac{x^3}{3}\right] = \pi \left[4x - x^2 + \frac{x^3}{12}\right]\Big|_1^2$$

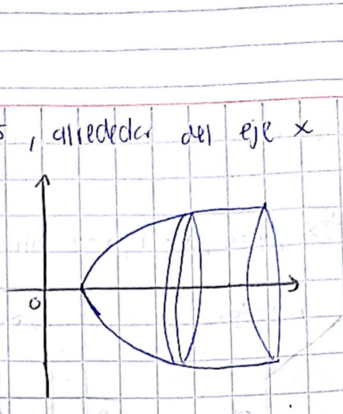
$$V = \pi\left[4(2) - 2^2 + \frac{2^3}{12}\right] - \pi\left[4(1) - (1)^2 + \frac{1^3}{12}\right] = \pi\left(8 - 4 + \frac{8}{12}\right) - \pi\left(4 - 1 + \frac{1}{12}\right)$$

$$V = \pi\left[\frac{96}{12} - \frac{48}{12} + \frac{8}{12}\right] - \pi\left(\frac{48}{12} - \frac{12}{12} + \frac{1}{12}\right) = \pi\left(\frac{56}{12}\right) - \pi\left(\frac{37}{12}\right)$$

$$V = \pi\left(\frac{19}{12}\right)$$

b) $y = \sqrt{x-1}$, $y = 0$, $x = 5$, alrededor del eje x



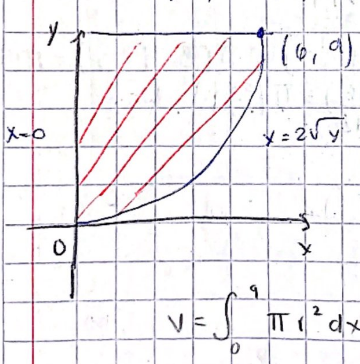

$$V = \int_1^5 \pi r^2 dx = \pi \int_1^5 (\sqrt{x-1})^2 = \pi \int_1^5 (x-1) = \pi \int_1^5 x\,dx - \int_1^5 1\,dx$$

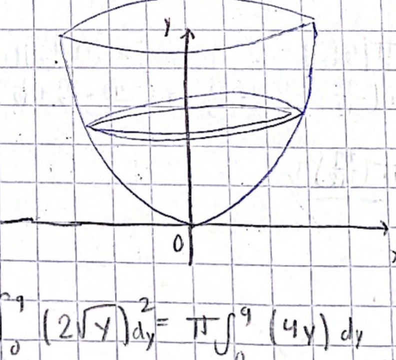
$$V = \pi \left(\frac{x^2}{2}\right) - (x) \Big|_1^5 \qquad V = \pi \left[\left(\frac{5^2}{2}\right) - (5)\right] - \pi\left(\frac{1^2}{2} - 1\right)$$

$$V = \pi\left(\frac{25}{2} - \frac{10}{2}\right) - \pi\left(\frac{1}{2} - \frac{2}{2}\right) = \pi\left(\frac{15}{2}\right) - \pi\left(-\frac{1}{2}\right) = \pi\left(\frac{15}{2}\right) + \pi\left(\frac{1}{2}\right)$$

$$V = \pi\left(\frac{16}{2}\right) = \underline{\pi 8}$$

c) $x = 2\sqrt{y}$, $x = 0$, $y = 9$   $x = 2\sqrt{y}$



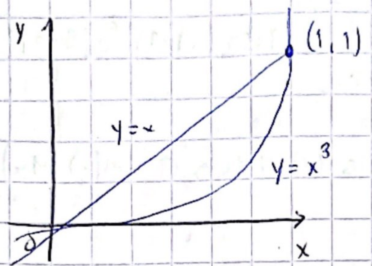

$$V = \int_0^9 \pi r^2 dx = \pi \int_0^9 (2\sqrt{y})^2 dy = \pi \int_0^9 (4y)\,dy$$

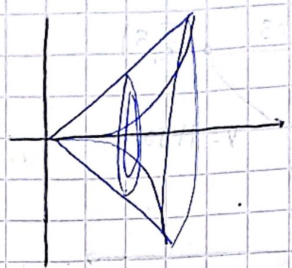
$$V = \pi\left[4 \int y\,dy\right] = \pi\left[4\left(\frac{y^2}{2}\right)\right] = \pi\left[4\,\frac{9^2}{2}\right] - 0$$

$$V = \left[4\,\frac{9^2}{2}\right] = \left[\frac{8}{2}\,\frac{81}{2}\right]^{\pi} = \underline{162\pi}$$

d) $y = x^3$, $y = x$, $x \geq 0$; alrededor del eje x

$f(y)$ $g(y)$



Metodo Arandelas

$V = V_F - V_g$

$V_T = \pi \{ \int_a^b f^2(x)\,dx - \int_a^b g^2(x)\,dx$

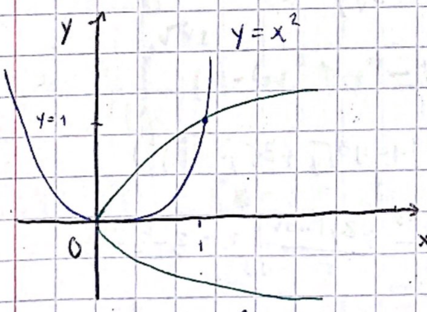
$V_T = \pi \int_a^b (f^2(x) - g^2(x)\,dx)$

$V_F = \pi \int_0^1 (x)^2 dx - \pi \int_0^1 (x^3)^2 dx$

$V_F = \pi \int_0^1 x^2\,dx - \pi \int_0^1 x^6\,dx$

$V_F = \pi \left[\frac{x^3}{3}\right]\Big|_0^1 - \pi \left[\frac{x^7}{7}\right]\Big|_0^1 = \pi\left(\frac{1}{3}\right) - \pi\left(\frac{1}{7}\right) = \pi\left(\frac{4}{21}\right)$

e) $y = x^2$, $x = y^2$; alrededor de $y = 1$




$y = x^2$, $x = y^2$, $y = 1$, $y = x^2$, $\sqrt{x} = \sqrt{y^2}$, $\sqrt{x} = y$

$y = \sqrt{x}$

$V = \int_0^1 \pi(1 - x^2)^2 dx - \int_0^1 \pi(1 - \sqrt{x})^2 dx$

$V = \int_0^1 \pi(1 - 2(x^2) + x^4)\,dx - \int_0^1 \pi(1 - 2\sqrt{x} + x)\,dx$

$V = \pi \int_0^1 1\,dx - 2\int x^2 dx + \int_0^1 x^4\,dx - [\pi \int_0^1 1\,dx - 2\int_0^1 x^{1/2} + \int_0^1 x\,dx]$

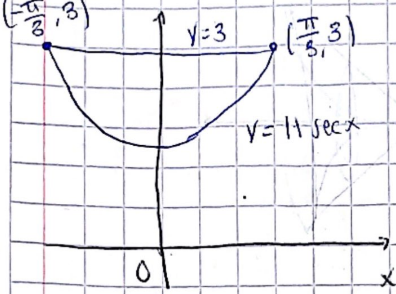
$V = \pi \left[x - \frac{2x^3}{3} + \frac{x^5}{5}\right] - \left[\pi\left[x - 2\left[\frac{x^{3/2}}{3/2}\right] + \frac{x^2}{2}\right]\right]$

$V = \pi\left[x - \frac{2x^3}{3} + \frac{x^5}{5}\right] - \left[\pi\left[x - \frac{2x^{3/2}}{3} + \frac{x^2}{2}\right]\right]\Big|_0^1$

$V = \pi\left[1 - \frac{2(1)^3}{3} + \frac{1^5}{5}\right] - \pi\left[1 - \frac{2(1)^{3/2}}{3} + \frac{1^2}{2}\right]$

$V = \frac{8}{15}\pi - \frac{\pi}{6} = \frac{11\pi}{30}$

f) $y = 1 + \sec(x)$, $y = 3$, alrededor de $y = 1$




$$V = \pi \int_{-\pi/3}^{\pi/3} (1 + \sec(x) - 1)^2 - (3-1)^2$$

$$V = \pi \int_{-\pi/3}^{\pi/3} \sec^2(x) - 4\, dx = \pi \tan(x) - 4x \Big|_{-\pi/3}^{\pi/3}$$

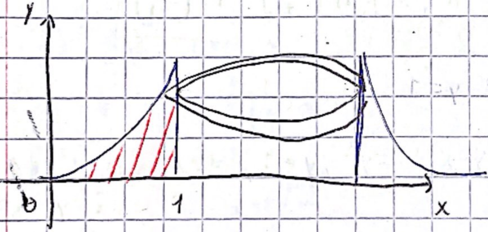
$$V = \pi\left(\left(\tan(\pi/3) - 4(\pi/3)\right) - \left(\tan(-\pi/3) - 4(-\pi/3\right)\right)$$

$$= \pi\left(\left(\sqrt{3} - \frac{4\pi}{3}\right) - \left(-\sqrt{3} + 4\pi/3\right)\right) = \pi(2\sqrt{3} - 8\pi/3)$$

$$= 2\pi(\sqrt{3} - 4\pi/3) = 2\pi(4\pi/3 - \sqrt{3})$$

V =

g) $y = x^3$, $y = 0$, $x = 1$, alrededor de $x = 2$




$f(x) = x^3$

$g(x) = 1$

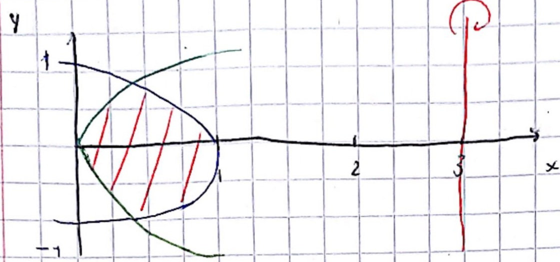
$\sqrt[3]{y} = x$

$$V = \pi \int_0^1 (2 - \sqrt[3]{y})^2 - (2-1)^2 = \pi \int_0^1 (4 - 4\sqrt[3]{y} + \sqrt[3]{y^2} - (1)^2)$$

$$= \pi \int_0^1 3 - 4y^{1/3} + y^{2/3} = \pi\left(\left(3x - \frac{12y^{4/3}}{4} + \frac{3y^{(5/3)}}{5}\right)\Big|_0^1 = 3 - 3 + \frac{3}{5}\right.$$

$$V = \frac{3}{5}\pi$$

h) $x = y^2$, $x = 1 - y^2$, alrededor de $x = 3$




$x = y^2$

| | | | | |
|---|---|---|---|---|
| x | 1/4 | 0 | 1 | 9/4 |
| y | .5 | 0 | 1 | 1.8 |

$x = 1 - y^2$

| | | | | |
|---|---|---|---|---|
| x | 3/4 | 1 | 0 | -5/4 |
| y | .5 | 0 | 1 | 1.5 |

$$V_T = \pi \int_0^{\sqrt{\frac{1}{2}}} (3-y^2)^2 - (2-(1-y^2))^2 \, dy$$

$$V_T = \pi \int_0^{\sqrt{1/2}} (9 - 6y^2 + y^4 - 1 - 2y^2 - y^4) \, dy$$

$$V_T = \pi \int_0^{\sqrt{1/2}} (8 - 8y^2) \, dy = \pi \left[ 8y - 8/3 y^3 \right] \Big|_0^{\sqrt{1/2}}$$

$$= \pi \left[ 8(\sqrt{1/2} - \frac{8}{3}(\sqrt{1/2})^3 \right] - [0]$$

$$= \pi \left( 4\sqrt{2} - \frac{2\sqrt{2}}{3} \right) = \frac{10\sqrt{2}}{3} \pi$$

# TRABAJO

1) Se requiere una fuerza de 40N para sostener un resorte que está estirado desde su posición natural de 10cm a una longitud de 15cm. ¿Cuánto trabajo se hace al estirar el resorte de 15 a 18cm?

$$f(x) = k(x) \qquad f(x) = 8x \qquad W = \int_5^8 8x\,dx$$

$$40N = k(5) \qquad W = \int_5^8 f(x)\,dx$$

$$k = 8 \qquad = 8\int_5^8 x\,dx$$

$$= 8[x^2]dx \Big|_5^8 = 8(8^2) - 8(5^2)dx = 512 - 200 = \underline{312}$$

2) Una partícula se mueve a lo largo del eje x debido a la acción de una fuerza de $f(x)$ libra, cuando la partícula está a x pies del origen. Si $f(x) = x^2+4$, calcule el trabajo realizado conforme la partícula se mueve del punto donde $x=2$ hasta $x=4$

$$W\int_2^4 (x^2+4)\,dx = \frac{x^3}{3} + 4x \Big|_2^4 = \frac{4^3}{3} + 4(4) - \frac{2^3}{3} + 4(2)$$

$$= 37.333 - 10.666 = \underline{26.667}$$

3) Un gorila de 360lb trepa a un arbol a una altura de 20 pies. Encuentre el trabajo realizado si el gorila alcanza esa altura en 10 segundos y 5 segundos

$W = F \cdot D$

$W = 360lb \cdot 20p$

$W = 7200\ lb\text{-}p$

La respuesta para los 2 tiempos es la misma ya que el tiempo no afecta en el trabajo.

4) ~~...~~

Una fuerza variable de $5x^{-2}$ libras mueve un objeto a lo largo de una linea recta cuando está a x pies del origen. Calcule el trabajo realizado para mover el objeto desde x = 1 pie a x = 10 pies

$$W = \int_1^{10} 5x^{-2}\,dx$$

$$W = 5\left[x^{-1}\right]\Big|_1^{10} = 5(10)^{-1} - 5(1)^{-1} = 0.5 - 5$$

$$= -4.5$$

5) Se requiere una fuerza de 10lb para mantener estirado un resorte de 4inch mas de su longitud natural. ¿Cuanto trabajo se realiza al estirar el resorte desde su longitud natural hasta 6inch más de su longitud natural?

$f(x) = kx$ $\qquad f(x) = \frac{5}{2}x \qquad W = \int_0^6 \frac{5}{2}x\,dx$

$10lb = k(4)$

$\frac{10}{4} = k \qquad = \frac{5}{2}\int_0^6 x\,dx = \frac{5}{2}\left(\frac{x^2}{2}\right)\Big|_0^6$

$\frac{5}{2} = k \qquad = \frac{5}{2}\left(\frac{6^2}{2}\right) = \frac{5}{2}\left(\frac{18}{1}\right) = \frac{90}{2} = 45$

6) Un resorte tiene una longitud natural de 20cm. Compare el trabajo $W_1$ invertido en alargar un resorte desde 20cm a 30 con el trabajo $W_2$ realizado en estirarlo desde 30cm hasta 40cm. ¿Cómo se relacionan $W_2$ y $W_1$?

El trabajo es el mismo

Series (En los siguientes ejercicios utilice 5 términos en cada serie)

1) Utilice series para evaluar con exactitud de tres cifras decimales $\int_0^1 \sqrt{x}e^{-x}\,dx$

$f(x) = e^{-x}$ $\quad f(0) = 1$

$f'(x) = -e^{-x}$ $\quad f'(0) = -1 \qquad = 1 - x + \frac{x^2}{2} - \frac{x^2}{6} + \frac{x^4}{24}$

$f''(x) = e^{-x}$ $\quad f''(0) = 1$

$f'''(x) = -e^{-x}$ $\quad f'''(0) = 1$

$f^{IV}(x) = e^{-x}$ $\quad f^{IV}(0) = 1$

$$\int_0^1 \sqrt{x}\left(1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}\right)dx$$

$$= \int_0^1 \sqrt{x} - x^{3/2} + \frac{x^{5/2}}{2} - \frac{x^{7/2}}{6} + \frac{x^{9/2}}{24}\,dx$$

$$= \int_0^1 x^{1/2}dx - \int x^{3/2}dx + \frac{1}{2}\int x^{5/2}dx - \frac{1}{6}\int x^{7/2}dx + \frac{1}{24}\int x^{9/2}dx$$

$$= \frac{2x^{3/2}}{3} - \frac{2x^{5/2}}{5} + \frac{1}{2}\left(\frac{2x^{7/2}}{7}\right) - \frac{1}{6}\left(\frac{2x^{9/2}}{9}\right) + \frac{1}{24}\left(\frac{2x^{11/2}}{11}\right)$$

$$= \frac{2x^{3/2}}{3} - \frac{2x^{5/2}}{5} + \frac{x^{7/2}}{7} - \frac{x^{9/2}}{27} + \frac{x^{11/2}}{132}\Big|_0^1$$

$$= \left[\frac{2(1)^{3/2}}{3} - \frac{2(1)^{5/2}}{5} + \frac{(1)^{7/2}}{7} - \frac{(1)^{9/2}}{27} + \frac{(1)^{11/2}}{132}\right]$$

$$- \left[\frac{2(0)^{3/2}}{3} - \frac{2(0)^{5/2}}{5} + \frac{0^{7/2}}{7} - \frac{0^{9/2}}{27} + \frac{0^{11/2}}{132}\right]$$

$$= \left[\frac{2}{3} - \frac{2}{5} + \frac{1}{7} - \frac{1}{27} + \frac{1}{132}\right] - \left[\frac{0}{3} - \frac{0}{5} + \frac{0}{7} - \frac{0}{27} + \frac{0}{132}\right]$$

$$= \boxed{0.380063}$$

2) Obtenga la serie de Maclaurin para $f(x) = sen^2(x)$

$$f(x) = sen^2(x) = \frac{1 - \cos(2x)}{2}$$

| | |
|---|---|
| $f(x) = \cos(2x)$ | $f(0) = 1$ |
| $f'(x) = -2\,sen(2x)$ | $f'(0) = 0$ |
| $f''(x) = -4\,sen(2x)$ | $f''(0) = -4$ |
| $f'''(x) = 8\,sen(2x)$ | $f'''(0) = 0$ |
| $f^{IV}(x) = 16\cos(2x)$ | $f^{IV}(0) = 16$ |
| $f^{V}(x) = -32\,sen(2x)$ | $f^{V}(0) = 0$ |
| $f^{VI}(x) = -64\cos(2x)$ | $f^{VI}(0) = -64$ |
| $f^{VII}(x) = 128\,sen(2x)$ | $f^{VII}(0) = 0$ |
| $f^{VIII}(x) = 256\,sen(2x)$ | $f^{VIII}(0) = 256$ |

$$\cos(2x) = 1 + \cancel{0x}^0 + \left(\frac{(-4)x^2}{2!}\right) + \frac{0x^3}{3!} + \frac{16x^4}{4!} + \frac{0x^5}{5!} + \frac{(-64)x^6}{6!} + \frac{(0)x^7}{7!} + \frac{256}{8!}x^8$$

$$\cos(2x) = 1 - 2x^2 + \frac{16x^4}{24} - \frac{64x^6}{720} + \frac{256x^8}{40{,}320}$$

$$\cos(2x) = 1 + 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \frac{2}{315}x^8$$

$$sen^2(x) - \frac{1}{2}\left(1 - \left[1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \frac{2}{315}x^6\right]\right)$$

$$= \frac{1}{2}\left(0 + 2x^2 - \frac{2}{3}x^4 - \frac{4}{45}x^6 - \frac{2}{315}x^8\right) = x^2 - \frac{1}{3}x^4 + \frac{2}{45}x^6 - \frac{1}{315}x^8 + \frac{2}{14175}x^{10} \ldots$$

3) Determine las ~~serie de Maclaurin~~ tres primeros términos diferentes de 0 de la serie de Taylor para $\cot(x)$ en $1/2\,\pi$

$f(x) = \cot(x) = f(\pi/2) = 0$

$f'(x) = -\csc^2(x) = f'(\pi/2) = -1$

$f''(x) = 2\cot(x)\csc^2(x) = f''(\pi/2) = 0$

$f'''(x) = -2\csc^4(x) - 4\cot^2(x)\csc^2(x) = f'''(\pi/2) = -2$

$f^{IV}(x) = 12\cot(x)\csc^4(x) + 4\cot(x)\csc^2(x)\csc^2(x) + 2\cot^2(x)$

$f^{IV}(\pi/2) = 0$

$f^5(x) = -16\csc^6(x) - 88\cot^2(x)\csc^4(x) - 16\cot^4(x)\csc^2(x)$

$f^5(\pi/2) = -16$

$$\cot(x) = \frac{-1x}{1!} - \frac{2x^3}{3!} - \frac{16x^5}{5!}$$

$$= -x - \frac{1x^3}{3} - \frac{2x^5}{15}$$

4) $\int_0^1 \frac{\operatorname{Sen} x}{x}\,dx$

Serie senx

$f(0)+f'(0)x+\frac{f''(0)x^2}{2!} \dots \frac{f^n(c)x^n}{n!}$

$f(x)=\operatorname{sen} x$ $\quad f(0)=0$

$f'(x)=\cos x$ $\quad f'(0)=1$

$f''(x)=-\operatorname{sen} x$ $\quad f''(0)=0$

$f'''(x)=-\cos x$ $\quad f'''(0)-1$

$f^{IV}(x)=\operatorname{sen} x$ $\quad f^{IV}(0)=0$

$f^{5}(x)=+\cos x$ $\quad f^{5}(0)=1$

$$\frac{\operatorname{Sen}(x)}{x} = \cancel{0}+x+\cancel{\frac{(0)x^2}{2!}}+\frac{(-1)x^3}{3!}+\cancel{\frac{0(x^4)}{4!}}+\frac{(1)x^5}{5!}+\cancel{\frac{(0)x^6}{6!}}+\frac{(-1)x^7}{7!}$$

1 2 3 4

$$+\cancel{\frac{(0)x^8}{8!}}+\frac{(1)x^9}{9!}$$

5

$$=\left(x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+\frac{x^9}{362880}\right)dx = \int_0^1 \frac{1}{x}\left(x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+\frac{x^9}{362880}\right)$$

$$\int_0^1\left(1-\frac{x^2}{6}+\frac{x^4}{120}-\frac{x^6}{5040}+\frac{x^8}{362880}\right)dx=\left[x-\frac{x^3}{18}+\frac{x^5}{600}-\frac{x^7}{35280}+\frac{x^9}{3265920}\right]\Bigg|_0^1$$

$$=\left[1-\frac{1^3}{18}+\frac{1^5}{600}-\frac{1^7}{35280}+\frac{1^9}{3265920}\right]-(0)$$

$$=1-\frac{1}{18}+\frac{1}{600}-\frac{1}{35280}+\frac{1}{3265920}$$

$$\cong 0.946083$$

5) $\lim_{x \to 0} \frac{e - e^{\cos x}}{x^2}$

$f(x) = e - e^{\cos x}$ $\quad f(0) = 0$

$f'(x) = e^{\cos x}(\sin x)$ $\quad f'(0) = 0$

$f''(x) = -e^{\cos x}\cos x + e^{\cos x}\operatorname{sen}^2 x$ $\quad f''(0) = e$

$f'''(x) = -e^{\cos x}(2\cos x \operatorname{sen} x + \operatorname{sen} x) - e^{\cos x}\operatorname{sen} x(\operatorname{sen}^2 x - \cos x)$ $\quad f'''(0) = 0$

$f^{IV}(x) = e^{\cos x}\operatorname{sen}^4 x(\operatorname{sen}^2 x - 3\cos x - 1) - e^{\cos x}\cos x(\operatorname{sen}^2 x - 36x - 1) - e^{\cos x}$
$\operatorname{sen} x(2\cos x \operatorname{sen} x + 3\operatorname{sen} x)$ $\quad f^{IV}(0) = 4e$

$f^5(x) = e^{\cos x}(4\cos x \operatorname{sen}^3 x + 6\operatorname{sen}^3 x + 2(-6\cos x) - 4)\cos x \operatorname{sen} x - 6\cos x \operatorname{sen} x - \operatorname{sen} x)$
$- e^{\cos x}\operatorname{sen} x(\operatorname{sen}^4 x + (-6\cos x - 4)\operatorname{sen}^2 x + 3\cos^2 x + \cos x)$

$f^5(0) = 0$

$f^6(x) = e^{\cos x}(\operatorname{sen}^6 x + (-15x\cos x - 20)\operatorname{sen}^4 x + (45\cos^2 x + 75\cos x + 16)$
$\operatorname{sen}^2 x - 15\cos^2 x - 15\cos^2 x - \cos x)$

$f^6(0) = -31e$

$f^7(x) = e^{\cos x}\operatorname{sen} x(\operatorname{sen}^6 x + (-21\cos x - 35)\operatorname{sen}^4 x + (105\cos^2 x + 245\cos x + 91)\operatorname{sen}^2 x$
$-105\cos^3 x - 210\cos^2 x - 63\cos(x))$

$f^7(x) = 0$

$f^8(x) = e^{\cos x}(\operatorname{sen}^8 x + (-28\cos x - 56)\operatorname{sen}^6 x + (210\cos^2 x + 630\cos x + 336)$
$\operatorname{sen}^4 x + (-420\cos^3 x - 1260\cos^2 x - 756\cos x - 64)\operatorname{sen}^2 x + 105\cos^4 x$
$+ 210\cos^2(x) + 63\cos^2 x + \cos x)$

$f^8(0) = \boxed{379e}$

$e^{\cos x} = e - \frac{ex^2}{2!} + \frac{4ex^4}{4!} - \frac{31ex^6}{6!} + \frac{379ex^8}{8!}$

$= e - \frac{ex^2}{2} + \frac{ex^4}{6} - \frac{31ex^6}{270} + \frac{379ex^8}{40320}$

$=$

$$\lim_{x\to 0} \frac{e-\left(e-\frac{ex^2}{2}+\frac{ex^4}{6}-\frac{31ex^6}{270}+\frac{379ex^8}{40320}\right.}{x^2}$$

$$\lim_{x\to 0} \frac{\frac{604080ex^2-20160ex^4+(38880ex^6-1137ex^8)}{120960}}{x^2}$$

$$=\frac{\cancel{x^2}(604080e-20160ex^2+13888ex^4-1137ex^6)}{\cancel{x^2}(120460)}$$

$$\lim_{x\to 0} \frac{604080e-20160e(0)^2+13888e(0)^4-(137e(0)^6)}{120460}$$

$$=\frac{1}{2}e$$

6) $\lim_{x\to 0} \frac{\cosh x-\cos x}{\operatorname{senh} x-\operatorname{sen} x}$

$\cos x = 1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}$

$\operatorname{sen} x = x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+\frac{x^9}{362880}$

$f(x)=\cosh x \quad f(0)=1$
$f'(x)=\operatorname{senh} x \quad f'(0)=0$
$f''(x)=\cosh x \quad f''(0)=1$
$f'''(x)=\operatorname{senh} x \quad f'''(0)=0$
$f^{IV}(x)=\cosh x \quad f^{IV}(0)=1$

$=1+\frac{x^2}{2}+\frac{x^4}{24}+\frac{x^6}{720}+\frac{x^8}{40,320}$

$$\lim_{x\to 0} \frac{\left(1+\frac{x^2}{2}+\frac{x^4}{24}+\frac{x^6}{720}+\frac{x^8}{40320}\right)-\left(1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}\right)}{x+\frac{x^3}{6}+\frac{x^5}{5040}-\frac{x^7}{5040}-\left(x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+\frac{x^9}{362880}\right)}$$

$f(x)=\operatorname{senh} x \quad f(0)=0$
$f'(x)=\cosh x \quad f'(0)=1$
$f''(x)=\operatorname{senh} x \quad f''(0)=0$
$f'''(x)=\cosh x \quad f'''(0)=1$
$f^{IV}(x)=\operatorname{senh} x \quad f^{IV}(0)=0$
$f^{V}(x)=\cosh x \quad f^{V}(0)=1$

$x+\frac{x^3}{6}+\frac{x^5}{120}+\frac{x^7}{5040}+\frac{x^9}{362880}$

$$\lim_{x\to 0} \frac{x^2 + \frac{2x^6}{720}}{\frac{2x^3}{6} + \frac{2x^7}{5040}} = \frac{2520 + 1x^5}{840x + x^5} = \infty$$

7) $$\lim_{x\to 0} \frac{e^x - e^{\operatorname{sen} x}}{x^3}$$

$f(x) = e^{\operatorname{sen} x}$ $\quad f(0) = 1$

$f'(x) = e^{\operatorname{sen} x} \cos(x)$ $\quad f'(0) = 1$

$f''(x) = e^{\operatorname{sen} x} \cos^2(x) - e^{\operatorname{sen} x} \operatorname{sen}(x)$ $\quad f''(0) = 1$

$f'''(x) = -e^{\operatorname{sen} x}(2\cos x \operatorname{sen} x + \cos x) - e^{\operatorname{sen} x} \cos x(\operatorname{sen} x - \cos^2 x)$ $\quad f'''(0) = 0$

$f^{IV}(x) = -e^{\operatorname{sen} x} \cos x(2\cos x \operatorname{sen} x + 3\cos x) + e^{\operatorname{sen} x} \operatorname{sen} x(3\operatorname{sen} x - \cos^2 x + 1)$
$-e^{\operatorname{sen} x} \cos^2 x(3\operatorname{sen} x - \cos^2 x + 1)$

$f^4(0) = -3$

$f^5(x) = e^{\operatorname{sen}(x)}(12\cos x \operatorname{sen}^2(x) - 4\cos^3 x \operatorname{sen} x + 14\cos x \operatorname{sen} x + \cos x$
$(1 - 6\cos^2 x) + e^{\operatorname{sen} x} \cos x(3\operatorname{sen}^2 x + (1 - 6\cos^2 x)\operatorname{sen} x - 4\cos^2(x))$

$f^5(0) = -8$

$$\lim_{x\to 0} \frac{\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}\right) - \left(1 + x + \frac{x^2}{2} - \frac{3x^4}{24} - \frac{8x^5}{120}\right)}{x^3}$$

$$\lim_{x\to 0} \frac{x^3}{6} + \frac{4x^4}{24} + \frac{8x^5}{180} = x^3\left(\frac{1}{6} + \frac{4x}{24} + \frac{8x^2}{120}\right)$$

$$\lim_{x\to 0} = \frac{1}{6} + \frac{4x}{24} + \frac{8x^2}{120} = \frac{1}{6}$$

8) $\int_0^{\pi/2} \left(1 - \frac{1}{2}\operatorname{sen}^2 \phi\right)^{-1/2} d\phi$

Serie

$f(x) = \operatorname{sen}^2\theta \rightarrow f(c) = 0$

$f'(x) = 2\operatorname{sen}(x)\cos(x) \quad f'(c) = 0$

$f''(x) : 4\cos^2(x) - 2 \quad f''(c) : 0$

$f'''(x) = -8\operatorname{sen}(x)\cos(x) \quad f'''(c) = 0$

$\int_0^{\pi/2} \left(1 - \frac{x^2}{2} + \frac{1}{6}x^4 - \frac{x^6}{45} + \frac{x^8}{630}\right)^{-1/2} d\theta$

$x^2 - \frac{x^4}{3} + \frac{2x^6}{45} - \frac{x^8}{315}$

$\int_0^{\frac{\pi}{2}} \left(1 - \frac{\sqrt{2}}{\sqrt{x^2}} + \frac{\sqrt{6}}{\sqrt{x^4}} - \frac{3\sqrt{5}}{\sqrt{x^6}} + \frac{3\sqrt{70}}{\sqrt{x^8}}\right) d\theta$

$\int_0^{\frac{\pi}{2}} \left(1 - \sqrt{2}x^{-1} + \sqrt{6}x^{-2} - 3\sqrt{5}x^{-3} + 3\sqrt{70}x^{-4}\right) d\theta$

$= \left(x - \sqrt{2} + \sqrt{6}\frac{x^{-1}}{-1} - 3\sqrt{5}\frac{x^{-2}}{-2} + 3\sqrt{70}\frac{x^{-3}}{-3}\right)\Big|_0^{\pi/2}$

$= \left(\frac{\pi}{2} - \sqrt{2} - \sqrt{6}\left(\frac{\pi}{2}\right)^{-1} + \frac{3\sqrt{3}}{2}\left(\frac{\pi}{2}\right)^{-2} - \frac{3\sqrt{70}}{3}\left(\frac{\pi}{3}\right)^{-3}\right)$

$= \underline{1.854}$

9) $\int_0^1 \cos\sqrt{x}\, dx$

$f(x) = \cos\sqrt{x} \quad = f(0) = 1$

$f'(x) = \frac{\operatorname{sen}\sqrt{x}}{2\sqrt{x}} \quad = f'(0) = 1/2$

$= f''(0) = \frac{1}{120}$

$f''(x) = 1/4\left(\frac{\operatorname{sen}\sqrt{x}}{x^{3/2}}\right.$

Serie $= 1 - \frac{x}{2} + \frac{x^2}{24} - \frac{x^3}{720} + \frac{x^4}{40320}$

$= \int_0^1 \left(1 - \frac{x}{2} + \frac{x^2}{24} - \frac{x^3}{720} + \frac{x^4}{40320}\right) dx$

$= \left[x - \frac{1}{2}\frac{x^2}{2} + \frac{1}{24}\frac{x^3}{3} - \frac{1}{720}\frac{x^4}{4} + \frac{1}{40320}\frac{x^5}{5}\right]\Big|_0^1$

$= 1 - \frac{1}{4}(1) + \frac{1}{72}1^3 - \frac{1}{288800}(1)^4 + \frac{1}{201600}(1)^5$

$= \underline{0.7638}$

10) $\int_0^{0.5} \frac{dx}{1+x^4}$

$f(x) = \frac{1}{1+x^4} \to f(0) = 1$

$f'(x) = \frac{-4x^3}{(1+x^4)^2} \to f'(0) = 0$

$\int_0^{0.5} (1 - x^4 + x^8 - x^{12}) dx$

$f''(x) = \frac{10x^6 - 12x^2}{(1+x^4)^3} \to f''(0) = 0$

$= x - \frac{x^5}{5} + \frac{x^9}{9} - \frac{x^{13}}{13} \Big|_0^{0.5}$

$f'''(x) = \frac{-120x^9 + 240x^5 - 24x}{(1+x^4)^4} \to f(0) = 0$

$= 0.5 - \left(\frac{.5)^5}{5}\right) + \frac{(0.5)^9}{9} - \frac{(0.5)^{13}}{13} - (0)$

$= 0.49352$

12) Exprese $(1-\sqrt{x})^{3/2}$ como una serie de potencia y calcule con exactitud de tres cifras decimal el valor de $\int_0^4 (1-\sqrt{x})^{2/3}$

Serie $= 1 - \frac{2\sqrt{x}}{3} - \frac{x}{9} - \frac{4x^{3/2}}{81} - \frac{7x^2}{243}$

$\int_0^4 \left(1 - \frac{2\sqrt{x}}{3} - \frac{x}{9} - \frac{4x^{3/2}}{81} - \frac{7x^2}{243}\right.$

$= x - \frac{4x\sqrt{x}}{9} - \frac{x^2}{18} - \frac{8x^2\sqrt{x}}{405} - \frac{7x^3}{729} \Big|_0^4$

$= 4 - \frac{4(4)\sqrt{4}}{9} - \frac{4^2}{18} - \frac{8(4)^2\sqrt{4}}{405} - \frac{7(4)^3}{729} - 4$

$= -5.6910$

$\text{Sen}(x) = x - \frac{x^3}{3!} + \frac{1}{5!}x^5 - \frac{x^7}{7!} + \frac{x^9}{9!}$

13) Evalue

a) $\lim_{x \to 0} \frac{\text{Sen}(x) - \tan^{-1}(x)}{x^2 \ln(1+x)}$

$\tan^{-1}(x)$

$f(x) = \tan^{-1}(x) \rightarrow f(0) = 0$

$f'(x) = 1/1+x^2 = (1+x^2)^{-1} \rightarrow f'(0) = 1$

$f''(x) = -(1+x^2)^{-2}(2x) \rightarrow f''(0) = 0$

$f'''(x) = -2x(1+x^2)^{-2}$

$= (-2x)'(1+x^2)^{-2} + (-2x)((1+x^2)^{-2})'$

$= -2(1+x^2)^{-2} - (2x)(-2(1+x^2)^{-2}(2x))$

$= -2(1+x^2)^{-2} + 8x^2(1+x^2)^{-3}$

$f'''(0) = 2$

$f^{IV}(x) = -48x^3(1+x^2)^{-4} + 24x(1+x^2)^{-3} \rightarrow f^{IV}(0) = 0$

$f^{V}(x) = 384x^4(1+x^2)^{-5} - 144x^2(1+x^2)^{-4}$
$- 144x^2(1+x^2)^{-4} + 24(1+x^2)^{-3} \rightarrow f^{V}(0) = 24$

$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$

$\text{Sen}(x) - \tan^{-1}(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} - \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}\right)$

$= \frac{1}{6}x^3 - \frac{23}{120}x^5 + \frac{719}{5040}x^7$

Serie de $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$

$$\lim_{x\to 0} \frac{\operatorname{sen}(x)-\tan^{-1}(x)}{x^2 \ln(x)} = \lim_{x\to 0} \left( \frac{\frac{1}{6}x^3 - \frac{23}{120}x^5 + \frac{719}{5040}x^7}{x^2\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\right)} \right)$$

$$= \lim_{x\to 0} \frac{\cancel{x^3}\left(\frac{1}{6} - \frac{23}{120}x^2 + \frac{719}{5040}x^4\right)}{\cancel{x^3}\left(1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4}\right)} = \frac{\frac{1}{6}}{1} = \frac{1}{6}$$

6) $\lim_{x\to 0} \frac{\ln(x)}{x-1}$

Serie $\ln(x)$

$f(x) = \ln(x) \longrightarrow \infty$

$f'(x) = \frac{1}{x} = x^{-1} \longrightarrow \infty$

$f''(x) = -1x^{-2} \longrightarrow \infty$

$f'''(x) = 2x^{-3} \longrightarrow \infty$

$f^{IV}(x) = -6x^{-4} \longrightarrow \infty$

Serie de $x-1$

$f(x) = x-1 \longrightarrow 1$

$f'(x) = 1 \longrightarrow 1$

$f''(x) = 0 \longrightarrow 0$

$$\lim_{x\to 0} \frac{\infty}{x-1} = \infty$$

14)

a) $\int_0^{1/3} \sqrt{1+x^3}\,dx$

Serie $\sqrt{1+x^3}$

$= 1 + \frac{x^3}{3} - \frac{x^6}{8} + \frac{x^9}{16}$

$\int_0^{1/3} 1 + \frac{x^3}{2} - \frac{x^6}{8} + \frac{x^9}{16}$

$x = x + \frac{1}{2}\frac{x^4}{4} - \frac{1}{8}\frac{x^7}{7} + \frac{1}{16}\frac{x^{10}}{10}\Big|_0^{1/3}$

$= \frac{1}{3} + \frac{1}{8}(1/3)^4 - \frac{1}{56}(1/3)^7 + \frac{1}{160}(1/3)^{10}$

$= 0.334$ //

b) mediante serie de potencia

$\lim_{x \to 0} \frac{e - e^{\cos(x)}}{x^2}$

Serie

$e - \frac{ex^2}{2!} + \frac{ex^4}{6} - \frac{31ex^6}{720}$

$\lim_{x \to 0} \frac{e - e + \frac{ex^2}{2} - \frac{ex^4}{6} + \frac{31ex^6}{720}}{}$

$= \frac{\cancel{x^2}\left(\frac{e}{2} - \frac{ex^2}{6} + \frac{31ex^4}{720}\right)}{\cancel{x^2}\,(1)} = \frac{\frac{e}{2}}{1} = \frac{e}{2}$ //