Experiment Record: Conceptualization of Random Error and Propagation of Error

Experiment Title

Conceptualization of Random Error and Propagation of Error by Measuring the Dimensions of a Thin Metallic Rod Using Screw Gauge and Vernier Caliper

Objective

1. To measure the dimensions (diameter and length) of a thin metallic rod using a screw gauge and Vernier caliper.
2. To calculate the volume and surface area of the metallic rod.
3. To understand random errors and analyze the propagation of errors in measurements.

Apparatus Required

- Screw gauge
- Vernier caliper
- Thin metallic rod
- Calculator

Theory

Measurement Tools

- **Screw Gauge:** Measures diameter with high precision.

$$L.C. = \frac{\text{Pitch of screw}}{\text{Number of divisions on circular scale}}$$

- **Vernier Caliper:** Measures length.

  $L.C. =$ Value of 1 Main Scale Division (MSD)$-$Value of 1 Vernier Scale Division (VSD)

  Example: If 10 Vernier scale divisions = 9 main scale divisions, then

$$L.C. = 1\,\text{mm} - 0.9\,\text{mm} = 0.1\,\text{mm}.$$

Formulas

- **Volume of Rod:**
$$V = \pi r^2 h$$
where r = radius of rod, h = length of rod.

- **Surface Area of Rod:**
$$A = 2\pi r h + 2\pi r^2$$

Error Propagation

When we measure a set of data, We use standard deviation to estimate error.

$$\sigma_x = \sqrt{\overline{(x_i - \overline{x_i})^2}}$$

- For **multiplication or division**, relative errors are added:
$$\frac{\sigma_z}{z} = \frac{\sigma_x}{x} + \frac{\sigma_y}{y}$$

- For **addition or subtraction**, absolute errors are added:
$$\sigma_z = \sigma_x + \sigma_y$$

- Errors in this experiment:
  - **Volume Error:**
$$\frac{\sigma_V}{V} = 2\frac{\sigma_r}{r} + \frac{\sigma_h}{h}$$
$$\sigma_V = V\left(2\frac{\sigma_r}{r} + \frac{\sigma_h}{h}\right)$$
  - **Surface Area Error:**
$$\sigma_A = 2\pi\left(h\sigma_r + r\sigma_h\right) + 4\pi r\sigma_r$$

Procedure

1. **Measuring Diameter:**
   - Place the metallic rod in the jaws of the screw gauge.
   - Take three readings of the diameter (d) at different points along the rod.
   - Calculate the mean diameter and derive the radius ($r = d/2$).
2. **Measuring Length:**
   - Place the metallic rod in the jaws of the Vernier caliper.
   - Take three readings of the length (h) at different points along the rod.
   - Calculate the mean length.
3. **Calculations:**
   - Compute the volume and surface area using the measured dimensions.
   - Use the formulas for error propagation to calculate the uncertainties in the volume and surface area.

Observations

Screw Gauge Readings

Observation	Diameter (mm)	Error (mm)
1		
2		
3		

Mean Diameter: $\bar{d} = \ldots \pm \sigma_d$

Vernier Caliper Readings

Observation	Length (mm)	Error (mm)
1		
2		
3		

Mean Length: $\bar{h} = \ldots \pm \sigma_h$

Calculations

Volume

$$V = \pi r^2 h$$

Absolute error in volume:

$$\sigma_V = V \left(2\frac{\sigma_r}{r} + \frac{\sigma_h}{h} \right)$$

Surface Area

$$A = 2\pi r h + 2\pi r^2$$

Absolute error in surface area:

$$\sigma_A = 2\pi \left(h\sigma_r + r\sigma_h \right) + 4\pi r \sigma_r$$

Results

1. **Dimensions of the Rod:**
   - Diameter (d): $\ldots \pm \ldots$ mm
   - Length (h): $\ldots \pm \ldots$ mm
2. **Calculated Quantities:**
   - Volume (V): $\ldots \pm \ldots$ mm^3
   - Surface Area (A): $\ldots \pm \ldots$ mm^2

3. **Error Analysis:**
    - Propagated error in volume: ...
    - Propagated error in surface area: ...

Conclusion

The dimensions of the metallic rod were successfully measured using a screw gauge and Vernier caliper. The calculated volume and surface area were found with propagated uncertainties, providing insight into the impact of random measurement errors.

Explanation of Least Count (L.C.) of Vernier Caliper

The **Least Count (L.C.)** of a Vernier caliper is the smallest value that can be measured accurately using the instrument. It is derived based on the difference between the values of one main scale division (MSD) and one Vernier scale division (VSD).

Formula for Least Count

$L.C. = \text{Value of 1 Main Scale Division (MSD)} - \text{Value of 1 Vernier Scale Division (VSD)}$

Understanding the Vernier Scale Division (VSD)

The Vernier scale has a specific number of divisions that correspond to a fixed length on the main scale. For example:

- If 10 Vernier scale divisions = 9 main scale divisions,
- Then, the value of 1 Vernier scale division is:

$$\text{VSD} = \frac{\text{Value of 1 Main Scale Division (MSD)}}{\text{Number of Vernier Scale Divisions}}$$

  Substituting the values:

$$\text{VSD} = \frac{1\,\text{mm}}{10} = 0.1\,\text{mm}$$

Calculating the Least Count

The least count is then calculated as:

$$L.C. = \text{MSD} - \text{VSD}$$

If:

- $\text{MSD} = 1\,\text{mm}$,
- $\text{VSD} = 0.9\,\text{mm}$ (since 10 Vernier divisions = 9 main scale divisions),

Then:

$$L.C. = 1\,\text{mm} - 0.9\,\text{mm} = 0.1\,\text{mm}$$

This means the Vernier caliper can measure with a precision of 0.1 mm.

Conclusion

The least count of a Vernier caliper is the smallest measurable value and is derived using:

$L.C.$ = Value of 1 Main Scale Division (MSD)−Value of 1 Vernier Scale Division (VSD).

This method ensures accurate understanding of how the main scale and Vernier scale interplay to achieve fine measurements.