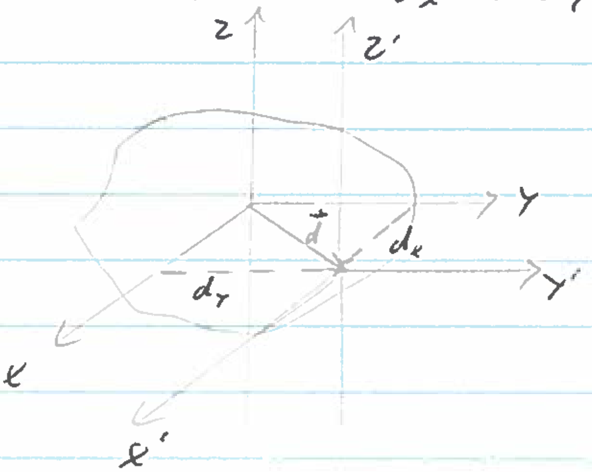
Parallel-Axis Theorem

- ~~[illegible]~~
- used to calculate I ~~[illegible]~~ when rotation is not through C-of-M.

Consider an object for which we know I through the C-of-M, but wish to know it for a rotational axis placed elsewhere. - shifted d_x & d_y in the x-y,



The transformation of the old to the new rotation axis is:

$$x' = x - d_x$$
$$y' = y - d_y$$
$$z' = z$$

Then the distance of any particular element from the new rotation axis is:

$$r'_\perp = \sqrt{x'^2 + y'^2}$$

In the new coordinate system, the new moment of inertia is:

$$I_{//} = \int_V (r_\perp')^2 \rho \, dV$$

So $(r_\perp')^2 = (x')^2 + (y')^2 = (x-d_x)^2 + (y-d_y)^2$

$$= x^2 + y^2 + d_x^2 + d_y^2 - 2xd_x - 2yd_y$$

$$= (x^2+y^2) + (d_x^2+d_y^2) - 2xd_x - 2yd_y$$

$$= r_\perp^2 + d^2 - 2xd_x - 2yd_y$$

So:

$$I_{//} = \int_V (r_\perp')^2 \rho \, dV$$

$$= \underbrace{\int_V r_\perp^2 \rho \, dV}_{I_{cm}} + \underbrace{d^2 \int_V \rho \, dV}_{d^2 M} - \underbrace{2d_x \int_V x\rho \, dV}_{X_{cm}} - \underbrace{2d_y \int_V y\rho \, dV}_{Y_{cm}}$$

As the origin for I_{cm} is placed at the centre of mass, $X_{cm} = Y_{cm} = 0$

$$\therefore \quad \underline{I_{//} = I_{cm} + d^2 M}$$