Computing risk group-specific years lost

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1 This document

This document accompanies the web application smikula.shinyapps.io/lostyears, which provides up to date estimates of years lost due to COVID-19 deaths, allowing for explicit specification of the risk group within individual age cohorts.[1] This facilitates computations of years lost under alternative assumptions about the selection mechanism of those who died from the rest of the population (surviving cohort).

This document describes how the risk groups are defined and how risk group-specific years lost are computed from the actuarial tables published by the Czech Statistical Office available and documented at www.czso.cz/csu/czso/umrtnostni_tabulky.

2 Risk groups

Let d_x be the number of individuals who die between the ages x and $x+1$, taken from the actuarial tables. Think of an individual who died exactly upon reaching the age x. Had she not died, she would have lived longer and dye at some age $x+y$, where $y \in \{0,\ldots,105-x\}$, 105 being the highest age of death in the actuarial tables.

Think of y's as identifiers of risk brackets from one of which a person who died at x was drawn and let $g_{x,y}$ denote the risk-group y given by the proportion of individuals dying between ages x and $x+y$

$$g_{x,y} = \frac{\sum_{i=0}^{y} d_{x+i}}{l_x},$$

where $l_x \equiv \sum_{i=0}^{105-x} d_{x+i}$ is the size of the cohort entering age x.

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[1] Full replication package is available at github.com/JosefMontag/years_lost_counter.

The highest-risk risk group $g_{x,0}$ is the proportion of individuals who would have died within one year. The lowest-risk risk group comprises of the whole cohort $g_{x,105-x} = l_x/l_x = 1$, covering all individuals dying between ages x and 105.

3 Risk group-specific years lost

Let a_x be the average number of person-years lived during the year of age at death, i.e. between x and $x+1$. According to the documentation of the actuarial tables, $a_x = 0.5$ for $x \in \{1, \ldots, 104\}$ and $a_{105} = 1/m_{105} \approx 1.49$ for males and ≈ 1.46 for females, where m_{105} is the mortality rate at age 105 from the actuarial tables.

A average person belonging to the risk group $g_{x,y}$, who died upon reaching the age of x, had potentially lost

$$e_{x,y} = \frac{\sum_{i=0}^{y} d_{x+i}(i + a_{x+i})}{\sum_{i=0}^{y} d_{x+i}}$$

years of life.

4 Extreme cases

For the highest-risk risk group $g_{x,0}$,

$$e_{x,0} = a_x,$$

i.e. 0.5 years for age at death $x < 105$.

For the whole cohort, i.e. the lowest-risk risk group $g_{x,105-x} = 1$,

$$\begin{aligned} e_{x,105-x} &= \frac{\sum_{i=0}^{105-x} d_{x+i}(i + a_{x+i})}{l_x} \\ &= e_x, \end{aligned}$$

i.e. the life expectancy at age x from the actuarial tables.