# Queueing Theory

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## Introduction

The M/M/1 and M/M/S models are fundamental in Queueing Theory. Both assume:

- Arrivals follow a Poisson process (exponential interarrival times).
- Service times are exponentially distributed.
- An infinite-capacity queue with FIFO discipline.

## 1 M/M/1 Model

### Parameters

- $\lambda$: arrival rate (customers per time unit).
- $\mu$: service rate (customers per time unit).

### Formulas

$$\begin{aligned}
\rho &= \frac{\lambda}{\mu}, && \text{(utilization)} \\
P_0 &= 1 - \rho, && \text{(empty system probability)} \\
L_s &= \frac{\lambda}{\mu - \lambda}, && \text{(avg. number in system)} \\
W_s &= \frac{1}{\mu - \lambda}, && \text{(avg. time in system)} \\
L_q &= \frac{\lambda^2}{\mu\,(\mu - \lambda)}, && \text{(avg. number in queue)} \\
W_q &= \frac{\lambda}{\mu\,(\mu - \lambda)}, && \text{(avg. waiting time in queue)} \\
P_n &= (1 - \rho)\,\rho^n, && \text{(probability of } n \text{ in system)}
\end{aligned}$$

## 2 M/M/S Model

### Parameters

- $\lambda$: arrival rate (customers per time unit).
- $\mu$: service rate per server (customers per time unit).
- $S$: number of servers.

### Formulas

$$
\begin{aligned}
\rho &= \frac{\lambda}{S\,\mu}, \\
P_0 &= \left[ \sum_{n=0}^{S-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^S}{S!\,(1-\rho)} \right]^{-1}, \\
L_q &= \frac{(\lambda/\mu)^S\,\rho}{S!\,(1-\rho)^2}\;P_0, \\
L_s &= L_q + \frac{\lambda}{\mu}, \\
W_q &= \frac{L_q}{\lambda}, \\
W_s &= W_q + \frac{1}{\mu}.
\end{aligned}
$$