Matlab Exercise:
Implementing a solver for solving systems of linear equations

MATVJII

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The purpose of this exercise is to create a solver for systems of linear equations with variable size.

The solver uses the Gauss-Jordan method to transform the original system of equations to a diagonal one from which the solution can be easily found.The pseudocode of the Gauss-Jordan implementation is given below:

Data: $\boldsymbol{b}$, $\mathbf{A}$
Result: $\boldsymbol{x}$, flag

IF: size $\mathbf{A}$ and size $\boldsymbol{b}$ are NOT correct $\rightarrow$ set $flag = -1$, $\boldsymbol{x} = nan$;
ELSE: $\rightarrow$ $\mathbf{Aa}$ = concatenation of $\mathbf{A}$ $\boldsymbol{b}$.
  FOR: each column of $\mathbf{Aa}$ $\rightarrow$ c
    $\rightarrow$ search the maximum absolute value entry and store the row of $\mathbf{Aa}$ $\rightarrow$ row$_p$;the row position in a vector $\rightarrow$ $\boldsymbol{v}_p$ and the pivot $\rightarrow$ pivot = row$_p$(c)
    IF: pivot ≈ 0 $\rightarrow$ flag = 1 if system is undetermined or flag = 2 if it is inconsistent
    ELSE: (the system of equations is consistent)
      FOR: every row $\rightarrow$ row$_k$ different to the row of the pivot row$_p$
        $\rightarrow$ row$_k$ = row$_k$ - (row$_p$) (Aa(k,c))/row$_p$(c);
  FOR: every row position $\rightarrow$ i
        x(i) = $\mathbf{Aa}$($\boldsymbol{v}_p$(i),end)/$\mathbf{Aa}$($\boldsymbol{v}_p$(i),i)

Algorithm 1: Gauss Jordan solver pseudo code

TO DO:

- Implement the above pseudocode in a matlab function.
- Solve the next linear system and compare the output with the results obtained in class

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \\ \boldsymbol{x} \\ \\ \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

- Do the same with the next system and justify the results

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 0 \\ 3 & 3 & 2 \end{pmatrix} \begin{pmatrix} \\ \boldsymbol{x} \\ \\ \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

- Load the attatched matlab wirk space `WS.mat`, solve the linear system $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$ with your solver and deliver the sum of the solution variables $S = \sum_i x_i$