

# TAREA MAXIMOS Y MINIMOS

① Obtenga los máximos y mínimos locales de las siguientes funciones.

a)  $f(x) = 4x - x^2$        $f'(x) = 4 - 2x$

$$f'(x) = 0$$

$$4 - 2x = 0$$

$$-2x = -4$$

$$x = -4 / -2$$

$$\boxed{x = 2}$$

$$f''(x) = (4 - 2x)^1$$

$$f''(x) = -2$$

MAXIMO LOCAL EN  $x = 2$   
CON VALOR DE

PUNTO CRITICO

$$\boxed{f(2) = 4(2) - (2)^2 = 8 - 4 = 4}$$

b)  $f(x) = x^2 - 4x + 4$

$$f'(x) = 2x - 4$$

$$f'(x) = 0$$

$$f''(x) = (2x - 4)^1$$

$$2x - 4 = 0$$

$$f''(1) = 2$$

$$2x = 4$$

$$x = 4 / 2$$

$$\boxed{x = 2}$$

MINIMO LOCAL EN  $x = 2$   
CON VALOR DE

PUNTO

CRITICO

$$\boxed{f(2) = (2)^2 - 4(2) + 4 = 4 - 8 + 4 = 0}$$

$$c) f(x) = x^2 - x - 20$$

$$f'(x) = 2x - 1$$

$$f'(x) = 0$$

$$2x - 1 = 0$$

$$2x = 1$$

$$\underbrace{x = \frac{1}{2}}$$

PUNTO CRÍTICO

$$f''(x) = (2x - 1)'$$

$$f''(x) = 2$$

MINIMO LOCAL EN  $x = 1/2$  CON VALOR

$$f(1/2) = (1/2)^2 - (1/2) - 20 = \underline{-81/4}$$

$$d) f(x) = x^3 - 9x$$

$$f'(x) = 3x^2 - 9$$

$$f'(x) = 0$$

$$3x^2 - 9 = 0$$

$$3(x^2 - 3) = 0$$

$$\sqrt{3} \sqrt{(x^2 - 3)} = 0$$

$$\sqrt{3} \sqrt{(x^2 - 3)} \cdot \sqrt{3} \sqrt{(x^2 + 3)} = 0$$

$$ab = 0, b = 0, a = 0$$

$$x - \sqrt{3} = 0$$

$$x_1 = \sqrt{3}$$

MAXIMO LOCAL EN

$$x = -\sqrt{3}$$
 VALOR DE

$$f(-\sqrt{3}) = 6(\sqrt{3})^3 - 9(-\sqrt{3})$$

$$= 6(\sqrt{3})^2\sqrt{3} - 9(-3^{1/2})$$

$$= 3(-\sqrt{3}) - 9(-\sqrt{3})$$

$$= -3\sqrt{3} + 9\sqrt{3}$$

$$\underline{= 6\sqrt{3}}$$

PUNTOS CRÍTICOS

MINIMO LOCAL EN  $x = \sqrt{3}$  CON VALOR

$$f(\sqrt{3}) = (\sqrt{3})^3 - 9(\sqrt{3}) = (\sqrt{3})^2\sqrt{3} - 9(\sqrt{3})$$
$$= 3\sqrt{3} - 9\sqrt{3} = \underline{-6\sqrt{3}}$$

$$c) f(x) = x^3 - 7x^2 + 12x$$

$$f'(x) = 3x^2 - 14x + 12$$

$$f'(x) = 0$$

$$3x^2 - 14x + 12 = 0$$

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(3)(12)}}{2(3)}$$

$$f''(x) = 6x - 14$$

(Maximo, o minimo?)

$$= 14 \pm \sqrt{196 - 144} / 6$$

$$f''\left(\frac{7+\sqrt{13}}{3}\right) = 6\left(\frac{7+\sqrt{13}}{3}\right) - 14$$

$$= 14 \pm \sqrt{52} / 6 = 14 \pm \sqrt{4(13)} / 6$$

$$= 2(7 + \sqrt{13}) - 14$$

$$= 14 \pm 2\sqrt{13} / 6$$

$$= 14 + 2\sqrt{13} - 14$$

$$= 2\sqrt{13}$$

$2\sqrt{13} > 0 \rightarrow \text{MINIMO LOCAL}$

$$x_1 = 7 + \sqrt{13} / 3$$

$$f''\left(\frac{7-\sqrt{13}}{3}\right) = 6\left(\frac{7-\sqrt{13}}{3}\right) - 14$$

$$x_2 = 7 - \sqrt{13} / 3$$

$$= 2(7 - \sqrt{13}) - 14$$

$$= 14 - 2\sqrt{13} - 14$$

$$= -2\sqrt{13}$$

$-2\sqrt{13} < 0 \rightarrow \text{MAXIMO LOCAL}$

MAXIMO LOCAL

MINIMO LOCAL

$$f\left(\frac{7-\sqrt{13}}{3}\right) = \left(\frac{7-\sqrt{13}}{3}\right)^2 - 7\left(\frac{7-\sqrt{13}}{3}\right)^2 + 12\left(\frac{7-\sqrt{13}}{3}\right)$$

$$f\left(\frac{7+\sqrt{13}}{3}\right) = \left(\frac{7+\sqrt{13}}{3}\right)^2 - 7\left(\frac{7+\sqrt{13}}{3}\right)^2 + 12\left(\frac{7+\sqrt{13}}{3}\right)$$

$$= 616 + 160\sqrt{13} + 3(434 + 48\sqrt{13})$$

$$+ 756 - 188\sqrt{13} / 27$$

$$= 70 + 26\sqrt{13}$$

$$\boxed{27}$$

$$= \frac{70 - 26\sqrt{13}}{27}$$

$$f) f(x) = 6x^2 - x^3$$

$$f'(x) = 12x - 3x^2$$

$$f'(x) = 0$$

$$12x - 3x^2 = 0$$

$$3x(4-x) = 0$$

$$\text{EN } x=0$$

$$\hookrightarrow f''(0) = 12 - 6(0)$$

$$= 12$$

$$\boxed{x=0}$$

$$\boxed{x=4}$$

MINIMO LOCAL EN  $x=0$  CON VALOR

$$\boxed{f(0) = 6(0)^2 - (0)^3 = 0}$$

$$\text{EN } x=4$$

$$\hookrightarrow f''(4) = 12 - 6(4) = -12$$

MAXIMO LOCAL EN  $x=4$  CON VALOR

$$\boxed{f(4) = 6(4)^2 - (4)^3 = 32}$$

$$g) f(x) = x^3 - 6x^2 + 9x$$

$$f'(x) = 3x^2 - 12x + 9$$

$$f'(x) = 0$$

$$f''(x) = 6x - 12$$

$$3x^2 - 12x + 9 = 0$$

$$\text{EN } x=1$$

$$\hookrightarrow f''(1) = 6(1) - 12 = -6$$

$$= -(-12) \pm \sqrt{12^2 - 4(3)(9)} / 2(3)$$

$$= 12 \pm \sqrt{36} / 6$$

$$= x_1 = 12 + 6 / 6 = 3$$

$$= x_2 = 12 - 6 / 6 = 1$$

MAXIMO LOCAL EN  $x=1$  VALOR

$$\boxed{f(1) = (1)^3 - 6(1)^2 + 9(1) = 4}$$

$$\text{EN } x=3 \rightarrow f''(3) = 6(3) - 12 = 6$$

MINIMO LOCAL EN  $x=3$  VALOR

$$\boxed{f(3) = 3^3 - 6(3)^2 + 9(3) = 0}$$

# TAREA LOGARITMOS

② Aplica propiedades de logaritmo para expresar con un solo logaritmo de las expresiones con un solo logaritmo

a)  $\ln(x+2) + \ln(x-2) - 3\ln(x)$

$$\begin{aligned} &= \ln \left[ \frac{(x+2)(x-2)}{x^3} \right] = \ln \left[ \frac{x^2-4}{x^3} \right] \\ &= \boxed{\ln(x^2-4) - 3\ln(x)} \end{aligned}$$

b)  $\frac{1}{2} \ln(x^2-4)^2 + \ln(x^2-5x+6) - 2\ln(x-2)$

$$= \ln \left[ \frac{(x^2-4)^{1/2} (x^2-5x+6)}{(x-2)^2} \right]$$

$$= \ln \left[ \frac{(x+2)(x-2)(x-2)(x-3)}{(x-2)^2} \right]$$

$$= \ln \left[ (x+2)(x-3) \right] =$$

$$= \boxed{\ln(x^2-x-6)}$$

# TAREA DERIVADAS LOGARITMOJ

- ③ Obtenga la primera y segunda de las siguientes funciones,

$$f(x) = \ln(x+2) + \ln(x-2) + 3\ln(x)$$

$$\begin{aligned} f'(x) &= \frac{1}{x+2} + \frac{1}{x-2} - 3 \cdot \frac{1}{x} - \frac{x-2+x+2}{(x+2)(x-2)} - 3x \\ &= \frac{2x}{x^2-4} - \frac{3}{x} = \frac{2x^2-3(x^2-4)}{x(x^2-4)} \end{aligned}$$

$$f'(x) = \frac{2x^2-3x^2+12}{x^3-4x} = \frac{-x^2+12}{x^3-4x}$$

$$f''(x) = \frac{-x^2+12}{x^3-4x} = \frac{(-x^2+12)'(x^3-4x) - (-x^2+12)(x^3-4x)'}{(x^3-4x)^2}$$

$$= (-2x)(x^3-4x) - (-x^2+12)(3x^2-4) / (x^3-4x)$$

$$= -2x^4 + 8x^2 - (-3x^4 + 4x^2 + 36x^2 - 48) / (x^3-4x)$$

$$= -2x^4 + 8x^2 + 3x^4 - 4x^2 - 36x^2 + 48 / (x^3-4x)$$

$$= \frac{x^4 - 32x^2 + 48}{(x^3-4x)^2}$$

$$g(x) = 2\ln(x^2+1) + 3\ln(x^3+1)$$

$$g'(x) = 2 \frac{1}{x^2+1} (2x) + 3 \frac{1}{x^3+1} 3x^2$$

$$= \frac{4x}{x^2+1} + \frac{9x^2}{x^3+1} = \frac{4x(x^3+1) + 9x^2(x^2+1)}{(x^2+1)(x^3+1)}$$

$$g''(x) = \frac{4x^4 + 4x + 9x^4 + 9x^2}{(x^2+1)(x^3+1)} = \frac{13x^4 + 9x^2 + 4x}{(x^5+x^2+x^3+1)}$$

$$g'''(x) = \frac{(13x^4 + 9x^2 + 4x)'(x^5+x^2+x^3+1) - (13x^4 + 9x^2 + 4x)(x^5+x^2+x^3+1)'}{(x^5+x^2+x^3+1)^2}$$

$$g'''(x) = \frac{-13x^8 - 14x^6 + 10x^5 - 9x^4 + 44x^3 - 4x^2 + 18x + 4}{(x^5+x^2+x^3+1)^2}$$