Question

Compute the work done for a bus to climb the hill from the ground if the bus tires are rolling without friction. The bus climbs from $x = -1$ to $x = 0$ along the curve $y = 1 - x^2$.

Answer

To compute the work done by integrating $F \cdot dl \cdot \cos(\theta)$:

Solution Steps

- **Curve Representation**:
  - The curve is $y = 1 - x^2$.
- **Force Analysis**:
  - Gravitational Force (F_g): Acts vertically downward (mg).
  - Component of F_g along the curve: $F_{g,\parallel} = mg\sin(\theta)$, where θ is the angle of the incline.
- **Displacement Analysis**:
  - $\frac{dx}{dl} = \cos(\theta)$, We know from figure
  - $\frac{dy}{dx} = \tan(\theta) = -2x$, We know $tan(\theta)$, θ is the angle made by tangent and x-axis is the slope of the curve.
  - Therefore, $\sin(\theta) = \frac{-2x}{\sqrt{1+(2x)^2}}$
  - The differential displacement dl is:

$$dl = \sqrt{dx^2 + dy^2} = \sqrt{1 + 4x^2}\, dx$$

- **Work Done Calculation**:
  - The work done by the gravitational force component along the path is:

$$W = \int_{-1}^{0} F_{g,\parallel}\, dl$$

  - Substituting the expressions:

$$W = \int_{-1}^{0} mg\sin(\theta)\, dl = \int_{-1}^{0} mg\left(\frac{-2x}{\sqrt{1+4x^2}}\right)\cdot\sqrt{1+4x^2}\, dx = mg\int_{-1}^{0} -2x\, dx$$

  - Solving the integral:

$$W = mg\left[-x^2\right]_{-1}^{0} = mg\left(0 - (-1)^2\right) = mg \cdot 1$$

Thus, the work done W is:

$$W = m \cdot g \cdot 1$$

where $g \approx 9.81\,\mathrm{m/s}^2$.

Force Diagram

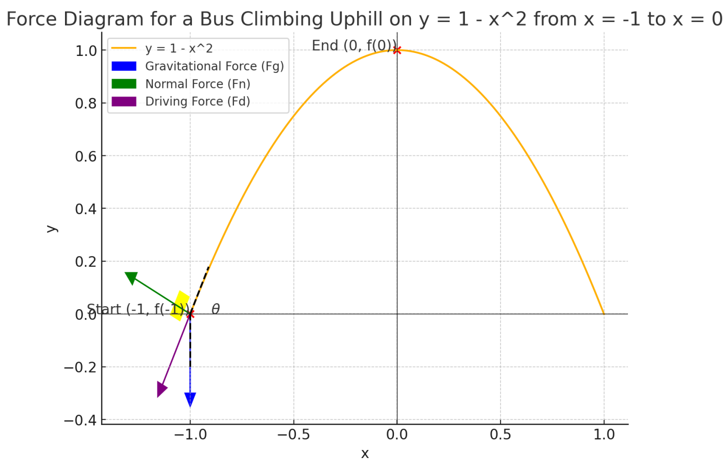

Figure 1: Force Diagram for a Bus Climbing Uphill on $y = 1 - x^2$ from $x = -1$ to $x = 0$

1 Further Questions for discussion in class

- Does this answer match with the answer we get from change in potential energy?
- Why didn't we consider the effect of friction at all?