UNIVERSIDAD AUTONOMA DE AGUASCALIENTES

DEPARTAMENTO DE MATEMÁTICAS Y FÍSICA

Materia: **CÁLCULO INTEGRAL**

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TAREA 7 PARCIAL 2

INTEGRALES QUE CONDUCEN A FUNCIONES TRIGONOMETRICAS INVERSAS

$2a = 1 \quad a = 1/2 \qquad F = x^2 + 2ax + a^2 = (x+a)^2$

Evalua

① $\int \frac{1}{x^2+x+1} = \int \frac{1}{x^2+x+\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1} = \int \frac{1}{(x+\frac{1}{2})^2 + 1 - \left(\frac{1}{2}\right)^2}$

$= \int \frac{1}{(x+\frac{1}{2})^2 + 3/4} = \int \frac{1\,dx}{(x+1/2)^2 + 3/4} = \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)$

$a^2 = 3/4, \; a = \sqrt{3}/2$

$u^2 = (x+1/2)^2, \; (x+1/2) = u$

$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\frac{2x+1}{2}}{\frac{\sqrt{3}}{2}}\right) = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2(2x+1)}{2\sqrt{3}}\right)$

$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

② $\int \frac{dx}{4x^2+4x+3}\, dx = \int \frac{dx}{4(x^2+x+3/4)} = \int \frac{dx}{4(x^2+x+\frac{3}{4} + (\frac{1}{2})^2 - (1/2)^2}$

$= \int \frac{dx}{4(x+1/2)^2 + 3/4 - (1/2)^2} = \int \frac{dx}{4(x+1/2)^2 + 1/2}$

$= \frac{1}{4} \int \frac{dx}{(x+1/2)^2 + 2}$ $\quad \tan^{-1}$

$a^2 = 1/2$

$a = 1/\sqrt{2}$

$u^2 = (x+1/2)^2$

$u = (x+1/2)$

$= 1/4 \frac{1}{\frac{1}{\sqrt{2}}} \tan^{-1}\left(\frac{x+1/2}{\frac{1}{\sqrt{2}}}\right) + C$

$= \frac{\sqrt{2}}{4} \tan^{-1}\left(\frac{2\sqrt{2}x + \sqrt{2}}{2}\right) + C$

(5) $\int \frac{2x^3}{2x^2-4x+3} = 2\int \frac{x^3}{2x^2-4x+3} = \int \left(\frac{5x-6}{2(2x^2-4x+3)} + \frac{x+2}{2} \right) dx$

$= \frac{1}{2}\int \frac{5x-6}{2x^2-4x+3}\, dx + \frac{1}{2}\int x\, dx + \int 1\, dx$

$= \int \left(\frac{5(4x-4)}{4(2x^2-4x+3)} - \frac{1}{2x^2-4x+3} \right) dx = 5\int \frac{x-1}{2x^2-4x+3}\, dx - \int \frac{1}{2x^2-4x+3}$

$\int \frac{x-1}{2x^2-4x+3}\, dx = \frac{1}{4}\int \frac{1}{u}\, du = \frac{\ln(u)}{4} = \ln(2x^2-4x+3)/4$

$u = \sqrt{2}(x-1)$

$u = 2x^2-4x+3$

$du = 4x-4$

$\int \frac{1}{2x^2-4x+3} dx = \int \frac{1}{(\sqrt{2}x-\sqrt{2})^2+1} dx = \frac{1}{\sqrt{2}}\int \frac{1}{u^2+1}\, du$

$= \frac{1}{\sqrt{2}}\int \frac{1}{u^2+1} = \arctan(u)/\sqrt{2} = \tan^{-1}(\sqrt{2}(x-1))/\sqrt{2}$

$= \frac{5\ln(2x^2-4x+3)}{4} - \frac{\tan^{-1}(\sqrt{2}(x-1))}{\sqrt{2}} + \frac{x^2}{2} + x + C$

(7) $\int \frac{2x+3}{\sqrt{5-x^2-4x}}\, dx = \int \left(\frac{-2x-4}{\sqrt{-x^2-4x+5}} - \frac{1}{\sqrt{-x^2-4x+5}} \right) dx$

$2\int \frac{x+2}{\sqrt{-x^2-4x+5}}\, dx - \int \frac{1}{\sqrt{-x^2-4x+5}}\, dx$

$\int \frac{x+2}{\sqrt{-x^2-4x+5}} = -\frac{1}{2}\int \frac{1}{\sqrt{u}} = -\frac{1}{2}2\sqrt{u} = -\sqrt{u}$

$\int \frac{1}{\sqrt{-x^2-4x+5}}\, dx = \int \frac{1}{\sqrt{9-(x+2)^2}}\, dx = \frac{3}{\sqrt{9-9u^2}} = \frac{1}{\sqrt{1-u^2}} = \operatorname{sen}^{-1}(u) = \operatorname{sen}^{-1}\left(\frac{x+2}{3}\right)$

$u = \frac{x+2}{3}$

$= -\operatorname{sen}^{-1}\left(\frac{x+2}{3}\right) - 2\sqrt{-x^2-4x+5} + C$

TAREA 8 INTEGRALES QUE CONDUCEN A FUNCIONES LOGARITMICAS

① $\int \cot x \, dx$

$\int \cot x = \ln|\operatorname{sen} x| + c$

② $\int \frac{1}{\sqrt{x}(1+\sqrt{x})} dx = \frac{1}{\cancel{\sqrt{x}}(u)}\left(2(\cancel{\sqrt{x}})\right) = 2\int \frac{1}{u}$

$u = \sqrt{x} + 1$

$du = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}$

$= 2\ln(u) = 2\ln(\sqrt{x}+1) + c$

③ $\int x e^{x^2} = \int \cancel{x} e^{u}\left(\frac{1}{2\cancel{x}}\right) = \frac{1}{2}\int e^{u}$

$u = x^2$

$du = 2x \, dx$

$dx = \frac{1}{2x}$

$= \frac{1}{2} e^{x^2} + c$

④ $\int \frac{e^{\tan(x)}}{\cos^2(x)} = \int e^{\tan(x)} \sec^2(x) = \int e^{\tan(x)} \sec^2(x) \, dx$

$u = \tan(x)$

$du = \sec^2(x) \, dx$

$dx = \frac{1}{\sec^2(x)}$

$= \int e^{u} \cancel{\sec^2(x)}\left(\frac{1}{\cancel{\sec^2(x)}}\right)$

$= \int e^{u} = e^{u} = e^{\tan(x)} + c$

⑤ $\int \frac{5^{3x}}{5^{3x}+7}\,dx = \int \frac{5^{\cancel{3x}}}{u}\left(\frac{1}{3\ln(5)\cdot 5^{\cancel{3x}}}\right)$

$u = 5^{3x}+7$
$du = \ln(5)\cdot 5^{3x}\cdot(3x)'$
$du = \ln(5)\cdot 3\cdot(x)'\cdot 5^{3x}$
$du = 3\ln(5)\cdot 1\cdot 5^{3x}$
$du = 3\ln(5)\cdot 5^{3x}$
$dx = \frac{1}{3\ln(5)\cdot 5^{3x}}$

$= 3\ln(5)\int \frac{1}{u}$

$= 3\ln(5)\,\ln|u|$

$= \frac{\ln|u|}{3\ln(5)} = \frac{\ln(5^{3x}+7)}{3\ln|5|} + C$

⑥ $\int \frac{1}{\cos^2 x\tan x}\,dx = \int \frac{\sec^2(x)}{\tan x} = \int \frac{\cancel{\sec^2(x)}}{u}\left(\frac{1}{\cancel{\sec^2(x)}}\right)$

$u = \tan(x)$
$du = \sec^2(x)$
$dx = \frac{1}{\sec^2 x}$

$= \int \frac{1}{u} = \ln|u| = \ln(\tan x) + C$

⑦ $\int \frac{x+1}{x-5}\,dx = \int \frac{x+1}{x-5}\left(du\right) = \int \frac{u+6}{u}\,du$

$u = x-5$
$du = 1\,x$
$dx = du$

$= \int\left(\frac{6}{u}+1\right) = 6\int \frac{1}{u}du \int 1\,du$

$= 6\ln|u| + u = 6\ln|x-5| + (x-5)$

$6\ln|x-5| + x - 5 + C$

⑧ $\int \frac{e^x}{1+e^x} = \int \frac{e^x}{u}\left(\frac{1}{e^x}\right) = \int \frac{1}{u} = \ln|u| = \ln(1+e^x)+c$

$u = 1+e^x$
$du = e^x$
$dx\ \frac{1}{e^x}$

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TAREA 9

FUNCIONES HIPERBOLICAS

(1) $\int th^2 3x \, dx = \int th^2 u \, (1/3)$

$U = 3x$

$du = 3 \, du$

$dx = \frac{1}{3}$

$= \frac{1}{3}\int th^2 u \, du = \frac{1}{3}\int (1 - sech^2(u)) \, du$

$= \frac{1}{3}\left[\int 1 \, du - \int sech^2(u) \, du\right.$

$= \frac{1}{3}\left[u - tanh(u)\right] = \frac{1}{3}\left[3x - tanh(3x)\right]$

$= \frac{3x}{3} - \frac{tanh(3x)}{3} = x - \frac{tanh(3x)}{3} + C$

(2) $\int 2e^x \cosh x \, dx = 2\int e^x \cosh x \, dx$

$= \int 2e^x \cdot \frac{e^{-2x}(e^{2x}+1)}{4} \, dx = \frac{1}{4}\int \frac{u+1}{u} = \frac{1}{4}\int \frac{1}{u} + 1 \, du =$

$U = e^{2x}$

$du = 2e^{2x} \, dx$

$dx = \frac{e^{-2x}}{2}$

$\int 1/u \, du + \int 1 \, du = \ln(u) + u = \frac{1}{4}(\ln(e^{2x}) + e^{2x}$

$= \frac{e^{2x}}{4} + \frac{x}{2} = \frac{e^{2x}}{2} + x + C$

$$③\int \frac{dx}{\sqrt{9x^2-25}} = \int \frac{5}{3\sqrt{25u^2-25}} = \frac{1}{3}\int \frac{1}{\sqrt{u^2-1}}\,du$$

$u = \frac{3x}{5}$

$du = 3/5$

$dx = 5/3$

$$\int \frac{1}{\sqrt{u^2-1}}\,du = \int \frac{\sinh(v)}{\sqrt{\cosh^2(v)-1}} = \int 1 = v = \operatorname{arcosh}(u)$$

$u = \cosh(v)$

$v = \operatorname{arcosh}(u)$

$dv = \sinh(u)$

$$= \frac{1}{3}\operatorname{arcosh}(v) = \frac{1}{3}\operatorname{arcosh}\left(\frac{3x}{5}\right) + C$$

$$④\int \frac{dx}{x\sqrt{9-16x^2}} = \int \frac{1}{u^2-9} = \int \frac{1}{(u-3)(u+3)}$$

$U = \sqrt{9-16x^2}$

$du = \frac{-16x}{\sqrt{9-16x^2}}\,dx$

$dx = -\frac{\sqrt{9-16x^2}\,du}{16x}$

$$= \int \frac{1}{6(u-3)} - \frac{1}{6(u+3)}\,du$$

$$= \frac{1}{6}\int \frac{1}{u-3}\,du - \frac{1}{6}\int \frac{1}{u+3}\,du$$

$$\int \frac{1}{u-3} = \int \frac{1}{v} = \ln(v) = \ln|u-3) \quad \Big| \quad \int \frac{1}{u+3} = \ln(u+3)$$

$v = u-3$

$dv = 1$

$du = 1$

$$= \frac{\ln(\sqrt{9-16x^2}-3)}{6} - \frac{\ln(\sqrt{9-16x^2}+3)}{6} + C$$