From ΛCDM to EDE

Lecture 1: Theory

Laura Herold

CosmoVerse@Corfu May 13 - 18, 2024

Overview

Today:

- **Lecture 1**: Theory from ΛCDM to EDE (1:30h)
- **Hands-on session 1**: From theory to predictions (1h)

Tomorrow:

- **Lecture 2**: Observation — Can EDE solve the Hubble tension? (1h)
- **Hands-on session 2**: Let's analyse EDE with cosmic microwave background and supernovae (1:30h)

Outline: From ΛCDM to EDE

1. Introduction: basic equations in a homogeneous & isotropic universe (Friedmann equations, matter content in the universe, defining distances)

2. Hubble tension: How to solve the tension? How does CMB constrain H_0?

3. Early Dark Energy: Idea behind EDE, background dynamics of EDE

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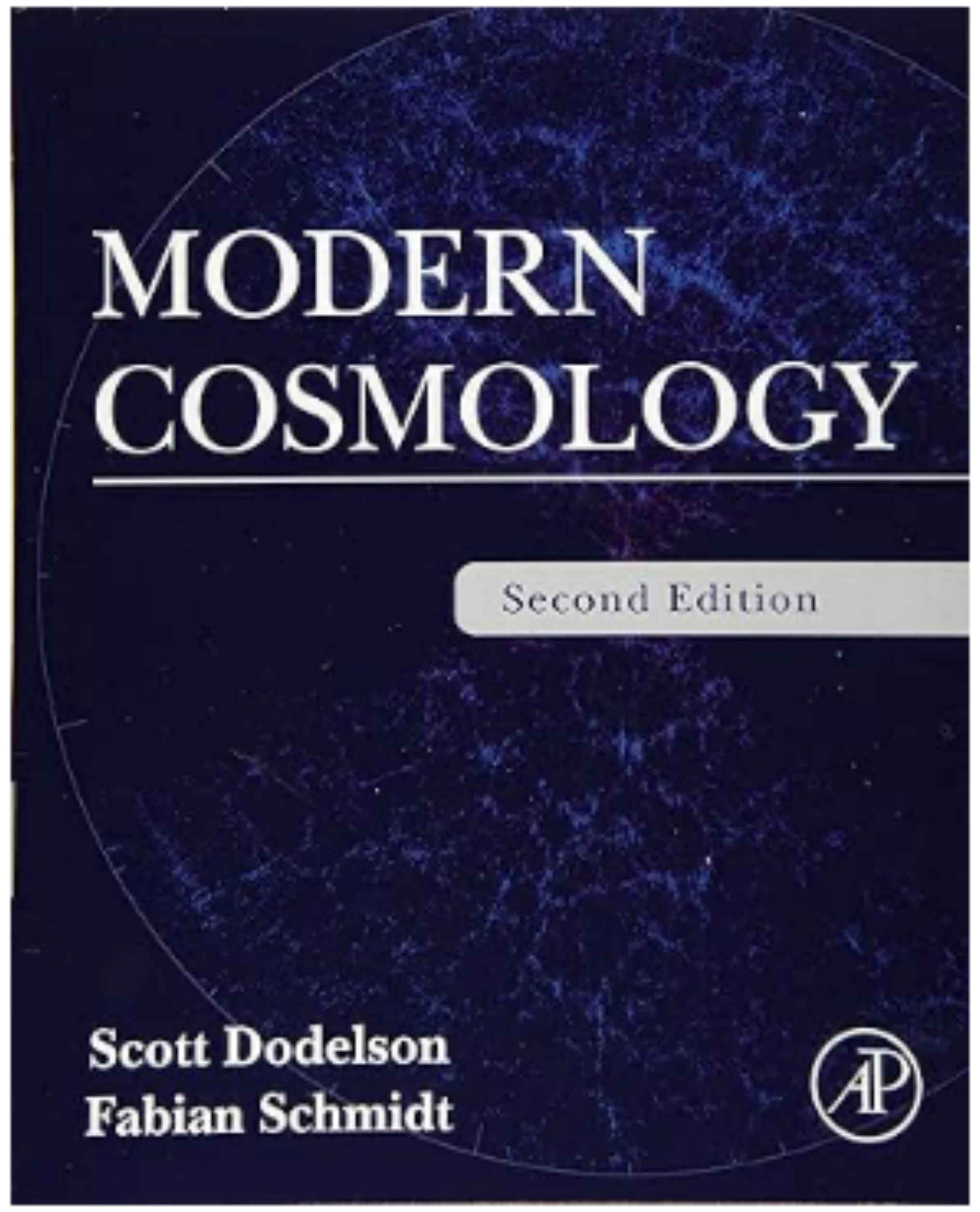
Please ask questions any time!

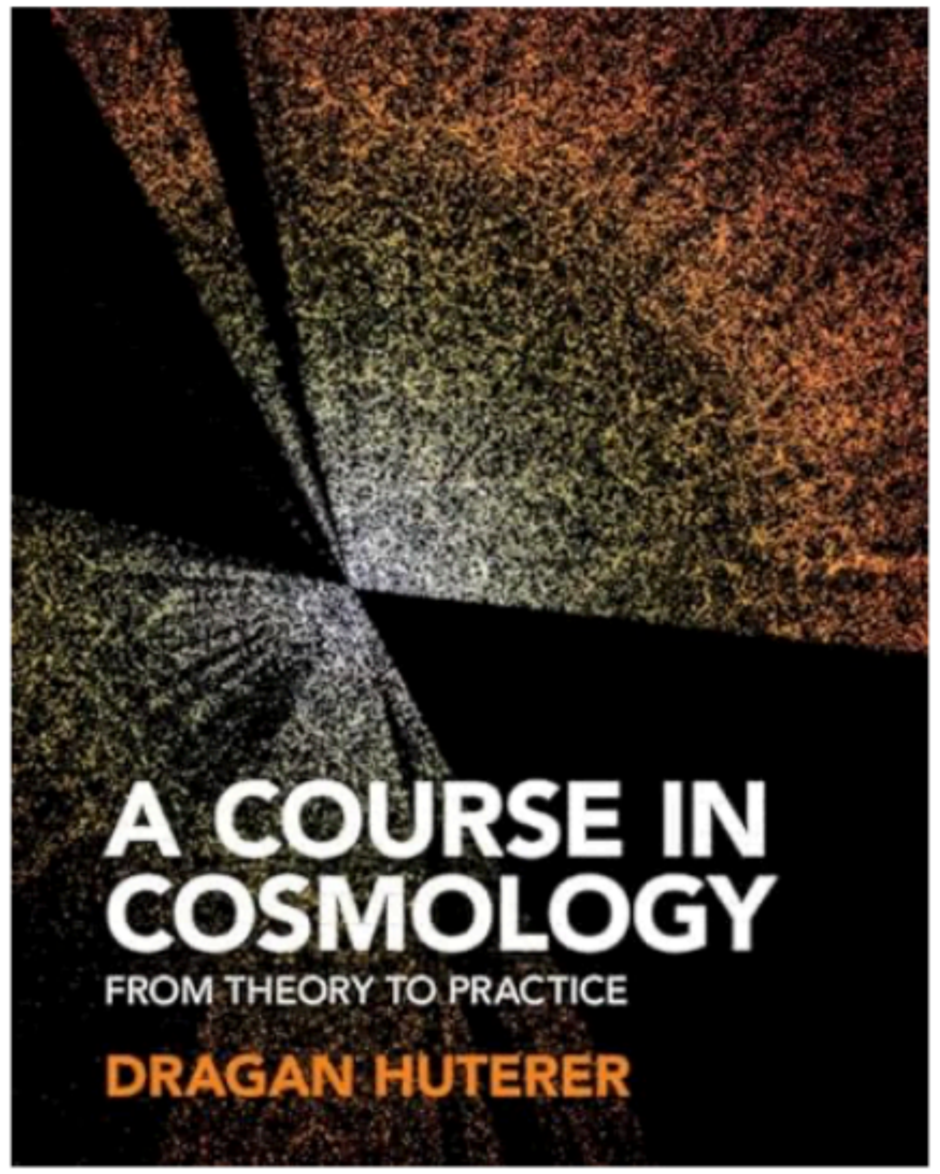
Introduction

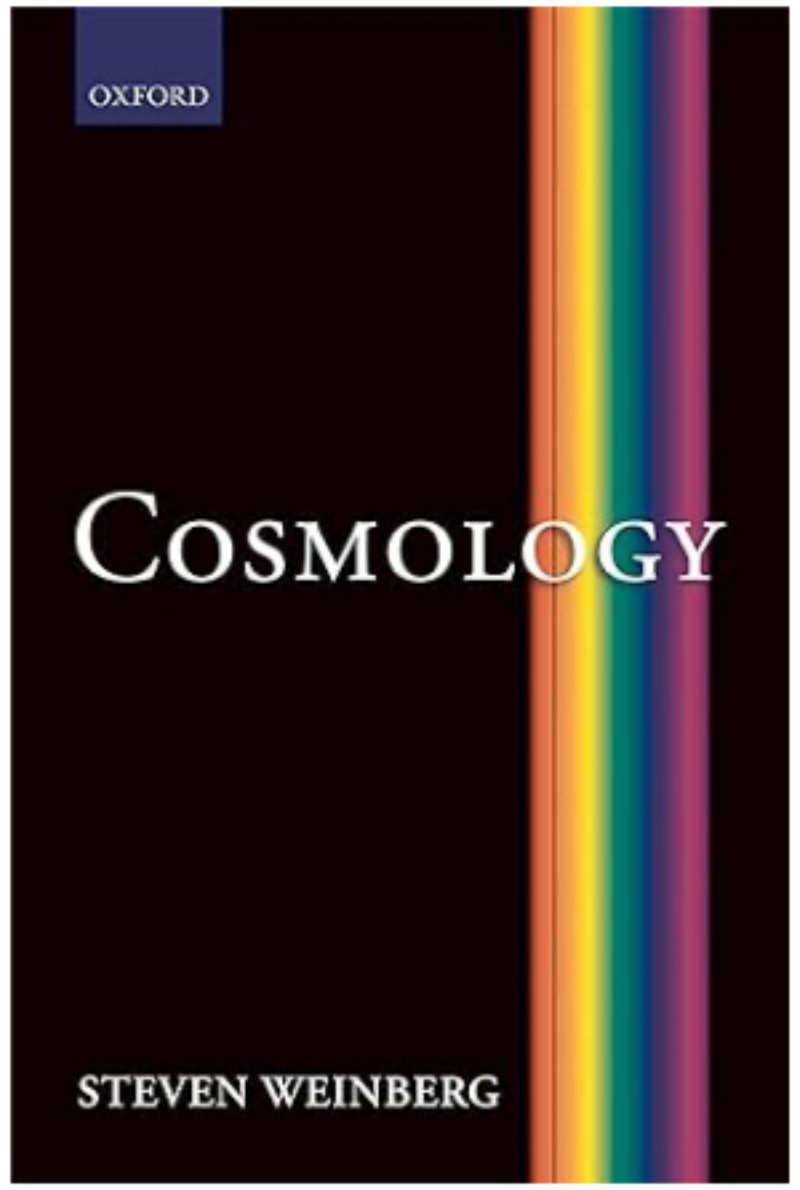
Short "crash course" to fix the notation

$\boxed{\text{Natural units: } c = 1}$

Resources

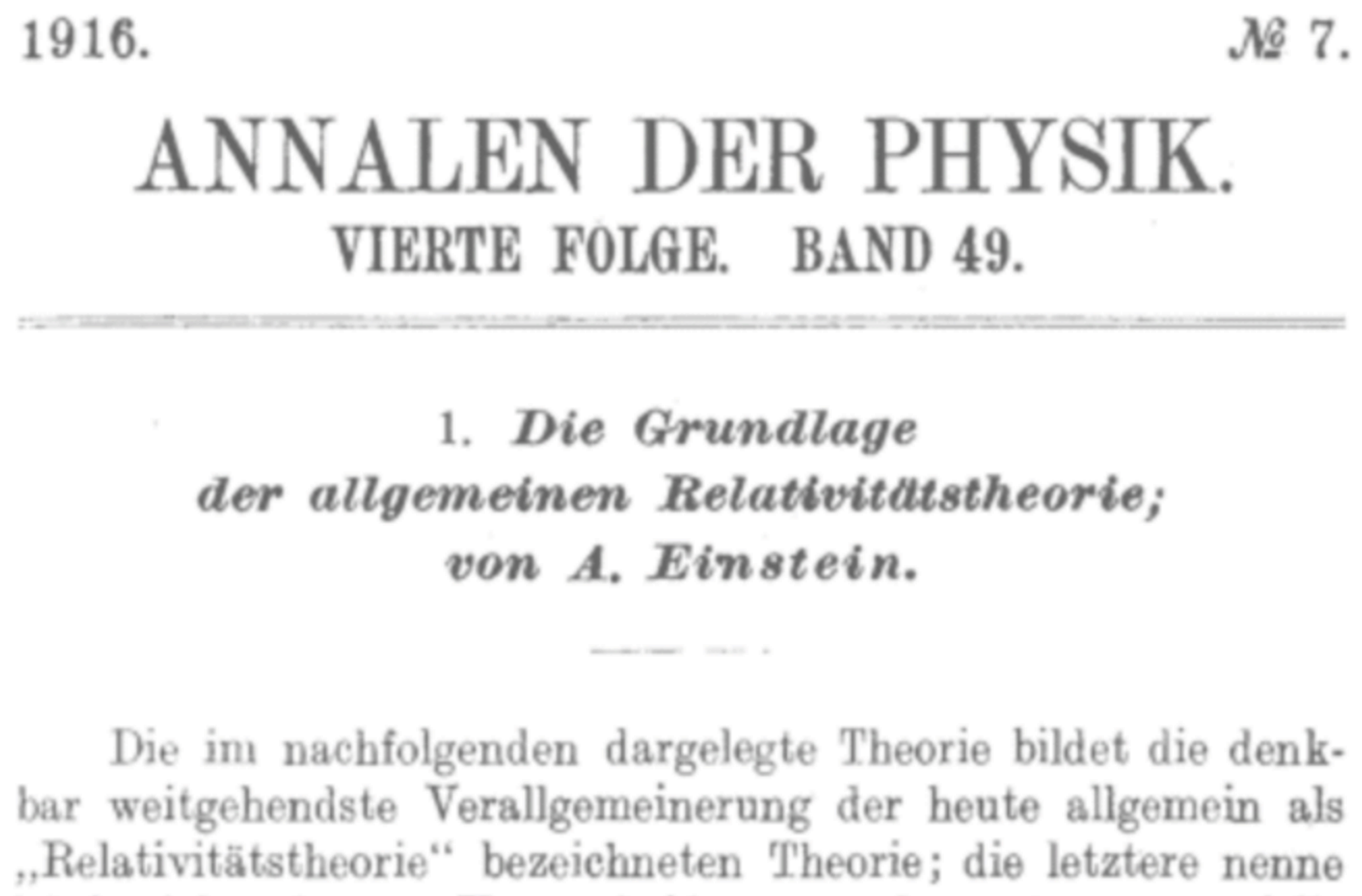

MODERN
COSMOLOGY
Second Edition
Scott Dodelson
Fabian Schmidt


A COURSE IN
COSMOLOGY
FROM THEORY TO PRACTICE
DRAGAN HUTERER


OXFORD
COSMOLOGY
STEVEN WEINBERG

General Relativity

- Let’s imagine it was 100 years ago:
  - We didn’t know about dark matter (DM)
  - We didn’t know about dark energy (DE)
  - But a few years ago, Albert Einstein had published the theory of General Relativity (GR)


1916. № 7.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 49.

1. *Die Grundlage der allgemeinen Relativitätstheorie;*
von A. Einstein.

Die im nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als „Relativitätstheorie“ bezeichneten Theorie; die letztere nenne

General Relativity

For more about GR, see Matteo Martinelli's lecture

- We will not go into details here but only sketch the rough idea

General Relativity

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- Einstein Equations:

$$R^{\mu\nu} - \frac{R}{2}g^{\mu\nu} + \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu},$$

Ricci curvature tensor

metric

cosmological constant

energy-momentum tensor

"Matter tells space how to curve, space tells matter how to move"
(Misner++ 1973)

General Relativity

- Solution for Einstein Equations for the universe as a whole?
- Cosmological Principle:

"On sufficiently large scales, the properties of the universe are the same for all observers."

or

"The universe is spatially homogeneous and isotropic on large scales."

General Relativity

- Solution for Einstein Equations for the universe as a whole?
- Cosmological Principle:

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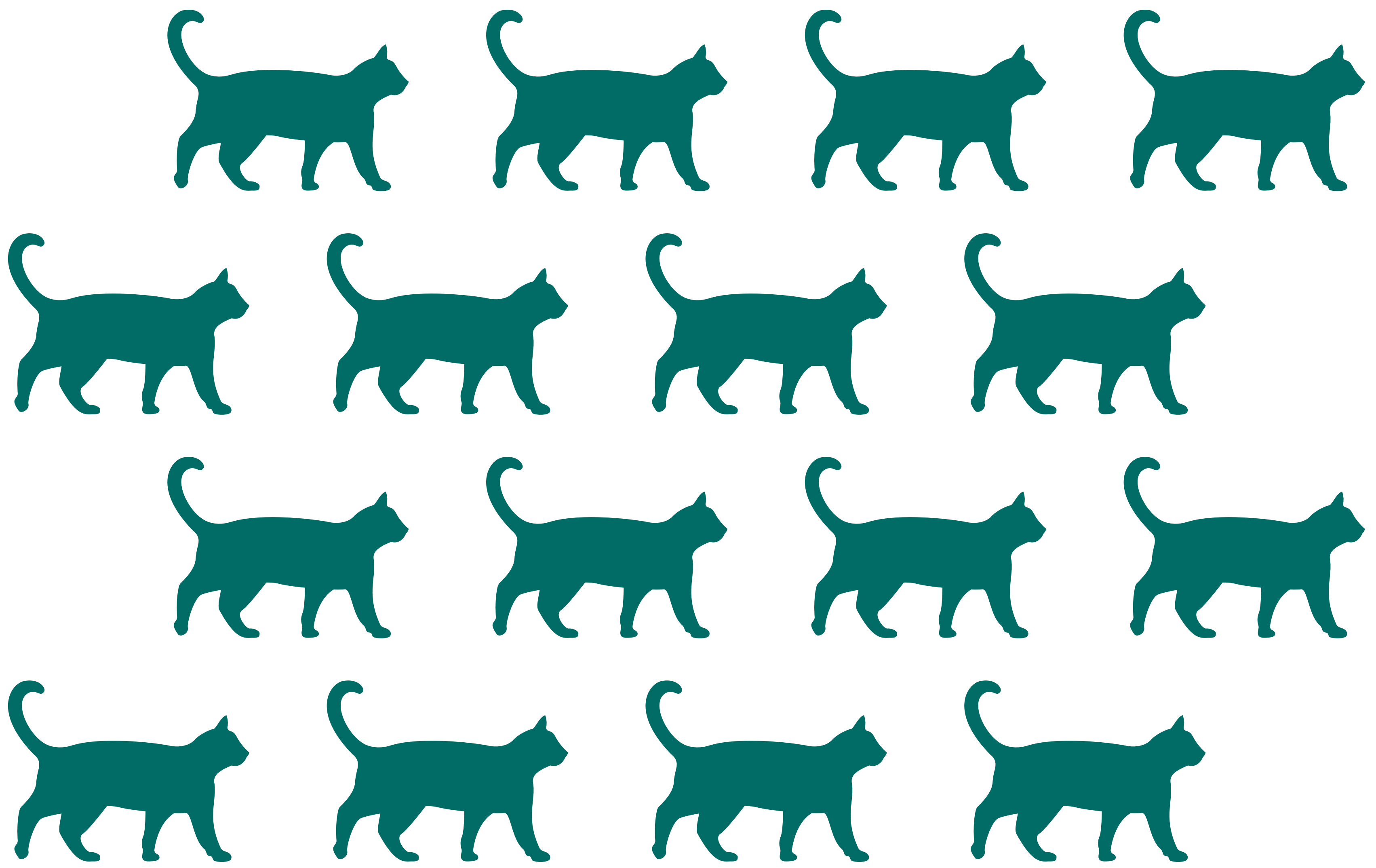
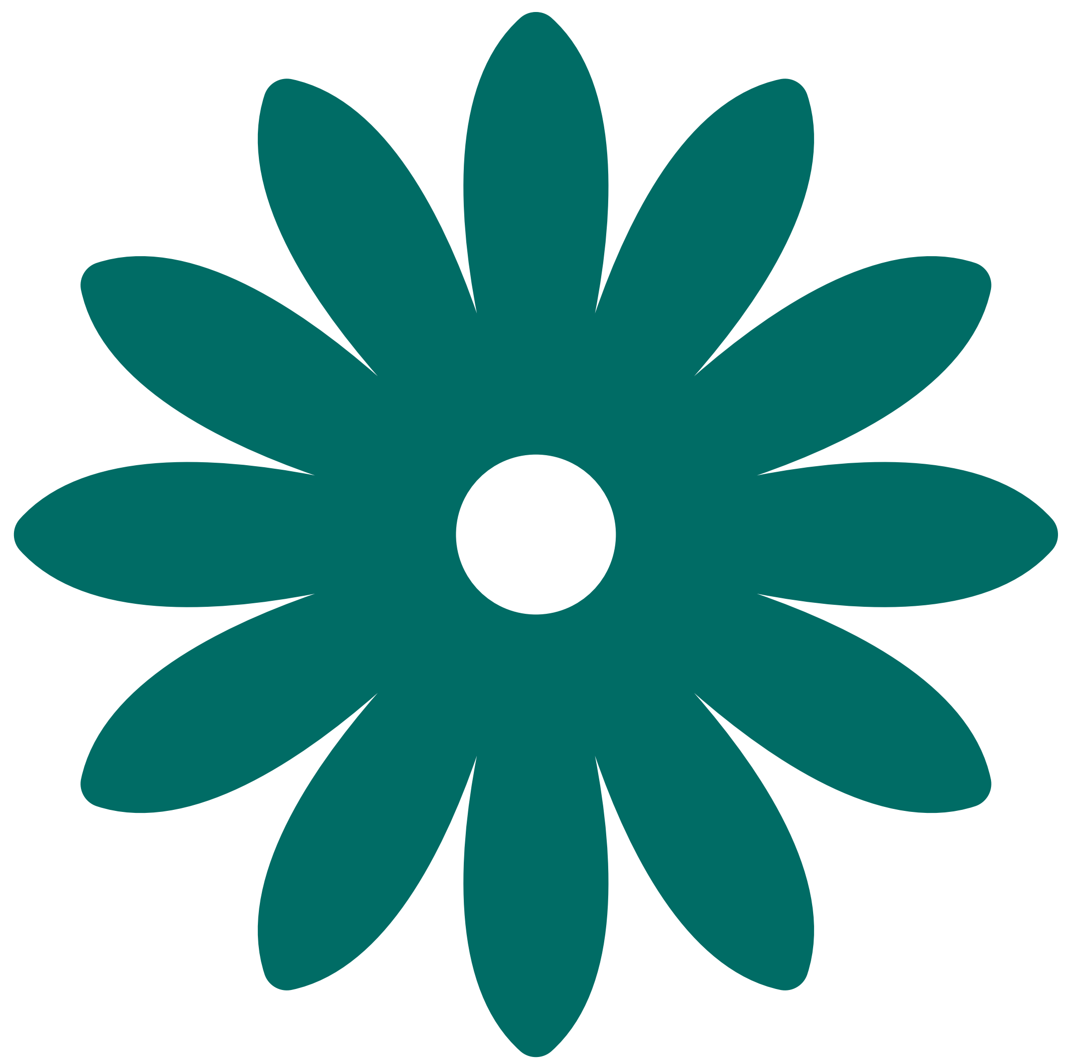
"The universe is spatially homogeneous and isotropic on large scales."

"the same in every point"

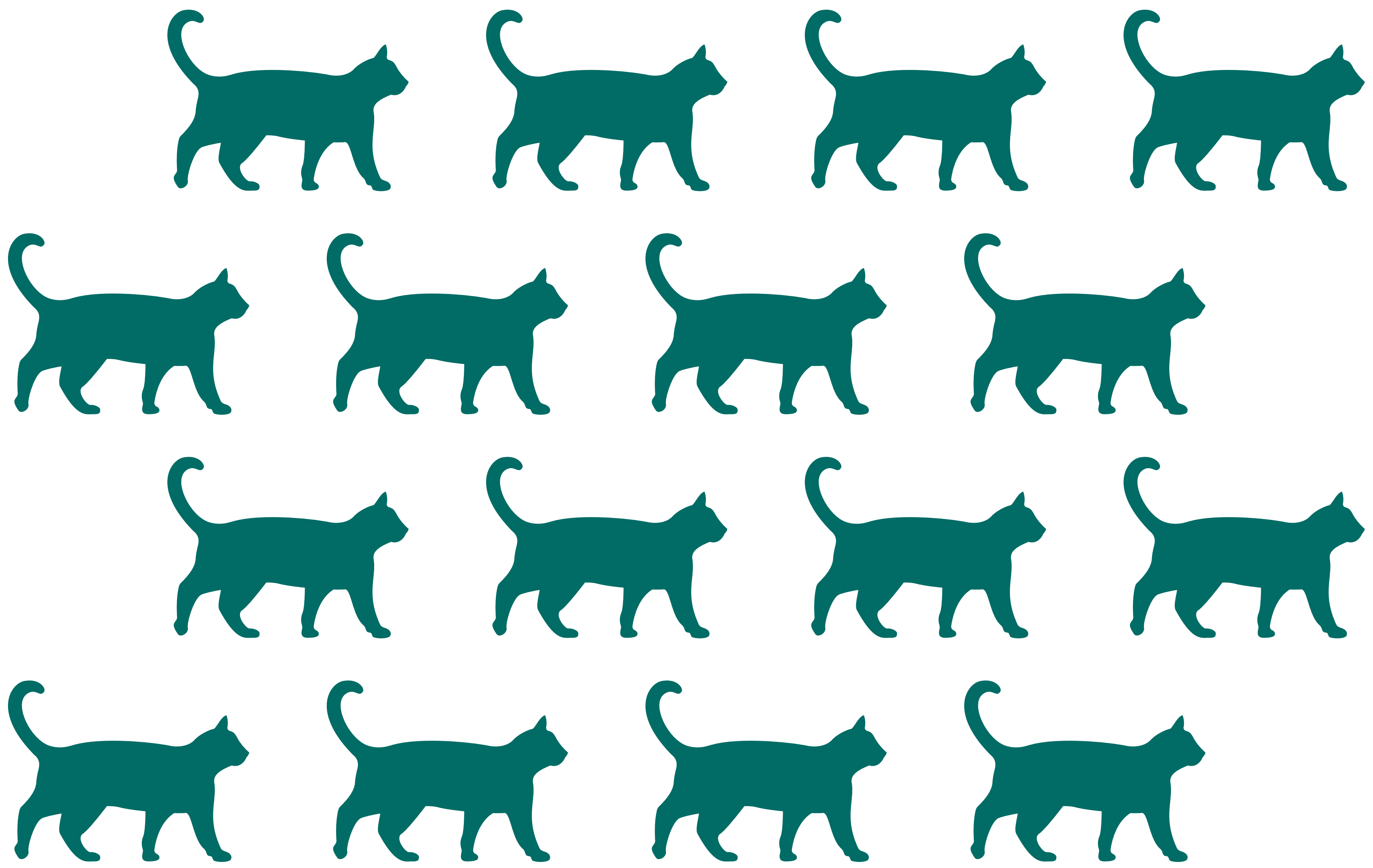
"the same in every direction"

Isotropic or homogeneous?

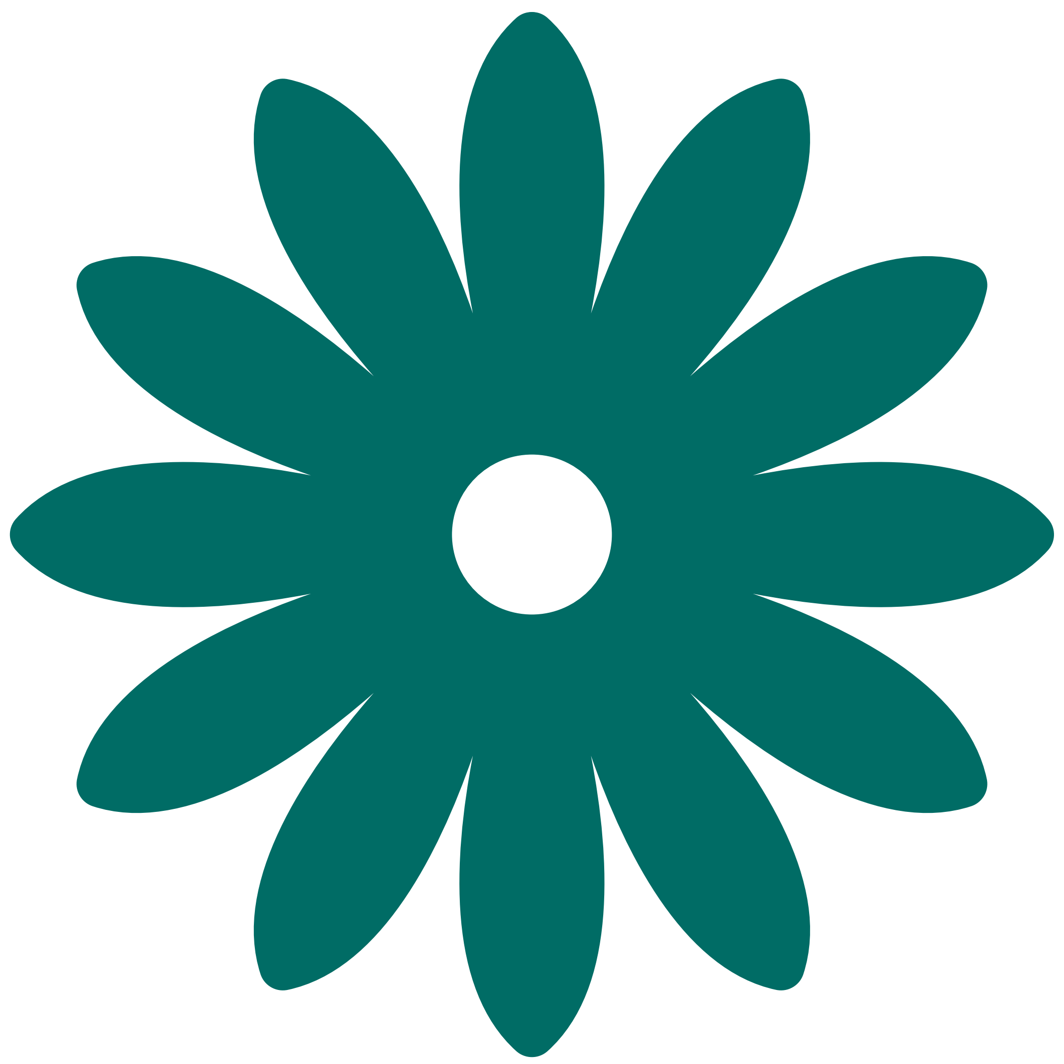




Isotropic or homogeneous?



homogenous but not isotropic



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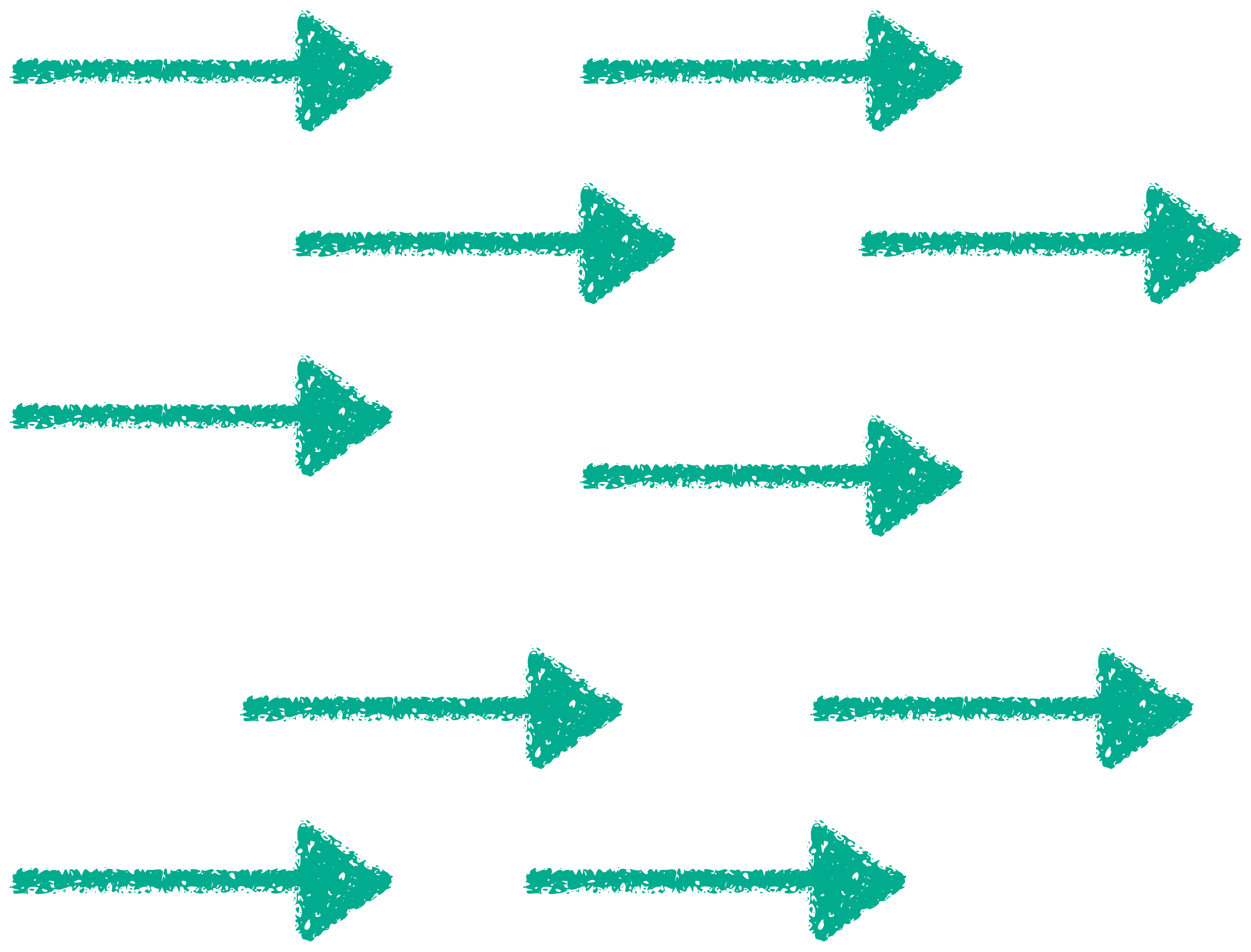
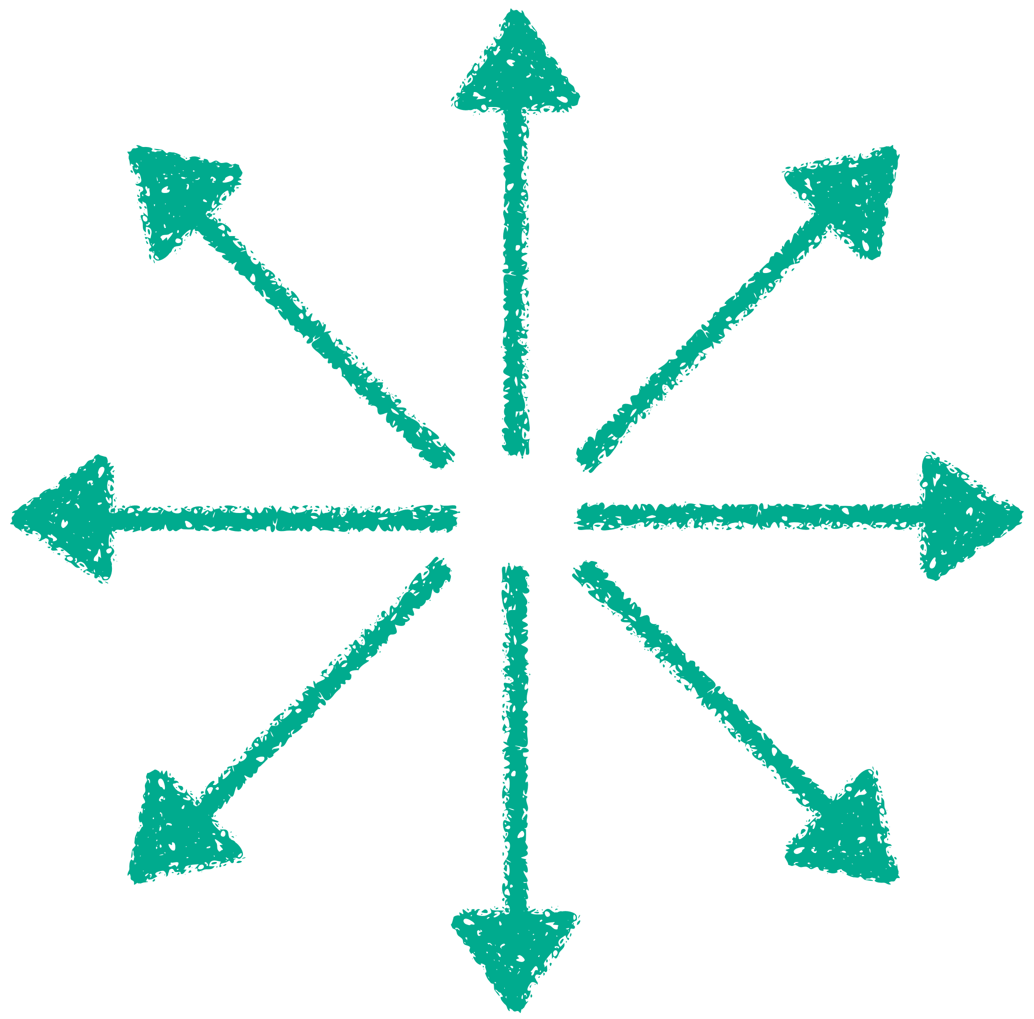


homogenous but not isotropic



isotropic but not homogenous

Friedmann Equations

- Friedmann 1922, Robertson 1935, Walker 1937: For a spatially homogeneous and isotropic universe, the metric simplifies to

$$ds^2 = -\,dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

spacetime line element

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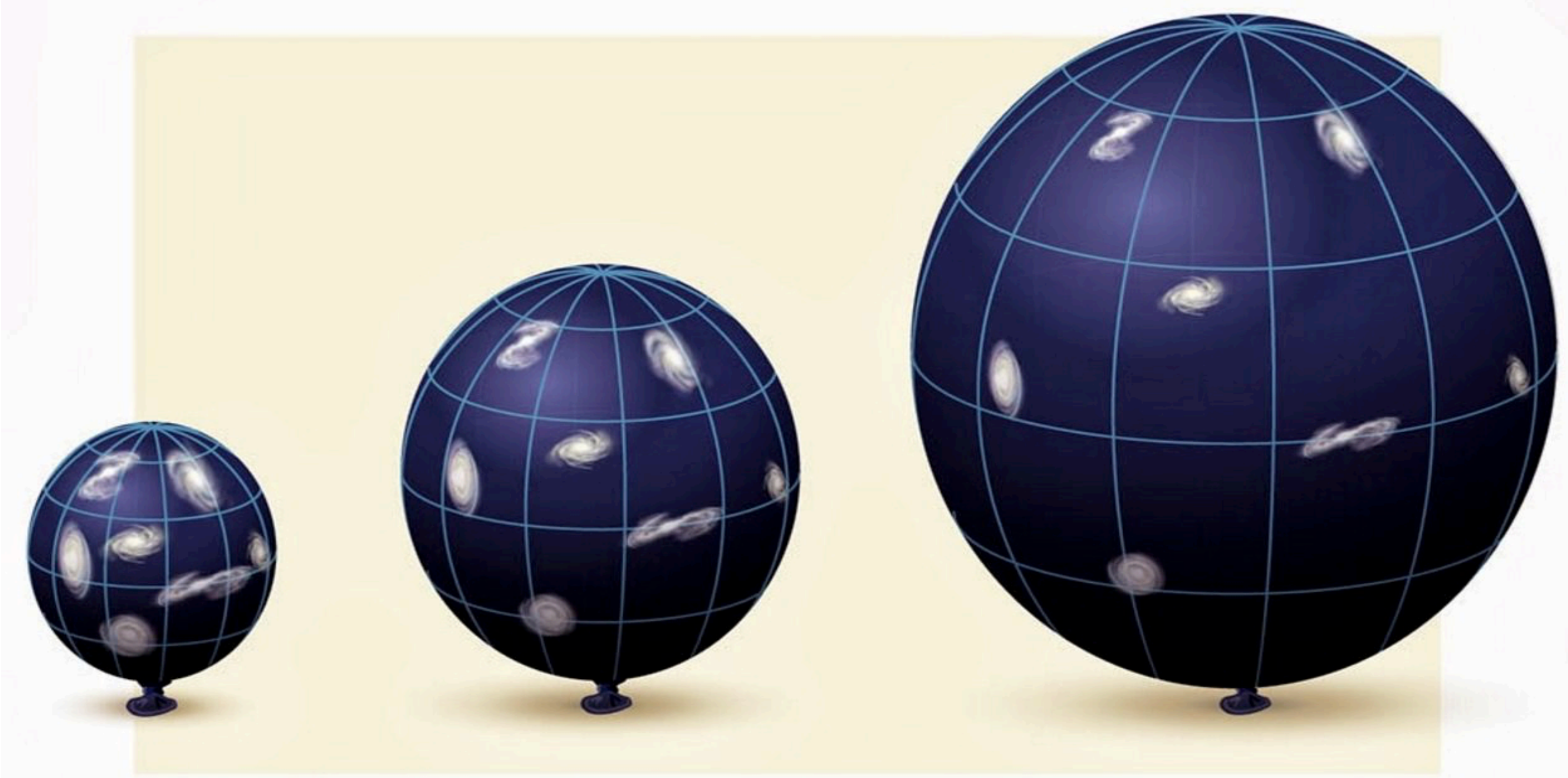
spacetime line element

scale factor



$$a(t_{\text{today}}) = a_0 = 1$$

Figure credit: Bianchi, Rovelli, Kolb

Friedmann Equations

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$$\underbrace{\mathrm{d}s^2}_{\text{spacetime line element}} = -\,\mathrm{d}t^2 + \underbrace{a^2(t)}_{\text{scale factor}}\left[\frac{\mathrm{d}r^2}{1-\underbrace{k}_{\text{curvature parameter}}r^2} + r^2\mathrm{d}\Omega^2\right]$$

$$k = \begin{cases} -1 & \text{hyperbolic} \\ 0 & \text{flat} \\ +1 & \text{spherical} \end{cases}$$

Friedmann Equations

- A perfect fluid is a fluid, which can be completely characterised by its (energy) density and pressure

- The energy momentum tensor of a perfect fluid is

$$T^{\mathrm{pf}}_{\mu\nu} = (\rho + p)\, u_\mu u_\nu + p g_{\mu\nu}$$

energy density

pressure

- $u_\mu = \dot{x}_\mu$ is the 4-velocity of the observer

- Inserting this in the Einstein Equations, yields the Friedmann Equations

Friedmann Equations

- Inserting the FLRW-metric and the energy momentum tensor of the perfect fluid into the Einstein equations, yields the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \quad \text{(i)}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad \text{(ii)}$$

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- Inserting (ii) into the temporal derivative of (i), yields the continuity equation:

$$\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a} \quad \text{(iii)}$$

Friedmann Equations

$$\boxed{\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} &= \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \quad \text{(i)} \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad \text{(ii)} \\ \dot{\rho} &= -3(\rho + p)\frac{\dot{a}}{a} \quad \text{(iii)} \end{aligned}}$$

- Where $\rho = \rho_{\rm tot}$ is the energy density of the universe.
- If we define $\rho_k = -\dfrac{3}{8\pi G}\dfrac{k}{a^2}$ and $\rho_\Lambda = \dfrac{\Lambda}{8\pi G}$, one can rewrite (i) as

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\rm tot} \quad \text{(i)}}$$

ΛCDM model $\rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda + \rho_k$

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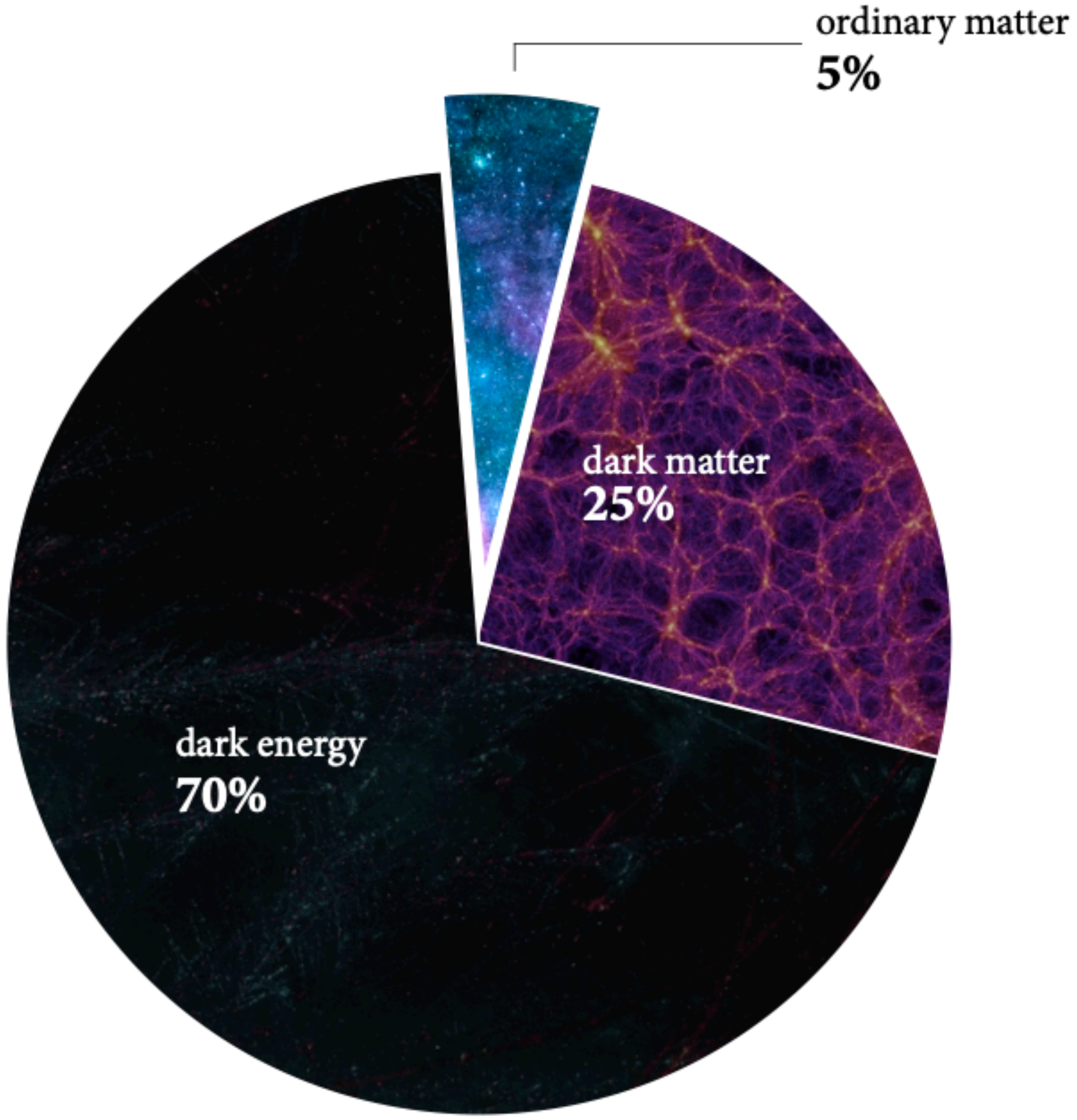
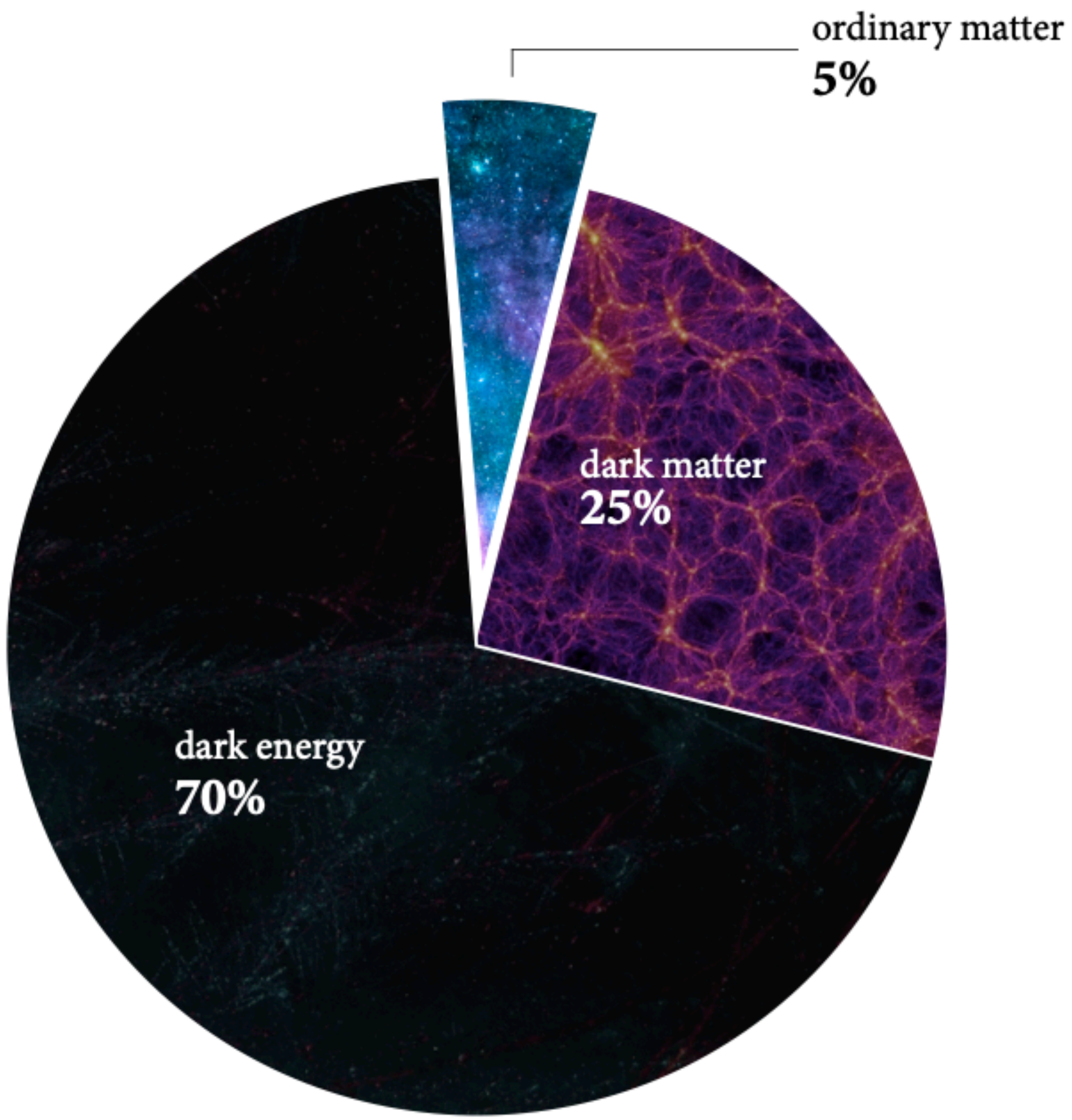


Figure credit: Florian Wolz

ΛCDM model $\rho_{\rm tot} = \rho_r + \rho_m + \rho_\Lambda$

- Equation of state: $p = w \cdot \rho$



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ΛCDM model $\rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda$

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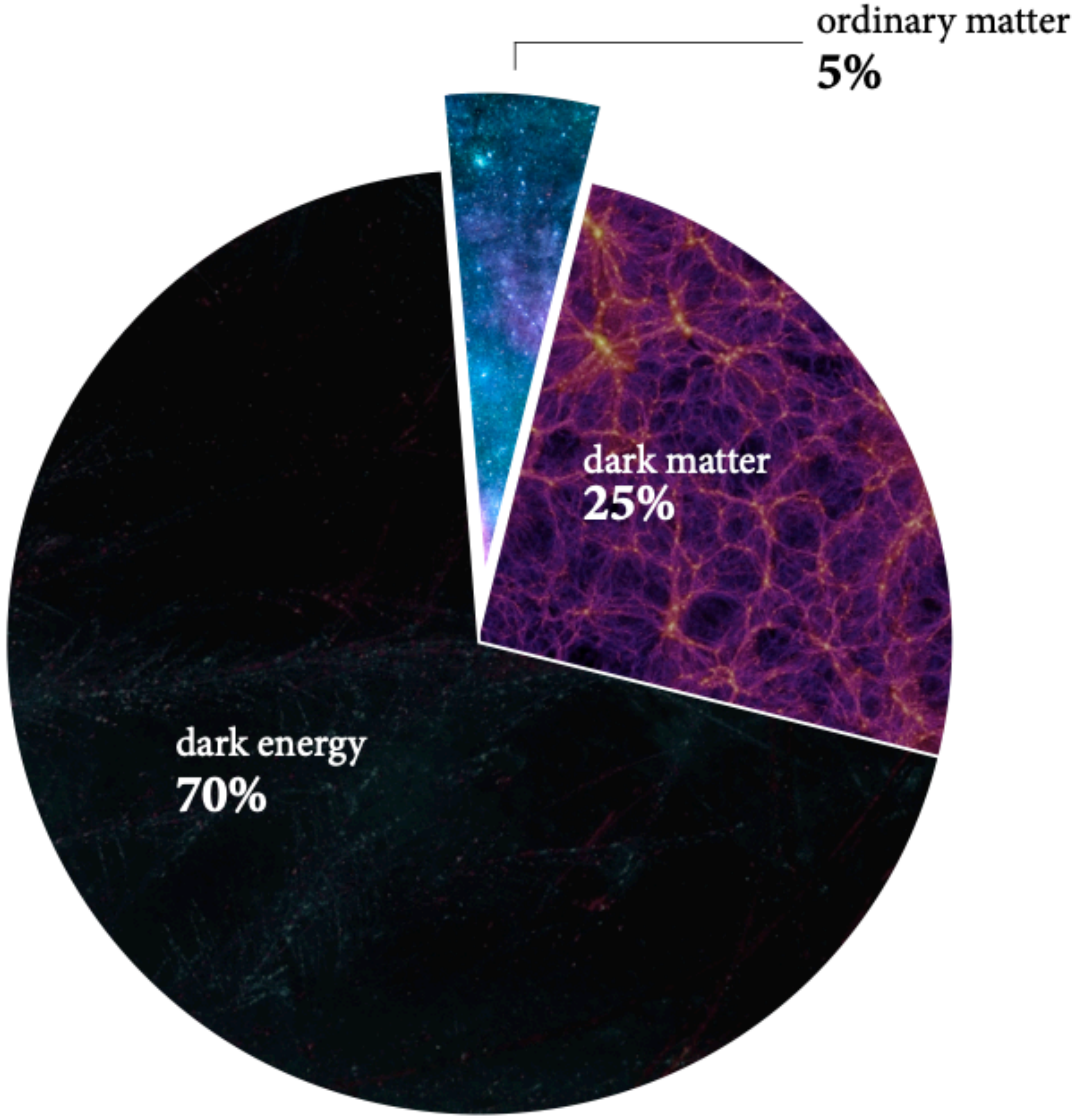
- EOS-parameter w for different matter species:
  - Dark matter and baryonic matter: $w = 0$



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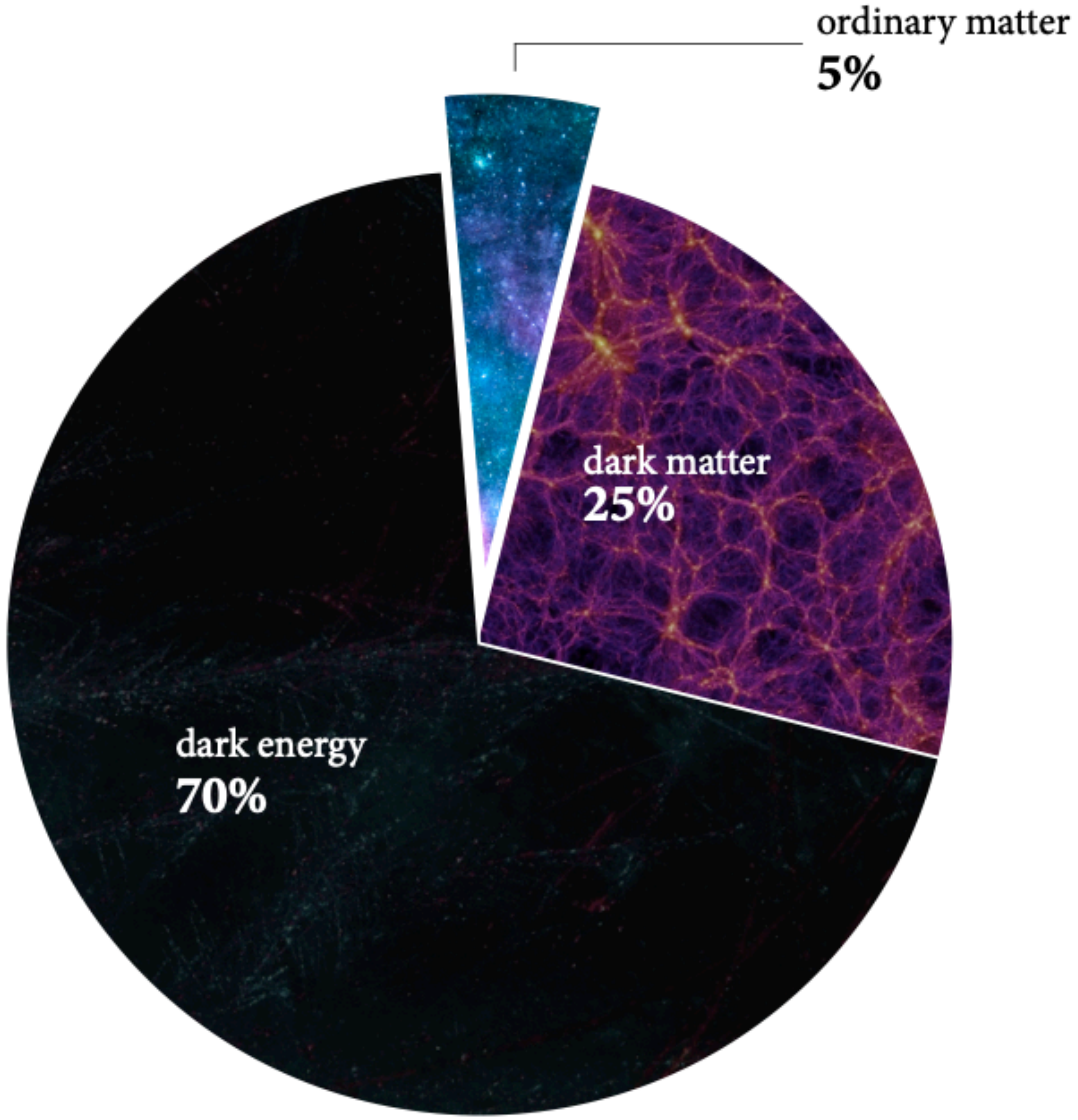

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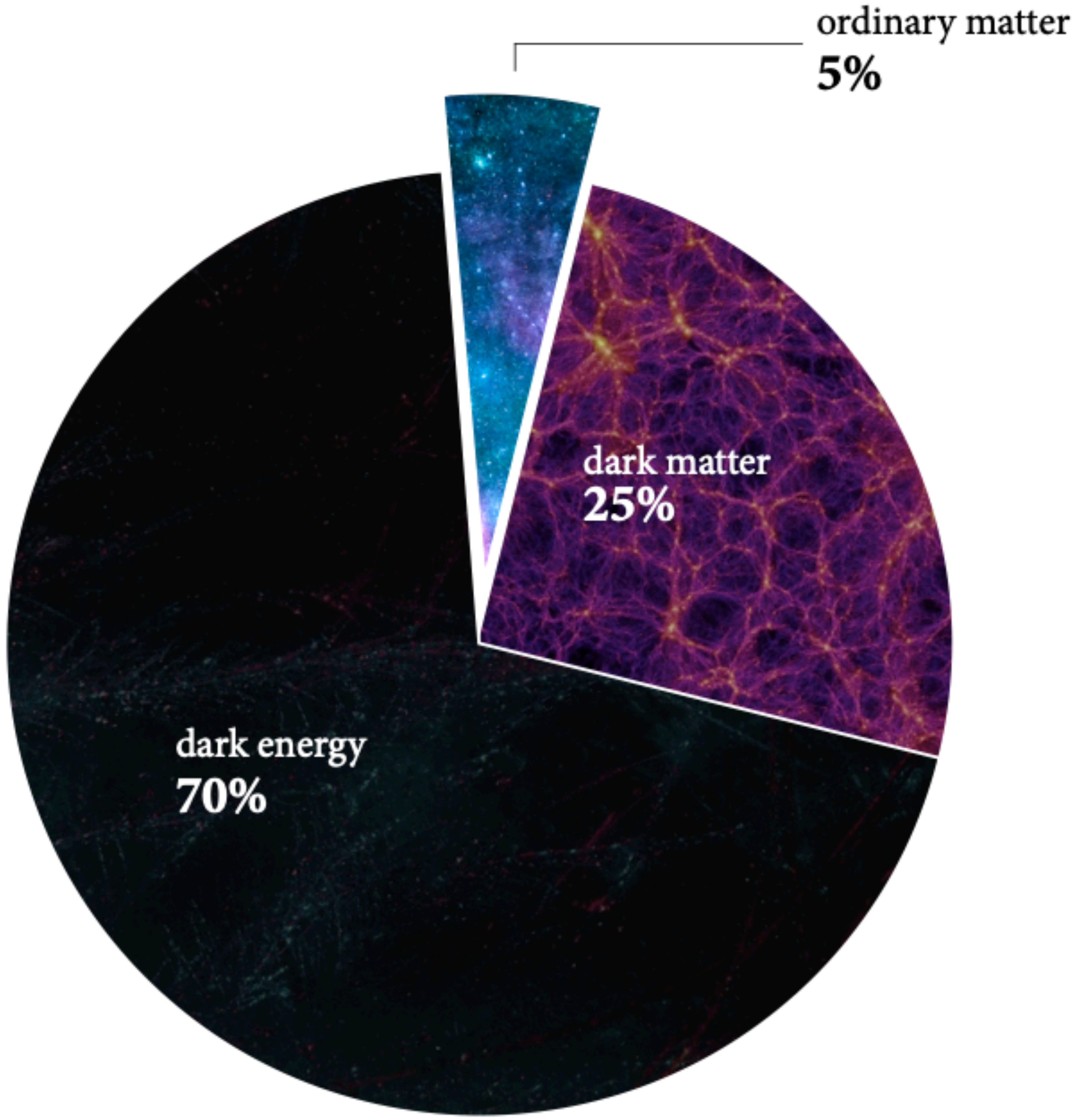
  - Dark energy: $w = -1$



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Friedmann Equations

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$$\dot{\rho} = -3\rho(1 + w)\frac{\dot{a}}{a}$$

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… (your exercise later)

$$\boxed{\frac{\rho(a)}{\rho_0} = \left(\frac{a}{a_0}\right)^{-3(1+w)}}$$

The subscript 0 refers to the current time “today”

The Hubble parameter

- The first Friedmann equation describes the rate of expansion as a function of the energy content ρ of the universe:

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\text{tot}} \quad \text{(i)}}$$

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- The Hubble parameter **today** is called the Hubble constant:

$$H_0 = H(t = t_0)$$

today

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The Hubble parameter

$$H^2(t) = \frac{8\pi G_N}{3}\rho_{tot}(t)$$

- Defining $\rho_{crit} = \dfrac{3H_0^2}{8\pi G_N}$

density of flat universe

The Hubble parameter

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- Defining $\rho_\mathrm{crit} = \dfrac{3H_0^2}{8\pi G_\mathrm{N}}$

$$\frac{H^2(a)}{H_0^2} = \frac{\rho_\mathrm{tot}(a)}{\rho_\mathrm{crit}}$$

The Hubble parameter

$$H^2(t) = \frac{8\pi G_\mathrm{N}}{3}\rho_\mathrm{tot}(t)$$

- Defining $\rho_\mathrm{crit} = \dfrac{3H_0^2}{8\pi G_\mathrm{N}}$ and the fractional energy densities $\Omega_I = \dfrac{\rho_I}{\rho_\mathrm{crit}}$:

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Defining the redshift $1 + z = \dfrac{a_0}{a}$, one can rewrite this in the famous form:

The Hubble parameter

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$$\Omega_{I,0} = \Omega_I(t_0) = \Omega_I$$

Distances in an expanding universe

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- In an expanding universe, consider two coordinate systems:
    - a comoving system that expands with the universe
    - a fixed coordinate system, in which objects drift apart with the expansion

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$$1 = c = \frac{\mathrm{d}r_p}{\mathrm{d}t} = a\frac{\mathrm{d}\chi}{\mathrm{d}t} \implies \boxed{\chi(t) = \int_{t_1}^{t_2} \frac{\mathrm{d}t'}{a(t')} = \int_{z_1}^{z_2} \frac{\mathrm{d}z'}{H(z')}}$$

Distances in an expanding universe

- The angular diameter distance D_A is defined such that it holds

$$\underbrace{\theta}_{\text{angle}} = \frac{\overbrace{s}^{\text{physical size of the object}}}{\underbrace{D_A}_{\text{ang. diam. distance}}}$$

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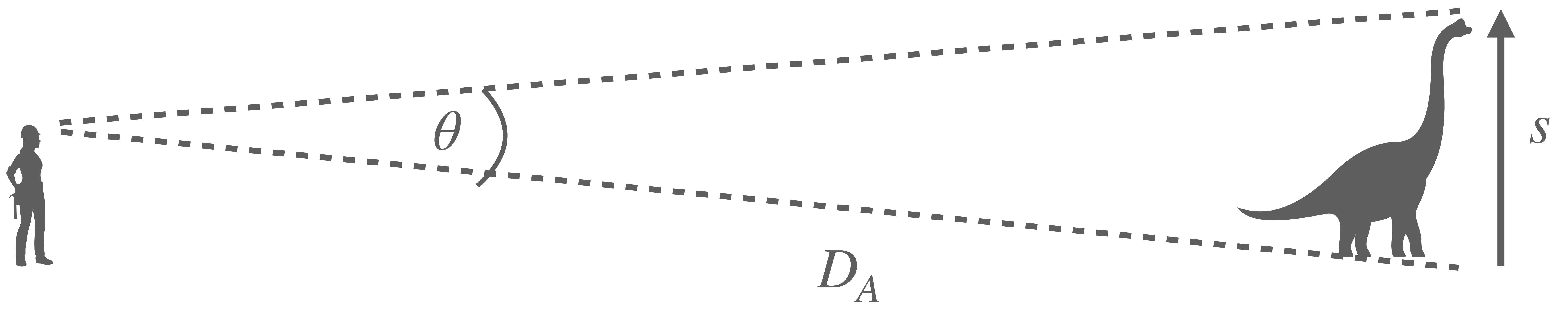
$$\theta = \frac{s}{D_A}$$

ang. diam. distance



Distances in an expanding universe

- The angular diameter distance D_A is defined such that it holds

$$\theta = \frac{s}{D_A}$$

$$\boxed{D_A(t) = a(t)\,\chi(t) = \frac{1}{1+z(t)}\int_0^{z(t)} \frac{\mathrm{d}z'}{H(z')}}$$

*To derive an equation for D_A, note that the proper size, s, of the object can also be expressed as $s = \chi(t)\,\theta \cdot a(t)$, where $\chi(t)\,\theta$ corresponds to the comoving size of the object.

Distances in an expanding universe

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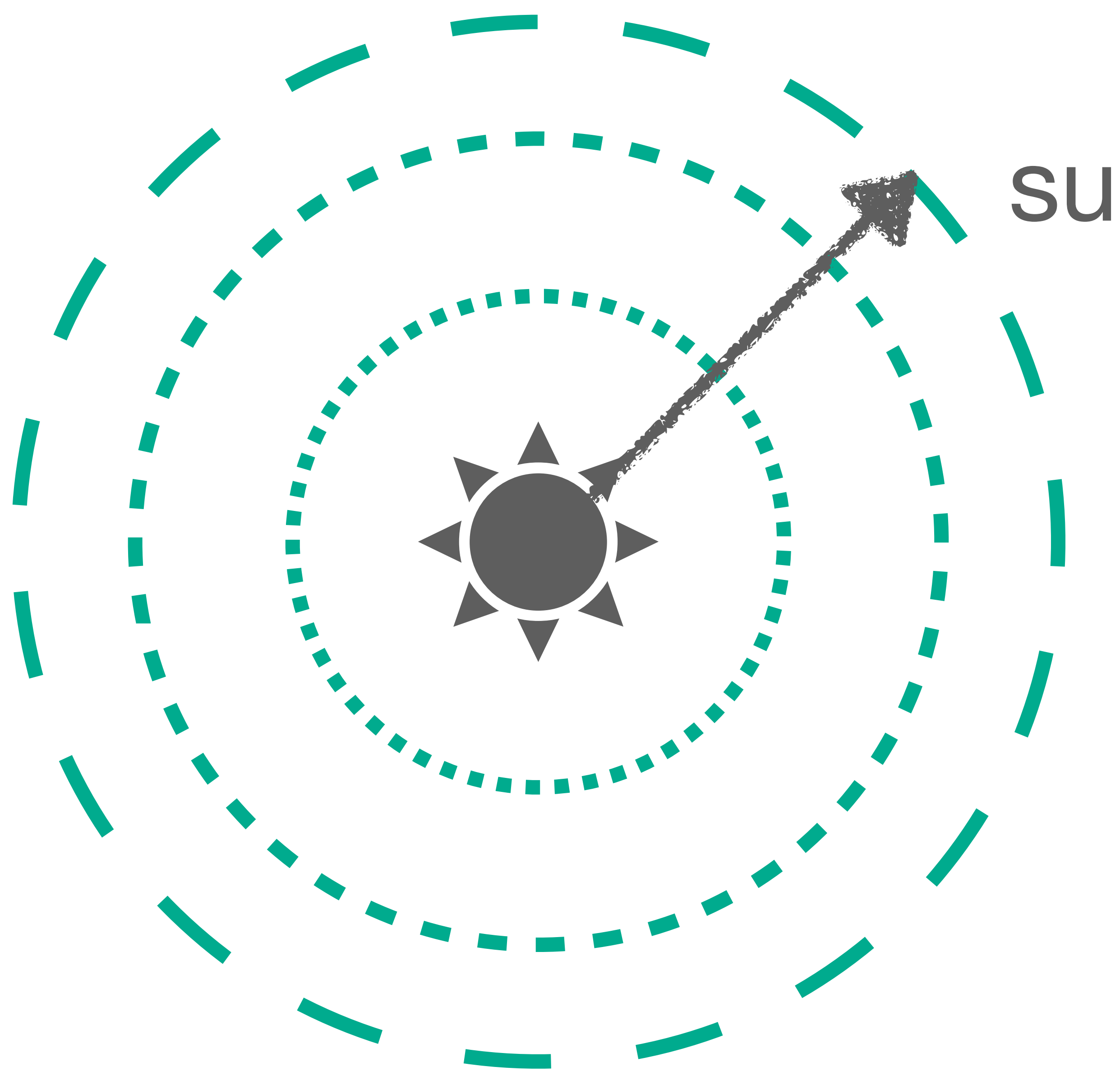


Distances in an expanding universe

- The luminosity distance D_L is defined such that it holds

$$F = \frac{L}{4\pi D_L^2}$$

$$\boxed{D_L(t) = \frac{\chi(t)}{a(t)} = [1 + z(t)] \int_0^{z(t)} \frac{c\,\mathrm{d}z'}{H(z')}}$$

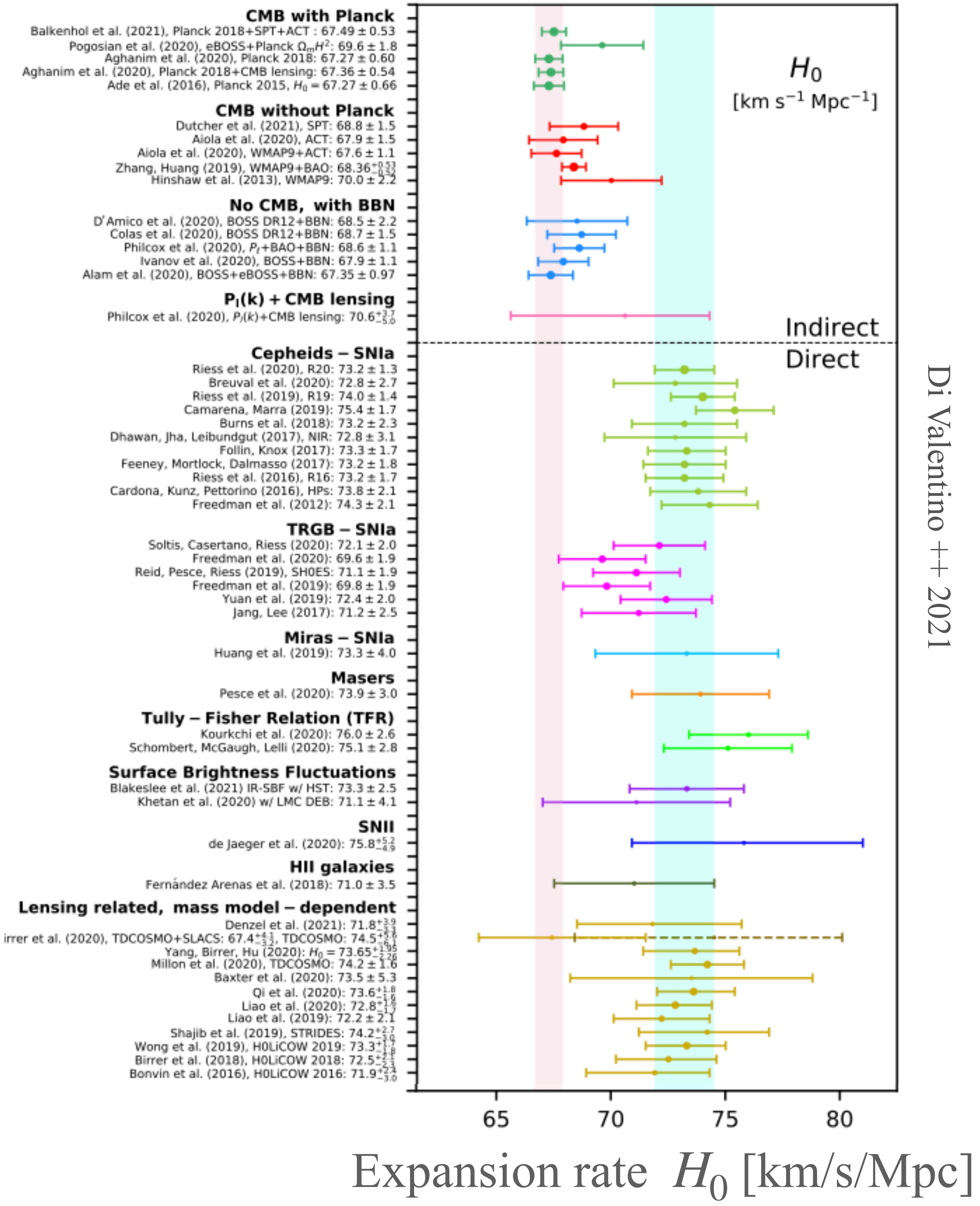
*Since the Universe is expanding, it holds that $F = \dfrac{L\,a^2}{4\pi\chi^2(a)}$, where the additional factor of a^2 comes from the fact that the expansion of the Universe leads to a dilution of photons ($\times\, a$) and to an increase in wavelength ($\times\, a$).

How to solve the Hubble tension?

The Hubble tension

- Indirect measurements: cosmic microwave background (CMB), baryon acoustic oscillations (BAO), galaxy clustering

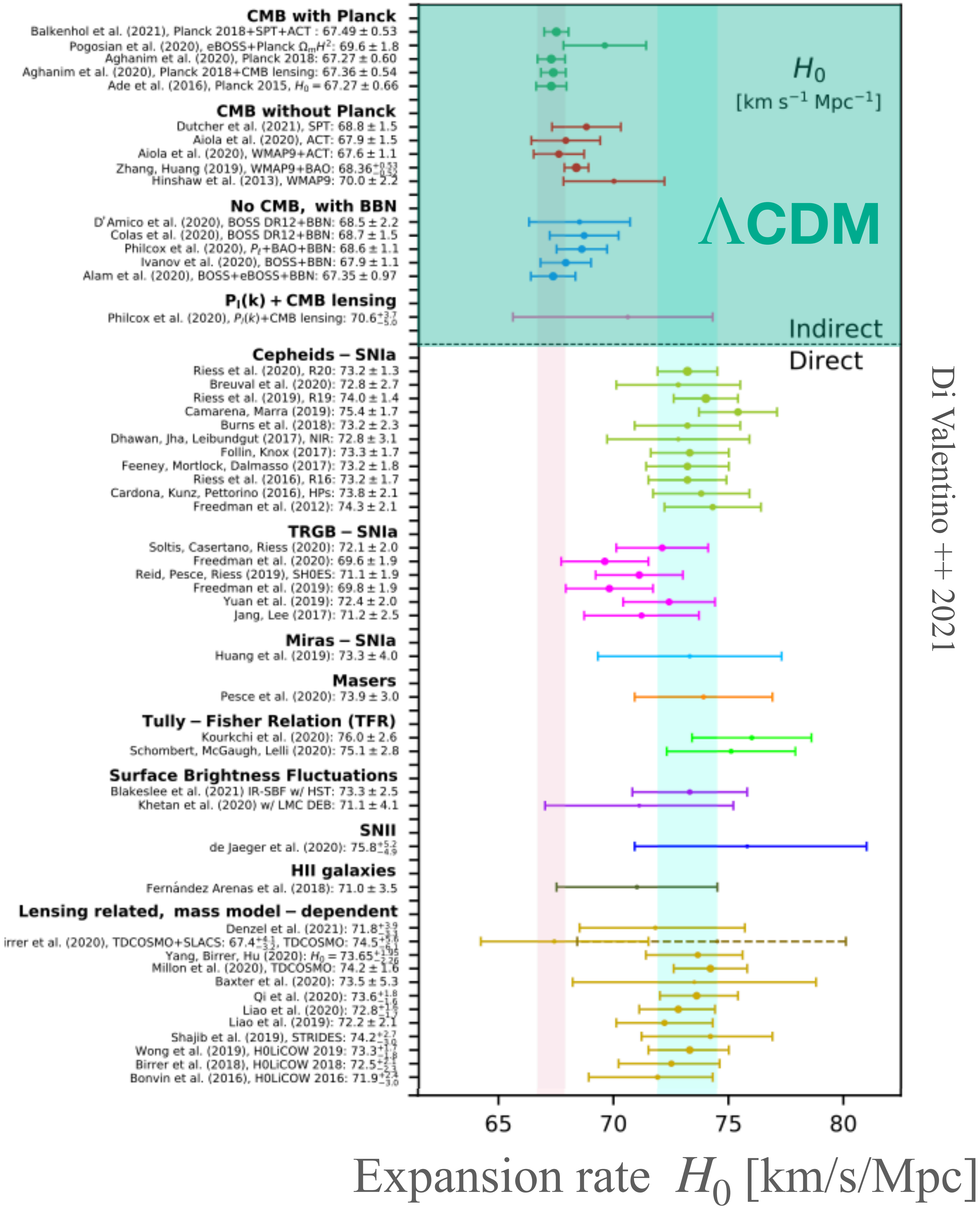
- Direct measurements: distance ladder (Cepheids, TRGB, SNe, …), gravitational lensing, …



The Hubble tension

- Indirect measurements: cosmic microwave background (CMB), baryon acoustic oscillations (BAO), galaxy clustering → precise but model dependent

- Direct measurements: distance ladder (Cepheids, TRGB, SNe, …), gravitational lensing, …

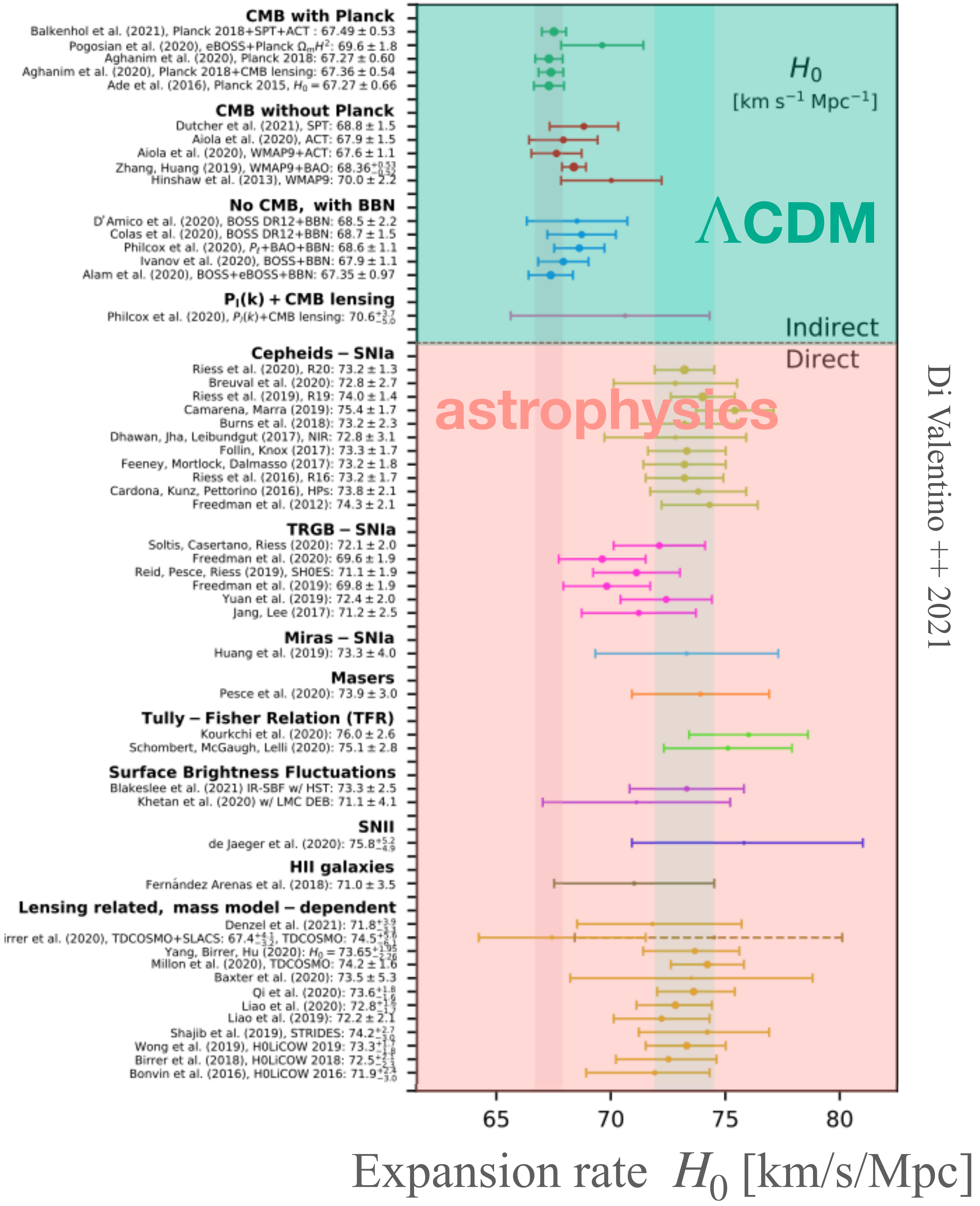


Di Valentino ++ 2021

The Hubble tension

- Indirect measurements: cosmic microwave background (CMB), baryon acoustic oscillations (BAO), galaxy clustering → precise but model dependent

- Direct measurements: distance ladder (Cepheids, TRGB, SNe, …), gravitational lensing, … → less precise due to astrophysical modelling but independent of cosmological model



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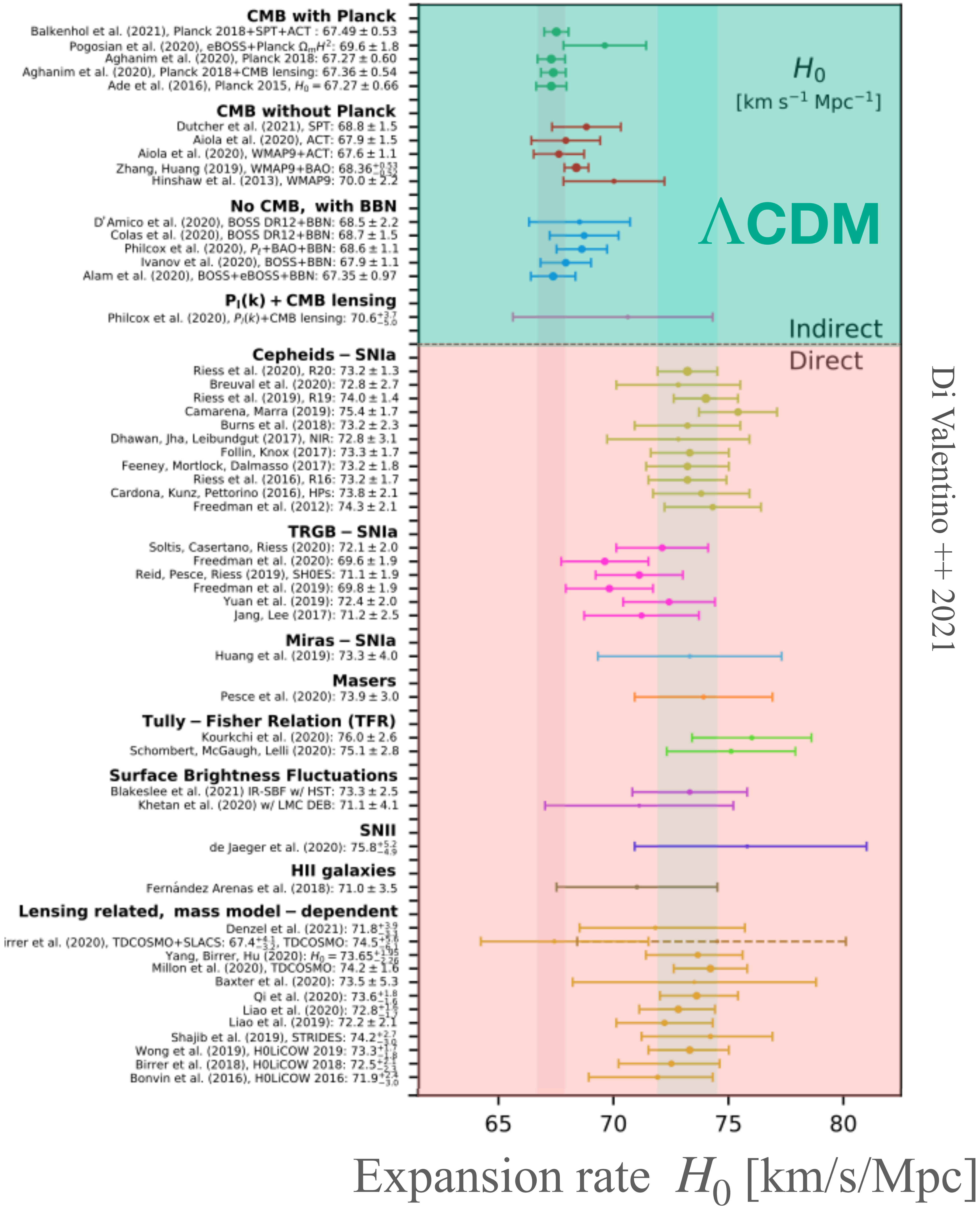
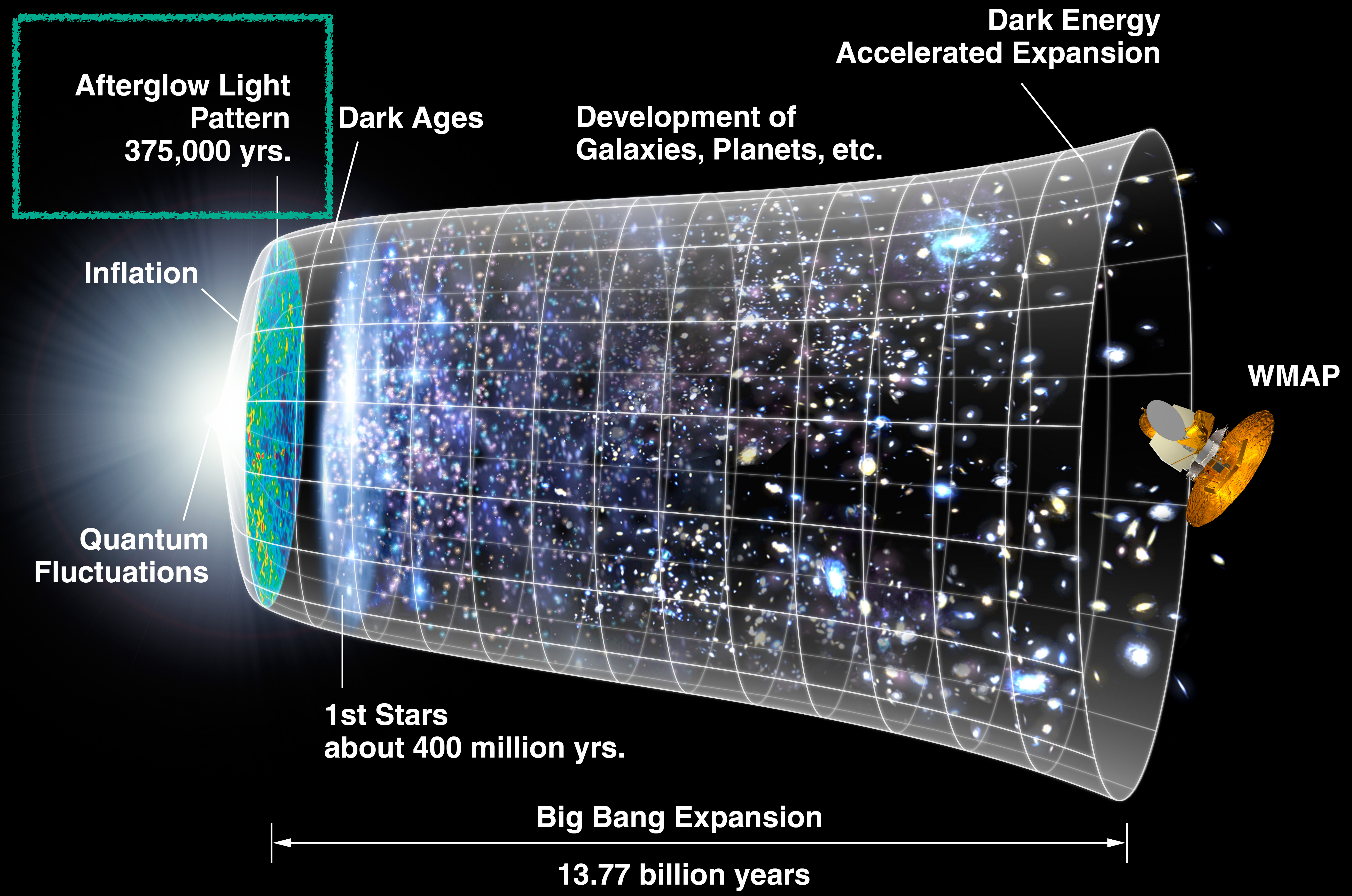
The Hubble tension

- General strategy to solve the H_0 tension: Assume direct measurements are correct and change cosmological model in order to *infer* a higher H_0
- Goal: “get CMB-H_0 to ~73 km/s/Mpc”



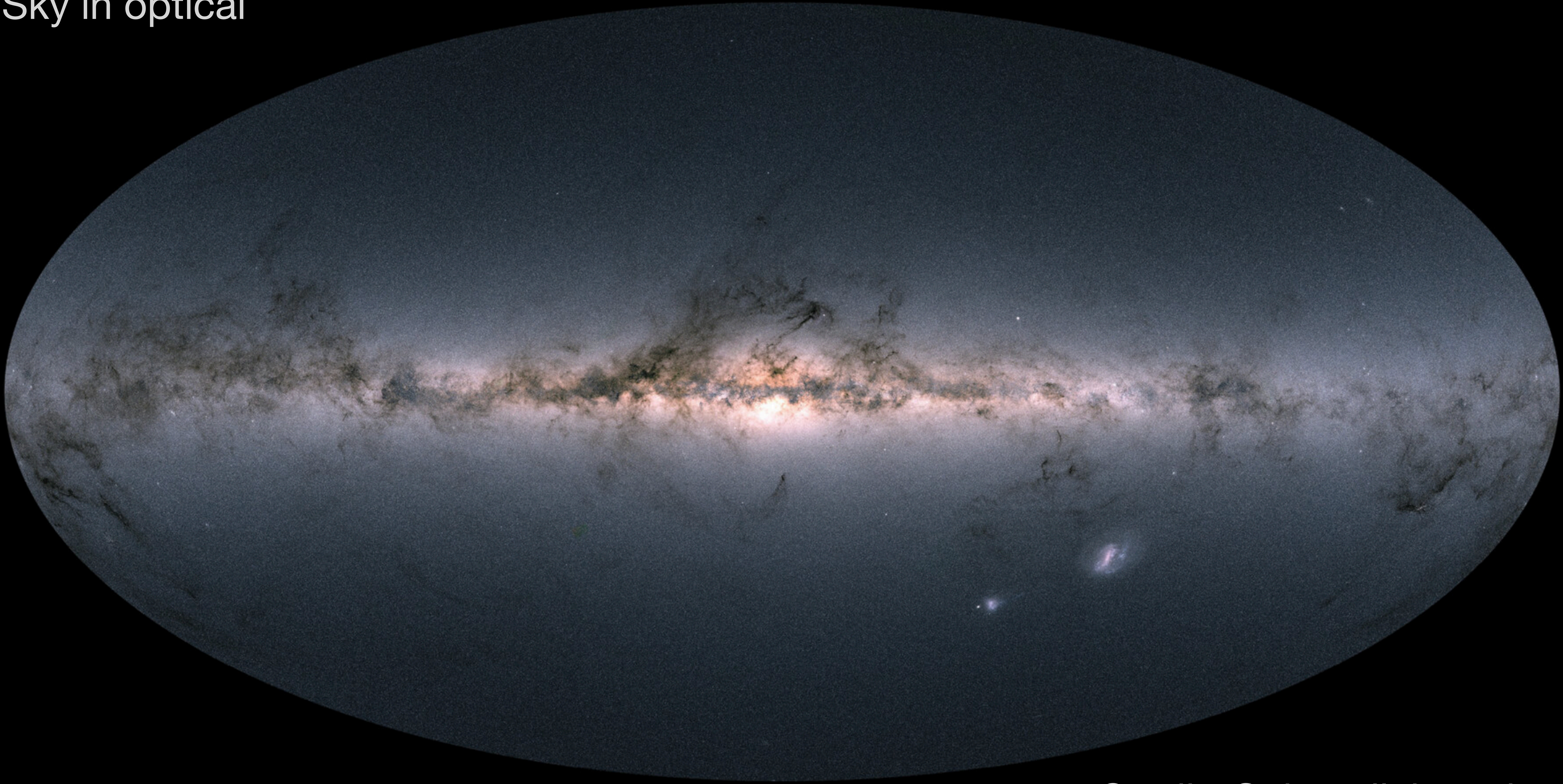
How does the CMB constrain H_0?

- In order to understand how we can solve the Hubble tension, we need to understand how the CMB constrains H_0
- The CMB provides the most important cosmological probe when it comes to constraining the parameters of the cosmological model with high accuracy
- However, it is an indirect probe of H_0: it depends on the cosmological model that is assumed $\rightarrow$ the CMB constrains the universe at early times and predicts H_0 today



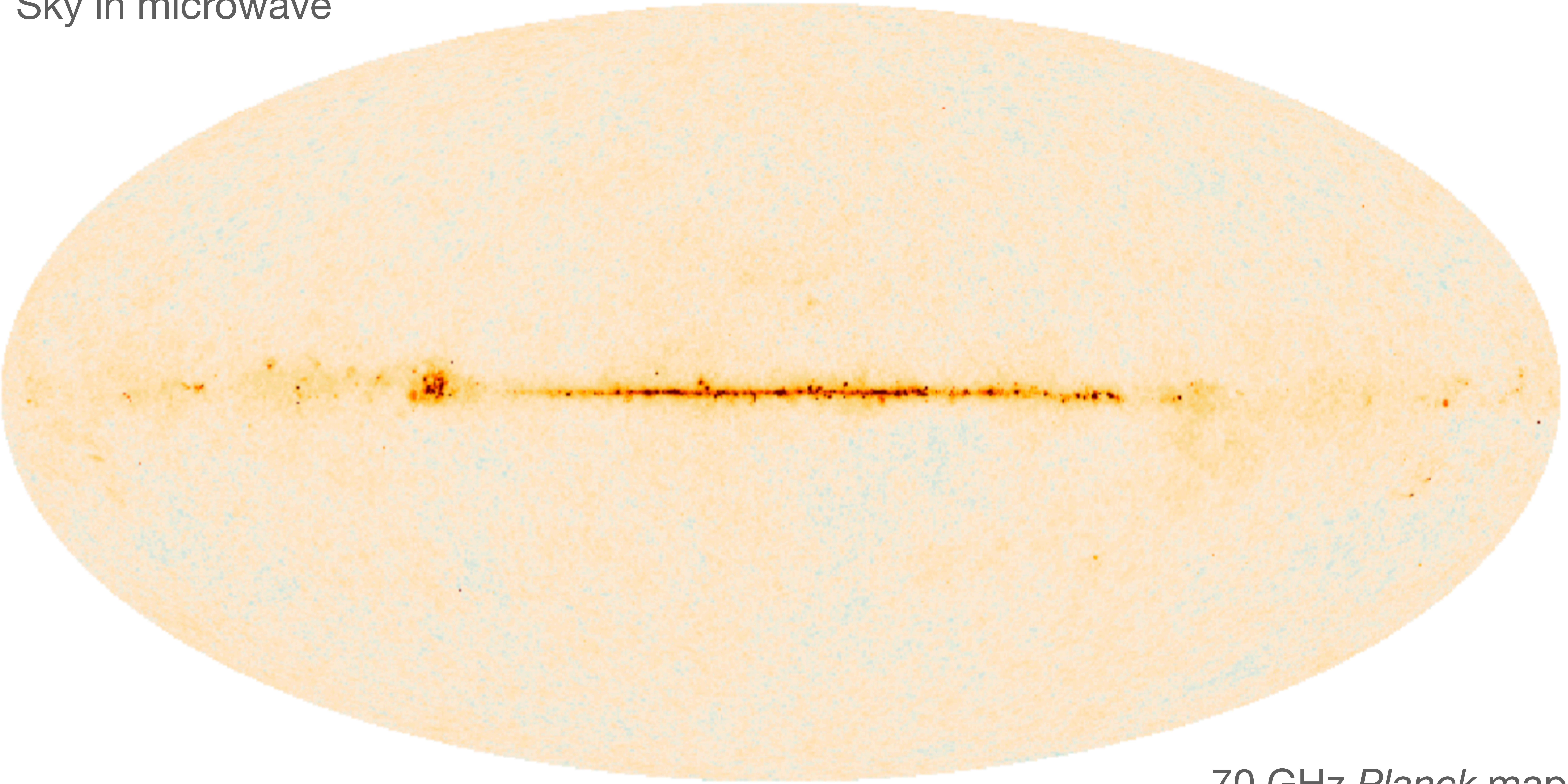
Credit: WMAP Collaboration

Sky in optical



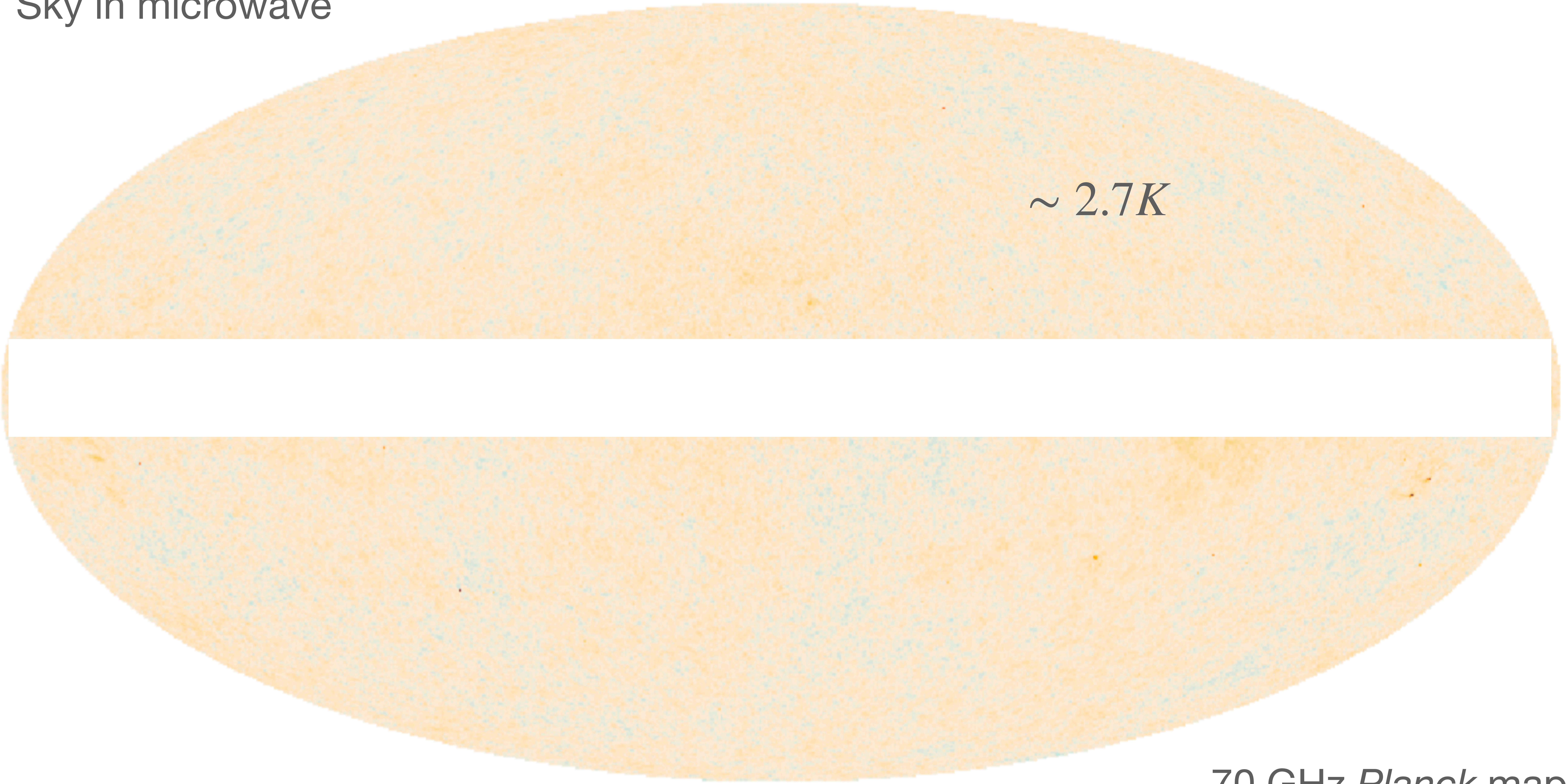
Credit: Gaia collaboration

Sky in microwave



70 GHz *Planck* map

Sky in microwave



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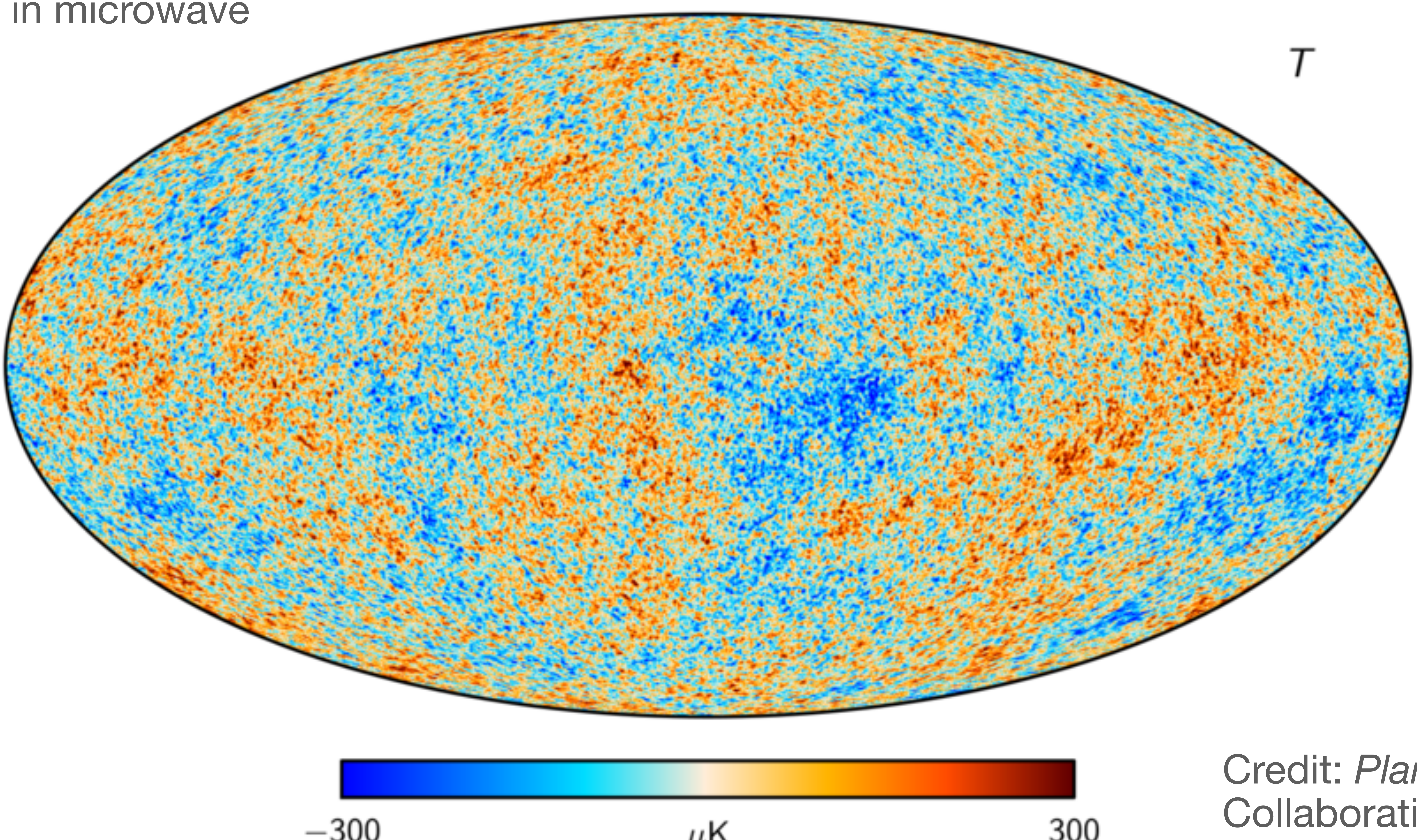
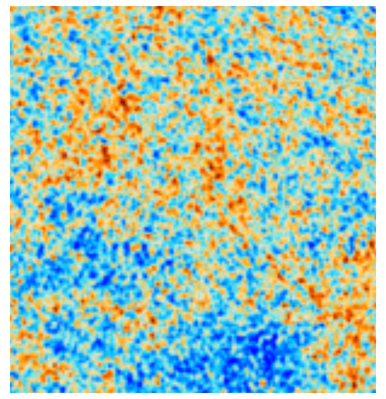


Credit: *Planck* Collaboration

The CMB

- How to analyse the CMB map with temperature fluctuations?
  - 1. Decompose the temperature fluctuations into a set of waves with various wavelength
  - 2. Plot the strength of each wavelength: Power spectrum
- In 1 dimension: decomposing into waves = Fourier transform
- On the unit sphere: decomposing into waves = Spherical harmonics decomposition

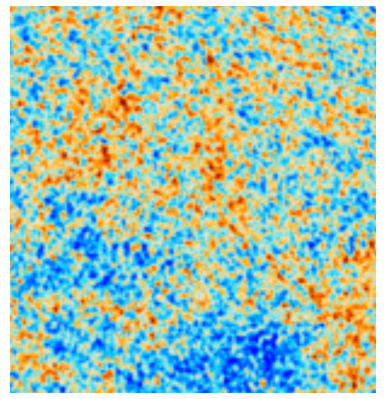
The CMB



- Decompose temperature fluctuations: $\Delta T(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} \, Y_{\ell m}(\theta, \phi)$

coefficients spherical harmonics

The CMB



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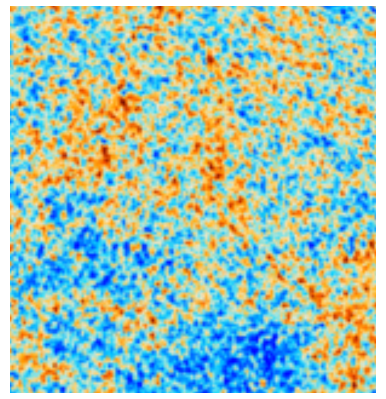
The CMB



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- ℓ defines the multipole, the higher ℓ the smaller the scale

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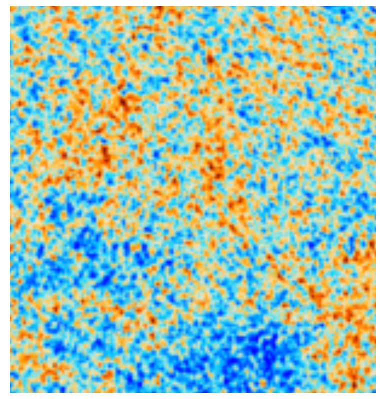

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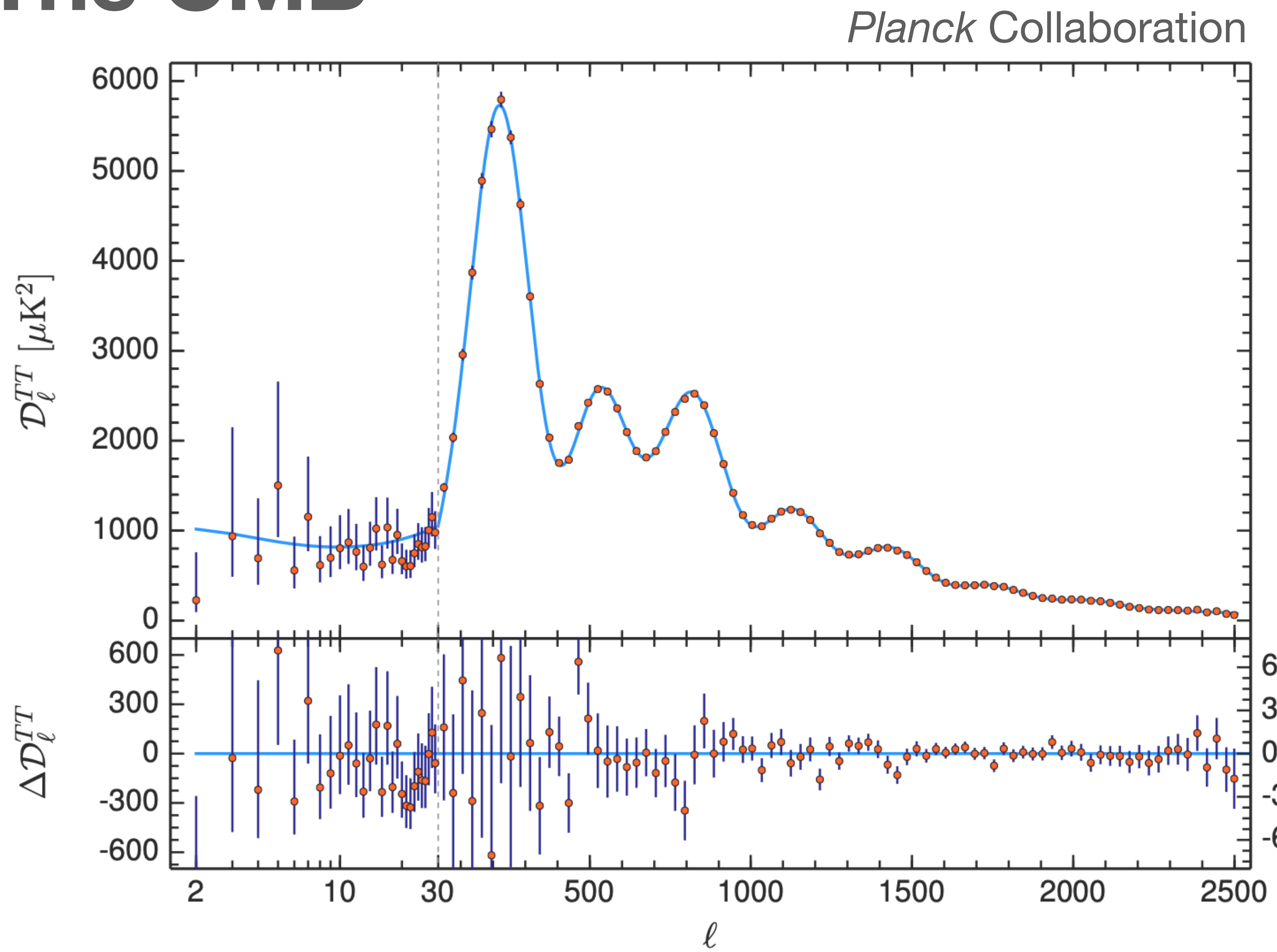
spherical harmonics

- The $a_{\ell m}$ contain the same information as the map itself
- ℓ defines the multipole, the higher ℓ the smaller the scale
- The power spectrum is then given by:

$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a_{\ell,m} \, a^*_{\ell,m}$$

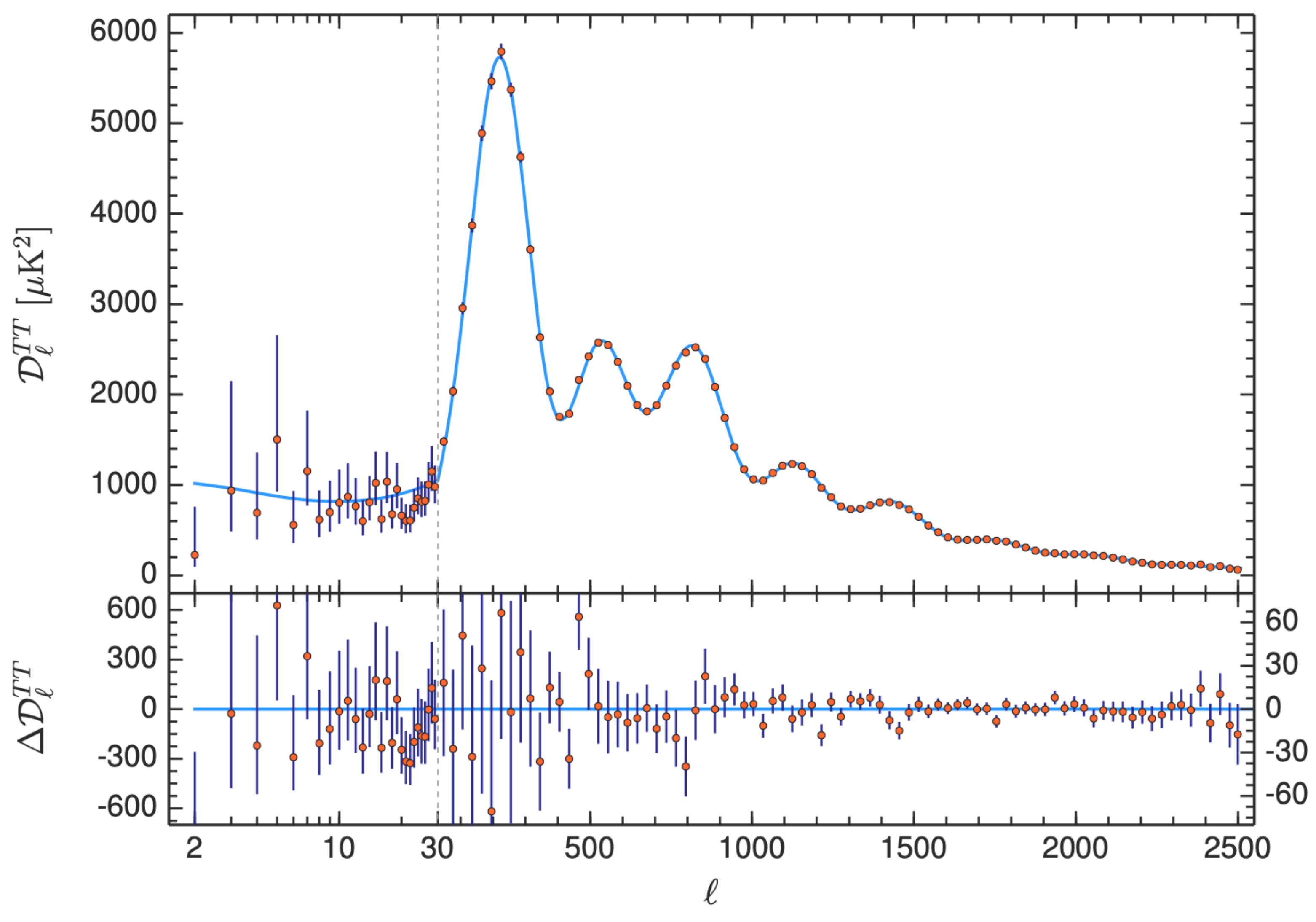
The CMB



$$D_\ell = \frac{\ell(\ell+1)}{2\pi} C_\ell$$

*You will need this in the hands-on session later

The CMB



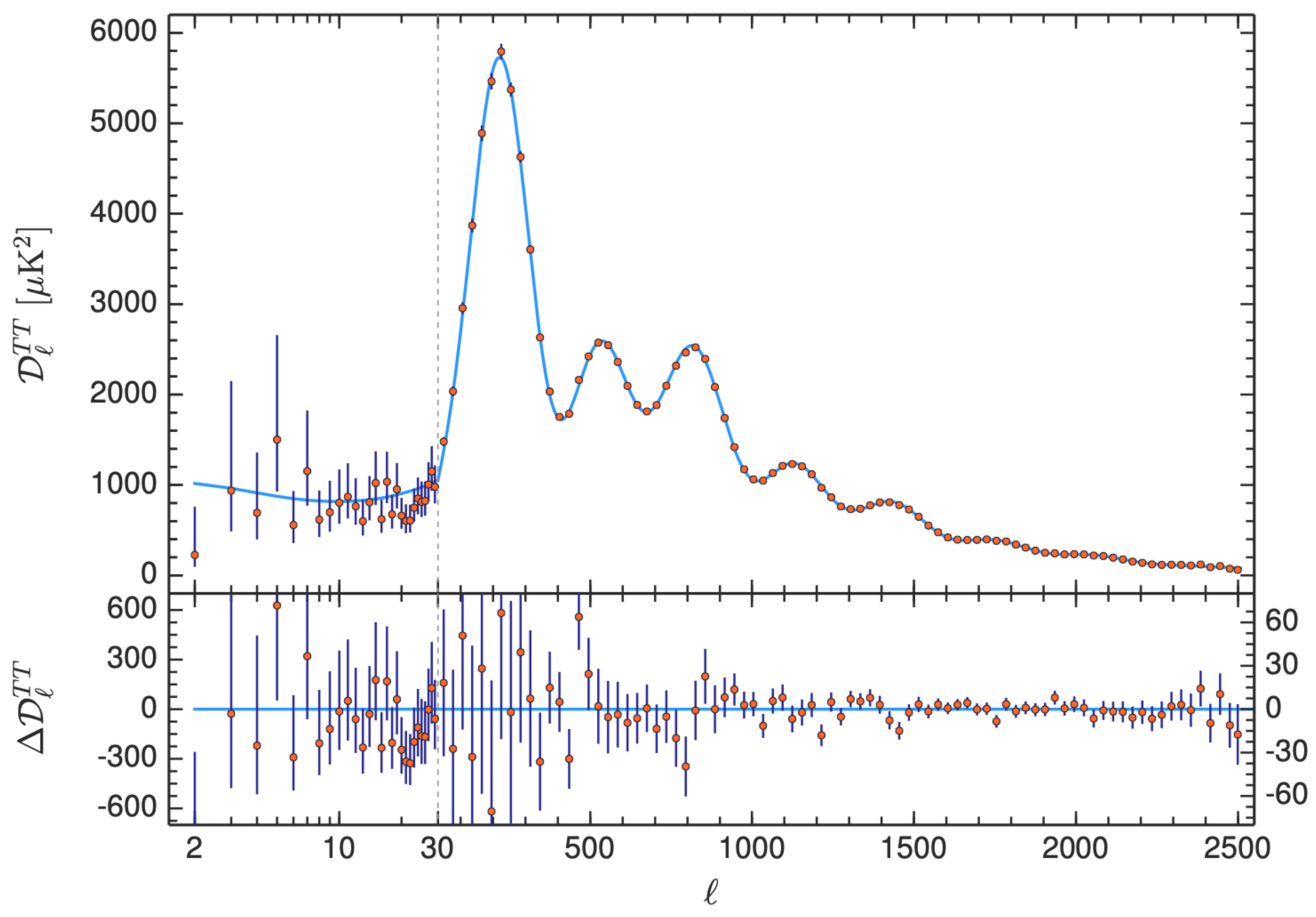
How to analyse data like this?

- To predict the CMB power spectrum: one needs to solve full fluid-equations (the Boltzmann equations) for all components of the universe
- This can be done with cosmological Boltzmann solvers like CAMB (Lewis&Bridle 2002) and CLASS (Blas, Lesgourgues, Tram 2011)
- These Boltzmann solvers take as input the ΛCDM parameters and can be sampled within MCMC samplers

*You will need this in the hands-on session later

The CMB

ΛCDM model: 6 parameters



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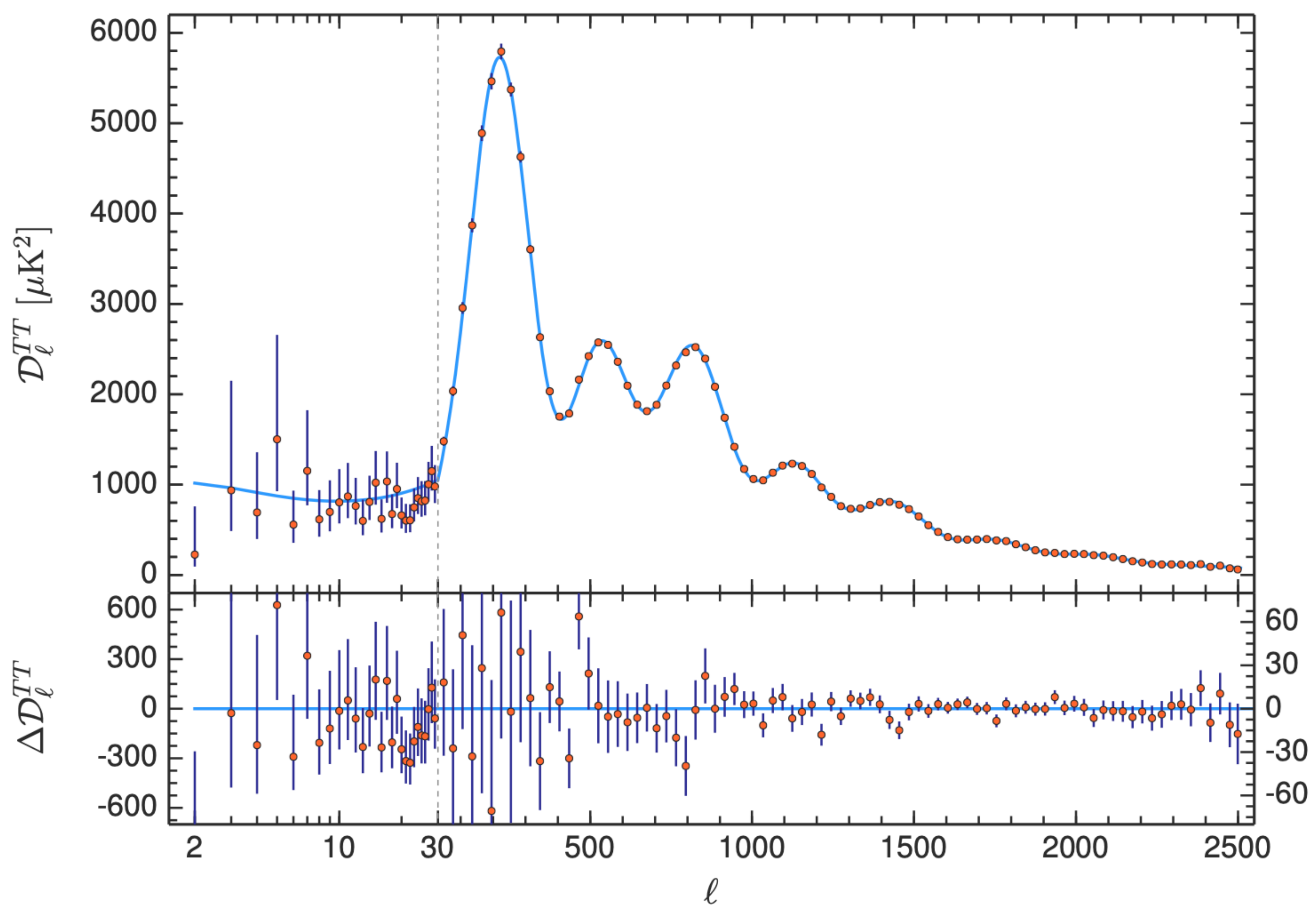

ΛCDM model: 6 parameters

- $\omega_{\rm cdm} = h^2\Omega_{\rm cdm}$:
- $\omega_b = h^2\Omega_b$:
- $h = H_0/(100\,{\rm km/s/Mpc})$:
- $\tau_{\rm reio}$:
- A_s:
- n_s:

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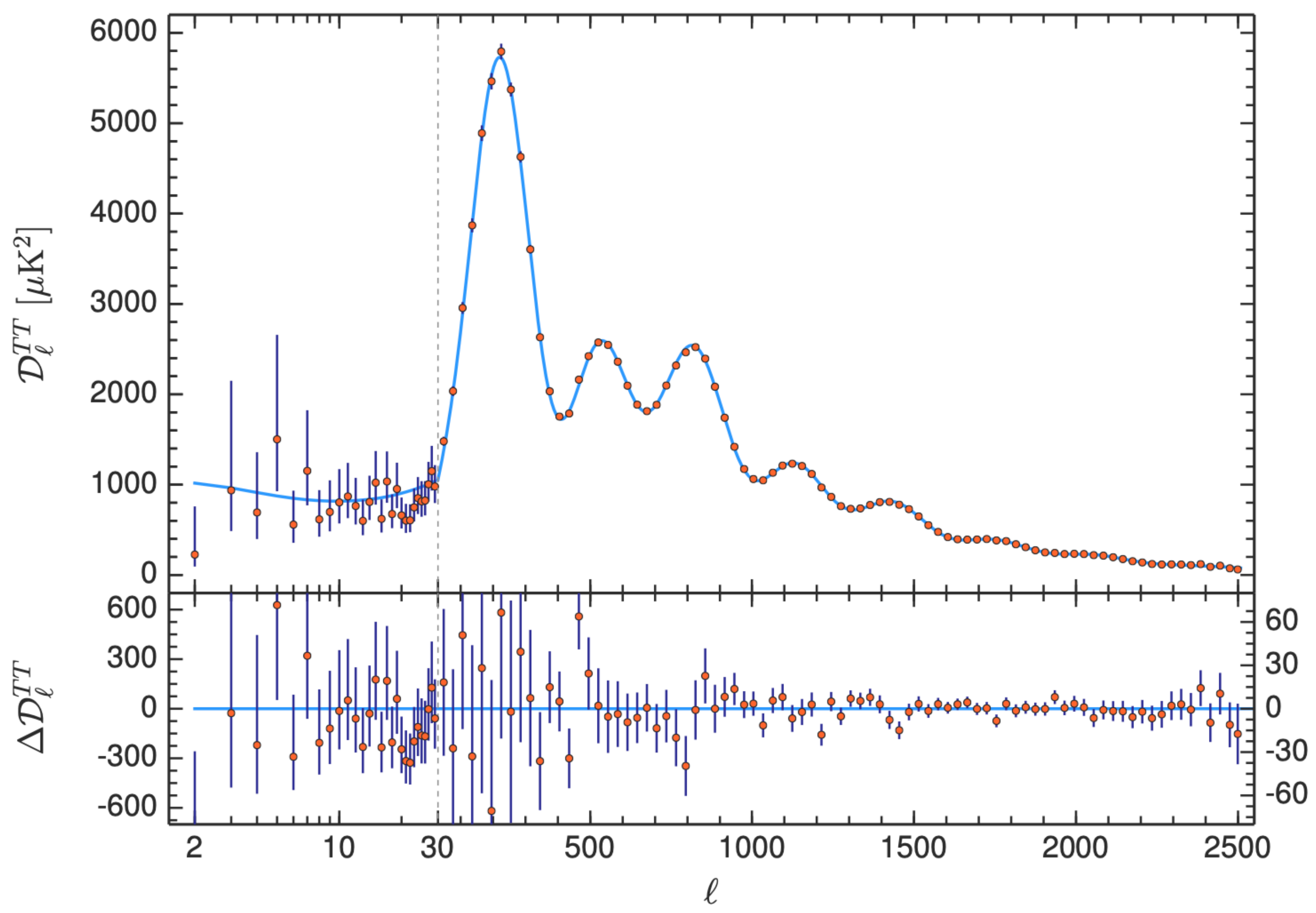

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- $\omega_{\rm cdm} = h^2\Omega_{\rm cdm}$: physical energy density in CDM
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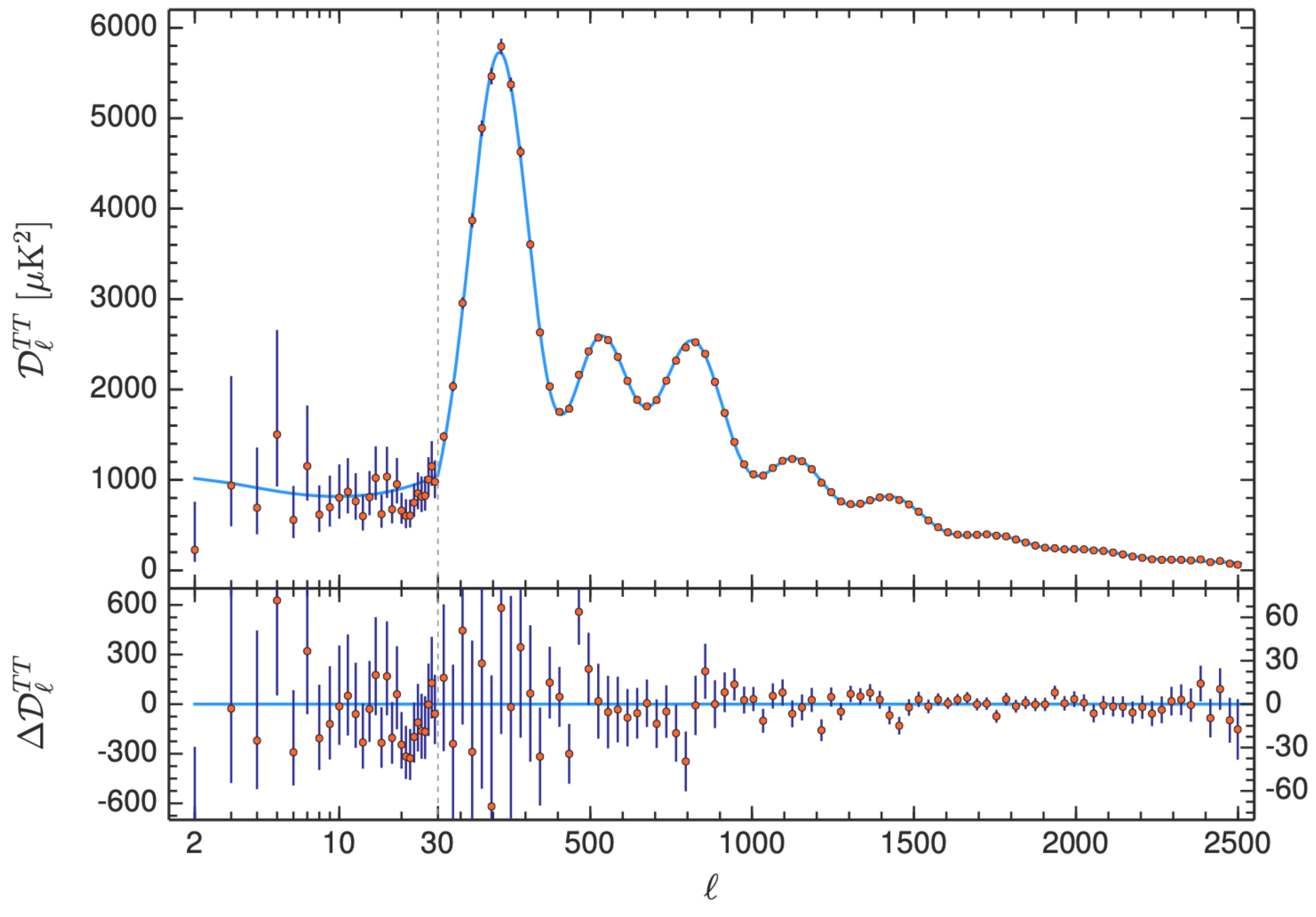

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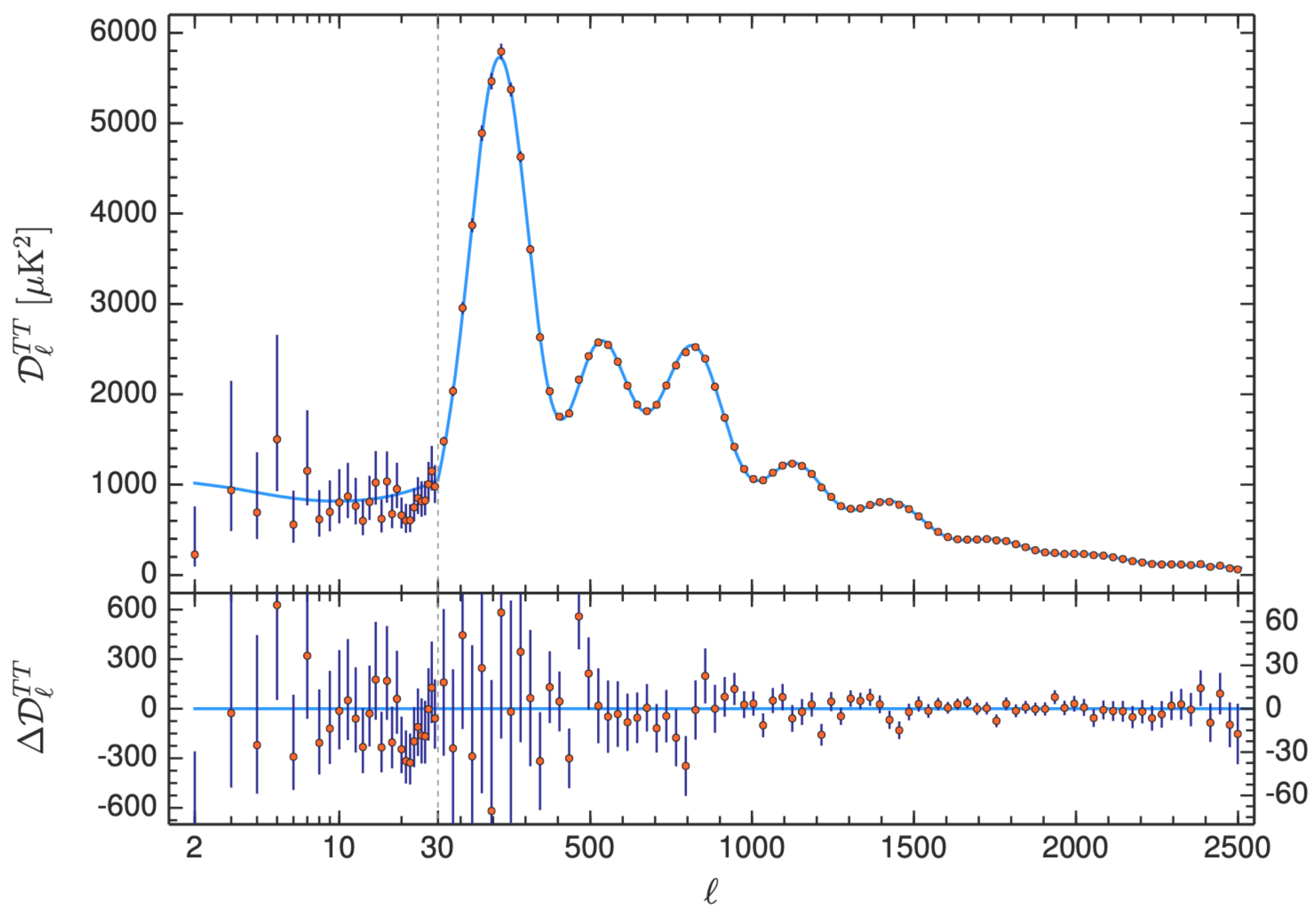

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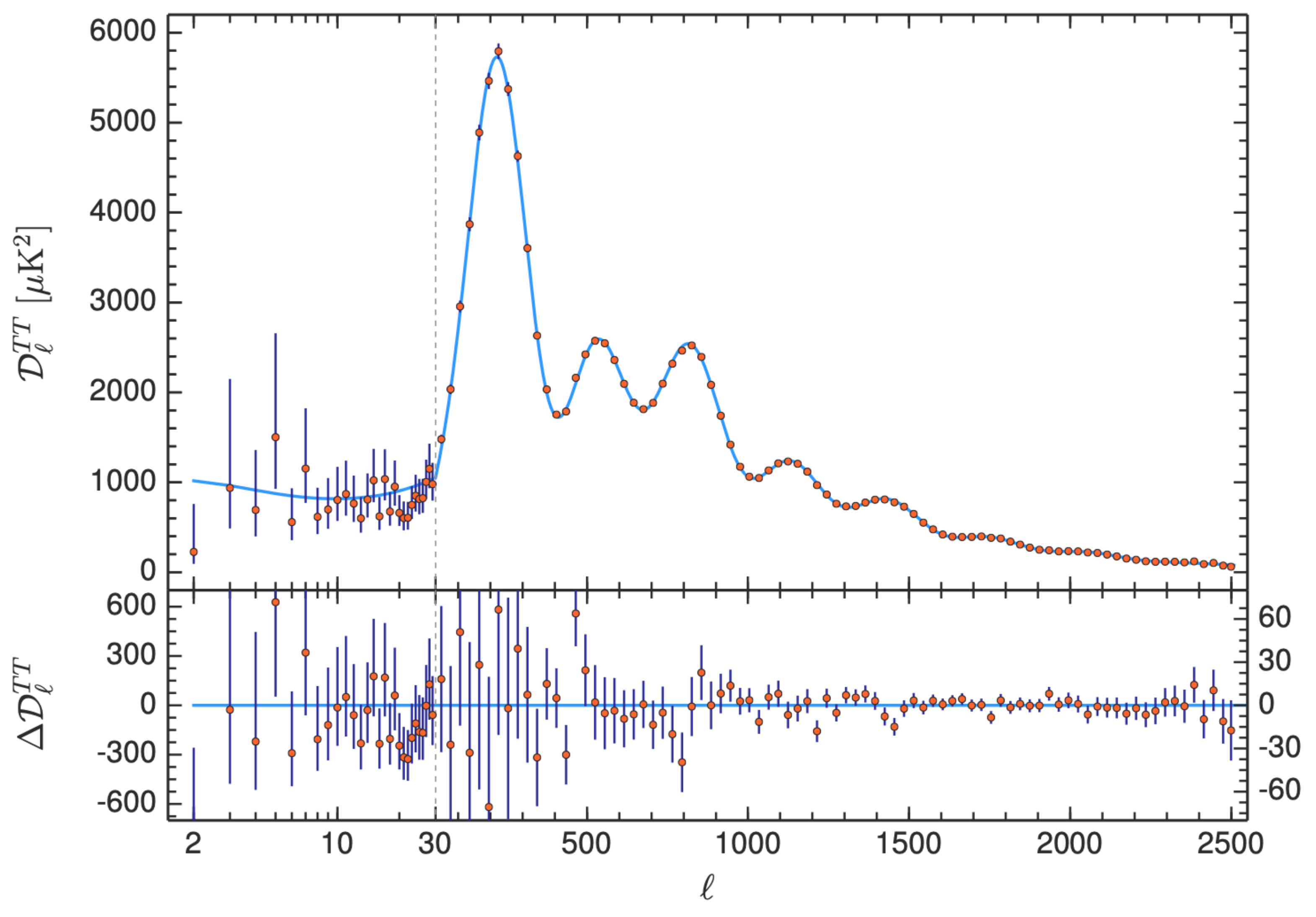

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- n_s:

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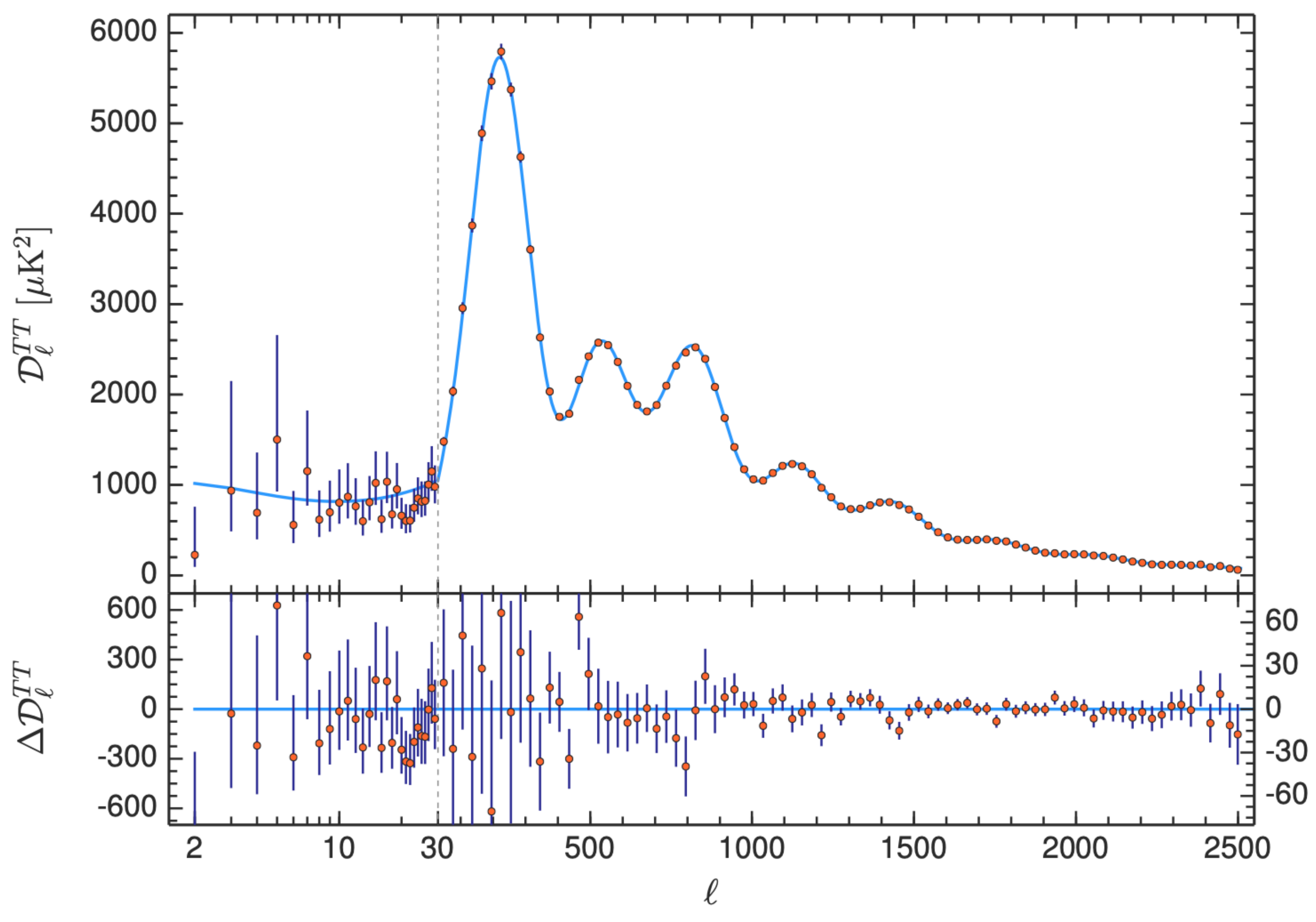

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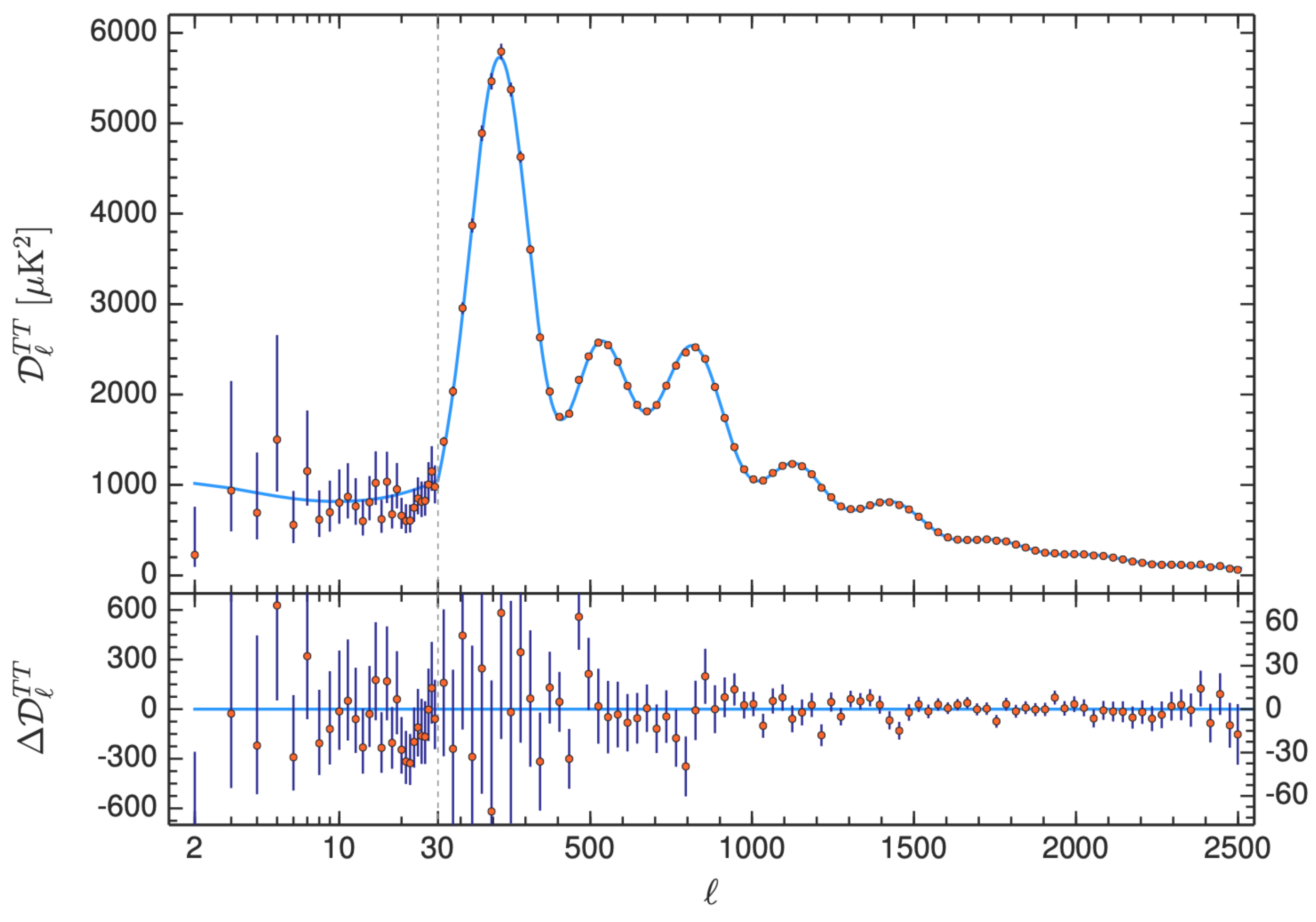

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- n_s: tilt of the primordial power spectrum

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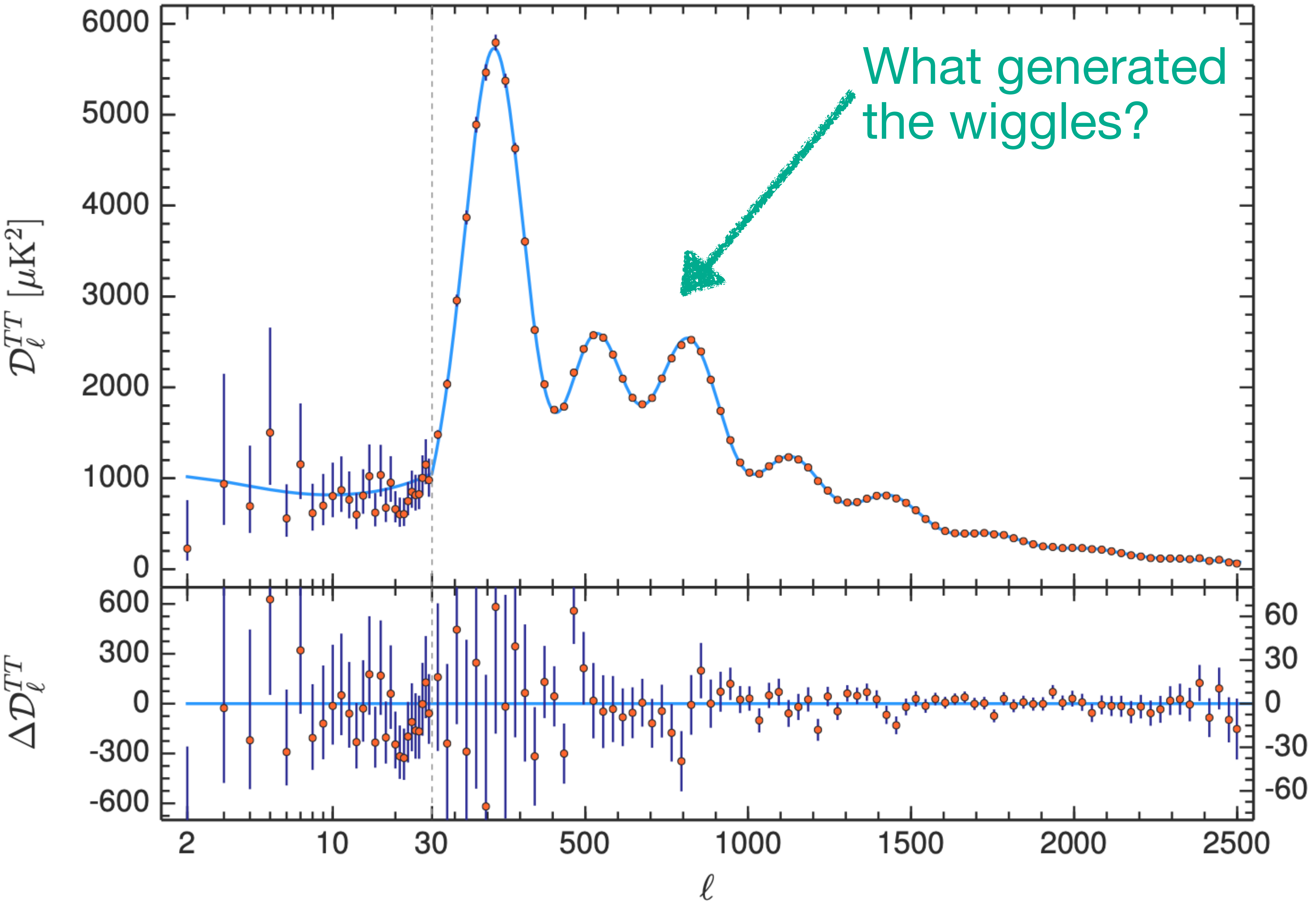
The CMB

Planck Collaboration



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The baryon acoustic oscillations (BAO)

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Credit: Mabel Amber

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- Small density fluctuations sourced pressure-density waves
  - Gravity as driving force
  - Photon pressure as restoring force
- Like rain drops on the water surface many waves overlap
- Once electrons and protons recombine, photons can travel freely → the density waves freeze

Credit: Mabel Amber

The baryon acoustic oscillations (BAO)

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- While the sound waves travel, the universe expands. This slows down the sound waves in the comoving coordinate frame

The baryon acoustic oscillations (BAO)

- Comoving sound horizon:

$$r_s = \int_0^{t^*} \frac{c_s(t)}{a(t)}\, \mathrm{d}t$$

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$$r_s = \int_0^{t*} \frac{c_s(t)}{a(t)}\,\mathrm{d}t = \int_0^{a*} \frac{c_s(a)}{\dot{a}a}\,\mathrm{d}a$$

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \dot{a}$$

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$$r_s = \int_0^{t*} \frac{c_s(t)}{a(t)}\,\mathrm{d}t = \int_0^{a*} \frac{c_s(a)}{\dot{a}a}\,\mathrm{d}a = -\int_\infty^{z*} \frac{c_s(z)a^2}{\dot{a}a}\,\mathrm{d}z$$

$$\frac{\mathrm{d}z}{\mathrm{d}a} = \frac{\mathrm{d}(1/a)}{\mathrm{d}a} = -\frac{1}{a^2}$$

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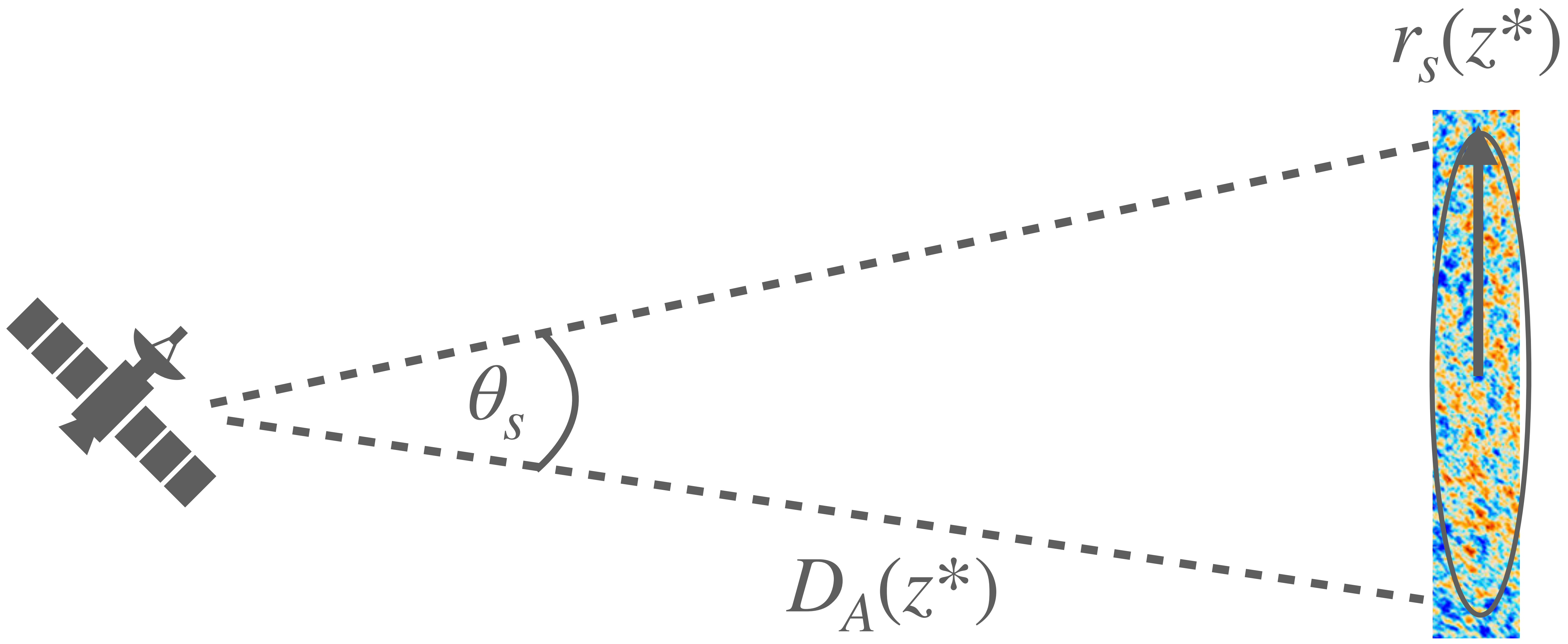
- The sound speed can be expressed as (e.g. Dodelson&Schmidt, 2020):

$$c_s(z) = \frac{1}{\sqrt{3(1 + R(z))}}$$

- With R being the baryon-photon ratio:

$$R(z) = \frac{3\omega_b}{4\omega_\gamma}\frac{1}{1+z}$$

The baryon acoustic oscillations (BAO)



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$$r_s^{\text{phys}}(z^*) = a(z^*)\, r_s(z^*)$$

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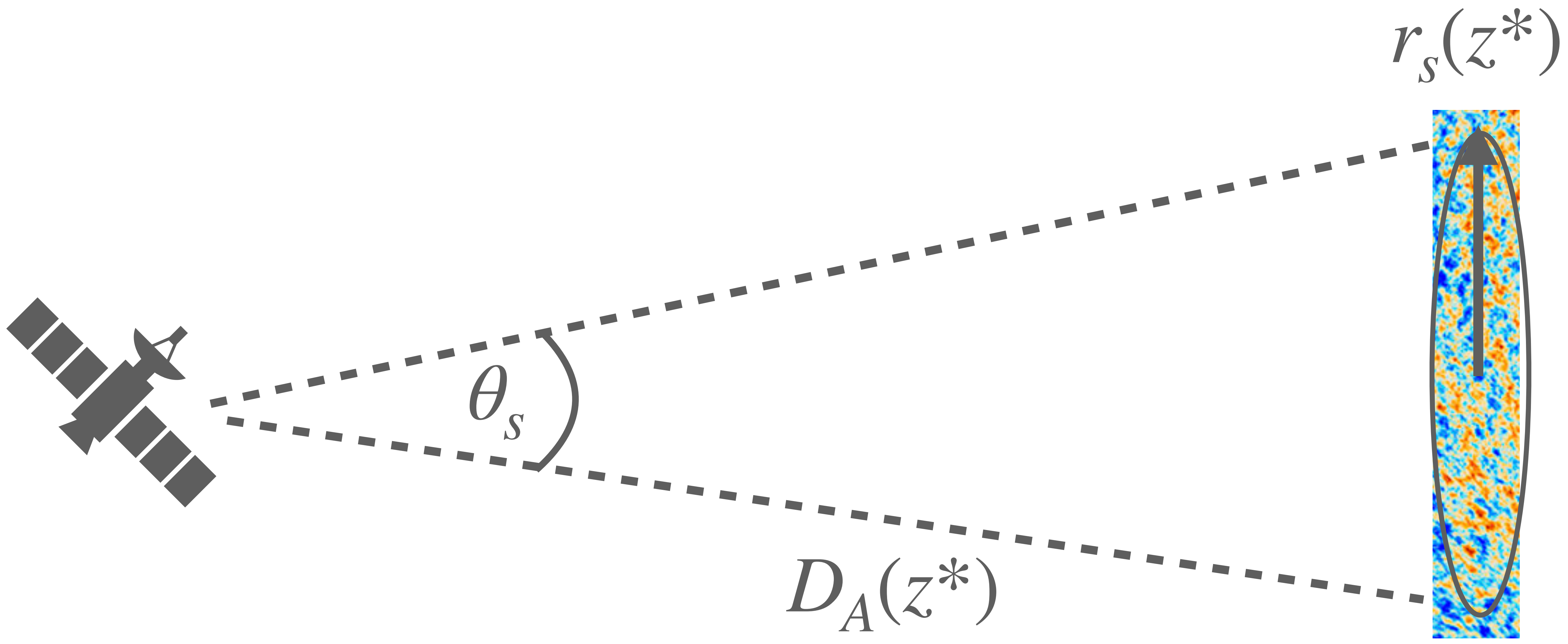

- What we observe is the *angular size of the sound horizon θ_s*
- In the small-angle approximation:

$$\theta_s = \frac{r_s^{\rm phys}(z^*)}{D_A(z^*)}$$

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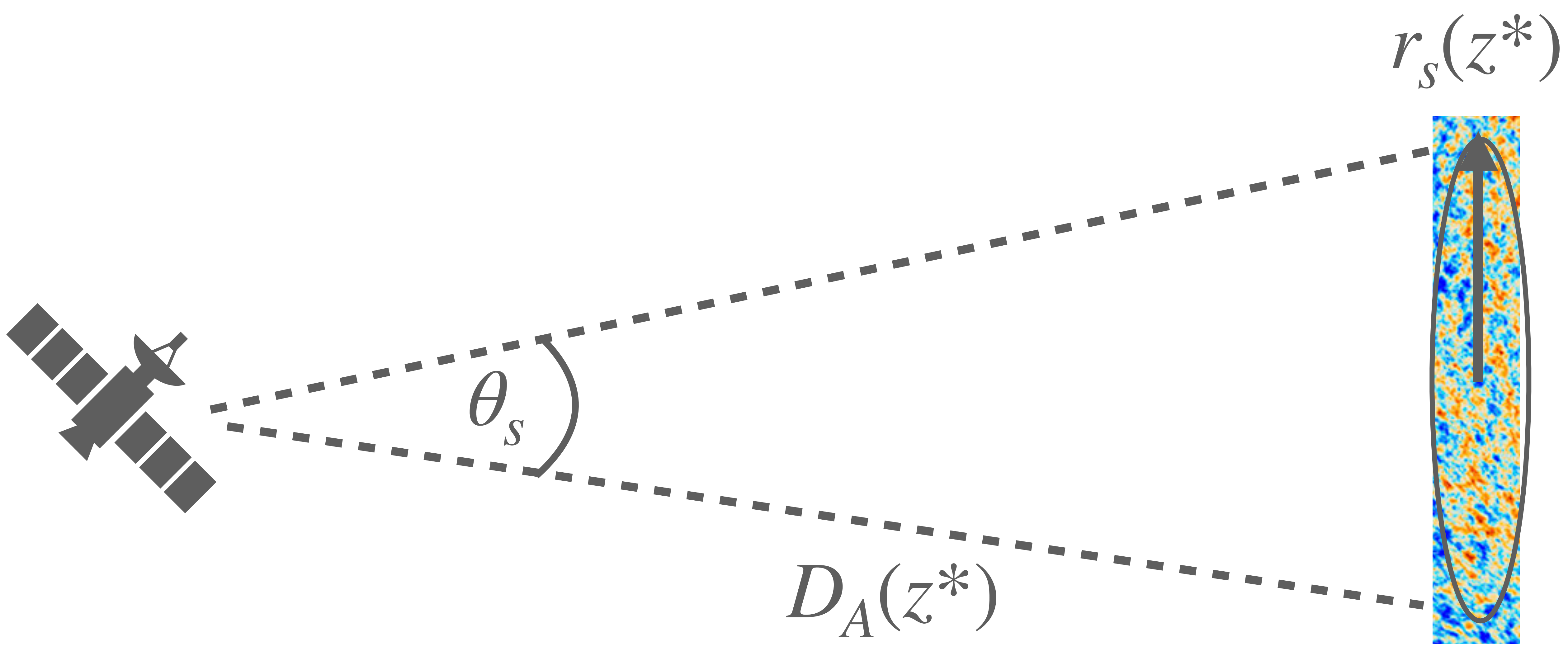
The baryon acoustic oscillations (BAO)



- What we observe is the *angular size of the sound horizon* θ_s
- In the small-angle approximation:

$$\theta_s = \frac{r_s^{\mathrm{phys}}(z^*)}{D_A(z^*)}$$

- where the angular diameter distance is (see slide earlier):

$$D_A(t) = \int_0^{z(t)} \frac{\mathrm{d}z'}{H(z')}$$

$$r_s^{\mathrm{phys}}(z^*) = a(z^*)\, r_s(z^*)$$

How does the CMB constrain H_0?

- We know that the CMB directly constrains

$$\theta_s = \frac{r_s^{\rm phys}(z*)}{D_A(z*)} = \frac{\int_{z*}^{\infty} c_s(z)\mathrm{d}z/H(z)}{\int_0^{z*} \mathrm{d}z/H(z)}$$

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$$H(z) = H_0\sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}$$

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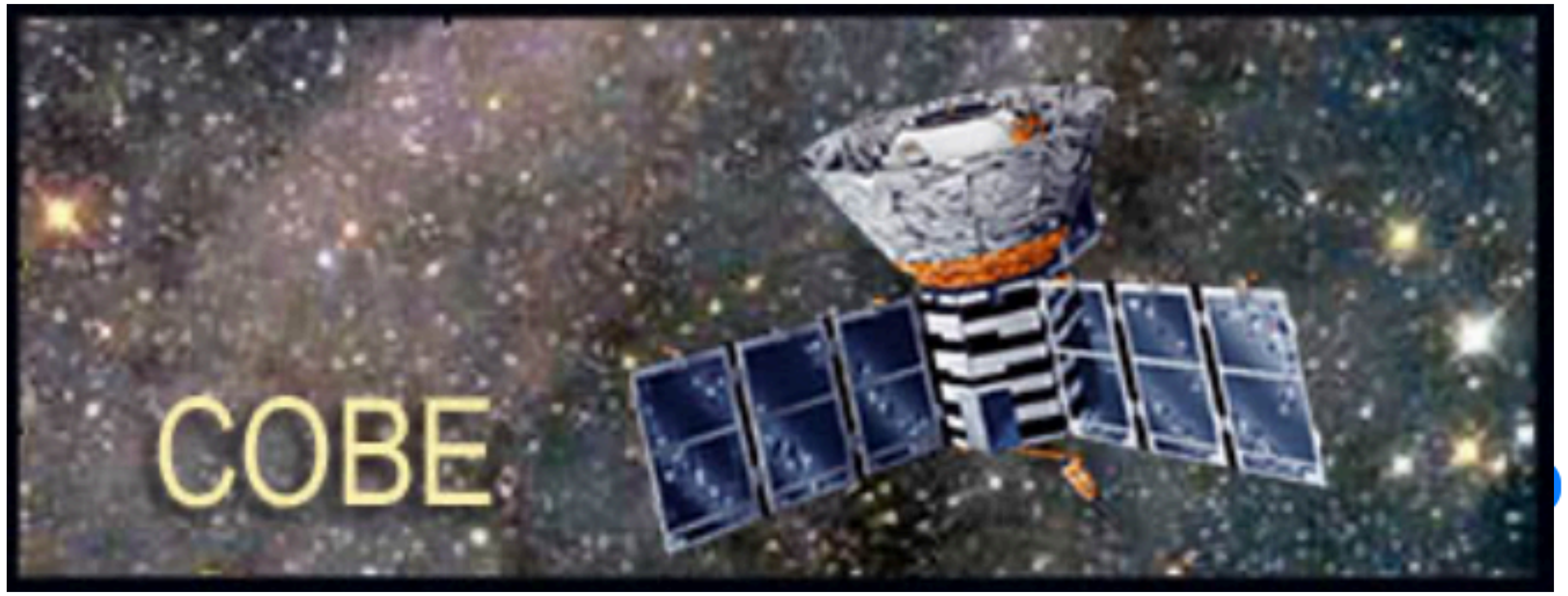
$$H(z) = H_0\sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}$$

- If we get Ω_r, Ω_m, Ω_Λ from somewhere else, this becomes an implicit equation for H_0 — How does the CMB constrain Ω_r, Ω_m, Ω_Λ?

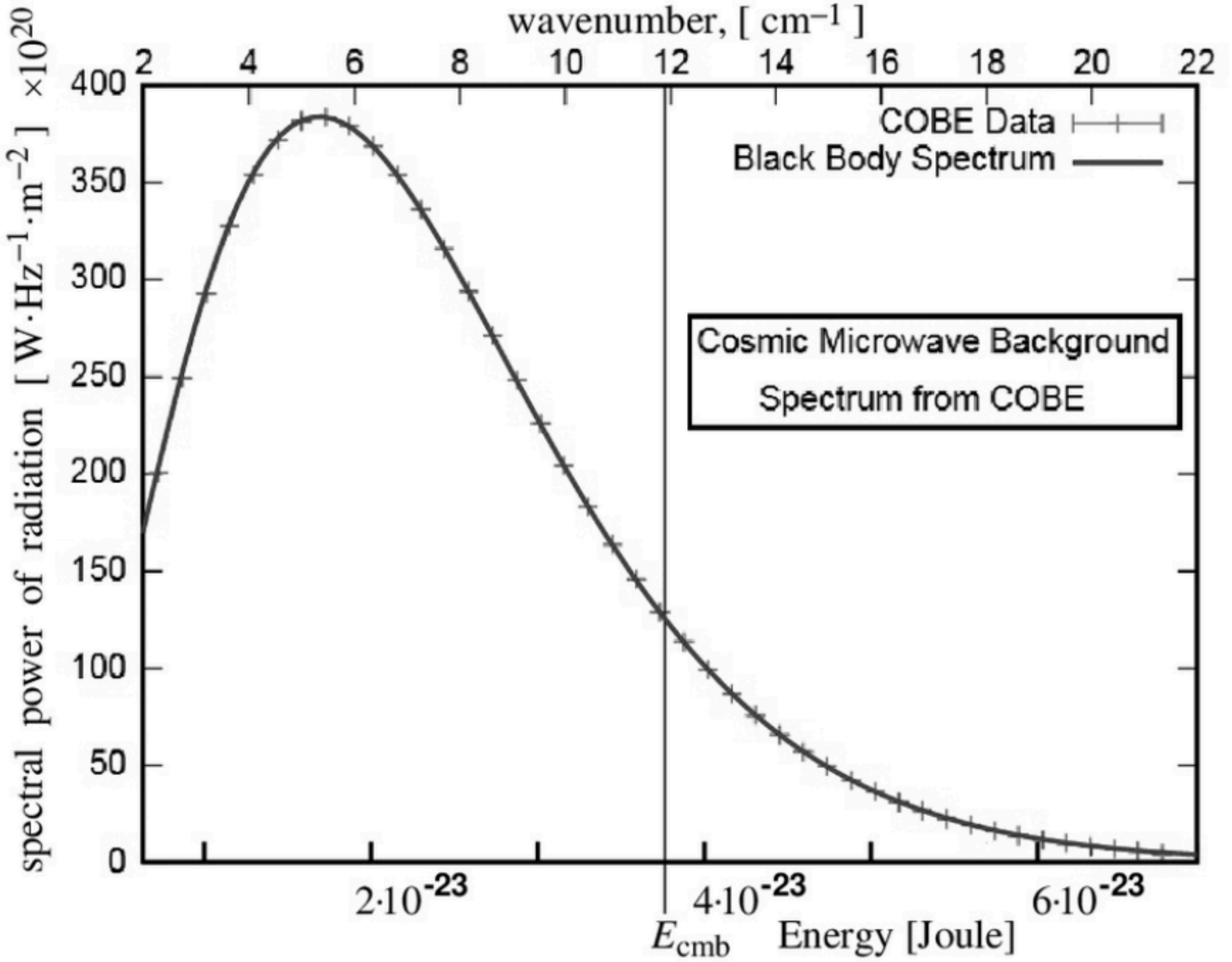
How does the CMB constrain Ω_r?

- The radiation density Ω_r is precisely measured by the CMB temperature

$$T = 2.725 \pm 0.002\,\mathrm{K}$$ (COBE - FIRAS measurement)



Credit: Berkeley Center for Cosmological Physics



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- For a black-body spectrum, the energy density in radiation is then given by (e.g. Weinberg 2008, Ch. 2.1):

$$\rho_{0,\mathrm{CMB}} = \int_0^\infty h\nu \cdot n(\nu)\mathrm{d}\nu = \frac{8\pi^5 k_B^4}{15h^3c^3}T^4 = 4.64 \cdot 10^{-34}\,\mathrm{g/cm^3}$$

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For more about neutrinos, see Olga Mena's lecture

- Taking into account the earlier time of decoupling of neutrinos, one finds: $\rho_\nu = 0.4\,\rho_{0,\mathrm{CMB}}$

- Since the CMB and neutrinos are by far the dominant contribution:

$$\Omega_r = 1.4\,\rho_{0,\mathrm{CMB}}/\rho_{\mathrm{crit}} = 4.15 \cdot 10^{-5}h^{-2}$$

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- It does not constrain Ω_m, but $\omega_m = h^2\Omega_m$, where $\omega_m = \omega_{\rm cdm} + \omega_b$

- ω_m is constrained by the height of the acoustic peaks:
  - a smaller ω_m leads to a later time of matter-radiation equality
  - this leads to a stronger decay of the gravitational potentials at recombination $\rightarrow$ early integrated Sachs-Wolfe effect (eISW)
  - Since the eISW adds in phase with the BAO, this leads to a boost of all peaks, particularly the first peak.

How does the CMB constrain Ω_Λ?

- The sum of the energy densities has to satisfy:

$$\Omega_r + \Omega_m + \Omega_\Lambda = \Omega_k$$

- For a flat universe:

$$\Omega_r + \Omega_m + \Omega_\Lambda = 1$$

- Hence, if we know Ω_r and $\omega_m = h^2\Omega_m$, we can compute

$$\Omega_\Lambda = 1 - \Omega_r - \omega_m/h^2$$

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- Since we can constrain Ω_r and ω_m independently, this becomes an implicit equation for h

$$\frac{H^2(z)}{(100\,\mathrm{km/s/Mpc})^2} = h^2\Omega_r(1+z)^4 + \omega_m(1+z)^3 + (h^2 - \omega_m - h^2\Omega_r)$$

How does the CMB constrain H_0?

- We know that the CMB directly constrains

$$\theta_s = \frac{r_s^{\mathrm{phys}}(z^*)}{D_A(z^*)} = \frac{\int_{z^*}^{\infty} c_s(z)\mathrm{d}z/H(z)}{\int_0^{z^*} \mathrm{d}z/H(z)}$$

* $c_s(z)$ and z^* also depend on cosmology but we neglect that here

- Cosmology enters mainly over the Hubble parameter here:

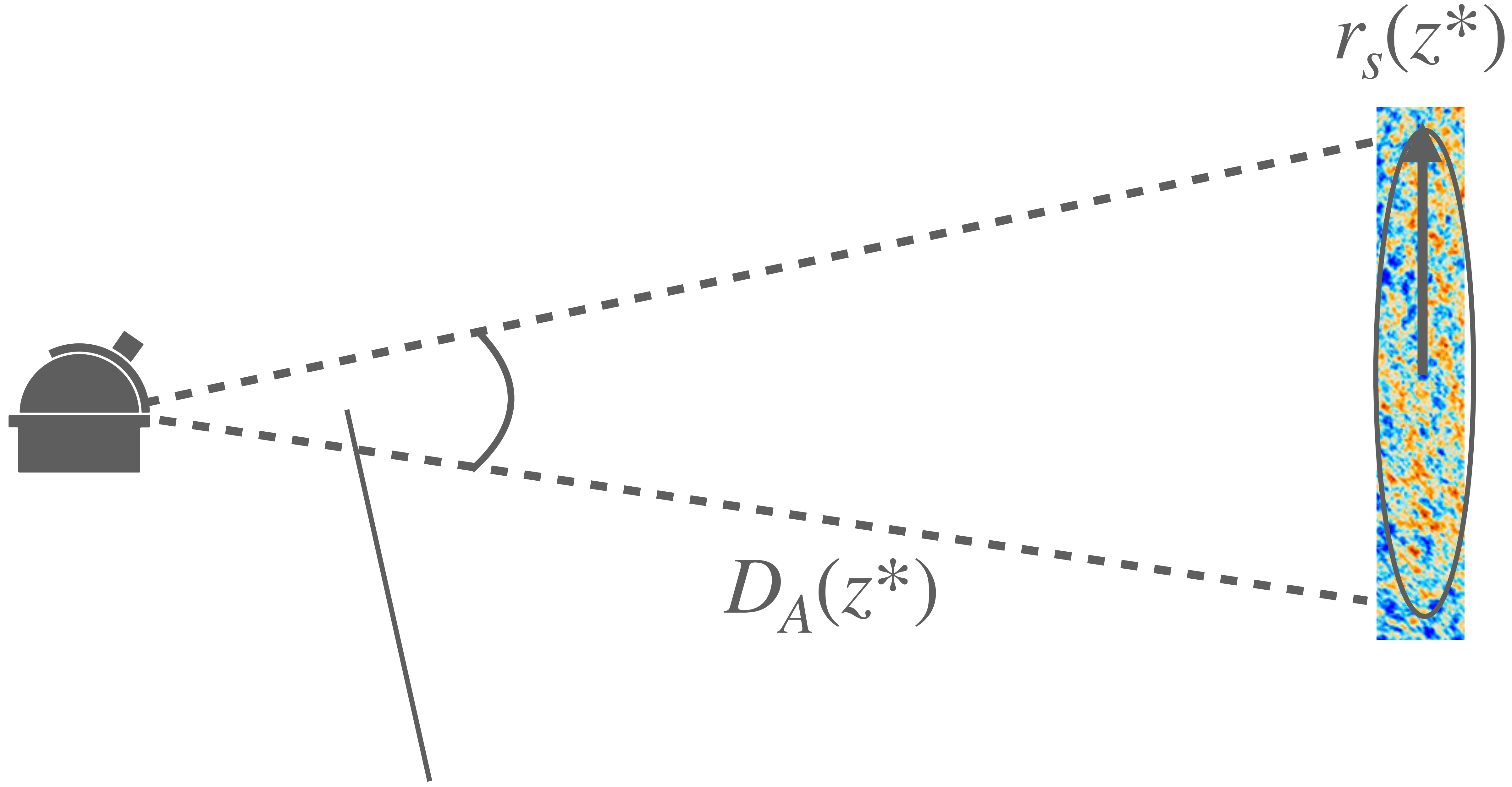
$$H(z) = H_0\sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}$$

- Since we can constrain Ω_r and ω_m independently, this becomes an implicit equation for h

$$\frac{H^2(z)}{(100\,\mathrm{km/s/Mpc})^2} = h^2\Omega_r(1+z)^4 + \omega_m(1+z)^3 + (h^2 - \omega_m - h^2\Omega_r)$$

Solutions to the Hubble tension

Solutions to the Hubble tension



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- θ_s is measured precisely by CMB $\rightarrow \theta_s$ fixed
- Two options to solve the Hubble tension:

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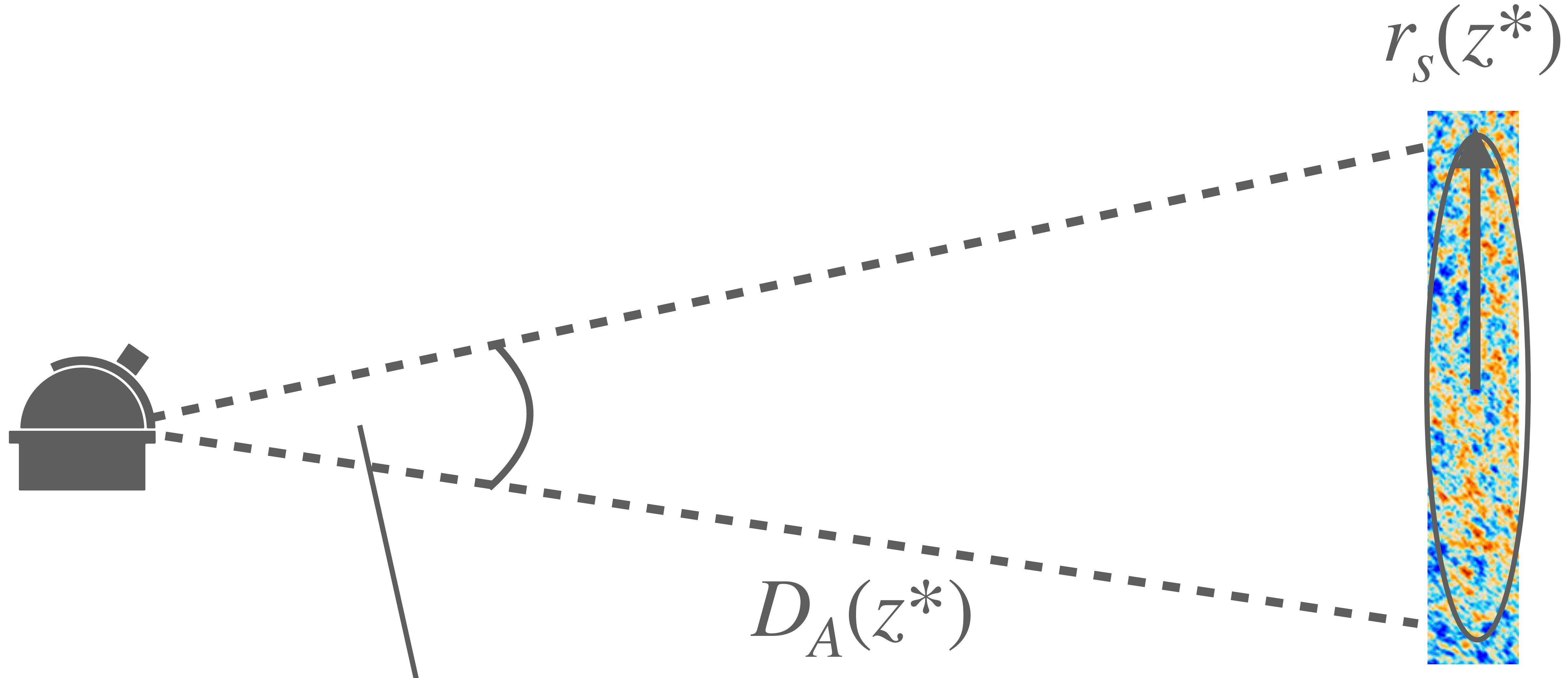

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Early-time solutions modify r_s

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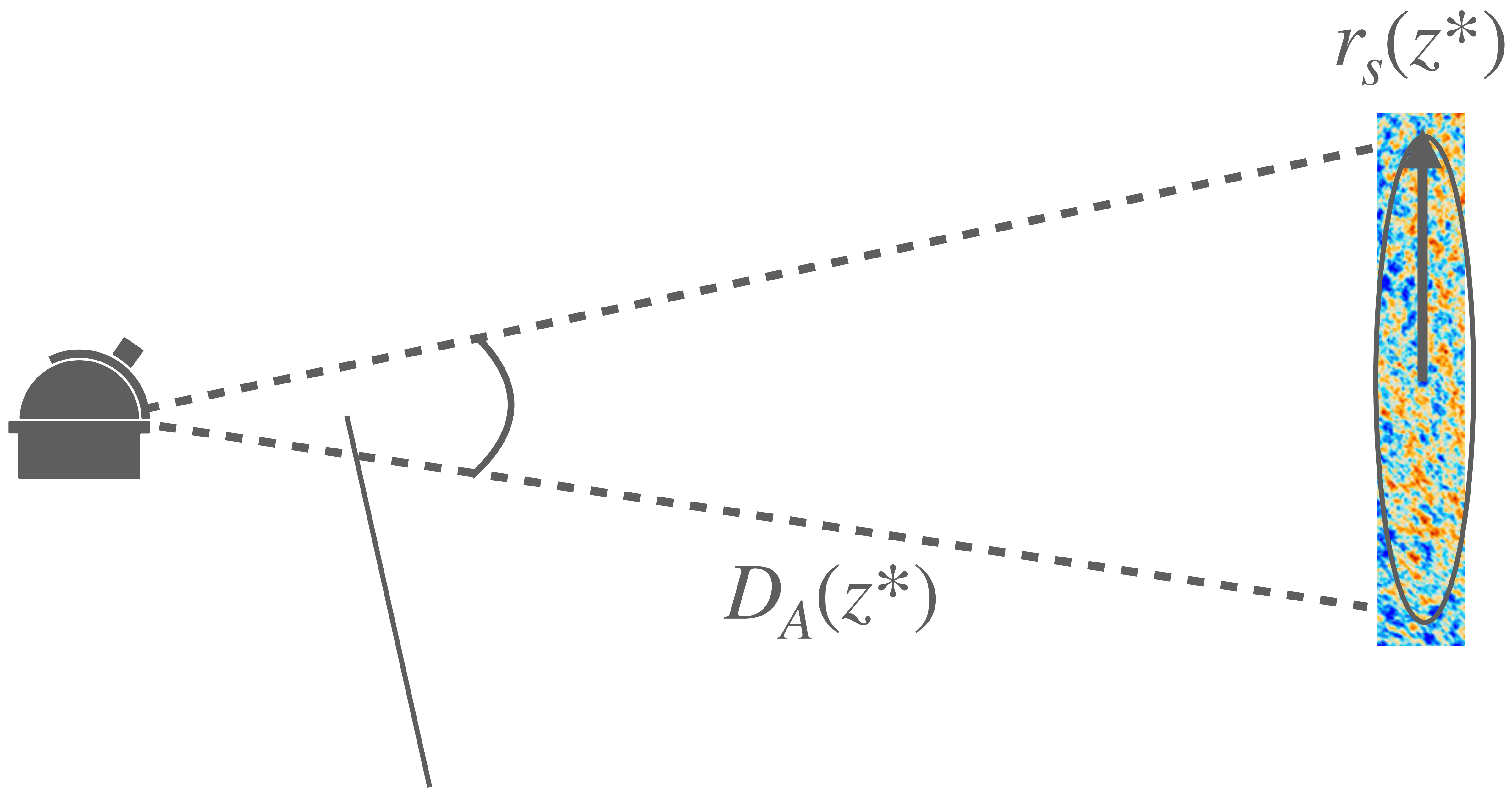

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Early-time solutions modify r_s

Late-time solutions modify D_A

Late-time solutions

- Modify $D_A(z^*) = \int_0^{z^*} \frac{\mathrm{d}z}{H(z)}$ by modifying the expansion rate between today (0) and recombination z^* (since $\Omega_r \approx 0$ at times after z^*):

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + 1 - \Omega_m}$$

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$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + 1 - \Omega_m}$$

- However, $H(z)$ is well constrained by galaxy BAO data and supernova data
  → It is challenging to increase H_0 enough

- But there are some promising approaches (see other lectures)

Early-time solutions

- Modify $r_s = \int_{z^*}^{\infty} \frac{c_s(z)\ \mathrm{d}z}{H(z)}$ → 3 options:
    - modify z^*
    - modify $c_s(z)$
    - modify $H(z)$

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  - **modify** z^* →
  - modify $c_s(z)$
  - modify $H(z)$

E.g. by modifying the mass of the electron m_e:

→ This shifts the energy levels of the atoms,

→ and changes the ionization energy,

→ which changes the time of recombination z^*

Early-time solutions

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  - modify $c_s(z)$
  - **modify** $H(z)$

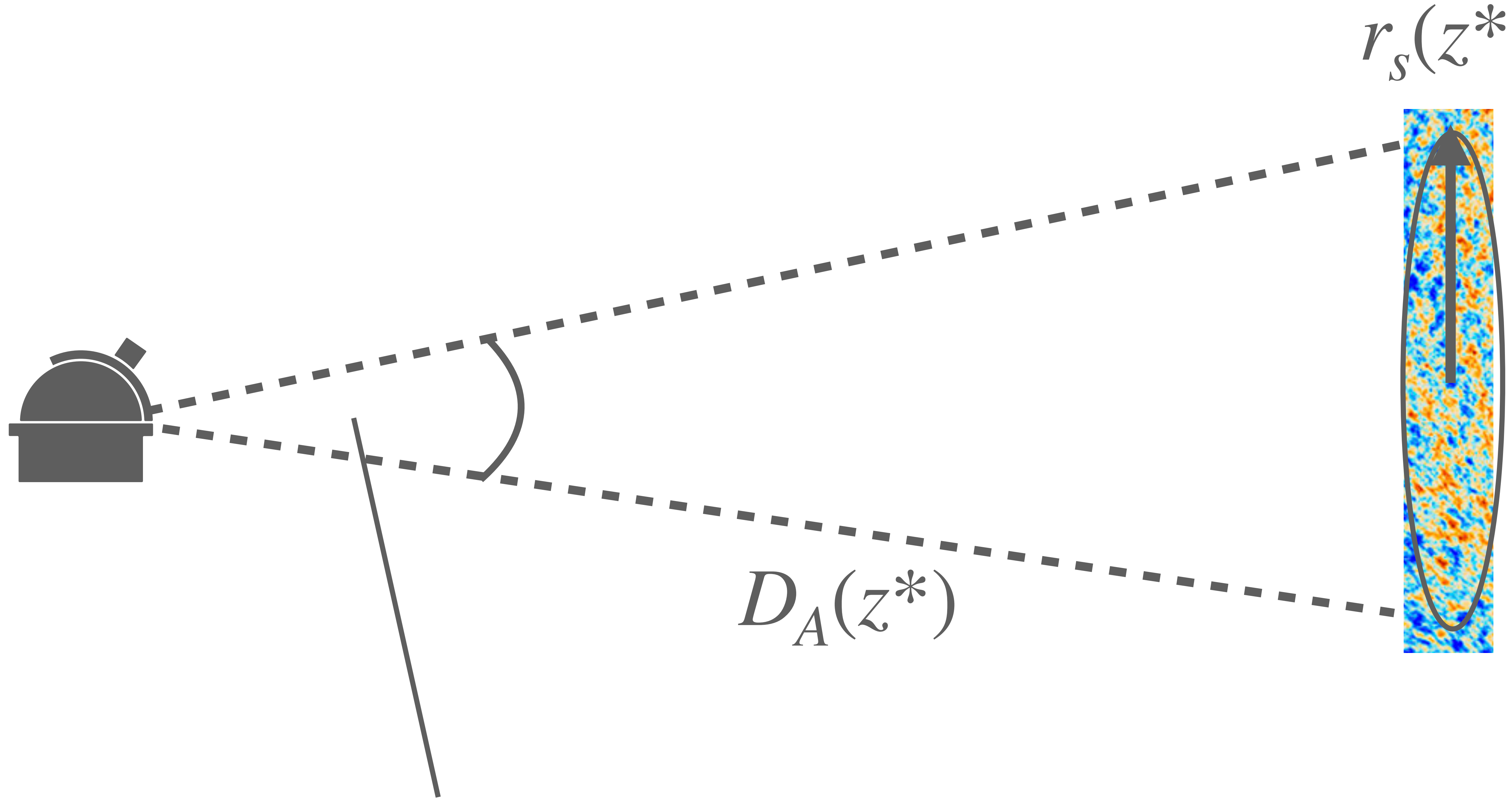
E.g. by introducing an energy density, which boosts the expansion rate before recombination z^* (at early times $\Omega_\Lambda \approx 0$):

$$H(z) = H_0\sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_?}$$

This leads to a smaller r_s.

The most successful of such ideas is EDE

Idea behind Early Dark Energy



Angular scale of sound horizon θ_s
measured with 0.03% precision by *Planck.*

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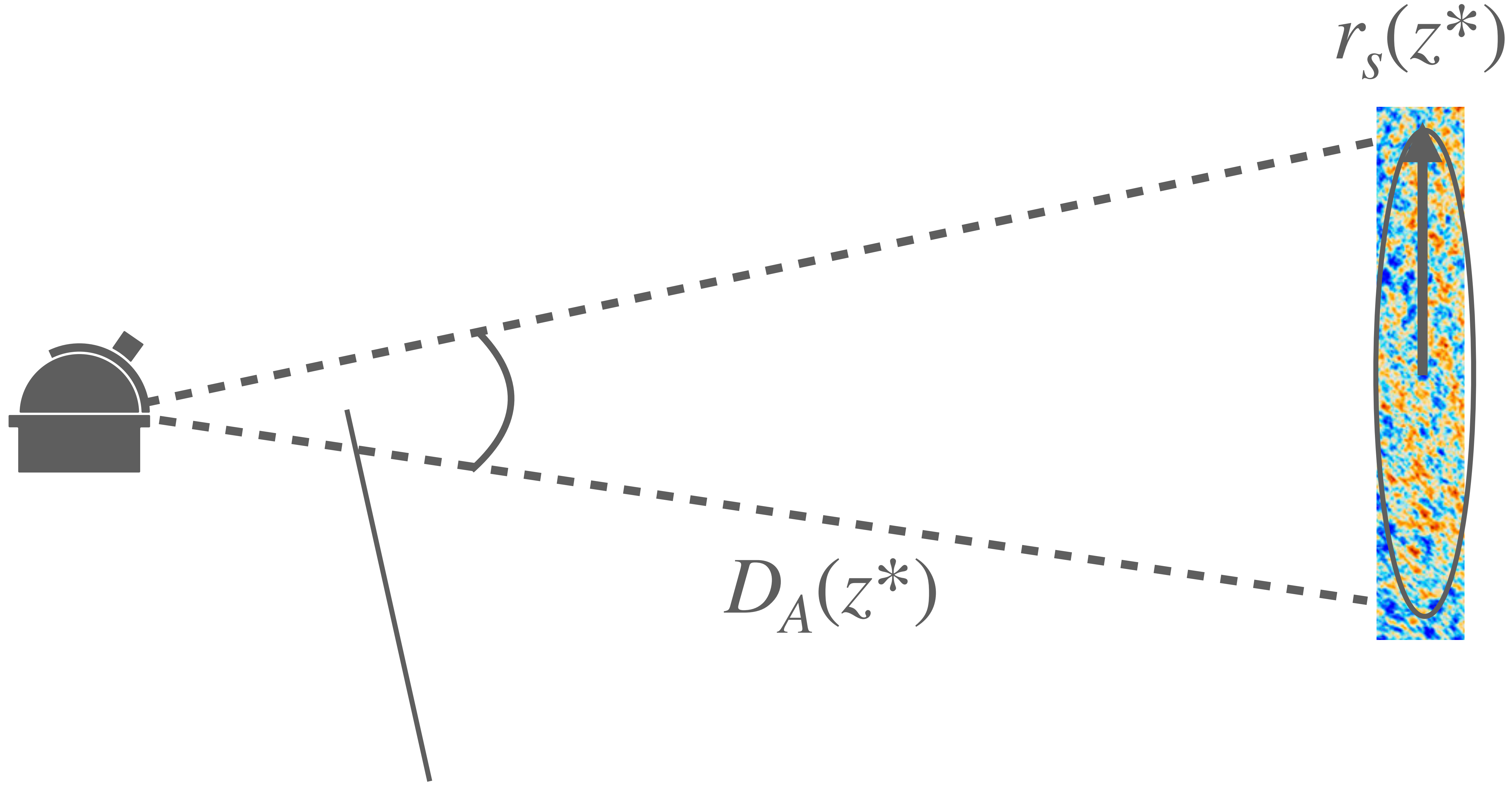

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Introducing an additional component before recombination reduces r_s.

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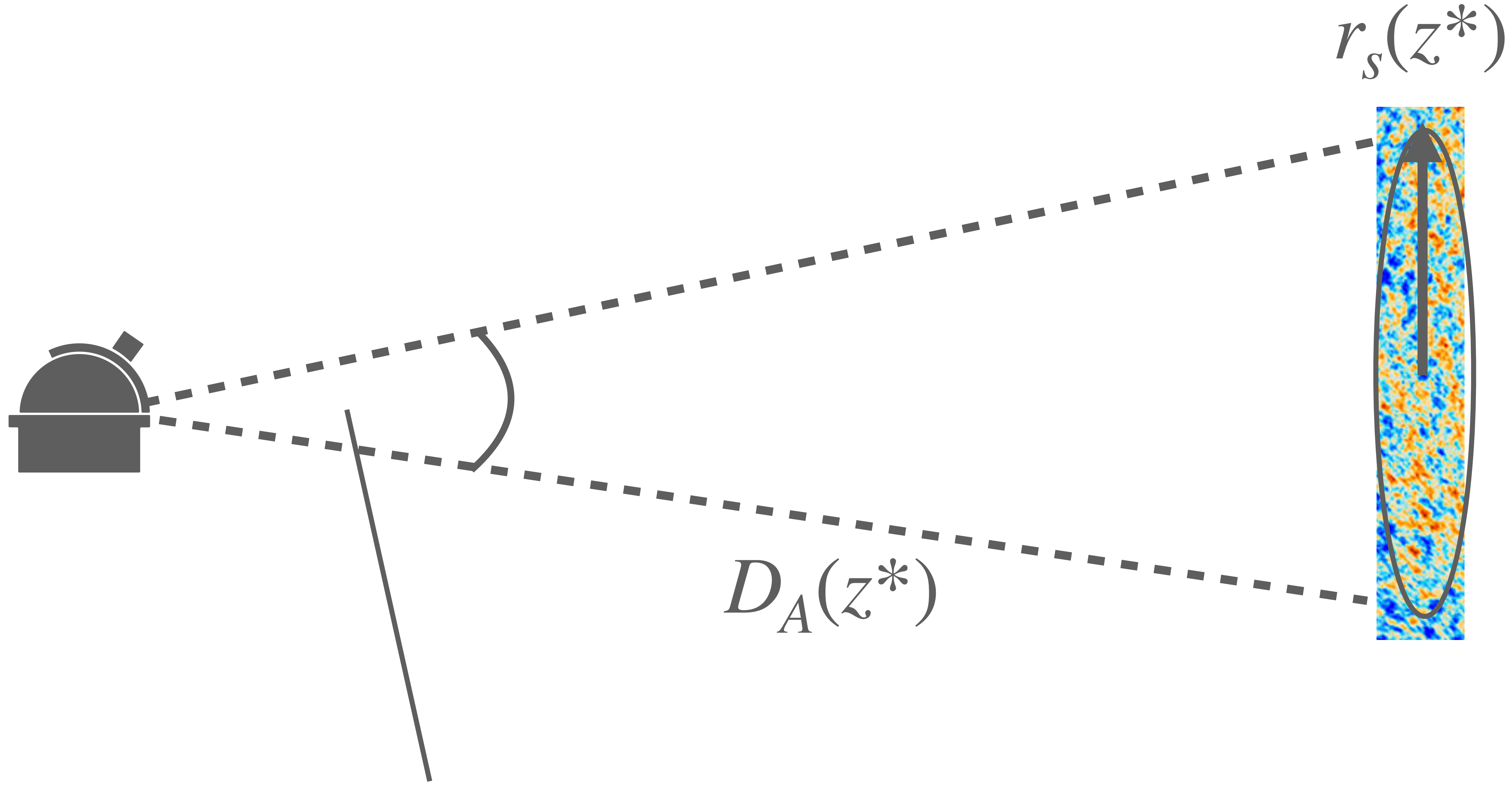

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Angular diameter distance D_A decreases.

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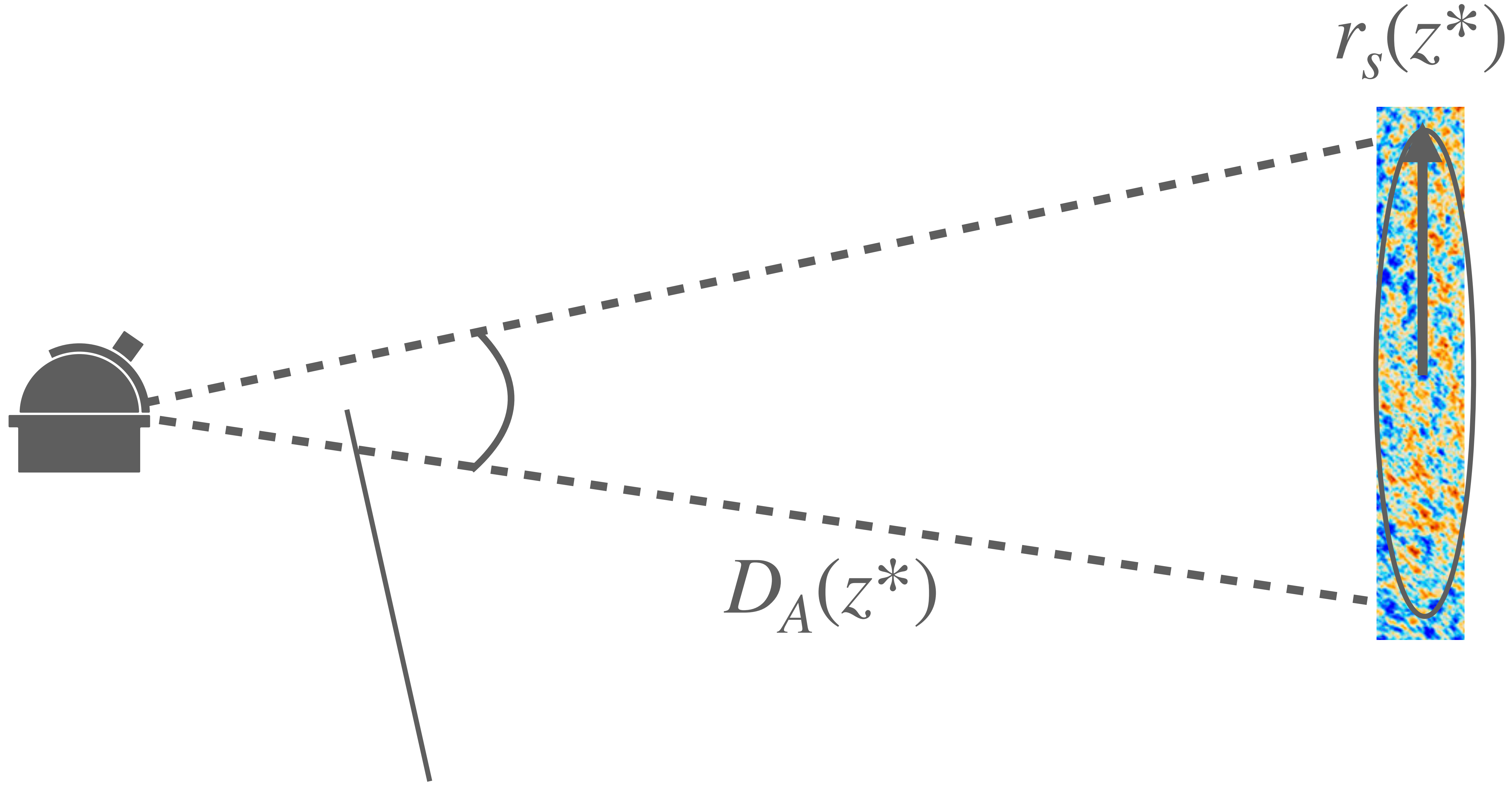

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Angular diameter distance D_A decreases.

↓ $H(z) = H_0\sqrt{\Omega_m(z) + \Omega_r(z) + \Omega_\Lambda}$

H_0 increases.

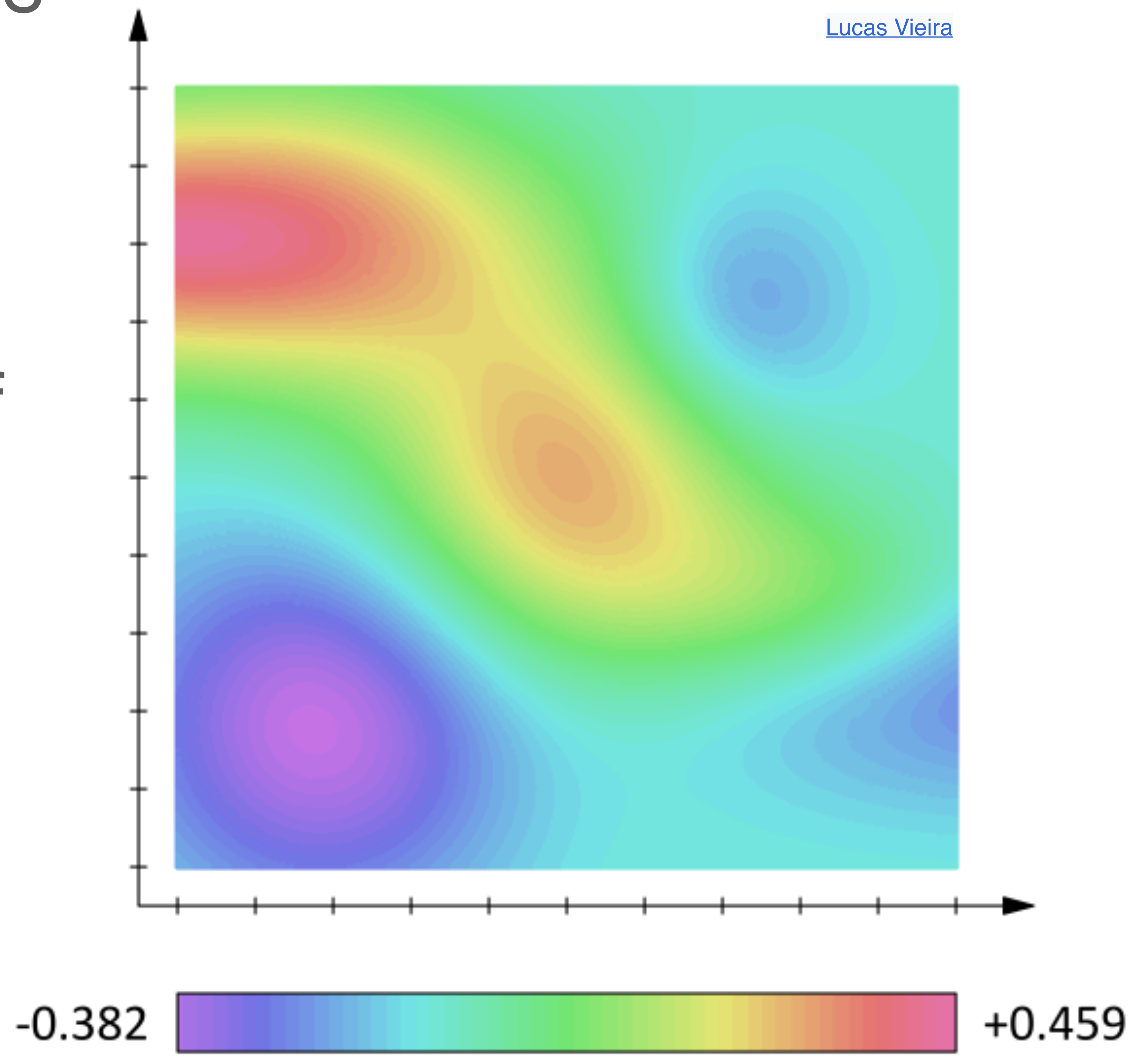
Short history of Early Dark Energy

- EDE has already been studied before the Hubble tension emerged in the context of quintessence models (Doran++ 2001, Wetterich++ 2004, Doran&Robbers 2006, Kamionkowski++ 2014)
- The axion-like EDE was proposed as a solution to the Hubble tension (Karwal&Kamionkowski 2016, Poulin++ 2018, 2019)
- There are many versions of the EDE model, but we will focus on the (most commonly studied) axion-like EDE model
- The axion-like EDE model is modelled as a scalar field
- Here: Only background equations

Scalar fields in an expanding spacetime

Intuition

- A scalar field $\phi(t, \vec{x})$ assigns each point in spacetime a single number
- Examples
  - Temperature, density, pressure,… as a function of position and time
  - Potential fields like the gravitational potential, electric potential
  - The inflaton driving inflation is most commonly a scalar field



temperature

Scalar field

Very similar to inflation: see William Giare's lecture

- The action of a scalar field minimally coupled to the metric is:

$$\mathcal{S}_\phi = \int \mathrm{d}^4x \sqrt{|\det(g)|} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right]$$

integral over whole spacetime

determinant of metric

kinetic term

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potential term

- One can compute the energy momentum tensor of the scalar field via:

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - V(\phi) \right)$$

Scalar field

- This gives for the energy-momentum tensor:

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)\right) = \dot{\phi}^2\delta^0_\mu\delta^0_\nu + \left[\frac{\dot{\phi}^2}{2} - V(\phi)\right]g_{\mu\nu}$$

Scalar field

- This gives for the energy-momentum tensor:

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- By comparing $T_{\mu\nu}$ to the energy momentum tensor of the perfect fluid: $T_{\mu\nu} = (\rho + p)\delta^0_\mu \delta^0_\nu + p g_{\mu\nu}$, one can read off the energy density and pressure of the scalar field:

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)\,.$$

Scalar field

- Inserting $\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$ into the first Friedmann equation and the continuity equation yields:

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$$\boxed{\begin{gathered} H^2 = \frac{8\pi G_\mathrm{N}}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right) \\ \ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0 \end{gathered}}$$

"Hubble friction"

Potential term

Scalar field

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- The second equation is the Klein-Gordon equation for a scalar field in an expanding space, where the second term $(3H\dot\phi)$ is the Hubble-drag term and the third term $(\frac{1}{2}\mathrm{d}V/\mathrm{d}\phi)$ is the potential-gradient term

Early Dark Energy

Early Dark Energy

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$$V(\phi) = V_0 \left[1 - \cos(\phi/f)\right]^n$$

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normalization of the potential: $V_0 = m^2 f^2$

m: mass

f: decay constant

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index n:

$n = 1$ "standard" axion; however: doesn't decay quickly enough

$n = 3$: is preferred value by the data (Poulin++ 2020) $\rightarrow$ decays fast enough (commonly fixed in EDE data analysis)

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Free parameters of the model:

- m mass ($V_0 = m^2 f^2$)
- f "decay constant"
- $\theta_i = \phi_i/f$ initial value of the field
- ($n = 3$)

Early Dark Energy

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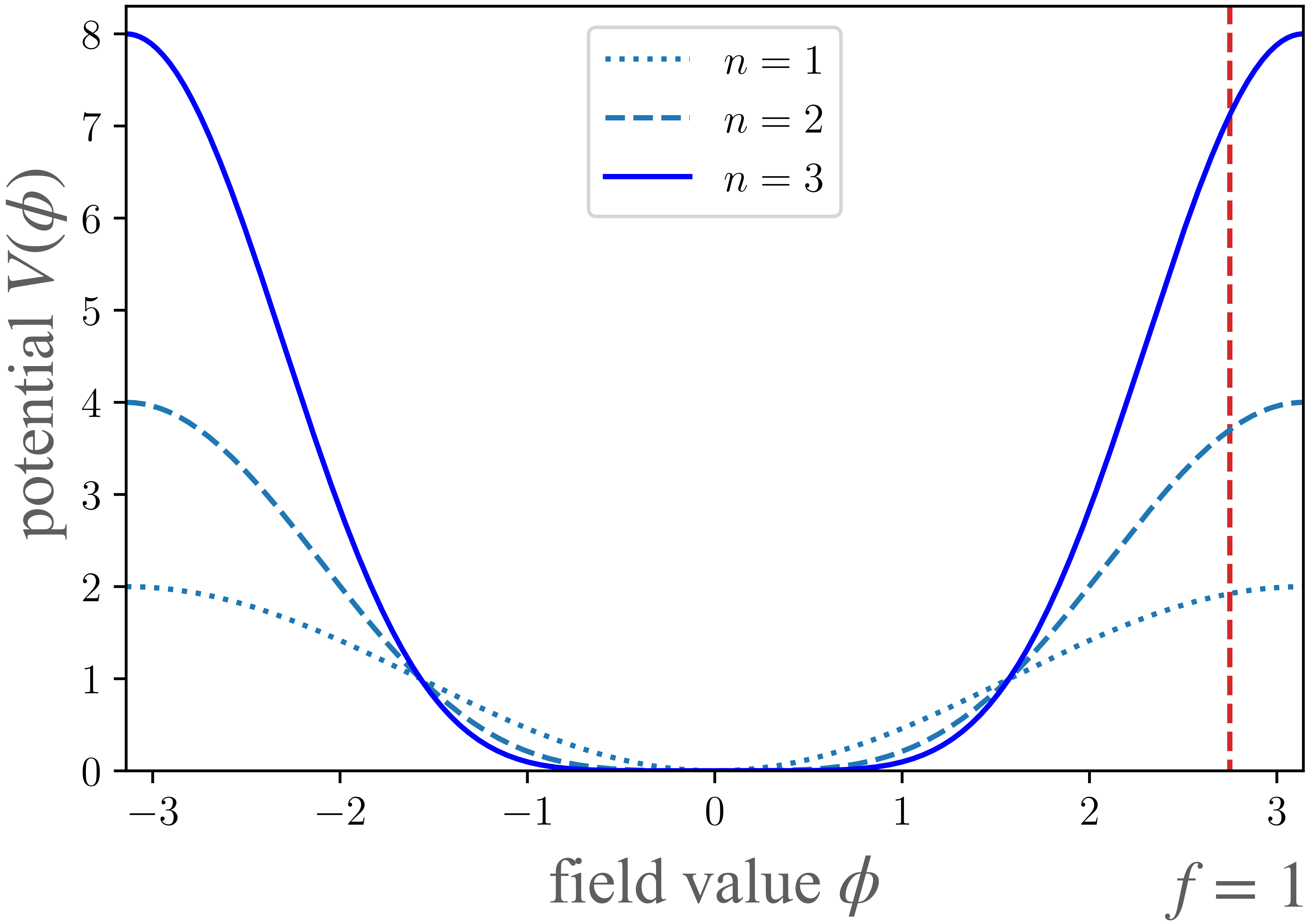

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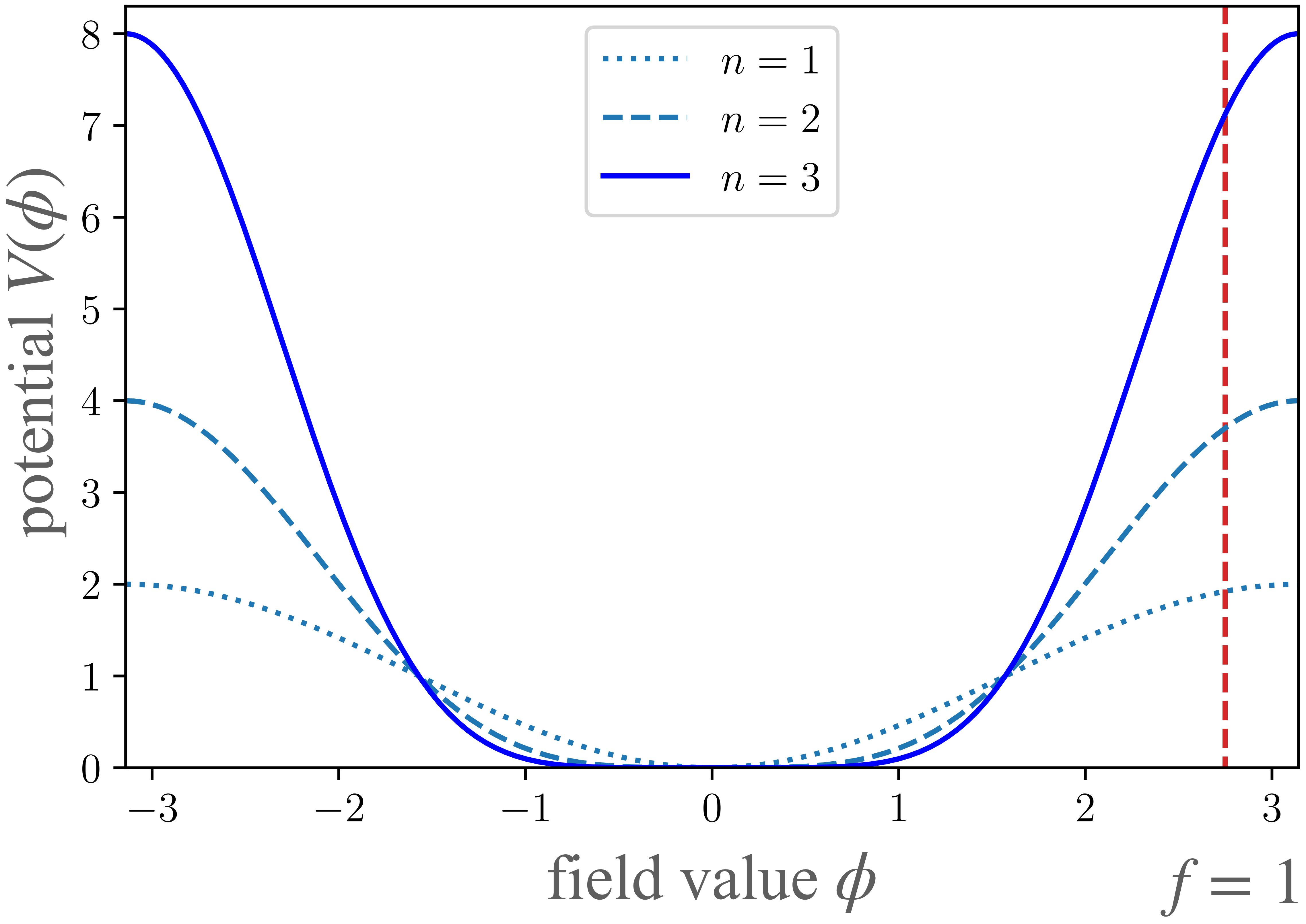

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Disclaimer: the following will include many "hand-wavy" approximations.

Analytical computations are too complicated otherwise and numerics is needed.

Early Dark Energy

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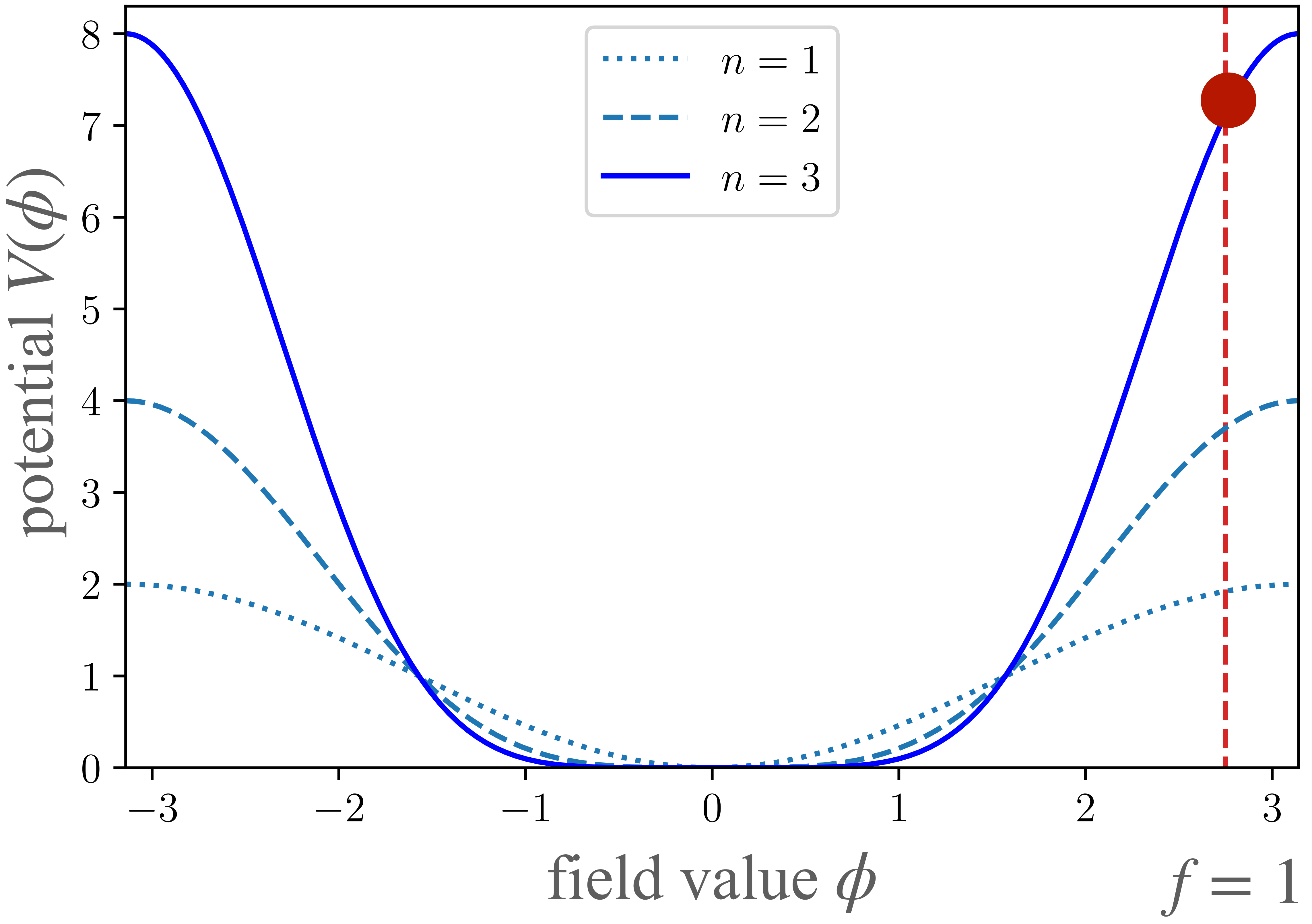

At early times:

- Dynamics governed by Klein-Gordon equation:

$$\ddot{\phi} + 3H\dot{\phi} + \cancel{\frac{\mathrm{d}V}{\mathrm{d}\phi}} = 0$$

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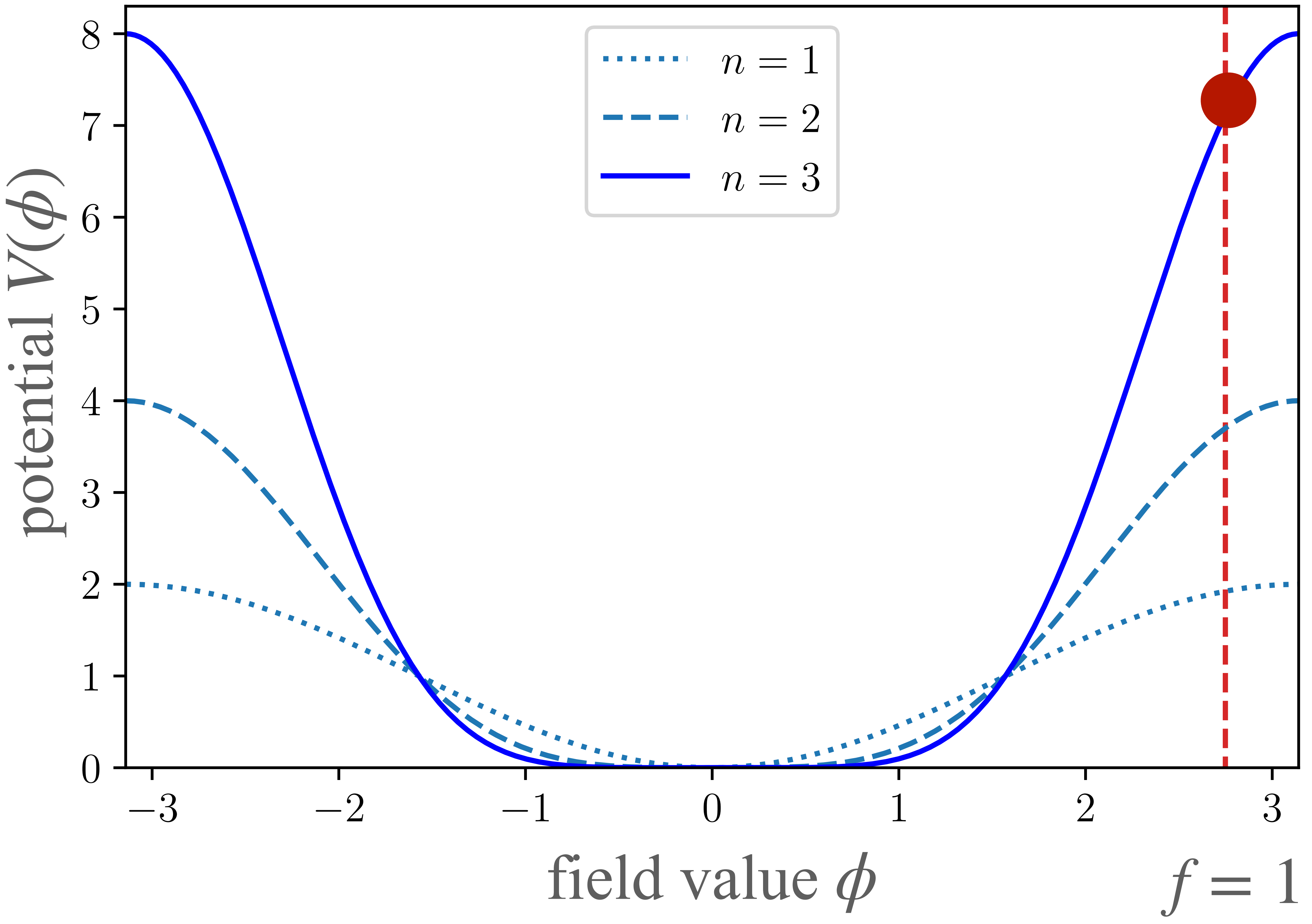

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$$\frac{\mathrm{d}V}{\mathrm{d}\phi} \approx 0$$

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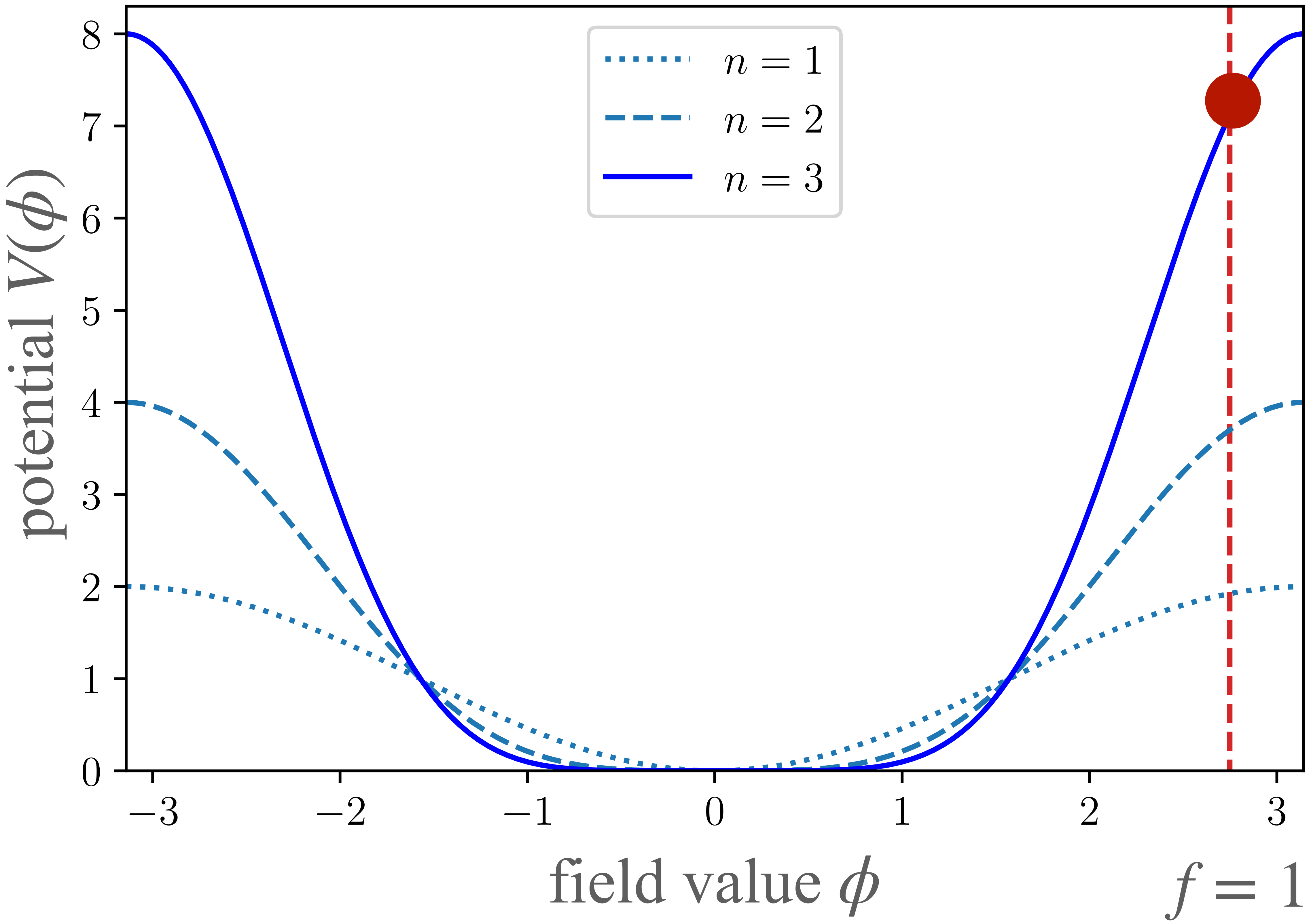

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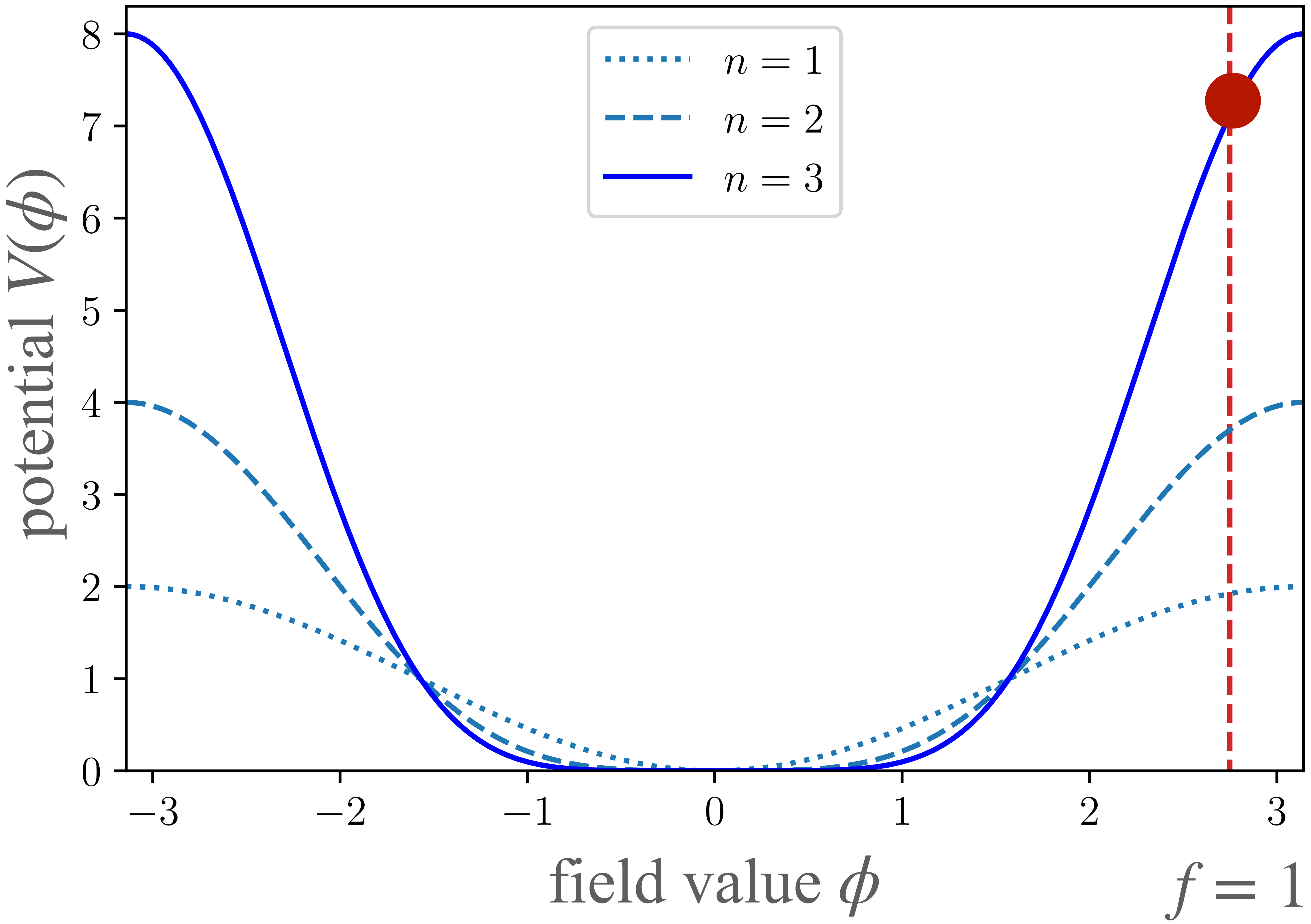
$$\frac{\mathrm{d}V}{\mathrm{d}\phi} \approx 0$$

- Moreover, at early times the expansion rate H is large; hence "Hubble friction" dominates:

$$3H\dot{\phi} \gg \frac{\mathrm{d}V}{\mathrm{d}\phi}$$

Early Dark Energy

$$V(\phi) = V_0 \, [1 - \cos(\phi/f)]^n$$



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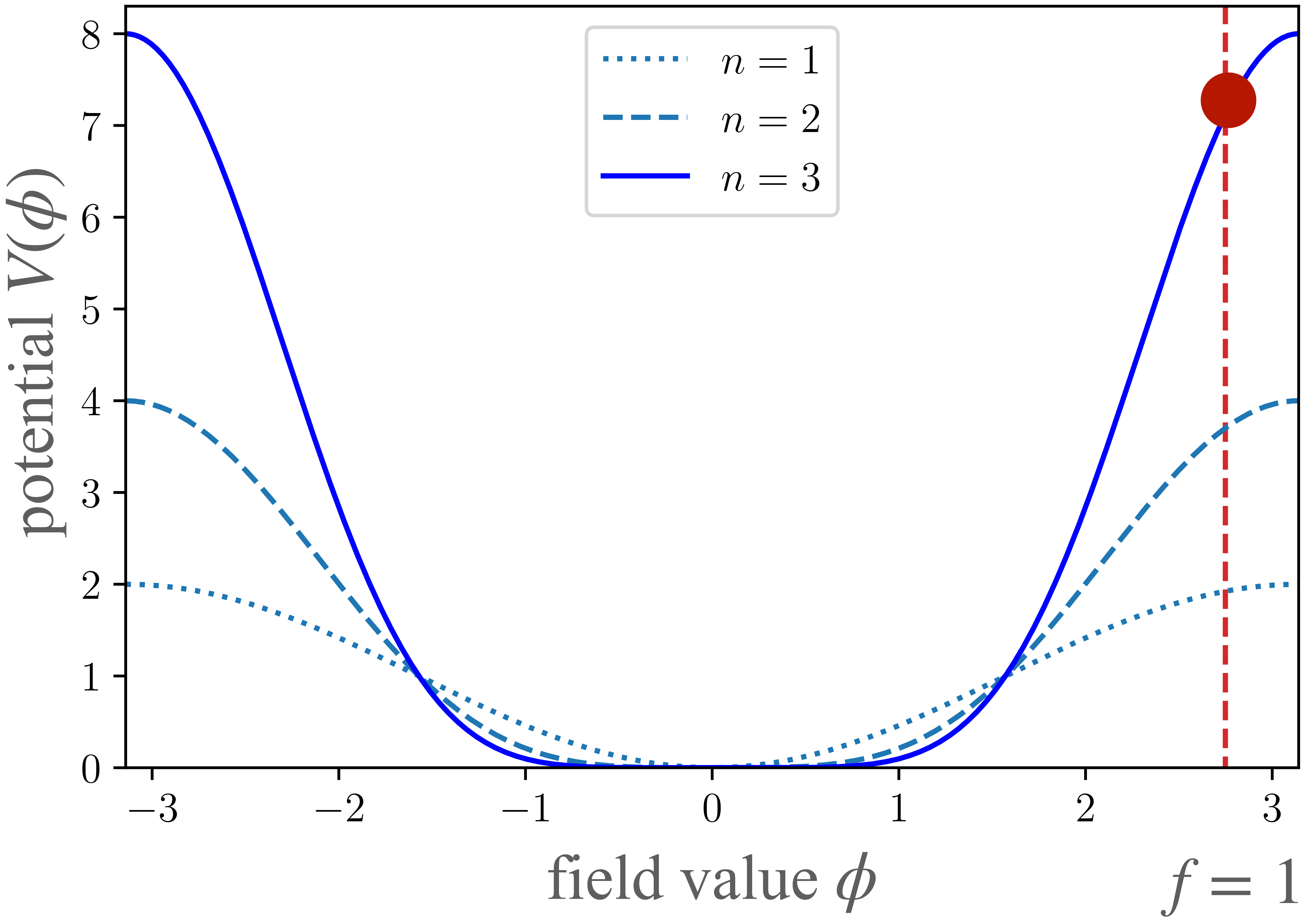

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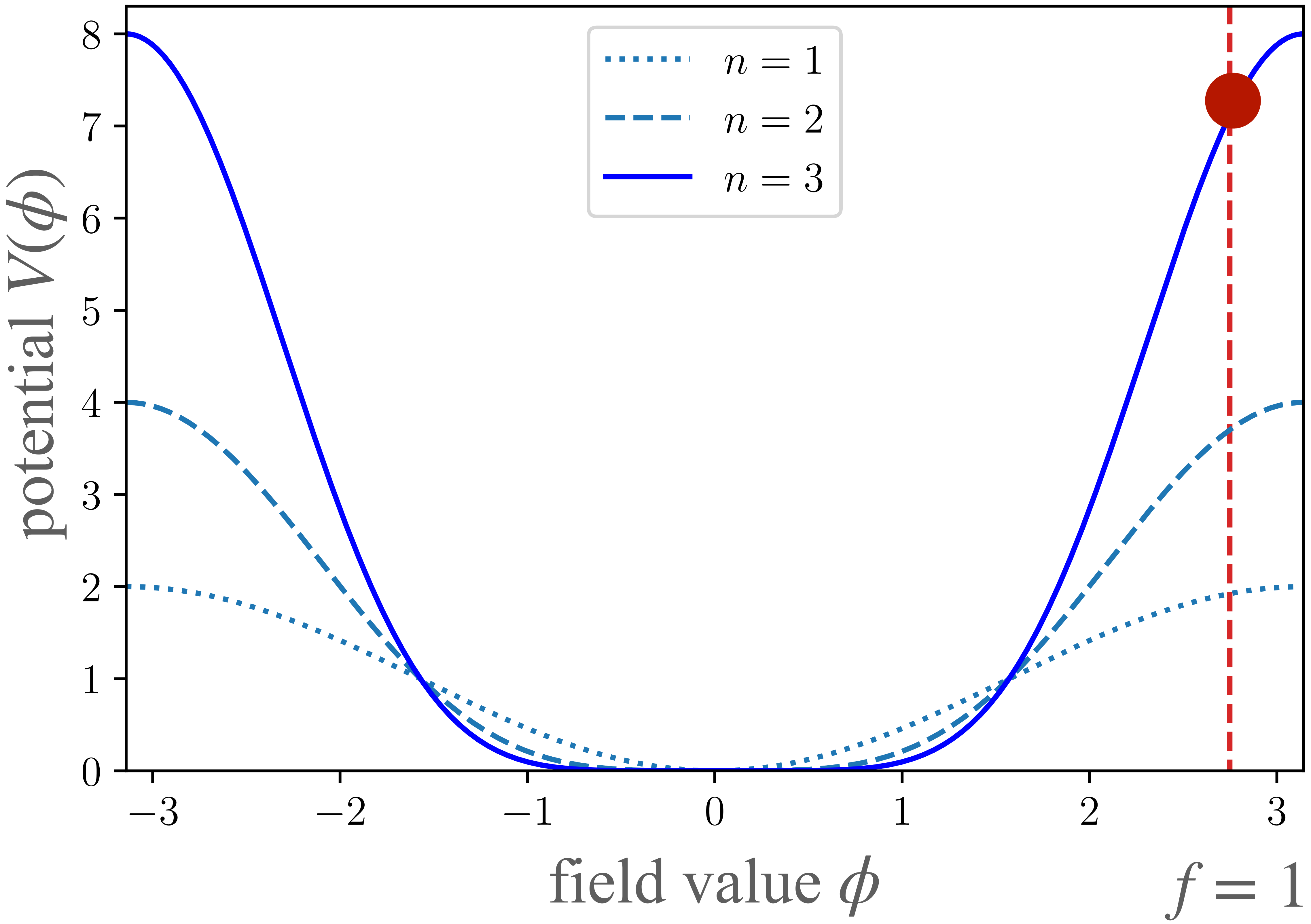

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- Then $\dot{\phi} = \psi \sim \mathrm{e}^{-3Ht}$

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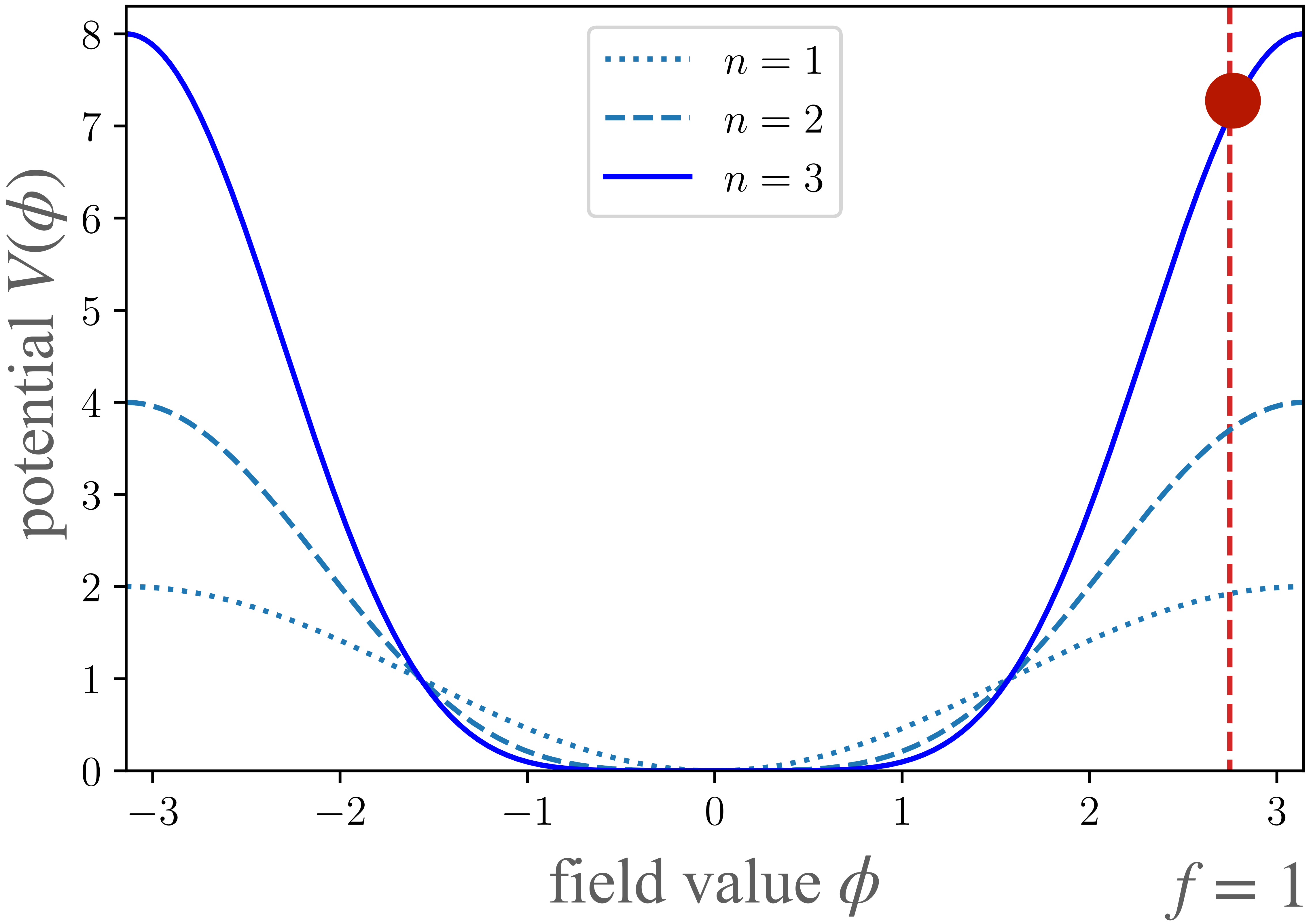

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H large at early times

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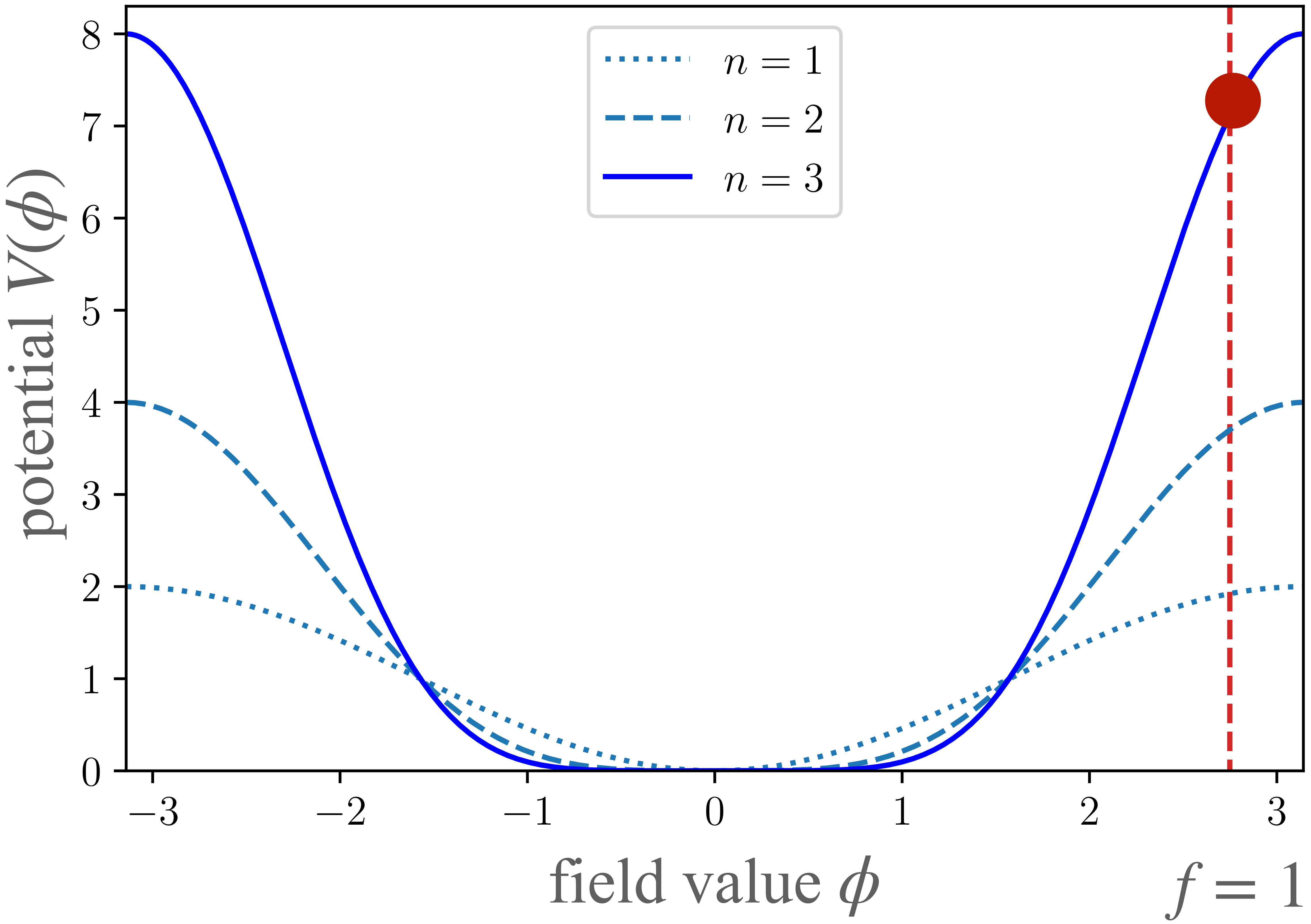

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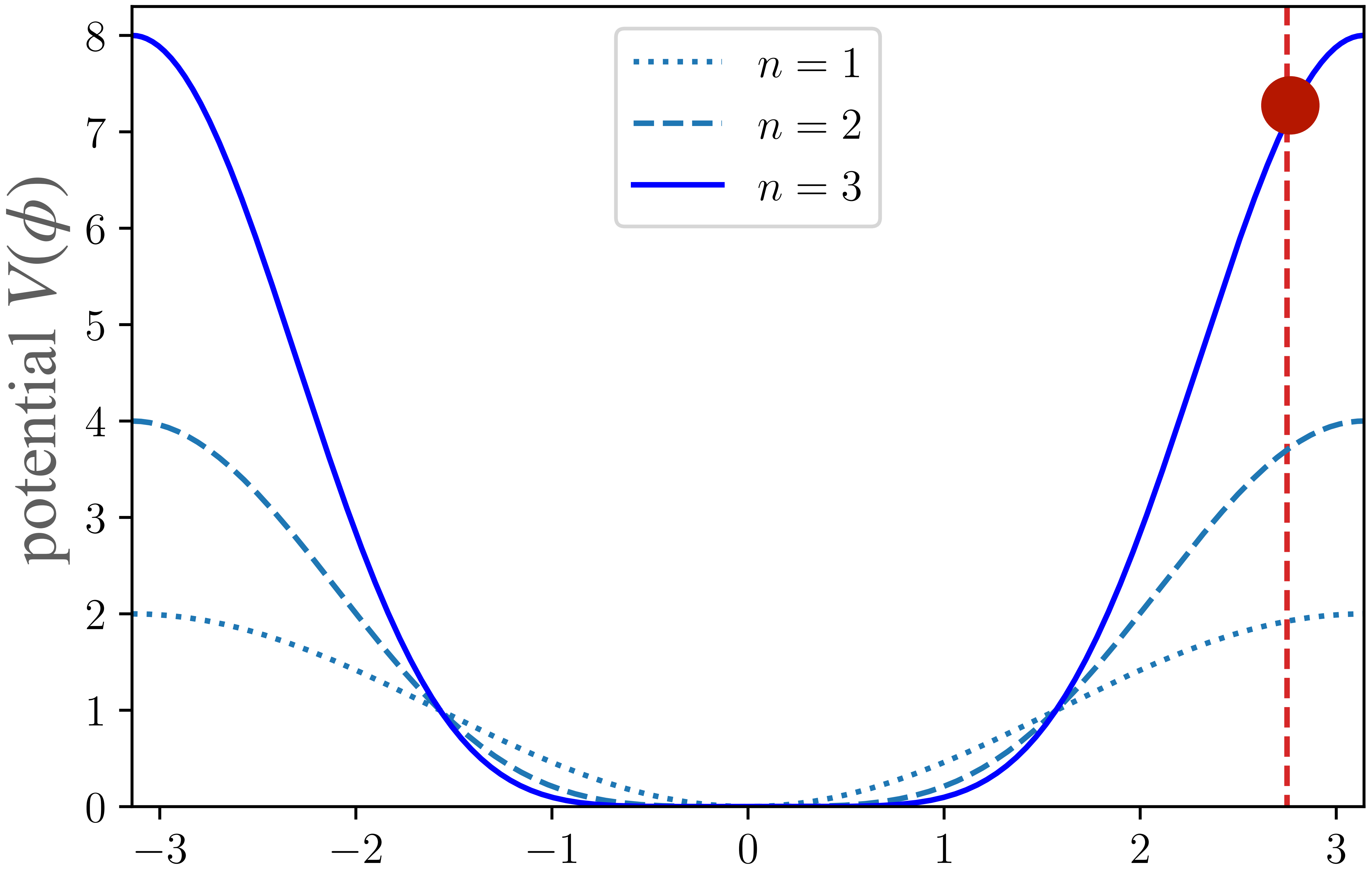

At early times:

- At early times: $\dot{\phi} \approx 0$
- This means for the energy density& pressure

$$\rho_\phi = \cancel{\frac{\dot{\phi}^2}{2}} + V(\phi), \quad p_\phi = \cancel{\frac{\dot{\phi}^2}{2}} - V(\phi)\,.$$

Early Dark Energy

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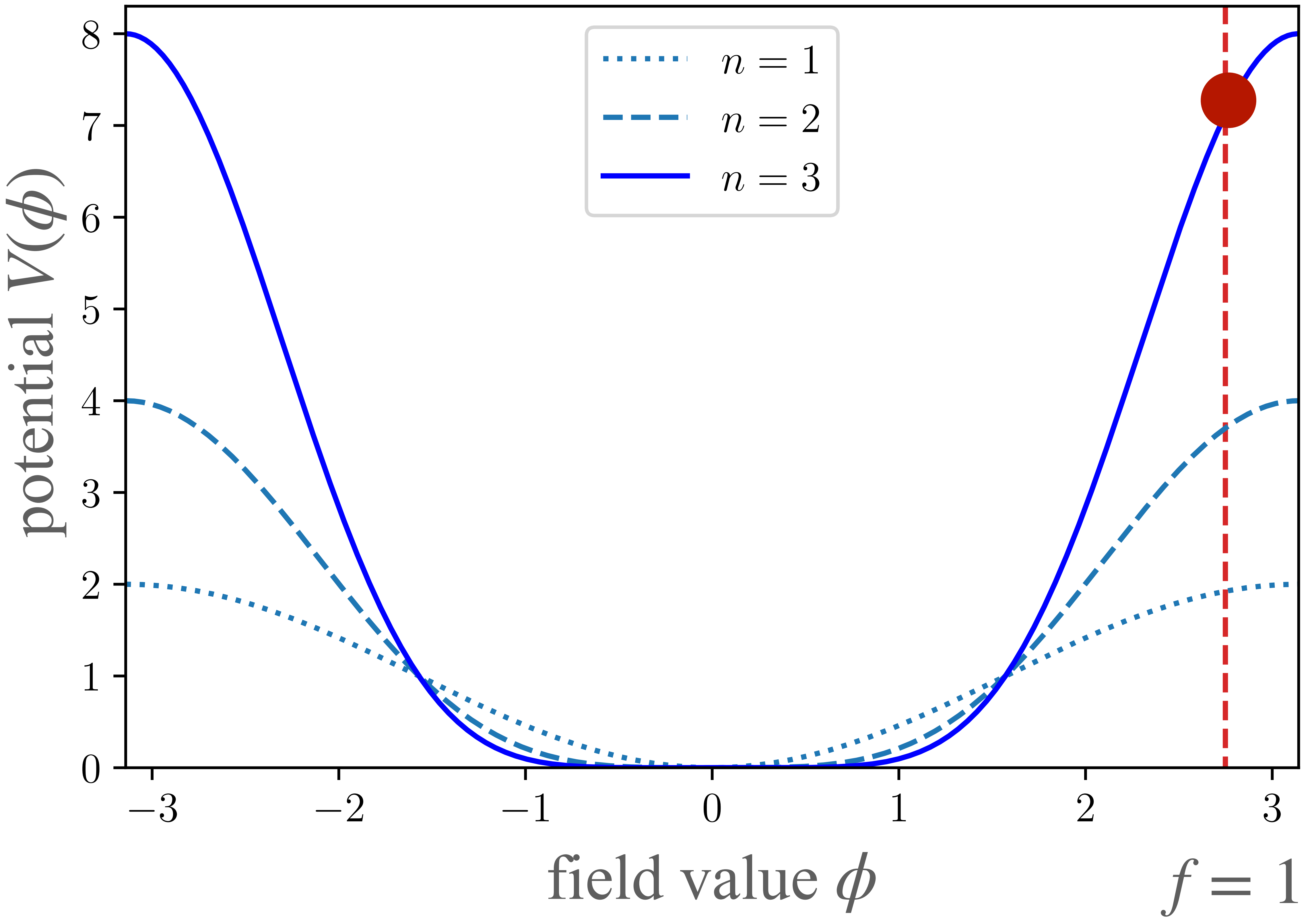

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$$\rho_\phi = \cancel{\frac{\dot{\phi}^2}{2}} + V(\phi), \quad p_\phi = \cancel{\frac{\dot{\phi}^2}{2}} - V(\phi)\,.$$

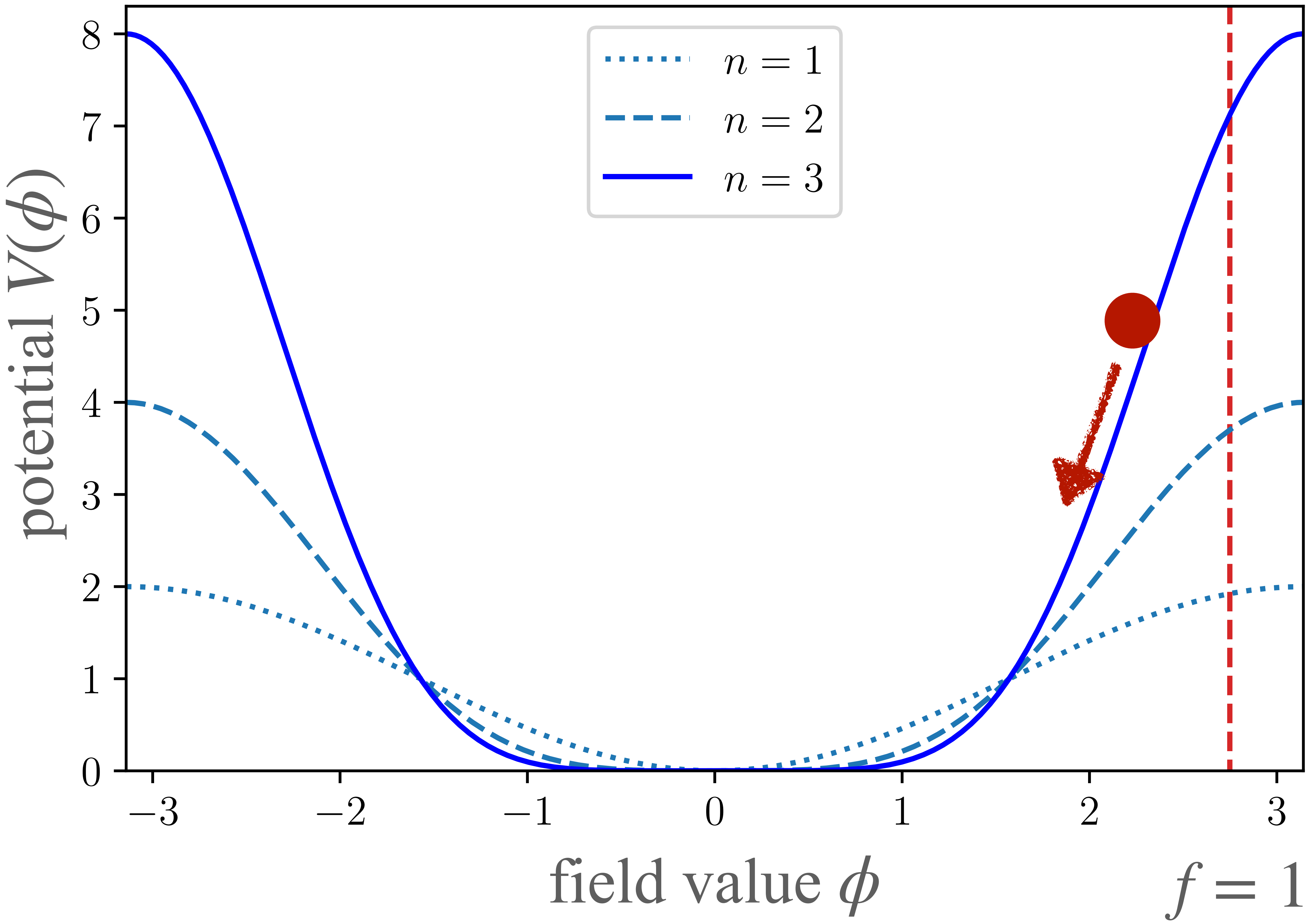
- The equation of state becomes:

$$w = \frac{p_\phi}{\rho_\phi} \approx -1$$

- Hence, at early times the field behaves like dark energy → “EDE”

Early Dark Energy

$$V(\phi) = V_0 \,[1 - \cos(\phi/f)]^n$$



$f = 1$

At the critical redshift:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0$$

- As time passes, $H(z)$ decreases, until at some critical redshift z_c:

$$3H\dot{\phi} \approx \frac{\mathrm{d}V}{\mathrm{d}\phi}$$

Early Dark Energy

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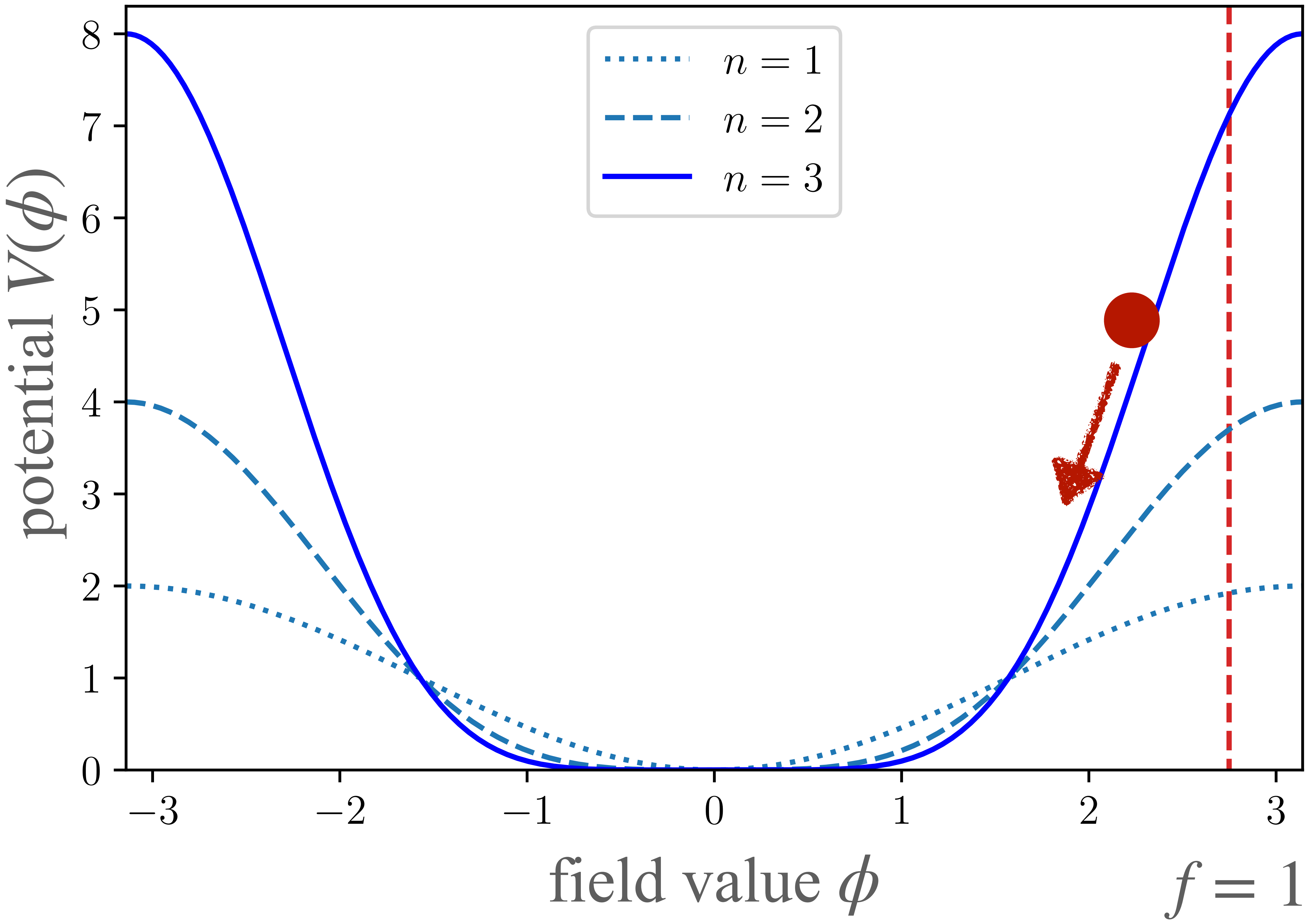

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$$3H\dot{\phi} \approx \frac{\mathrm{d}V}{\mathrm{d}\phi}$$

- Let's approximate:

- $V(\phi) = m^2 f^2 (1 - \cos(\phi/f))^n \approx$

For $n = 1$ and small ϕ: $1 - \cos(\phi/f) \approx \dfrac{\phi^2}{2f^2}$

Early Dark Energy

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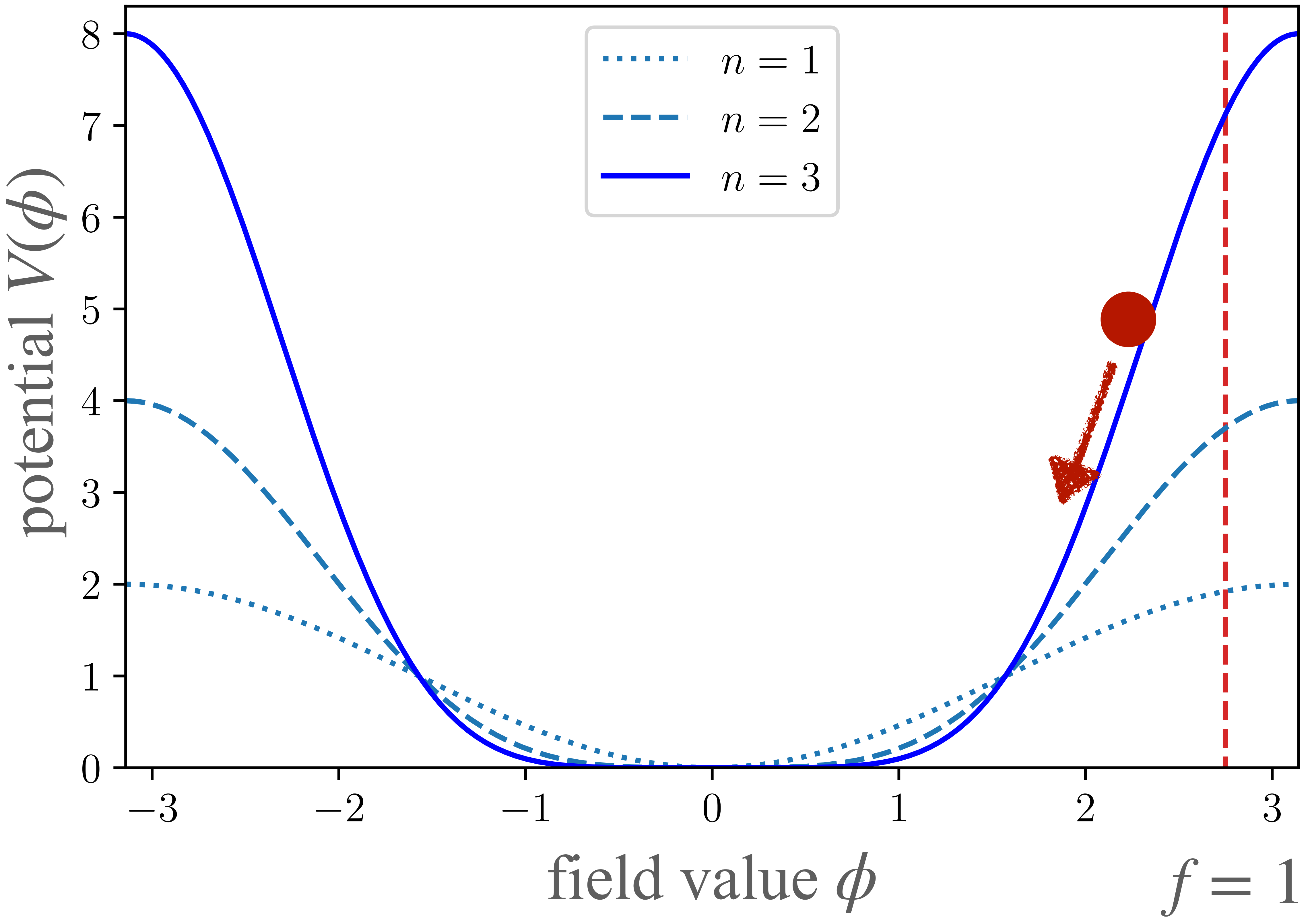

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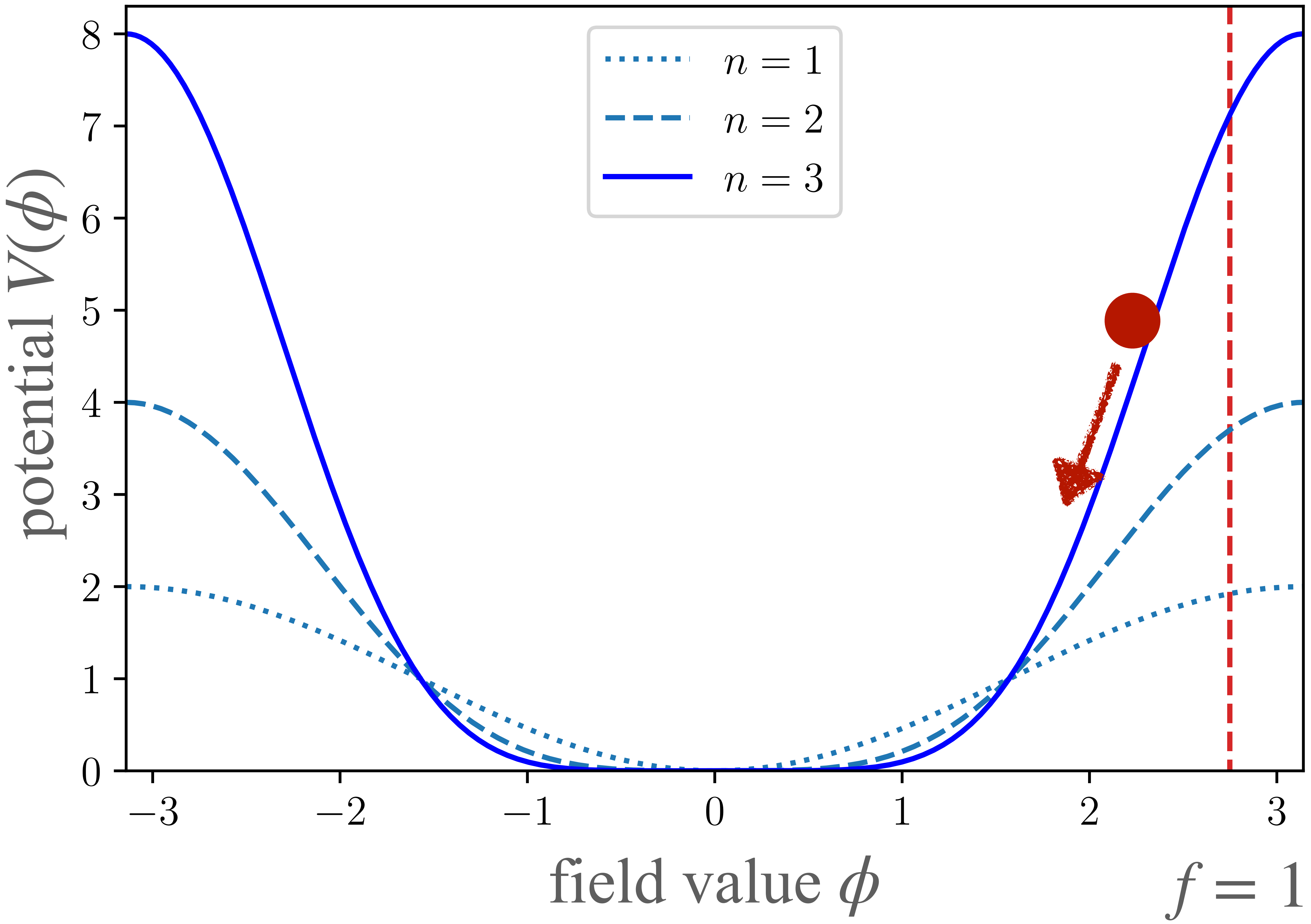

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Early Dark Energy

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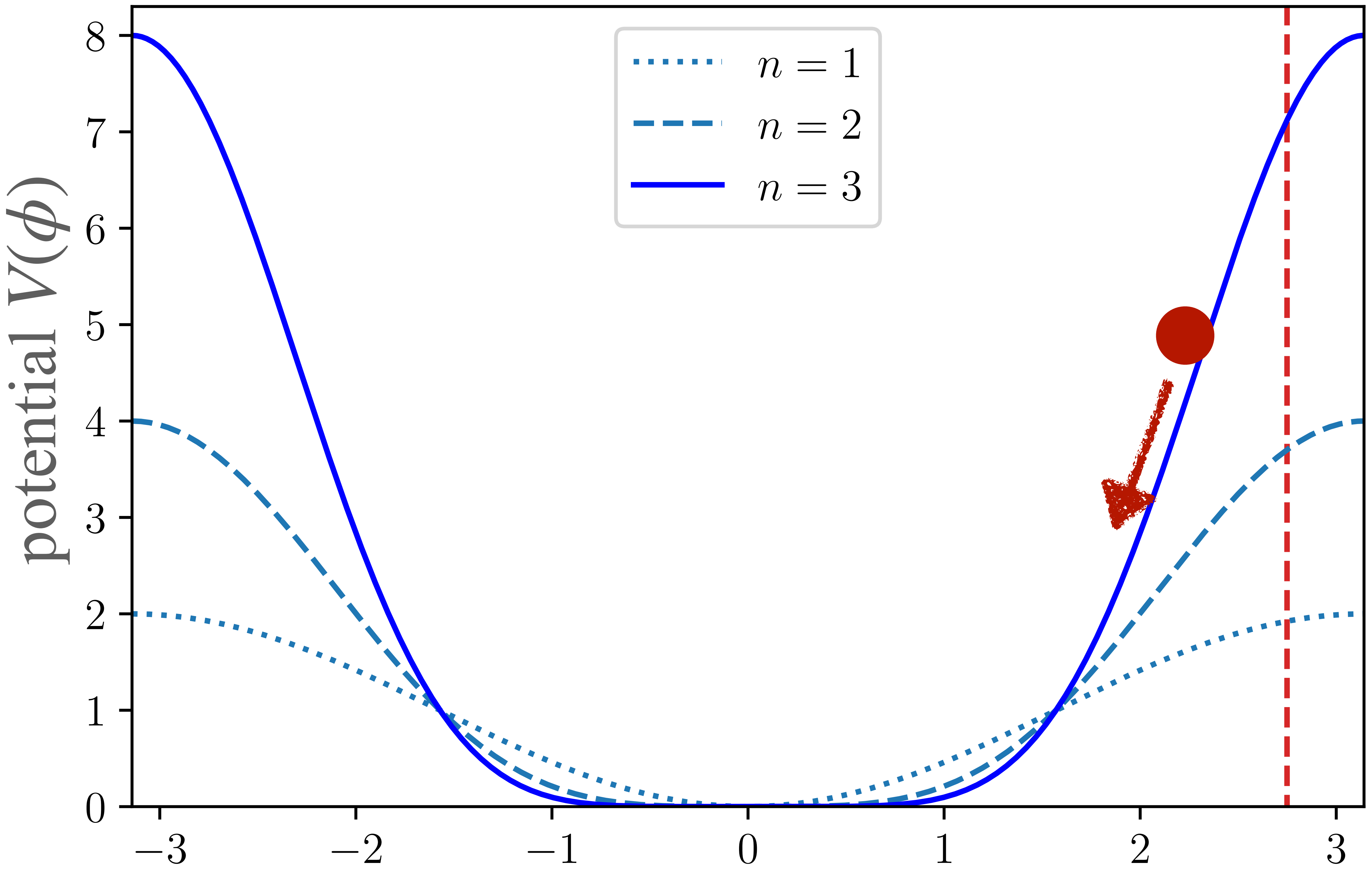

$f = 1$

At the critical redshift:

$$3H\dot{\phi} \approx \frac{\mathrm{d}V}{\mathrm{d}\phi}$$

- $V(\phi) \approx \dfrac{m^2}{2}\phi^2 \;\rightarrow\; \dfrac{\mathrm{d}V}{\mathrm{d}\phi} \approx m^2\phi$:

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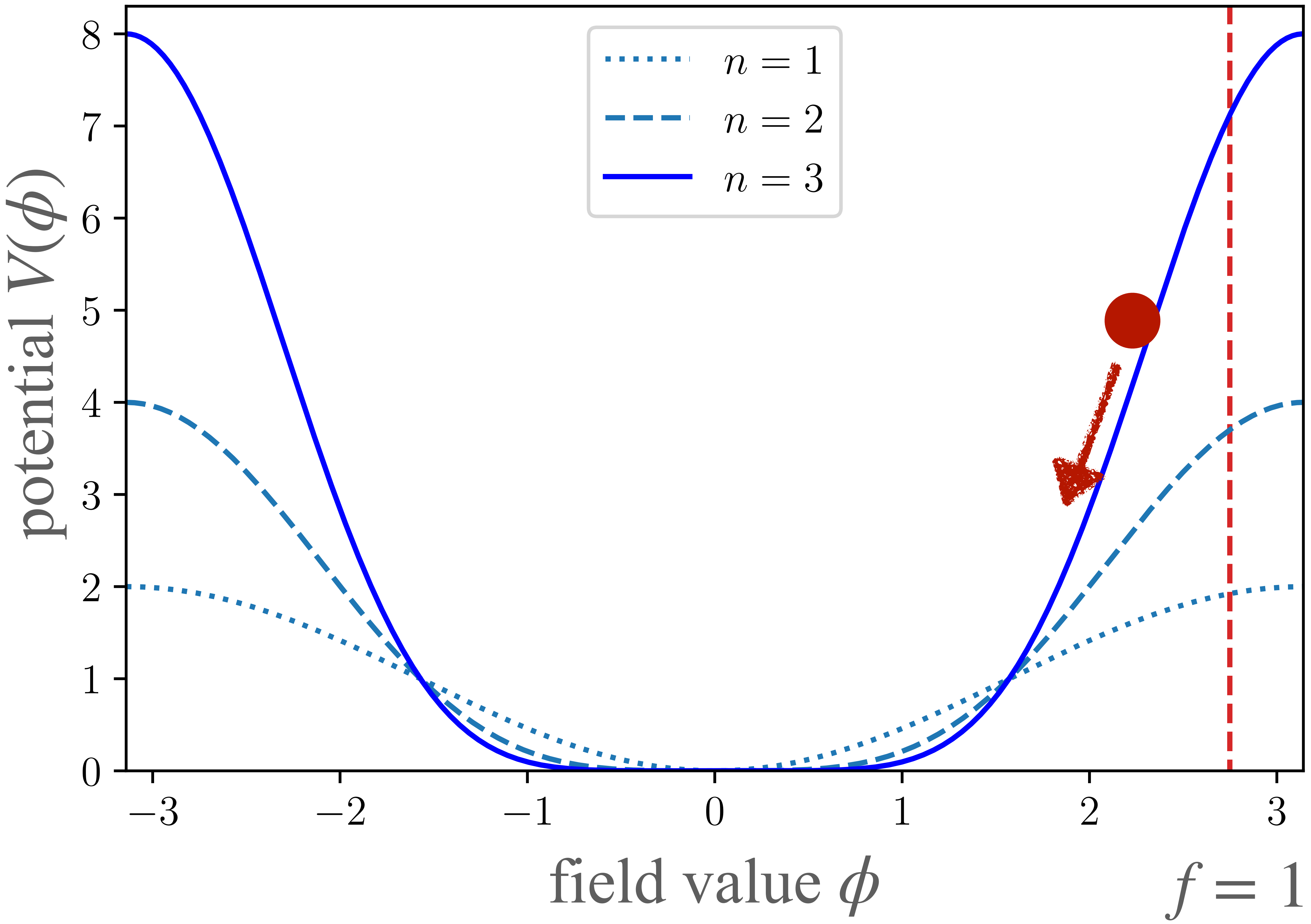

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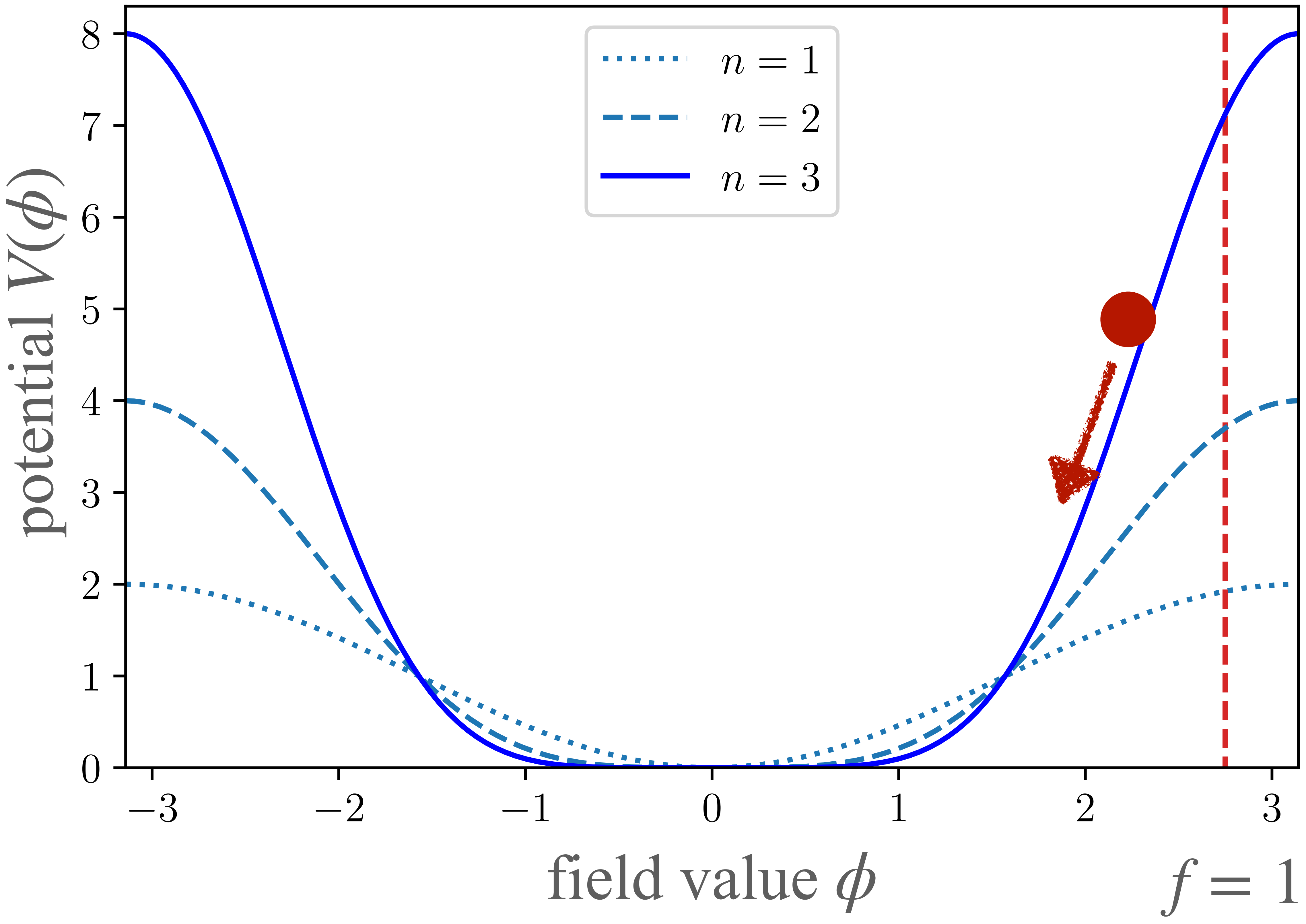

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- Using this, one can determine the "mass" of the EDE field:

$$m \sim 10^{-28}\ \mathrm{eV}$$

Early Dark Energy

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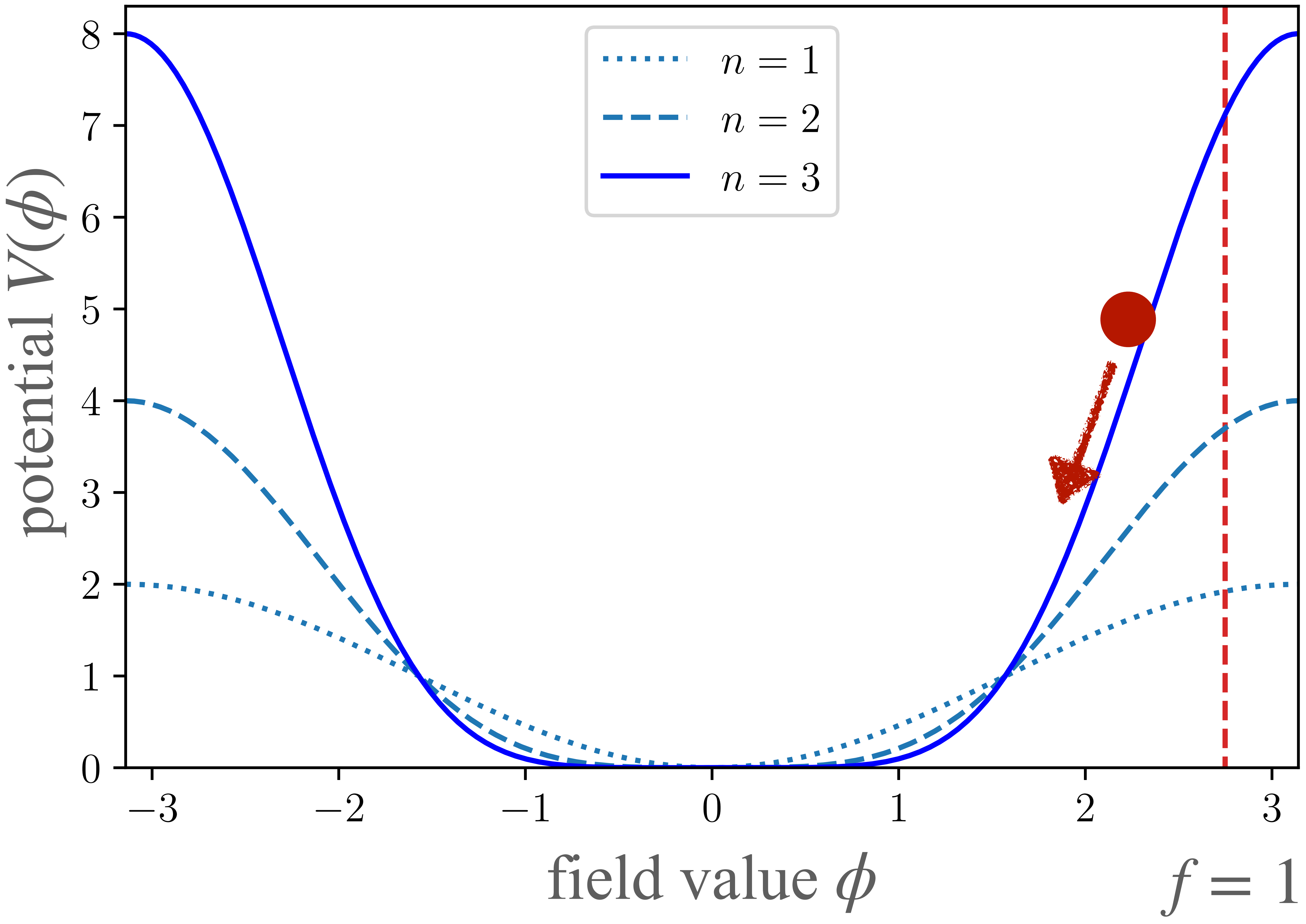

At times after z_c:

$$3H\dot{\phi} \ll \frac{\mathrm{d}V}{\mathrm{d}\phi}$$

- Hence the Klein-Gordon eq. is:

$$\ddot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} \approx 0$$

Early Dark Energy

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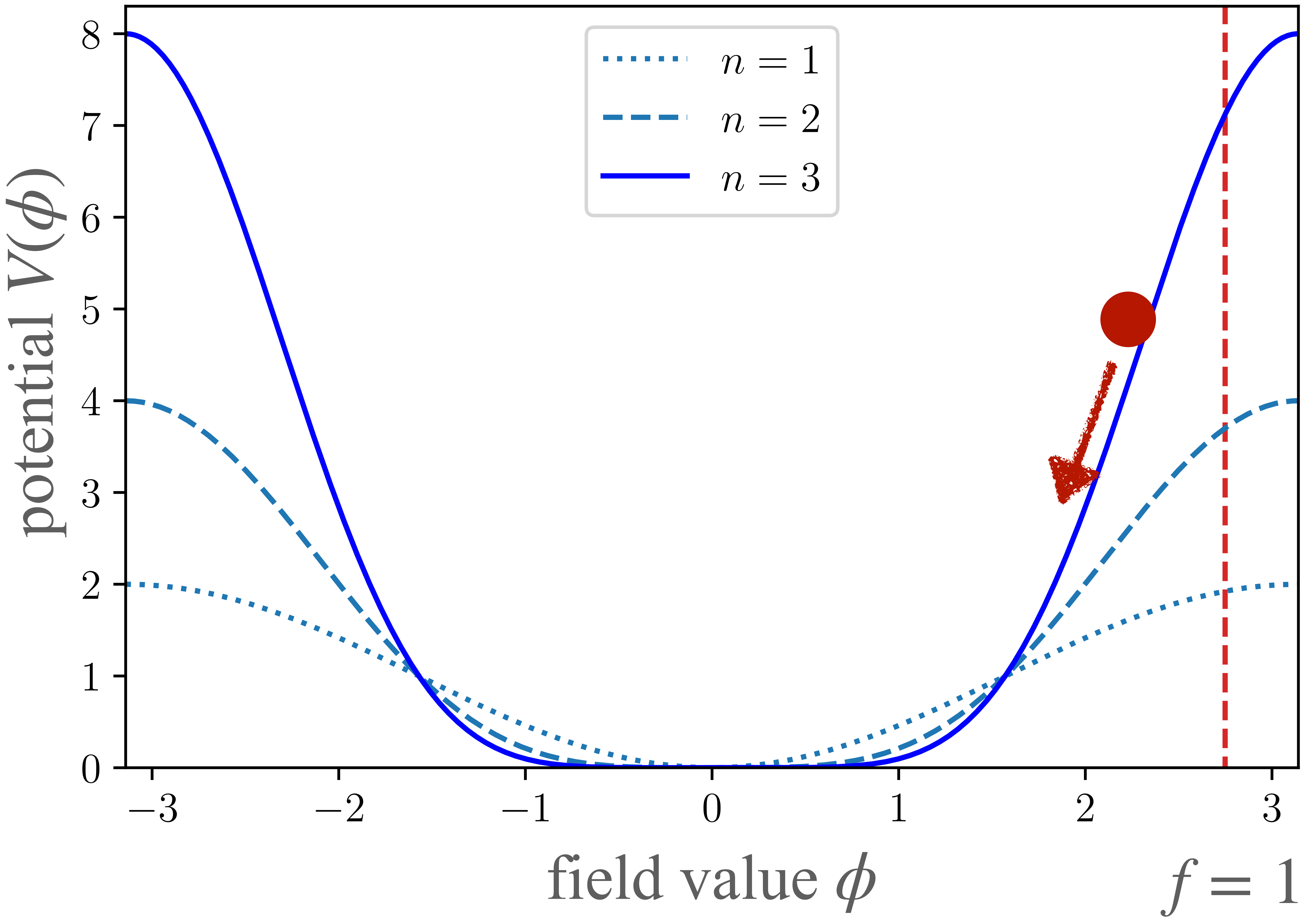

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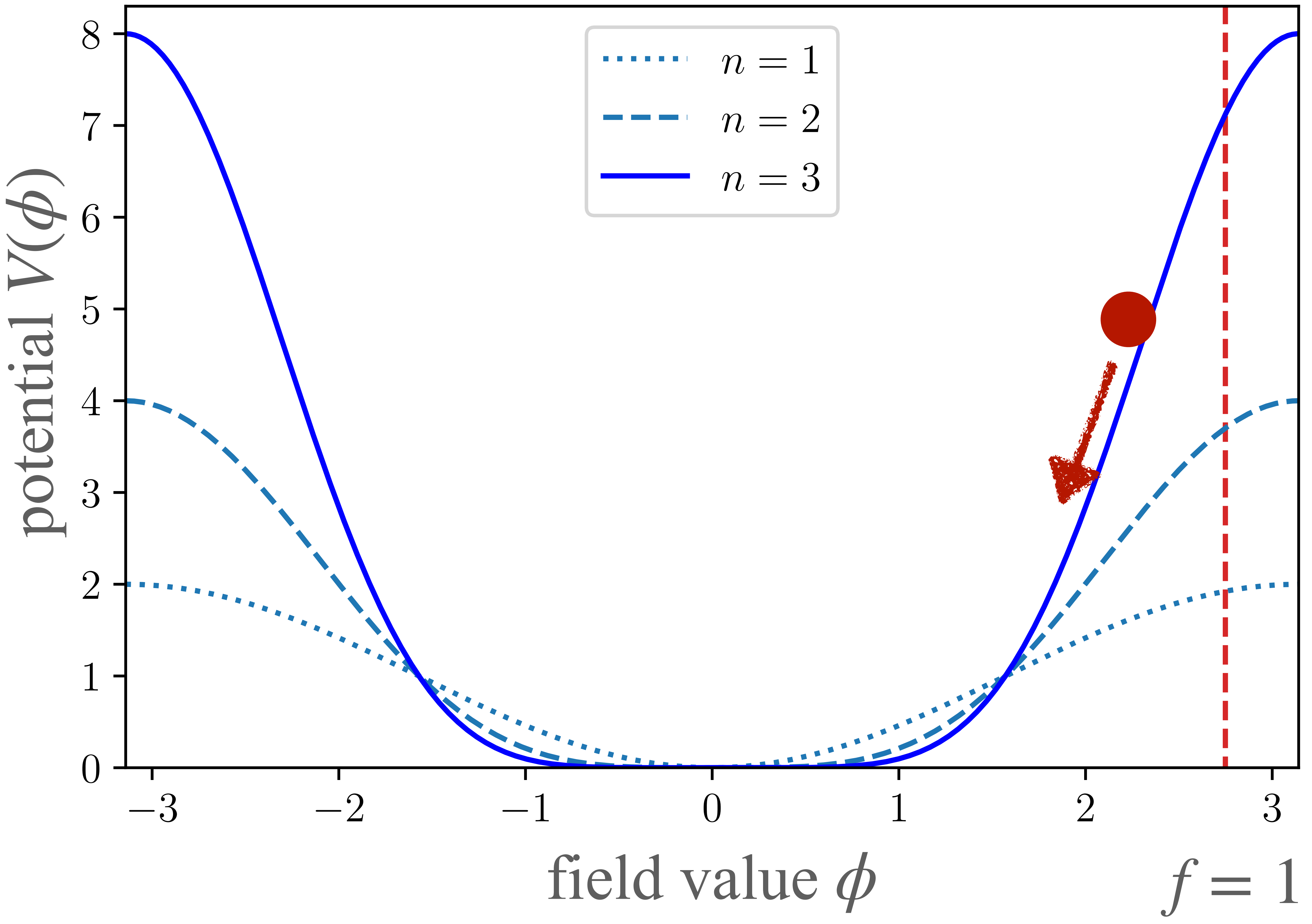

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Early Dark Energy

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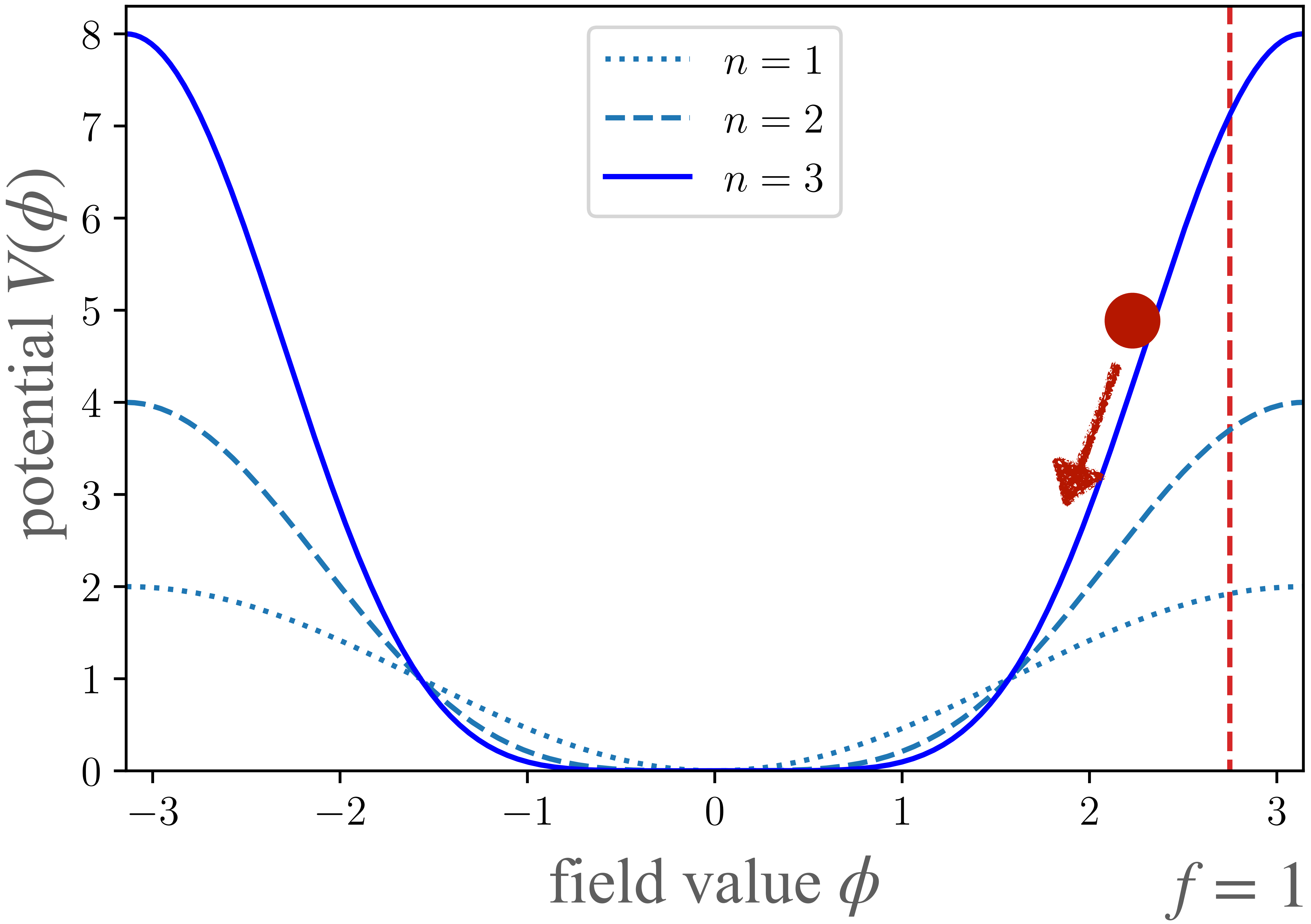

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Early Dark Energy

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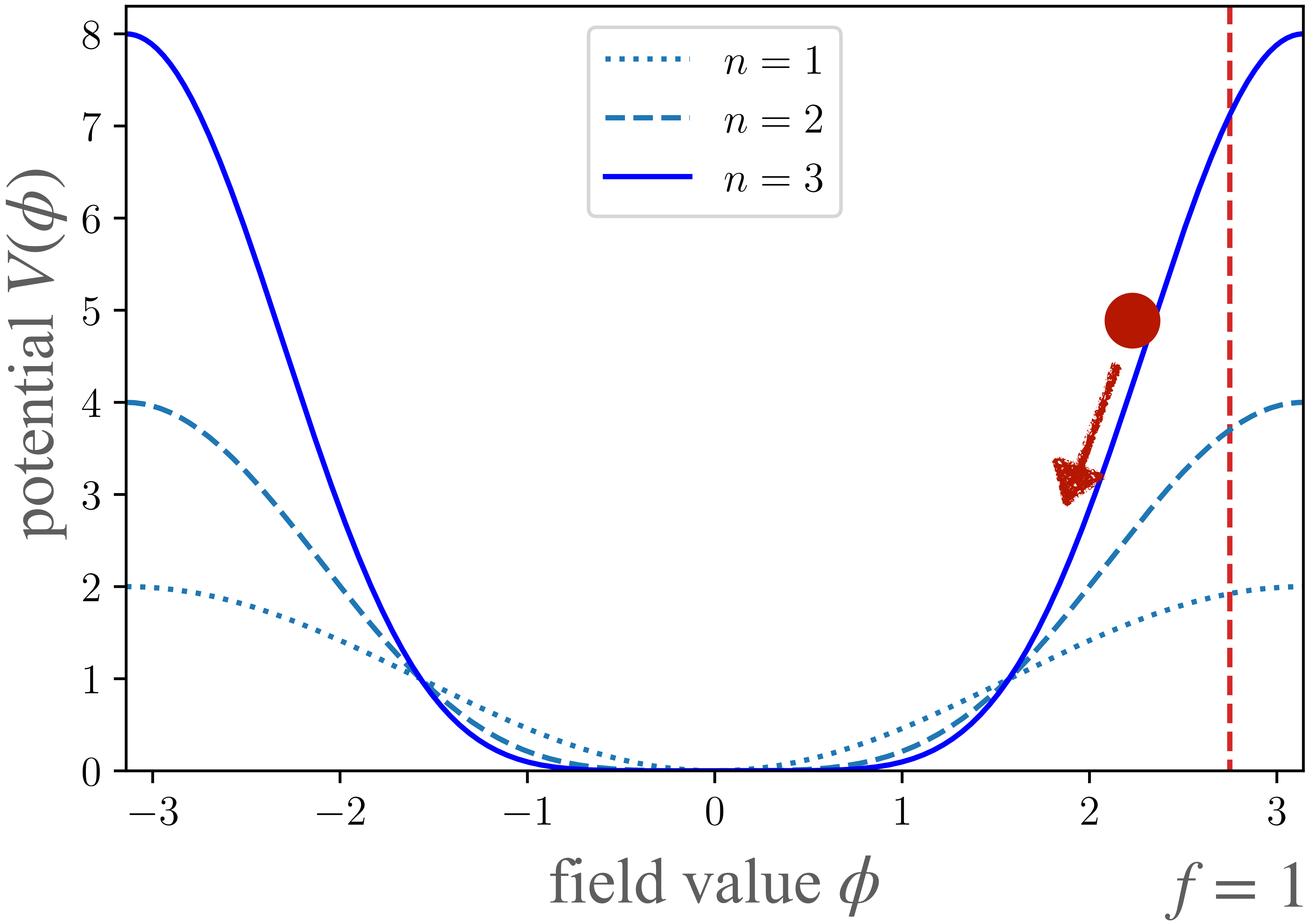

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At times after z_c:

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- This is an oscillation!

$$\phi(t) = \phi_0 \cos(mt)$$

Early Dark Energy

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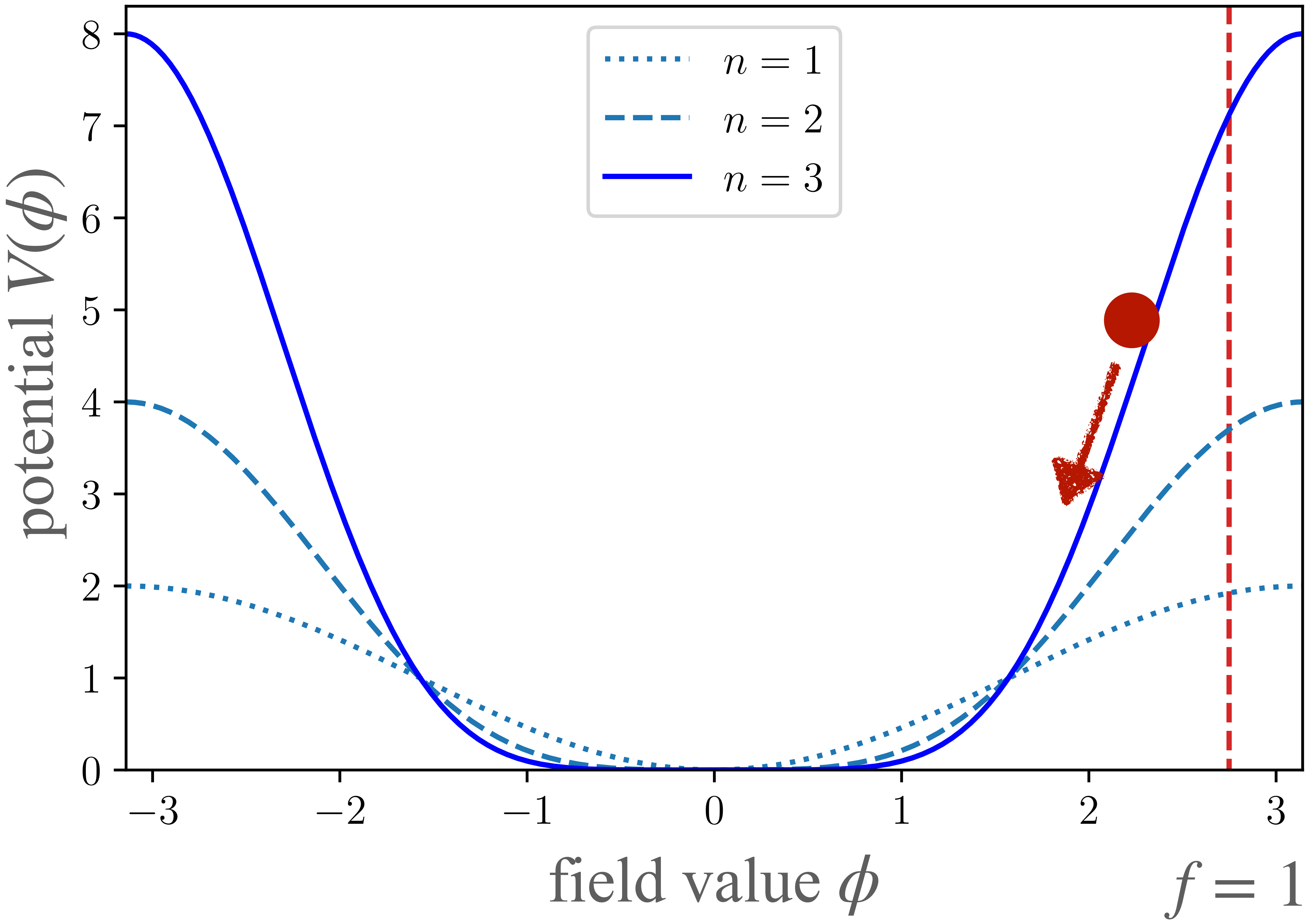


At times after z_c:

$$\boxed{\ddot{\phi} + m^2\phi \approx 0}$$

- This is an oscillation!

$$\phi(t) = \phi_0 \cos(mt)$$

- One can now numerically show that the EOS is:

$$\langle w \rangle = \frac{n-1}{n+1}$$

For $n = 3$: $\quad \langle w \rangle = \dfrac{1}{2}$

Early Dark Energy

Free "particle-physics" parameters:

- m mass ($V_0 = m^2 f^2$)
- f "decay constant"
- $\theta_i = \phi_i / f$ initial value of the field
- $(n = 3)$

Early Dark Energy

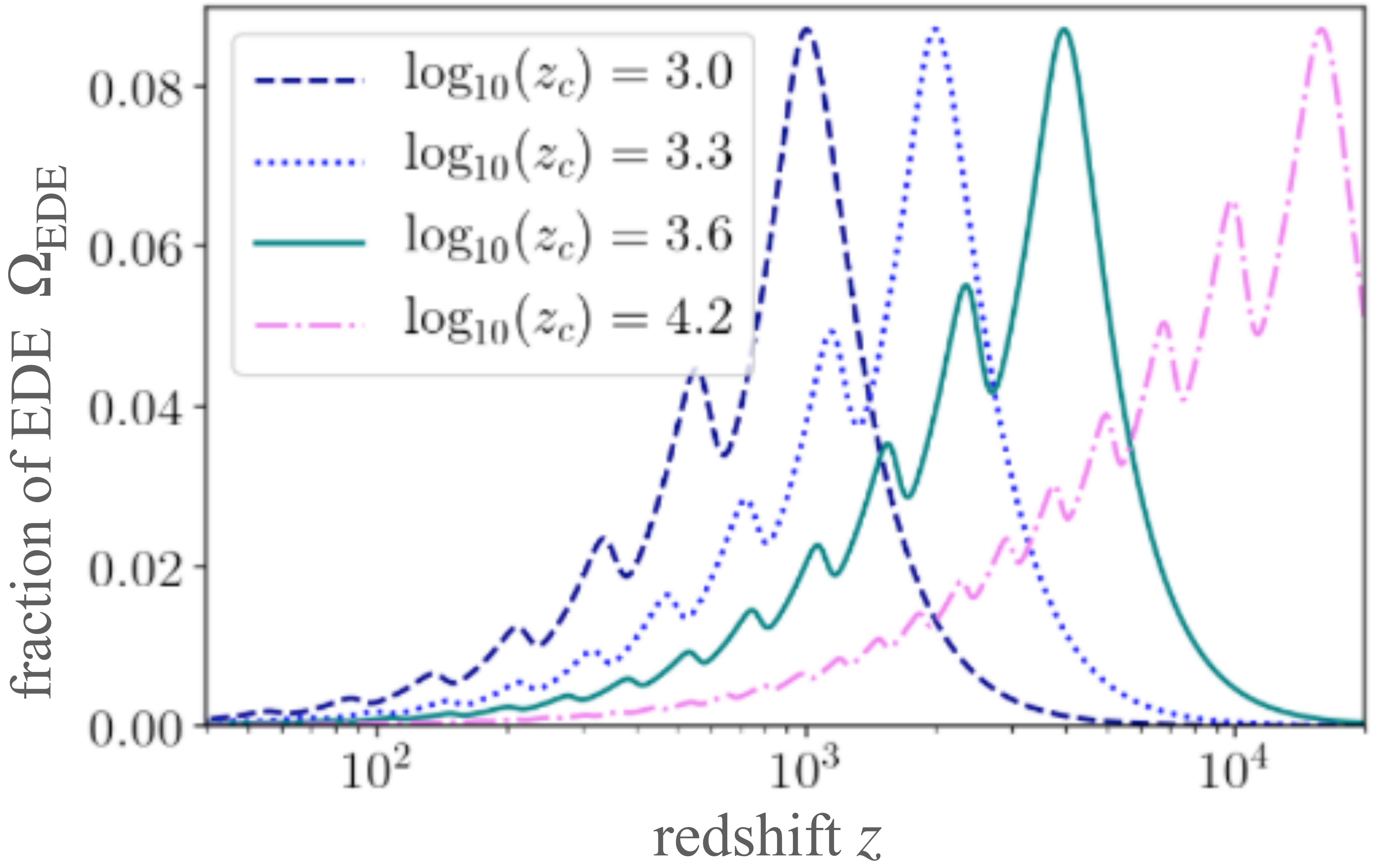
Free “particle-physics” parameters:

- m mass ($V_0 = m^2 f^2$)
- f “decay constant”
- $\theta_i = \phi_i / f$ initial value of the field
- $(n = 3)$

Free “phenomenological” parameters:

- $f_{\rm EDE}$ fraction of EDE at z_c
- z_c the critical redshift
- $\theta_i = \phi_i / f$ initial value of the field
- $(n = 3)$

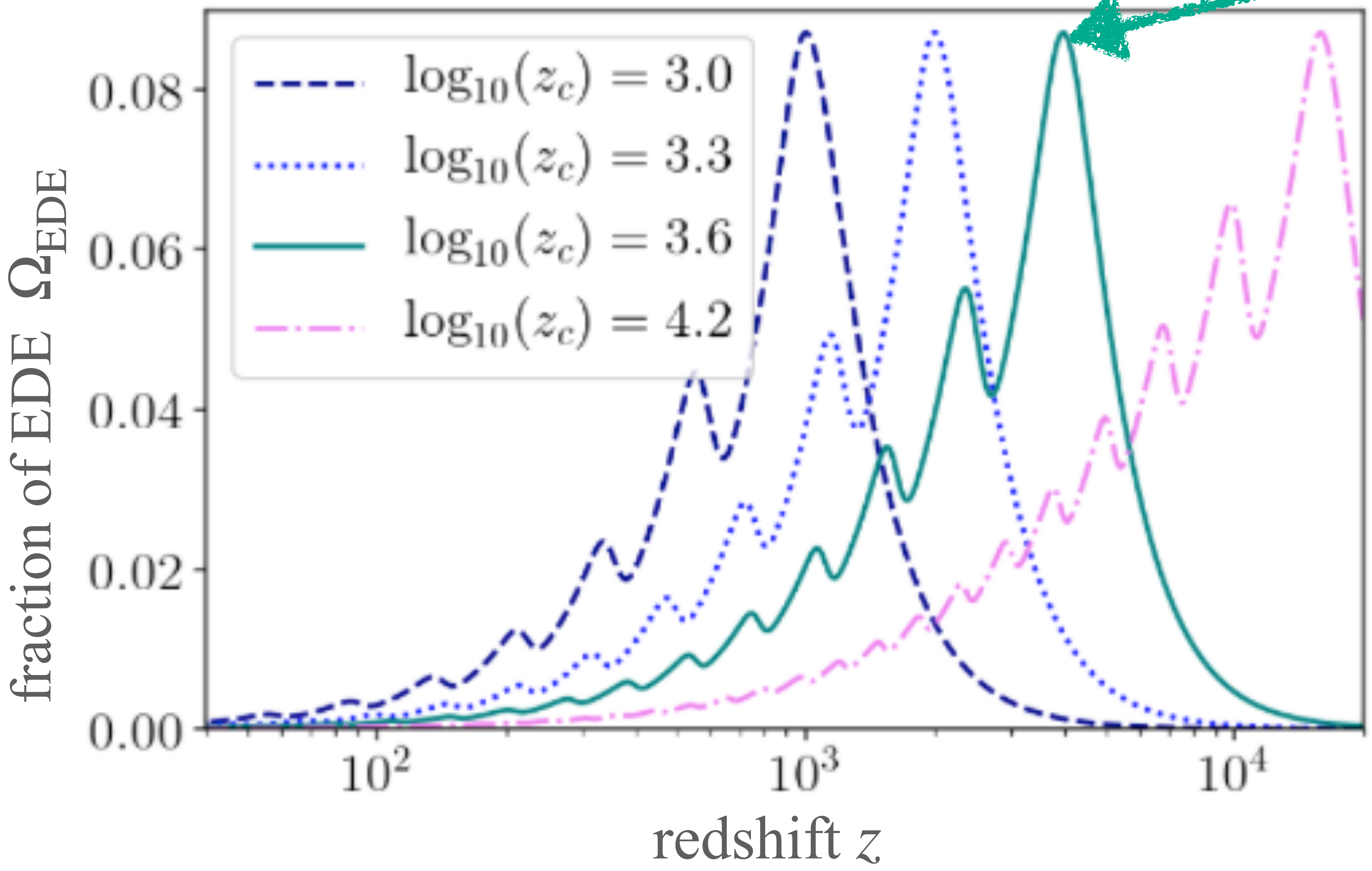
Early Dark Energy



The critical redshift z_c determines when the field starts oscillating, hence when $\Omega_{\rm EDE}$ peaks

Early Dark Energy

$f_{\mathrm{EDE}} = \Omega_{\mathrm{EDE}}(z_c)$: determines the height of the peak



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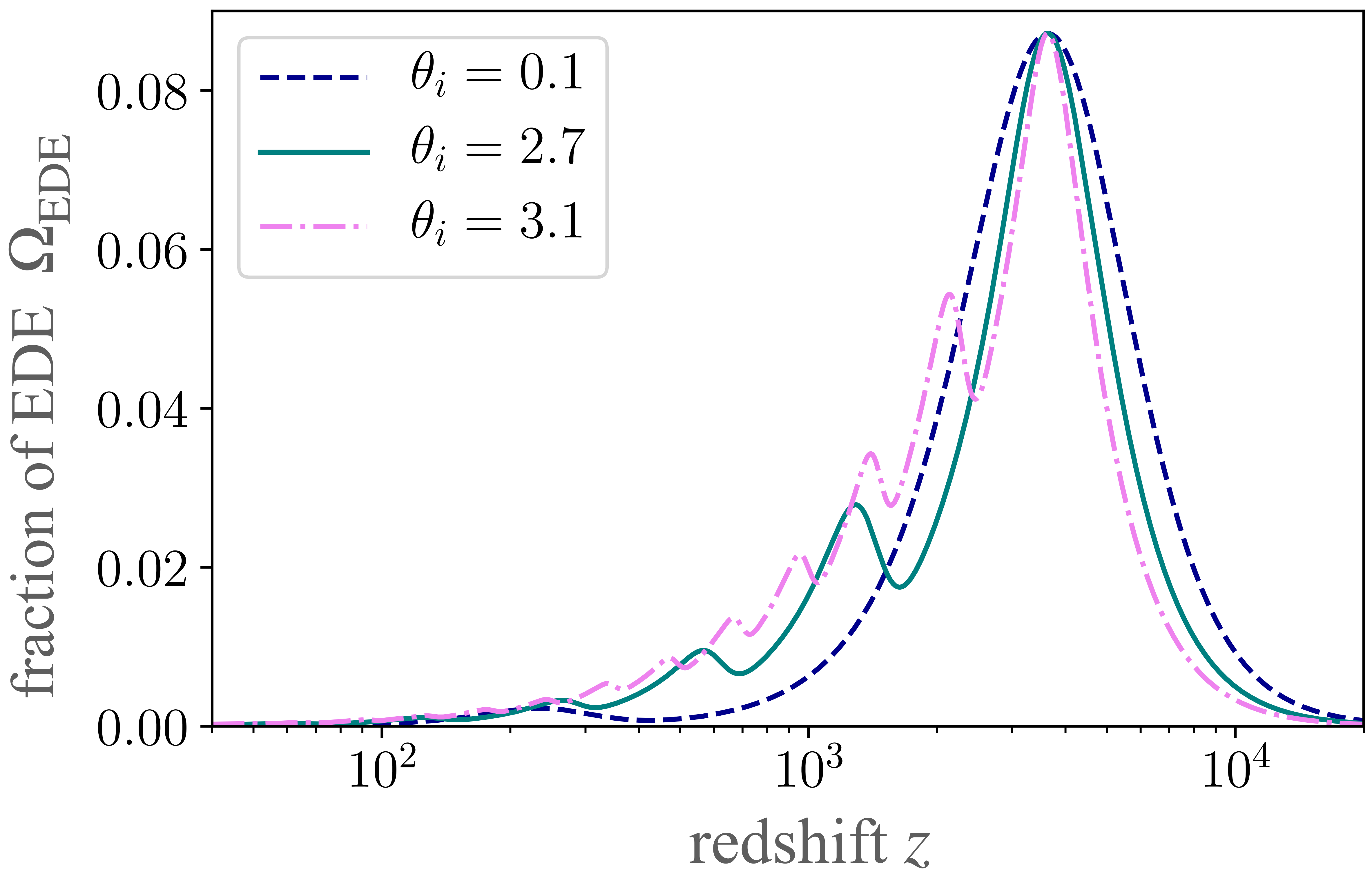
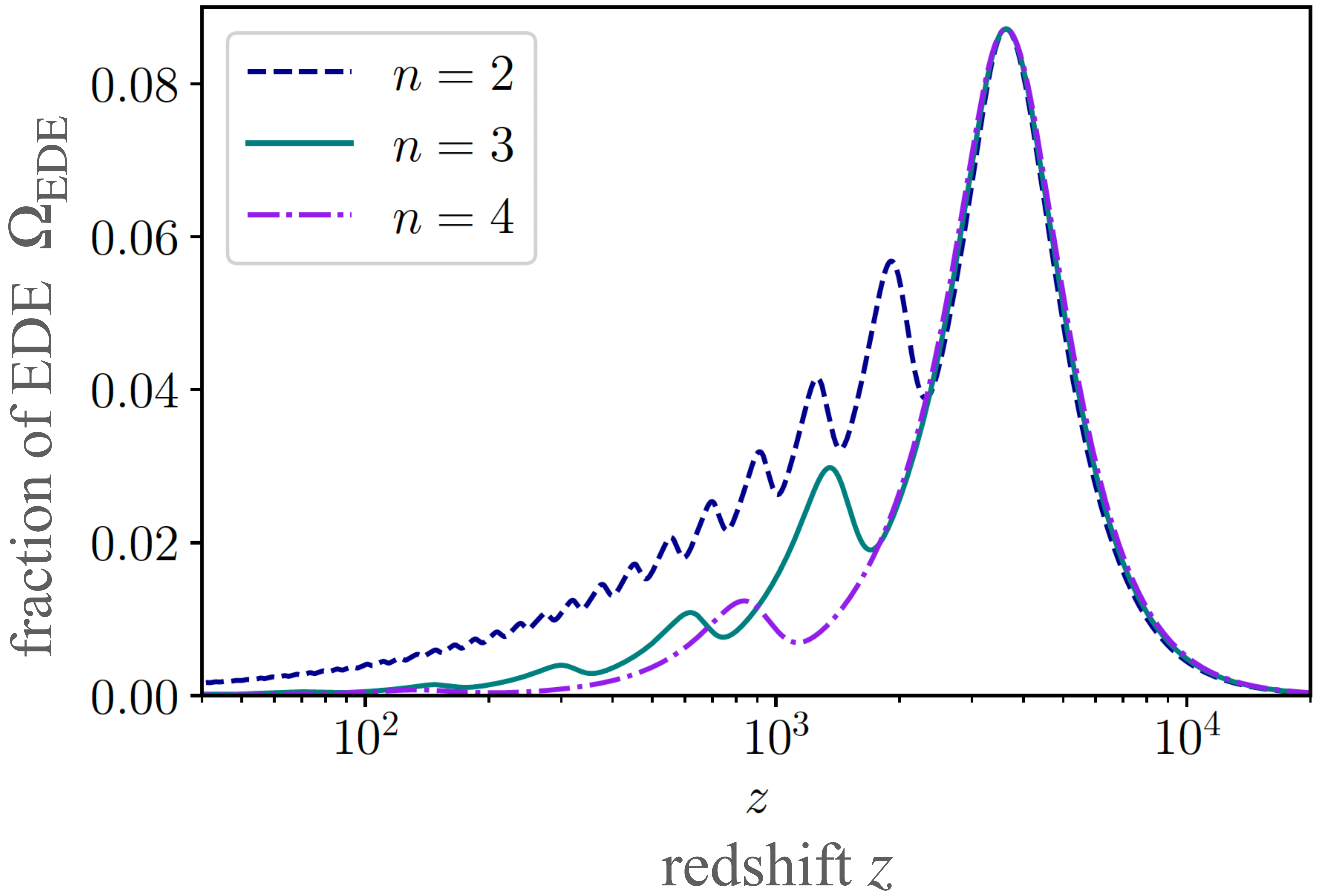
Early Dark Energy



The initial value $\theta_i \in [0,\pi]$ determines how fast the field oscillates when it decays:

The closer to π the faster the oscillations

Early Dark Energy



The index $n = 1, 2, 3, \ldots$ determines how fast the field decays:

The higher n, the faster the decay

$n = 3$ was shown to fit the data best

Recap Lecture 1

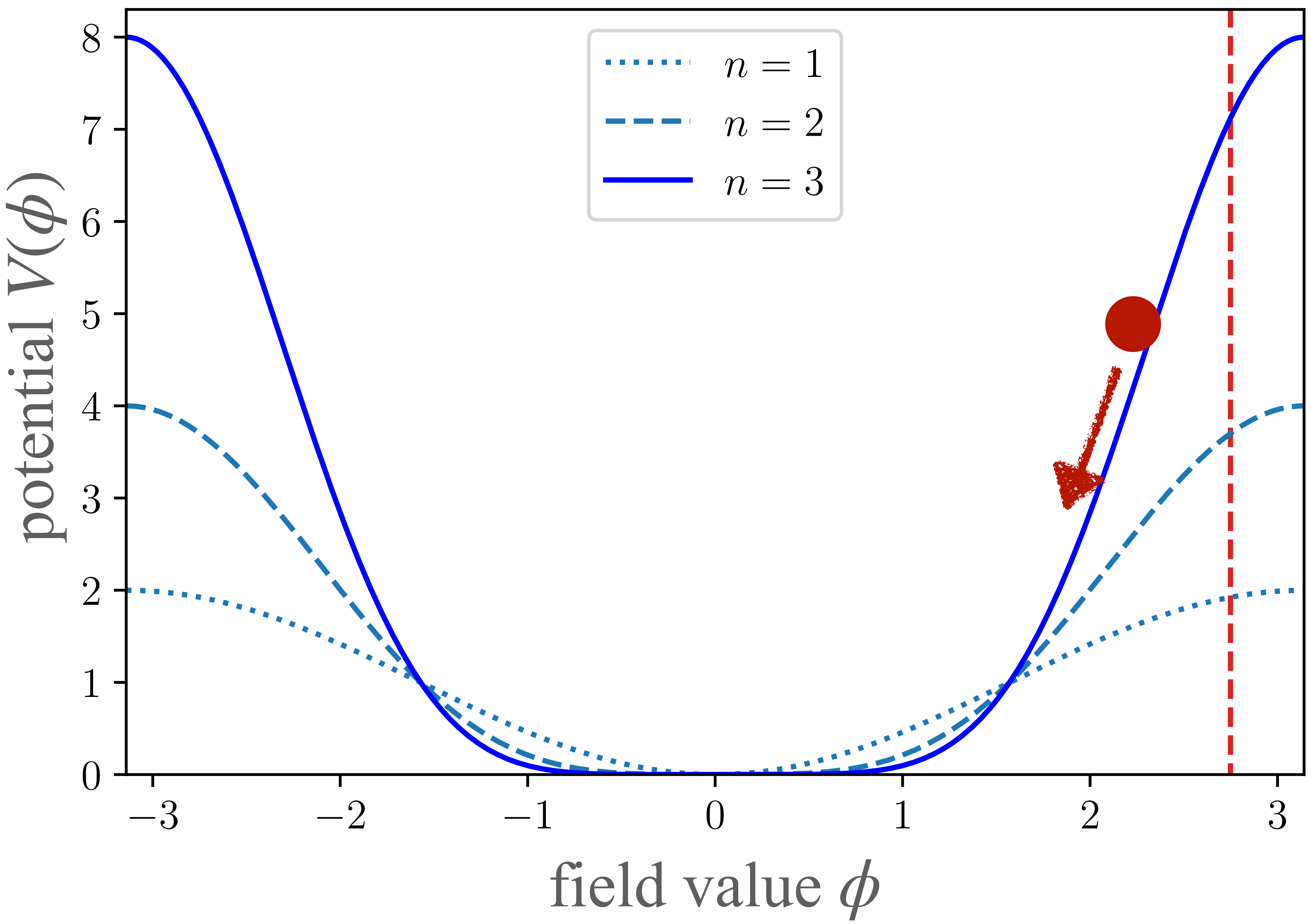
Introduction

- Friedmann equations describe (background) evolution of the components
- Hubble parameter
- distances
- CMB basics
- BAO
- How the CMB constrains H_0

$$\boxed{\begin{gathered}\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \quad \text{(i)}\\ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad \text{(ii)}\\ \dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a} \quad \text{(iii)}\end{gathered}}$$

Hubble tension and EDE

- Early- and late-universe solutions
  $$\theta_s = \frac{r_s}{D_A(z^*)}$$
- EDE: scalar field in an expanding universe with a “$1 - \cos$” potential
- EDE increases $H(z)$ before recombination and decays quickly after
- More about EDE: **tormorrow**



References

- Books: Dodelson&Schmidt “Modern Cosmology”, Weinberg: “Cosmology”, Huterer: “A course in cosmology”
- Komatsu “Physics of the Cosmic Microwave Background” (recorded lecture)
- Reviews about EDE:
    - Poulin et al. “The Ups and Downs of Early Dark Energy solutions to the Hubble tension: A review of models, hints and constraints circa 2023”
    - Kamionkowski&Riess: “The Hubble Tension and Early Dark Energy”