Ex 8.12
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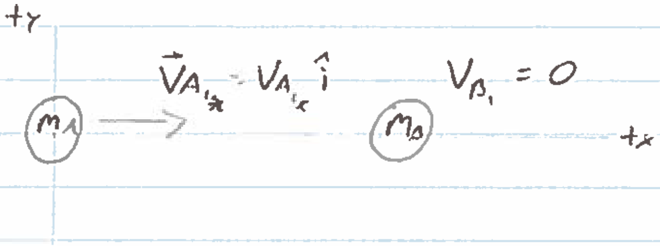
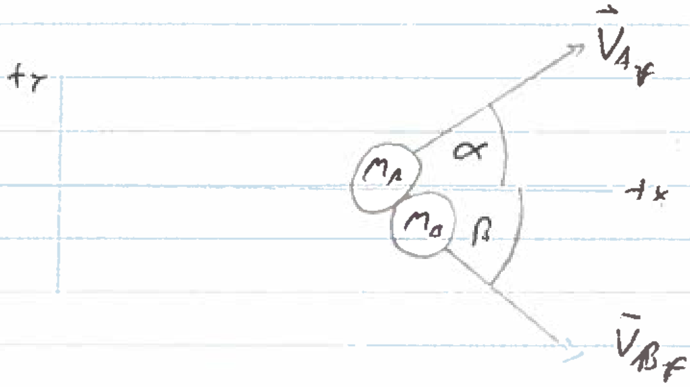
Before



m_A
m_B
$|\vec{V}_{A_{1x}}|$
$|\vec{V}_{B_1}|$

After



$|\vec{V}_{A_f}|$

To find $|\vec{V}_{B_2}|$ equate before & after KE, as collision is elastic

$$K_{A_i} + K_{B_i} = K_{A_f} + K_{B_f}$$

$$\tfrac{1}{2} m_A |\vec{V}_{A_i}|^2 + \tfrac{1}{2} m_B |\vec{V}_{B_i}|^2 = \tfrac{1}{2} m_A |\vec{V}_{A_f}|^2 + \tfrac{1}{2} m_B |\vec{V}_{\ldots}$$

$$|\vec{V}_{B_f}| = \sqrt{\frac{m_A}{m_B}\left(|\vec{V}_{A_i}|^2 - |\vec{V}_{A_f}|^2\right)}$$

$$= \sqrt{\frac{0.500\,kg}{0.300\,kg}\left((4.00\,ms^{-1})^2 - (2.00\,ms^{-1})^2\right)}$$

$$|\vec{V}_{B_f}| = 4.47\,ms^{-1}$$

Now conserve momentum in x & y:

$$\Delta p_{tot_x} = \Delta p_{A_x} + \Delta p_{B_x} = p_{A_{f_x}} - p_{A_{i_x}} + p_{B_{f_x}} - p_{B_{i_x}} = 0$$

$$\cancel{\Delta p_{tot_y}} \quad m_A V_{A_{f_x}} - m_A V_{A_{i_x}} + m_B V_{B_{f_x}} - m_B V_{i_{B_x}} = 0$$

$$m_A V_{A_f} \cos\alpha - m_A V_{A_i} + m_B V_B \cos\beta = 0 \qquad (1)$$

$$\Delta p_{tot_y} = \Delta p_{A_y} + \Delta p_{B_y} = p_{A_{f_y}} - p_{A_{i_y}} + p_{B_{f_y}} - p_{B_{i_y}} = 0$$

$$m_A V_{A_{f_y}} - m_A V_{A_{i_y}} + m_B V_{B_{f_y}} - m_B V_{B_{i_y}} = 0$$

$$m_A V_{A_f} \sin\alpha + m_B V_{B_f} \sin\beta = 0 \qquad (2)$$

Solve (1) for $\cos\beta$:

$$\cos\beta = \frac{m_A V_{A_i} - m_A V_{A_f}\cos\alpha}{m_B V_B}$$

Square:

$$\cos^2\beta = \frac{m_A^2 V_{A_i}^2 + m_A^2 V_{A_f}^2 \cos^2\alpha - 2 m_A^2 V_{A_i} V_{A_f} \cos\alpha}{m_B^2 V_{B_f}^2} \qquad (A)$$

Solve (2) for $\sin\beta$:

$$\sin\beta = -\frac{m_A V_{A_f}}{m_B V_{B_f}} \sin\alpha$$

Square:

$$\sin^2\beta = \frac{m_A^2 V_{A_f}^2}{m_B^2 V_{B_f}^2} \sin^2\alpha \qquad (B)$$

$$(A) + (B)$$

$$\sin^2\beta + \cos^2\beta = 1 = \frac{m_A^2 V_{A_f}^2 \sin^2\alpha}{m_B^2 V_{B_f}^2} + \frac{m_A^2 V_{A_i}^2 + m_A^2 V_{A_f}^2 \cos^2\alpha - 2m_A^2}{m_B^2 V_{B_f}^2}$$

$$m_B^2 V_{B_f}^2 = m_A^2 V_{A_f}^2 \sin^2\alpha + m_A^2 V_{A_f}^2 \cos^2\alpha + m_A^2 V_{A_i}^2 - 2m_A^2 V_{A_i}$$

$$m_B^2 V_{B_f}^2 = m_A^2 V_{A_f}^2 + m_A^2 V_{A_i}^2 - 2m_A^2 V_{A_i} V_{A_f} \cos\alpha$$

$$\frac{m_B^2}{m_A^2} V_{B_f}^2 = V_{A_f}^2 + V_{A_i}^2 - 2V_{A_i} V_{A_f} \cos\alpha$$

$$2V_{A_i} V_{A_f} \cos\alpha = V_{A_f}^2 + V_{A_i}^2 - \left(\frac{m_B}{m_A}\right)^2 V_{B_f}^2$$

$$2\cos\alpha = \frac{V_{A_f}}{V_{A_i}} + \frac{V_{A_i}}{V_{A_f}} - \left(\frac{m_B}{m_A}\right)^2 \frac{V_{B_f}^2}{V_{A_i} V_{A_f}}$$

$$\alpha = \cos^{-1}\left[\frac{1}{2}\left(\frac{V_{A_f}}{V_{A_i}} + \frac{V_{A_i}}{V_{A_f}} - \left(\frac{m_B}{m_A}\right)^2 \frac{V_{B_f}^2}{V_{A_i} V_{A_f}}\right)\right]$$

$$\alpha = \cos^{-1}\left[\frac{1}{2}\left(\frac{2ms^{-1}}{4ms^{-1}} + \frac{4ms^{-1}}{2ms^{-1}} - \left(\frac{0.3kg}{0.5kg}\right)^2 \frac{(4.47ms^{-1})^2}{(4ms^{-1})(2ms^{-1})}\right)\right]$$

$$\underline{\alpha = 36.9^\circ}$$

$$\sin \beta = -\frac{m_A V_{Af}}{m_B V_{Bf}} \sin \alpha$$

$$\beta = \sin^{-1}\left[-\frac{(0.5\,kg)(2\,ms^{-1})}{(0.3\,kg)(4.47\,ms^{-1})} \sin(36.9°) \right]$$

$$\beta = -26.6°$$

$\beta = 26.6°$ below x axis.