Numerical solutions for boson stars in $f(\mathcal{R})$ gravity using PINNs

José Perdiguero Gárate

December 26, 2023

1 Introduction

2 Dynamics in $f(\mathcal{R})$

In Palatini's formalisms, the manifold is endowed with two fundamental and independent fields: the metric tensor $g_{\mu\nu}$ and the affine connection $\Gamma^{\alpha}{}_{\beta\gamma}$, the former allow us to define the notion of distance, whereas the latter define the notion of parallelism. The action is written as follow

$$S[g,\Gamma] = \frac{1}{2k}\int \mathrm{d}^4x\sqrt{-g}f(\mathcal{R}) - \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\mathcal{P}(X,\Phi), \tag{1}$$

where the first term correspond to a generalization of the Einstein-Hilbert action, by replacing the functional with an arbitrary function of the Ricci scalar, and the second term stands for the matter sector defined as

$$\mathcal{P} = X - 2V(\Phi), \tag{2}$$

where $X = g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi$ and $V(\Phi) = -\frac{1}{2}\mu^2\Phi\Phi$, with μ as the mass of the complex scalar field. The field equations are obtained through varying the action with respect the fundamental fields

$$f_{\mathcal{R}}\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f_{\mathcal{R}} + \left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right)f_{\mathcal{R}} = k\mathcal{T}_{\mu\nu}, \tag{3}$$

$$\nabla_\lambda\left(\sqrt{-g}f_{\mathcal{R}}g^{\mu\nu}\right) = 0 \tag{4}$$

where Eqs.(3) and (4) are the field equation coming from varying the action with respect to the inverse metric tensor $g^{\mu\nu}$ and the affine connection $\Gamma^{\alpha}{}_{\beta\gamma}$ respectively. Additionally, the energy-momentum tensor is defined as

$$\mathcal{T}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{P}(X,\Phi)\right)}{\delta g^{\mu\nu}}. \tag{5}$$

An exact solution to Eq.(4) can be found by introducing a conformal transformation of the metric tensor

$$q_{\mu\nu} = f_{\mathcal{R}}g_{\mu\nu}. \tag{6}$$

where the conformal factor is written as $f_{\mathcal{R}}$. The conformal transformation, allow us to write an explicit relation between the affine connection with the conformal metric

$$\Gamma^{\lambda}{}_{\mu\nu} = \frac{1}{2} q^{\lambda\rho} \left(\partial_{\mu} q_{\rho\nu} + \partial_{\nu} q_{\rho\mu} - \partial_{\rho} q_{\mu\nu} \right), \tag{7}$$

this are the Christoffel symbols, and we can use it in Eq.(3) to find an equation that only involves the metric tensor and the matter.

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} q_{\mu\nu} \mathcal{R} = \frac{k}{f_{\mathcal{R}}} \mathcal{T}_{\mu\nu} - \frac{\mathcal{R} f_{\mathcal{R}} - f}{2 f_{\mathcal{R}}} q_{\mu\nu} - \frac{3}{2 f_{\mathcal{R}}^2} \left(\partial_{\mu} f_{\mathcal{R}} \partial_{\nu} f_{\mathcal{R}} - \frac{1}{2} q_{\mu\nu} (\partial f_{\mathcal{R}})^2 \right) + \frac{1}{f_{\mathcal{R}}} \left(\nabla_{\mu} \nabla_{\nu} f_{\mathcal{R}} - q_{\mu\nu} \Box f_{\mathcal{R}} \right) \tag{8}$$

3 Numerical analysis

4 Final remarks