An Introduction to

Inflationary Cosmology

(from theory to data analysis)

CosmoVerseSchool @Corfu

Lecture 2 of 2

— Theory Part —

Image: Planck's View of BICEP2/Keck Array Field. Credit: Jet Propulsion Laboratory, NASA and Caltech

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For any further materials/clarifications/questions/curiosities/feedback, feel free to Contact Me

PRIMORDIAL PERTURBATIONS

Primordial Scalar Modes

The quantum **fluctuations of the Inflaton field** can source irregularities in the **CMB**

$$\mathscr{P}_s = \left(\frac{1}{8\pi^2 M_{\rm pl}^2}\right)\left(\frac{H^2}{\epsilon}\right)$$

Primordial Tensor Modes

The quantum **fluctuations in the metric** could source a stochastic background of **Primordial Gravitational Waves,** imprinting the **CMB**

$$\mathscr{P}_{\rm t} = \left(\frac{2}{\pi^2 M_{\rm pl}^2}\right) H^2$$

Planck 2018

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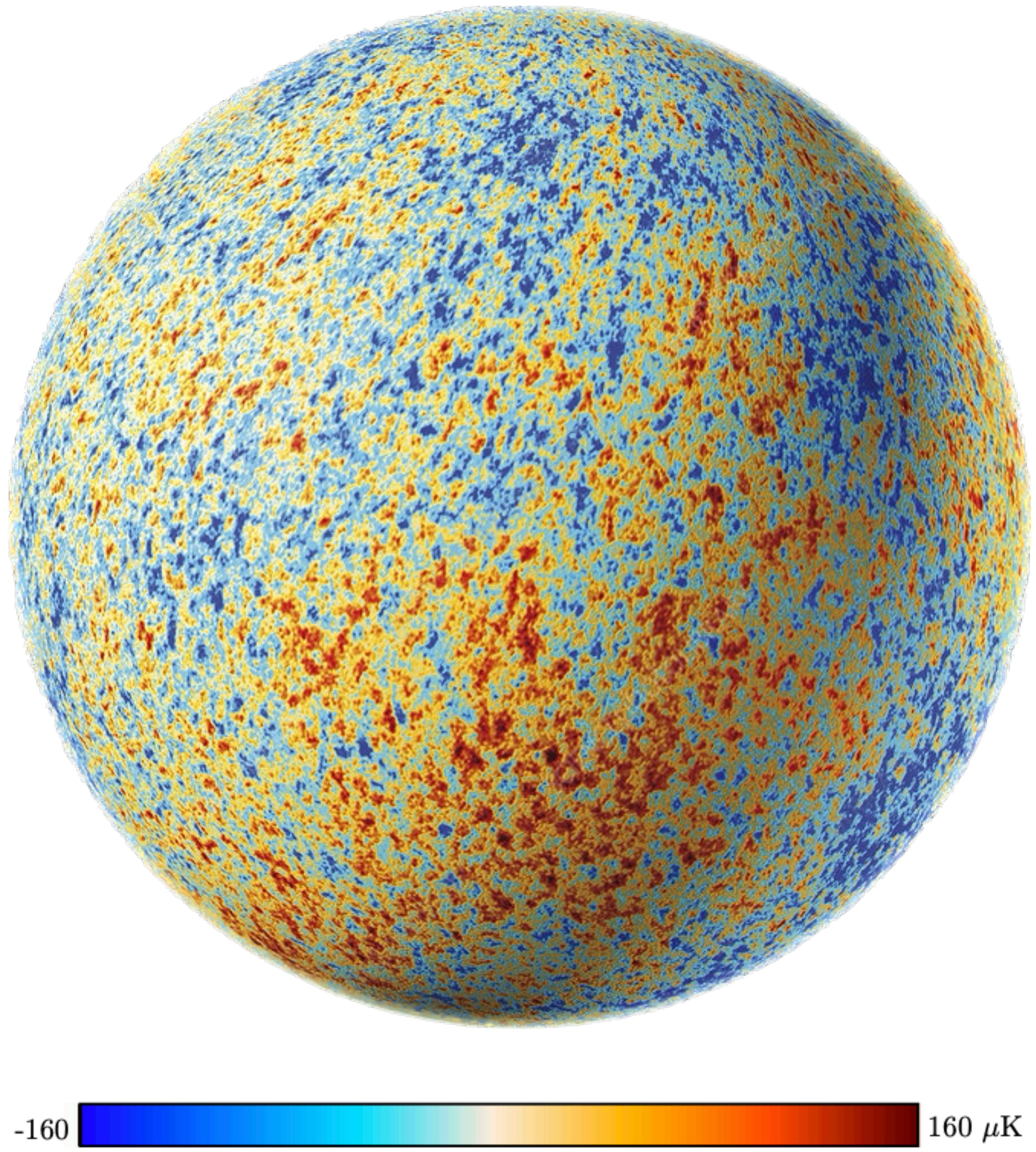

Primordial Perturbations

Primordial Scalar Modes

The quantum **fluctuations of the Inflaton field** can source irregularities in the **CMB**

$$\mathscr{P}_s = \left(\frac{1}{8\pi^2 M_{\rm pl}^2}\right)\left(\frac{H^2}{\epsilon}\right) = \left(\frac{1}{12\pi^2 M_{\rm pl}^6}\right)\left(\frac{V^3}{V_\phi^2}\right)$$

Primordial Tensor Modes

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$$\mathscr{P}_{\rm t} = \left(\frac{2}{\pi^2 M_{\rm pl}^2}\right) H^2 = \left(\frac{2}{3\pi^2 M_{\rm pl}^4}\right) V$$

Step-by-step derivation

- We start from:

$$\mathcal{P}_s = \left(\frac{1}{8\pi^2 M_{\rm pl}^2}\right)\left(\frac{H^2}{\epsilon}\right)$$

$$\mathcal{P}_{\rm t} = \left(\frac{2}{\pi^2 M_{\rm pl}^2}\right) H^2$$

- We Use the slow-roll relations:

$$\epsilon \simeq \frac{M_p^2}{2}\left(\frac{V_\phi}{V}\right)^2 \qquad H^2 \simeq \frac{V(\phi)}{3M_p^2}$$

- After straightforward manipulations we get:

$$\mathcal{P}_s = \left(\frac{1}{12\pi^2 M_{\rm pl}^6}\right)\left(\frac{V^3}{V_\phi^2}\right)$$

$$\mathcal{P}_{\rm t} = \left(\frac{2}{3\pi^2 M_{\rm pl}^4}\right) V$$

Primordial Perturbations

Primordial Scalar Modes

The quantum **fluctuations of the Inflaton field** can source irregularities in the **CMB**

$$\mathscr{P}_s(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} \qquad n_s = \left.\frac{d \ln \mathscr{P}_s}{d \ln k}\right|_{k=k_*} = 2\eta - 6\epsilon$$

Primordial Tensor Modes

The quantum **fluctuations in the metric** could source a stochastic background of **Primordial Gravitational Waves,** imprinting the **CMB**

$$\mathscr{P}_T(k) = r A_s \left(\frac{k}{k_*}\right)^{n_T} \qquad n_T = \left.\frac{d \ln \mathscr{P}_T}{d \ln k}\right|_{k=k_*} = -\frac{r}{8} = -2\epsilon$$

Important Note

- We can use the following relations:

$$k = a\, H$$

$$\ln k = \ln a + \ln H$$

$$d \ln k = d \ln a + d \ln H = H\, dt + \frac{\dot{H}}{H} dt$$

$$d \ln k \approx H\, dt = \frac{H}{\dot{\phi}} d\phi$$

$$\frac{d}{d \ln k} \approx \frac{\dot{\phi}}{H} \frac{d}{d\phi}$$

$$\boxed{\frac{d}{d \ln k} \approx -M_p^2 \frac{V'}{V} \frac{d}{d\phi},}$$

Primordial Perturbations

Step-by-step derivation

- For n_s we have:

$$
\begin{aligned}
n_s - 1 \equiv \frac{d\ln \mathcal{P}_s}{d\ln k} &= \frac{1}{\mathcal{P}_s}\frac{d\mathcal{P}_s}{d\ln k} \\
&= \left(12\pi^2 M_p^6\right)\left(\frac{V'^2}{V^3}\right)\left[-M_p^2 \frac{V'}{V}\frac{d}{d\phi}\left(\frac{1}{12\pi^2 M_p^6}\frac{V^3}{V'^2}\right)\right] \\
&= -M_p^2\left(\frac{V'^3}{V^4}\right)\frac{d}{d\phi}\left(\frac{V^3}{V'^2}\right) \\
&= -M_p^2\left(\frac{V'^3}{V^4}\right)\left(3\frac{V^2}{V'} - 2\frac{V^3\,V''}{V'^3}\right) \\
&= \Bigg[\underbrace{-3M_p^2\left(\frac{V'}{V}\right)^2}_{-6\epsilon_{\rm v}} + \underbrace{2\,M_p^2\left(\frac{V''}{V}\right)}_{\eta_{\rm v}}\Bigg] \\
&= -6\epsilon_{\rm v} + 2\eta_{\rm v}.
\end{aligned}
$$

- For n_t we have:

$$
\begin{aligned}
n_t \equiv \frac{d\ln \mathcal{P}_t}{d\ln k} &= \frac{1}{\mathcal{P}_t}\frac{d\mathcal{P}_t}{d\ln k} \\
&= \left(\frac{3\pi^3 M_p^4}{2}\right)\frac{1}{V}\left[-M_p^2\frac{V'}{V}\frac{d}{d\phi}\left(\frac{2}{3\pi^3 M_p^4}V\right)\right] \\
&= -M_p^2\left(\frac{V'}{V}\right)^2 \\
&= -2\epsilon_{\rm v}.
\end{aligned}
$$

Primordial Scalar Modes

The quantum **fluctuations of the Inflaton field** can source irregularities in the **CMB**

$$
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Primordial Tensor Modes

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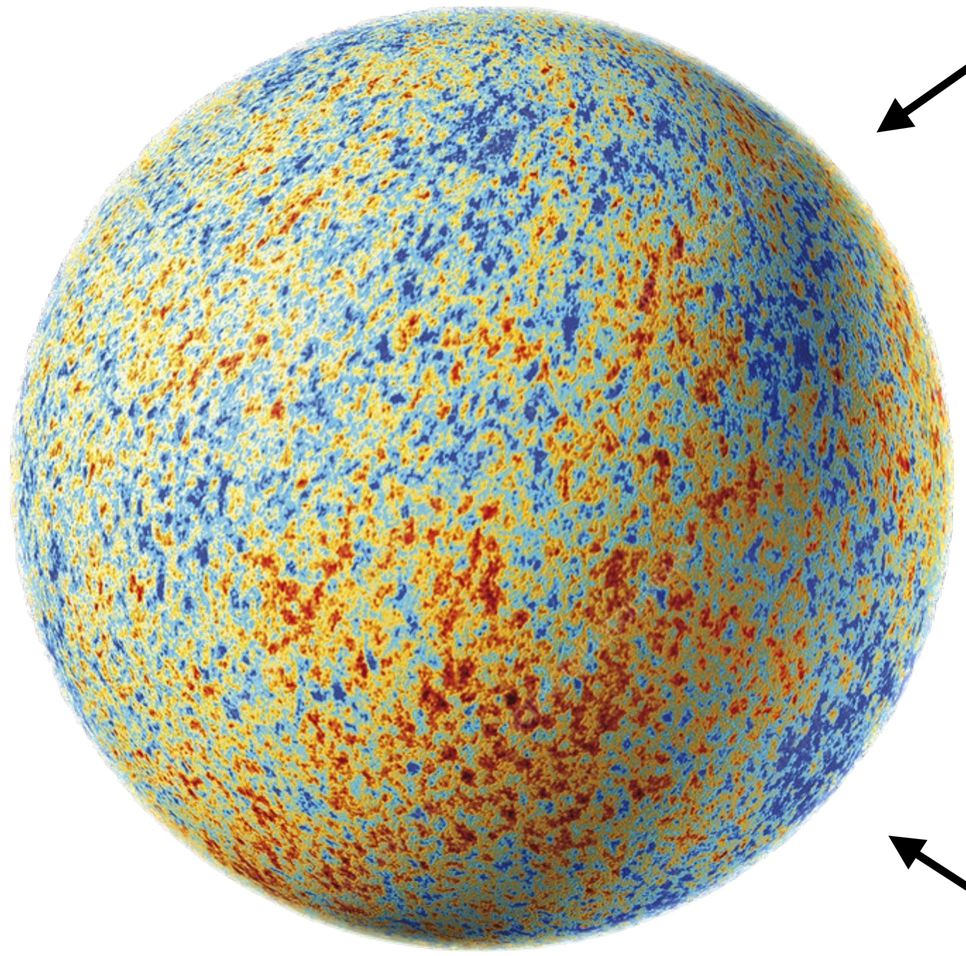
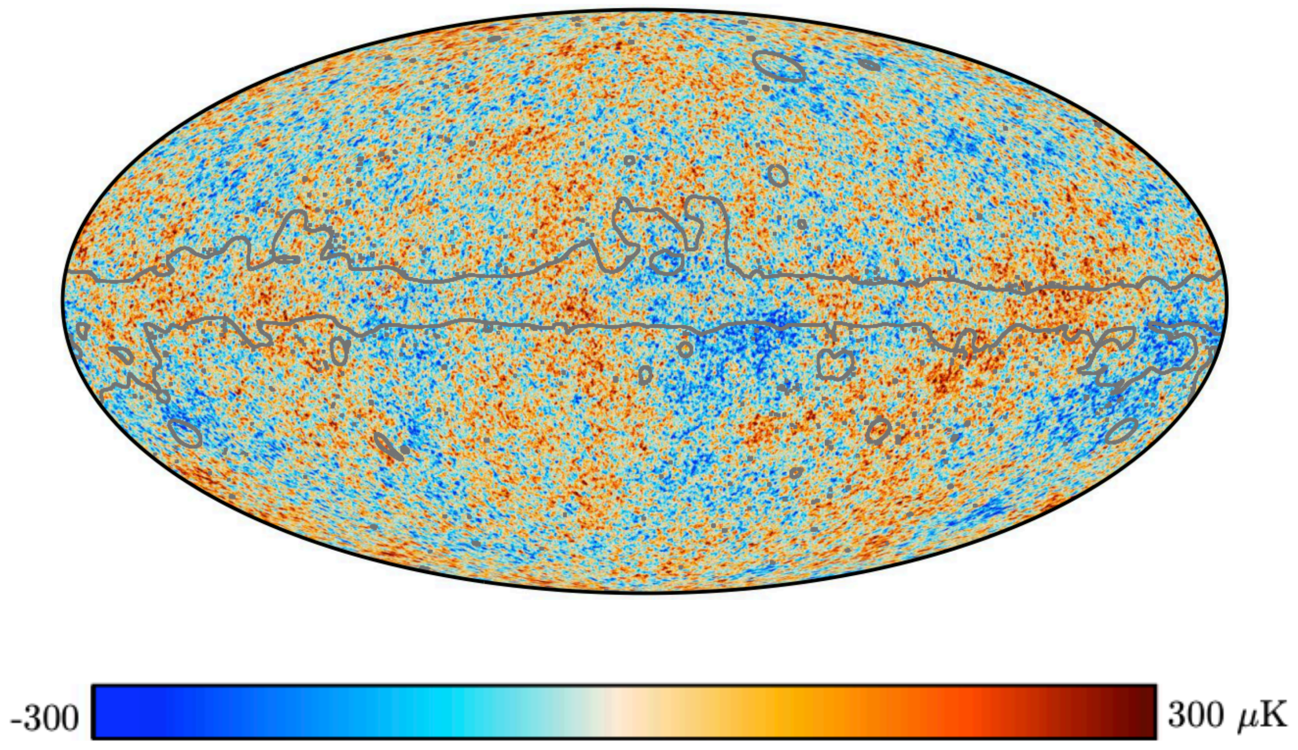
$$
\mathscr{P}_T(k) = r\,A_s\left(\frac{k}{k_*}\right)^{n_T} \qquad n_T = \left.\frac{d\ln \mathscr{P}_T}{d\ln k}\right|_{k=k_*} = -\frac{r}{8} = -2\epsilon
$$

Primordial Perturbations

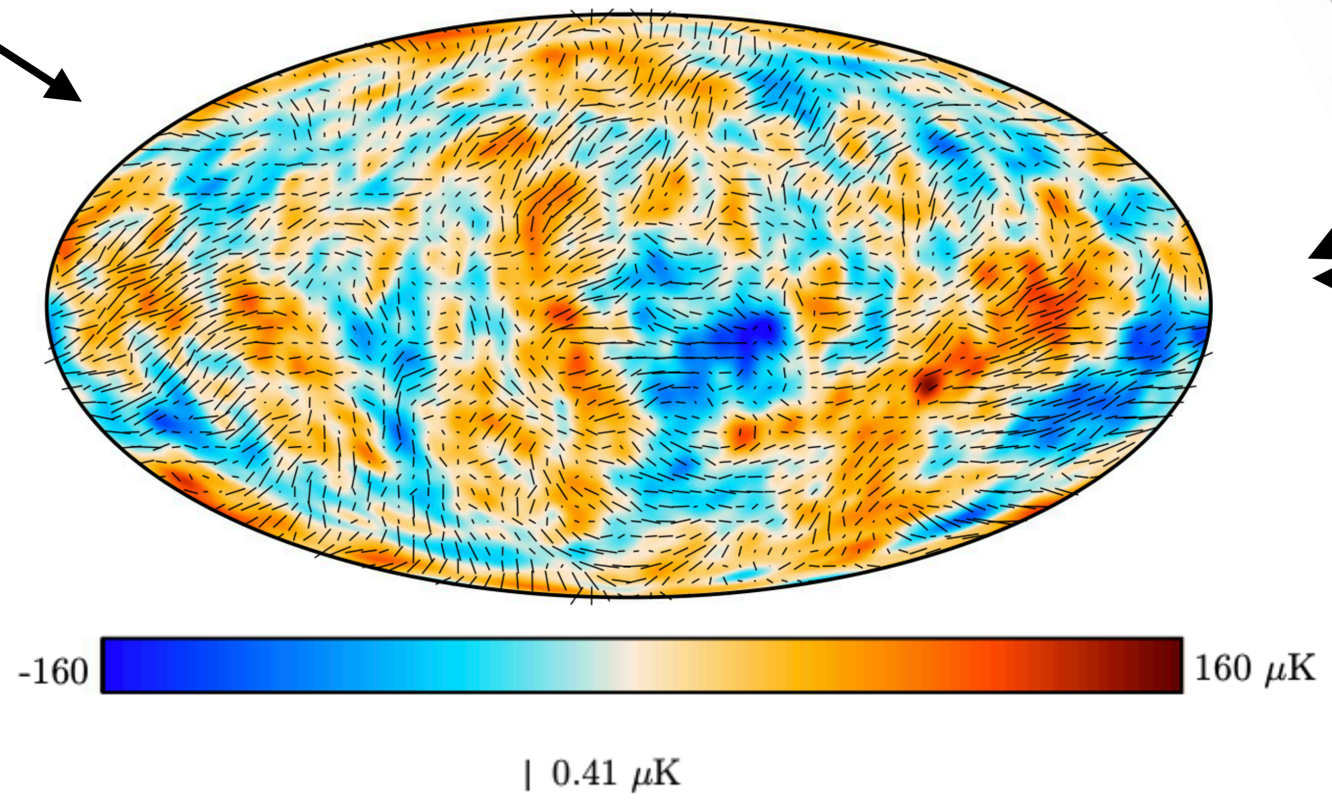
Relic Photons from the Big Bang

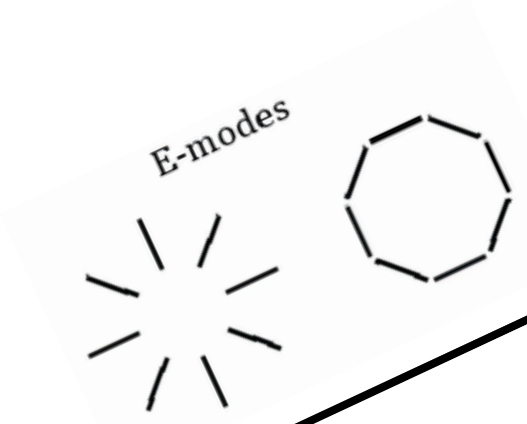


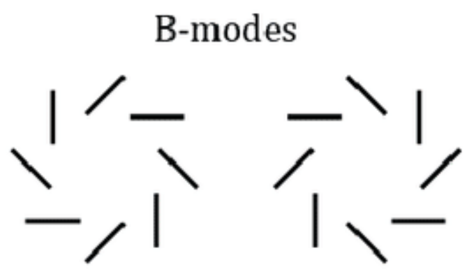
Temperature anisotropies



Polarization







We can extract 4 independent observables

(note: assuming that parity is conserved)

1) Angular power spectrum of temperature anisotropies C_ℓ^{TT}
(**TT spectrum**)

2) Temperature and E-mode cross-spectrum C_ℓ^{TE}
(**TE spectrum**)

3) Angular power spectrum of E-mode polarisation C_ℓ^{EE}
(**EE spectrum**)

4) Angular power spectrum of B-mode polarisation C_ℓ^{BB}
(**BB spectrum**)

Primordial Perturbations

Note: $\ell \propto 1/\theta \propto 1/\mathrm{R}$

Scalar and Tensor modes contribution to CMB spectra:

TT spectrum: Scalar > Tensor at any ℓ

TE spectrum: Scalar > Tensor at any ℓ

EE spectrum: Scalar > Tensor at any ℓ

BB spectrum: Tensor > Scalar at $\ell \lesssim 100$ (i.e., at large scales)

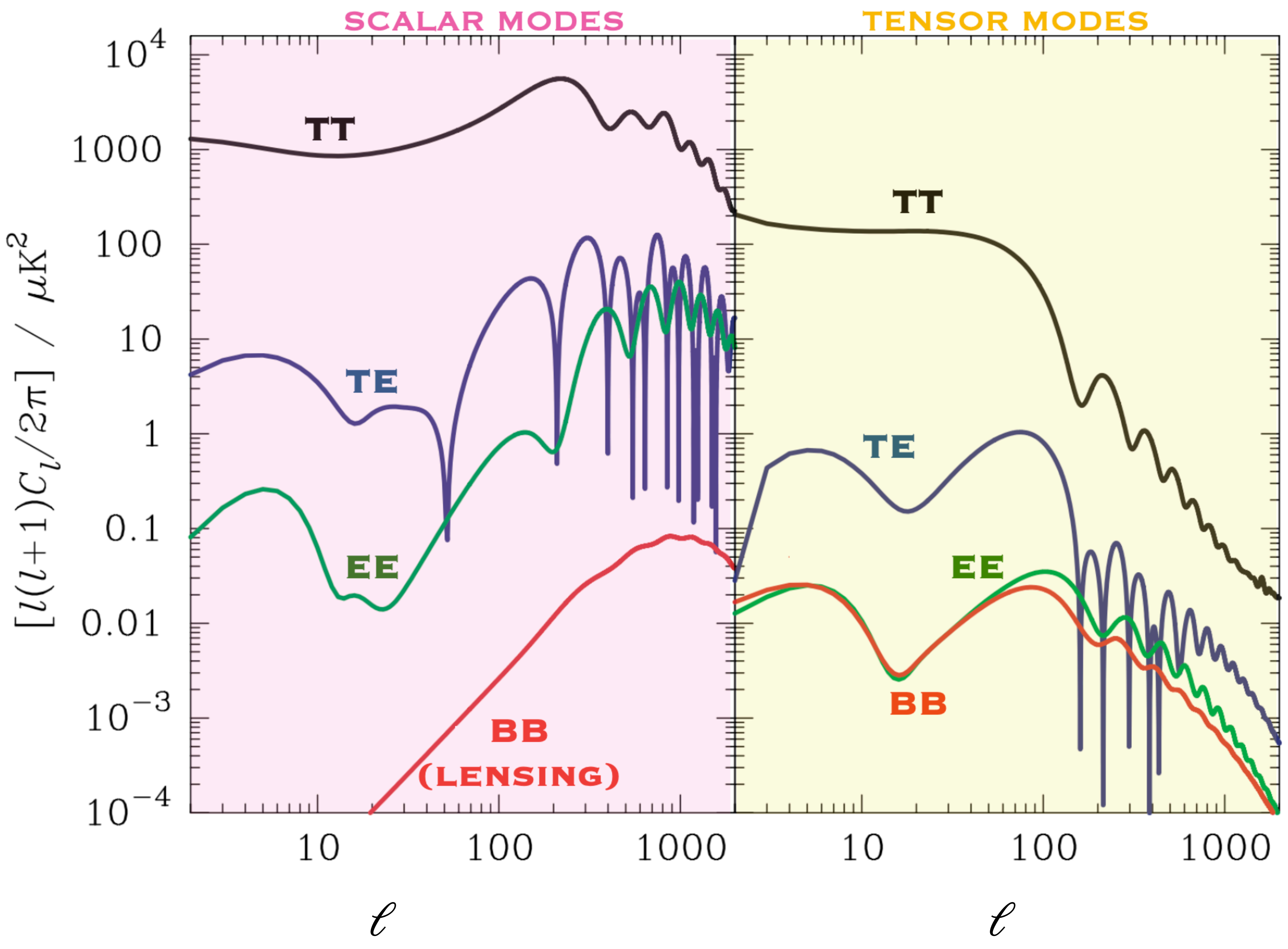


Figure inspired by Gorbunov & Rubakov
"Cosmological Perturbations and Inflationary Theory", Chapter 10
See also A. Challinor arXiv:astro-ph/0606548

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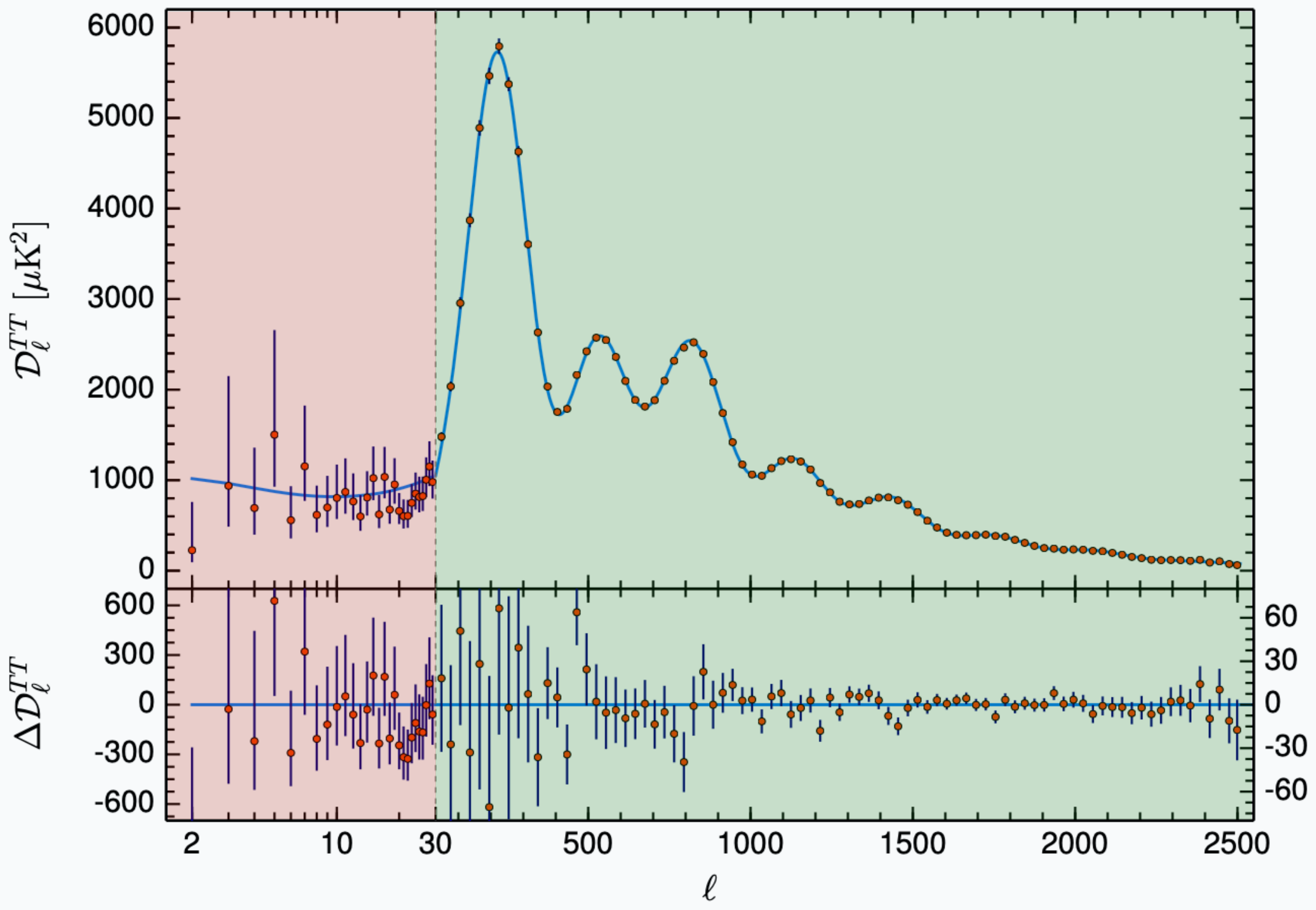
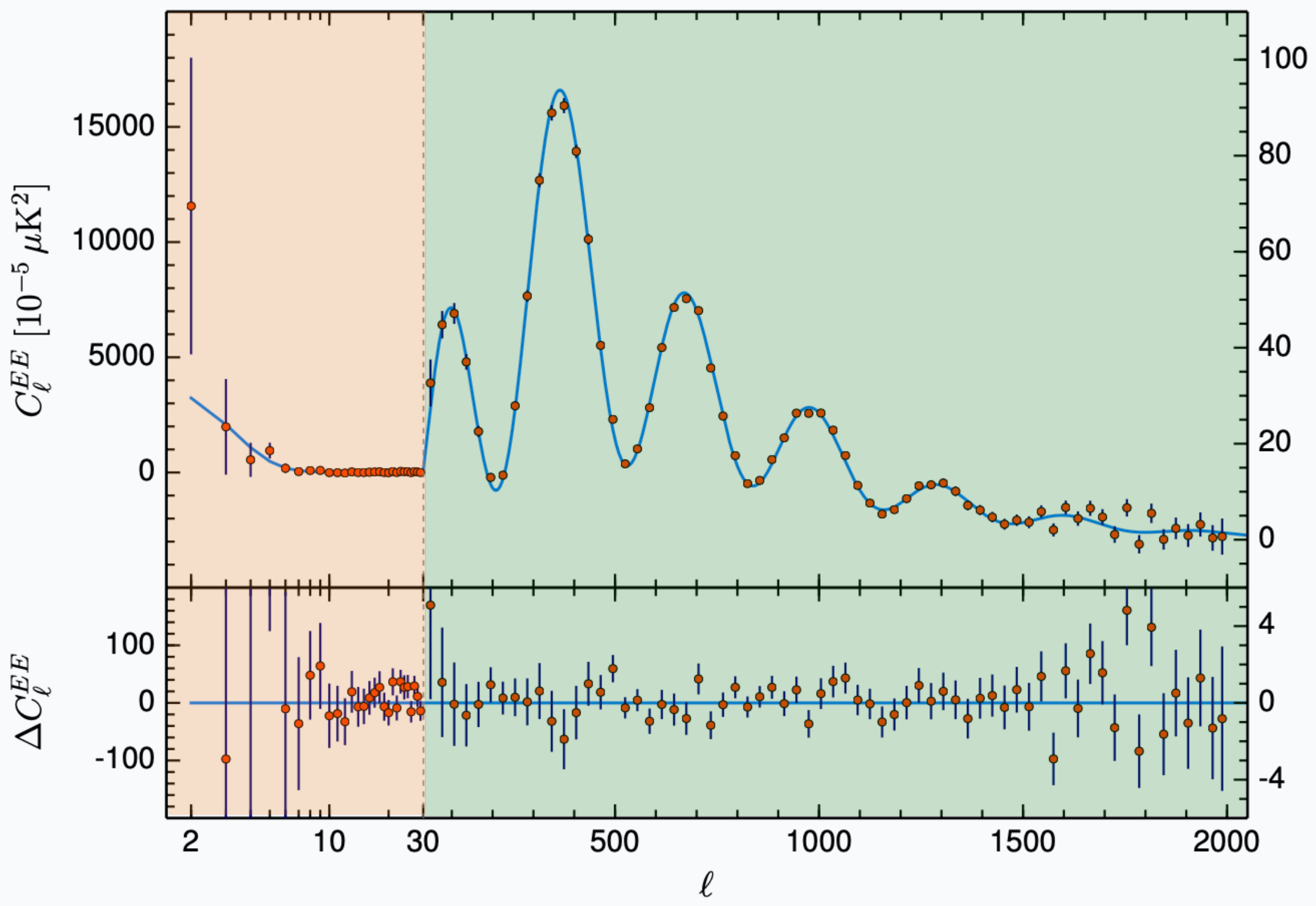
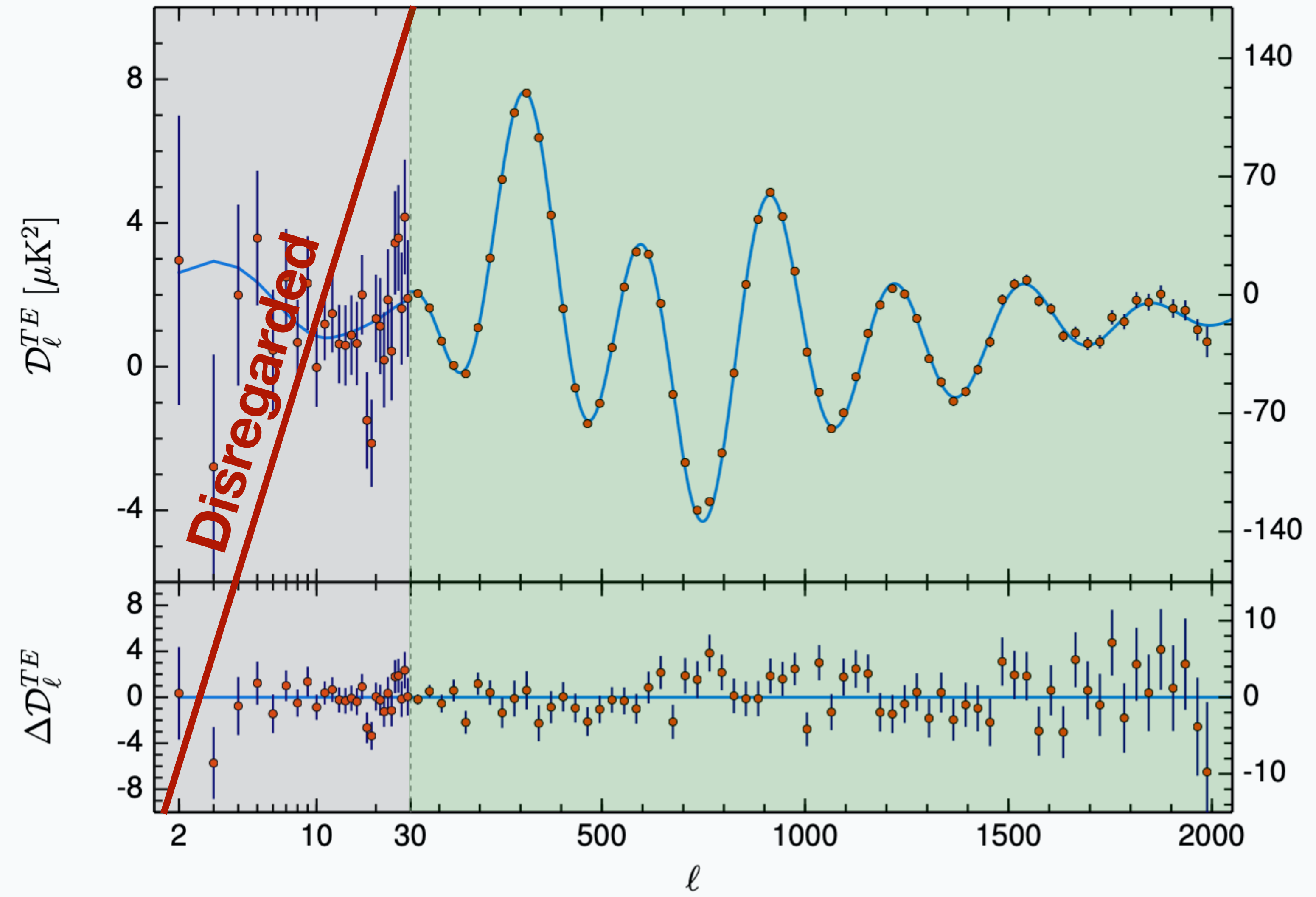
TT Spectrum



EE Spectrum



TE Cross-Spectrum



Low-multipole temperature data

$2 \leq \ell \leq 30$ in the TT Spectrum

Low-T

Low-multipole Polarization data

$2 \leq \ell \leq 30$ in the EE Spectrum

Low-E

Disregarded

Low-multipole ~~TE data~~

~~$2 \leq \ell \leq 30$ in the TE Spectrum~~

The low-TE data show excess of variance compared to simulations at low multipoles, for reasons that are not understood

High-multipole temperature data

$30 < \ell \lesssim 2500$ in the TT Spectrum

High-multipole EE Polarization data

$30 < \ell \lesssim 2000$ in the EE Spectrum

High-multipole TE data

$30 < \ell \lesssim 2000$ in the TE Spectrum

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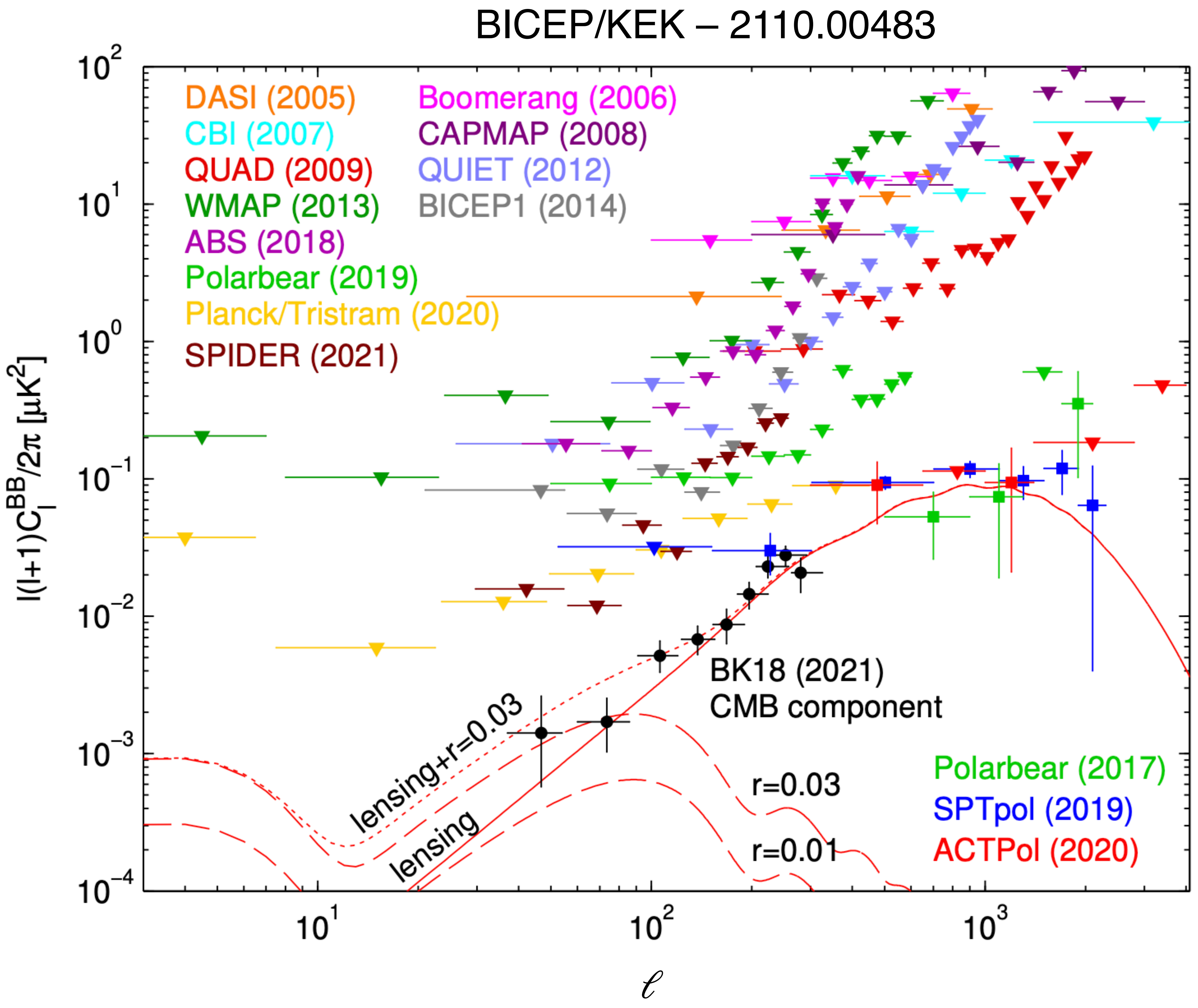
B-Modes Polarization

Tensor > Scalar at $\ell \lesssim 100$

To constraint primordial tensor modes we need large-scale B-mode polarization

Many experiments have been (and will be) collecting data

BICEP/KEK-2018 most precise data so far



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Joint analysis of Planck and BICPE/KEK:

1) $n_s \neq 1$ **at 8.5σ :** $n_s = 0.9678 \pm 0.0036$ (at 68% CL)

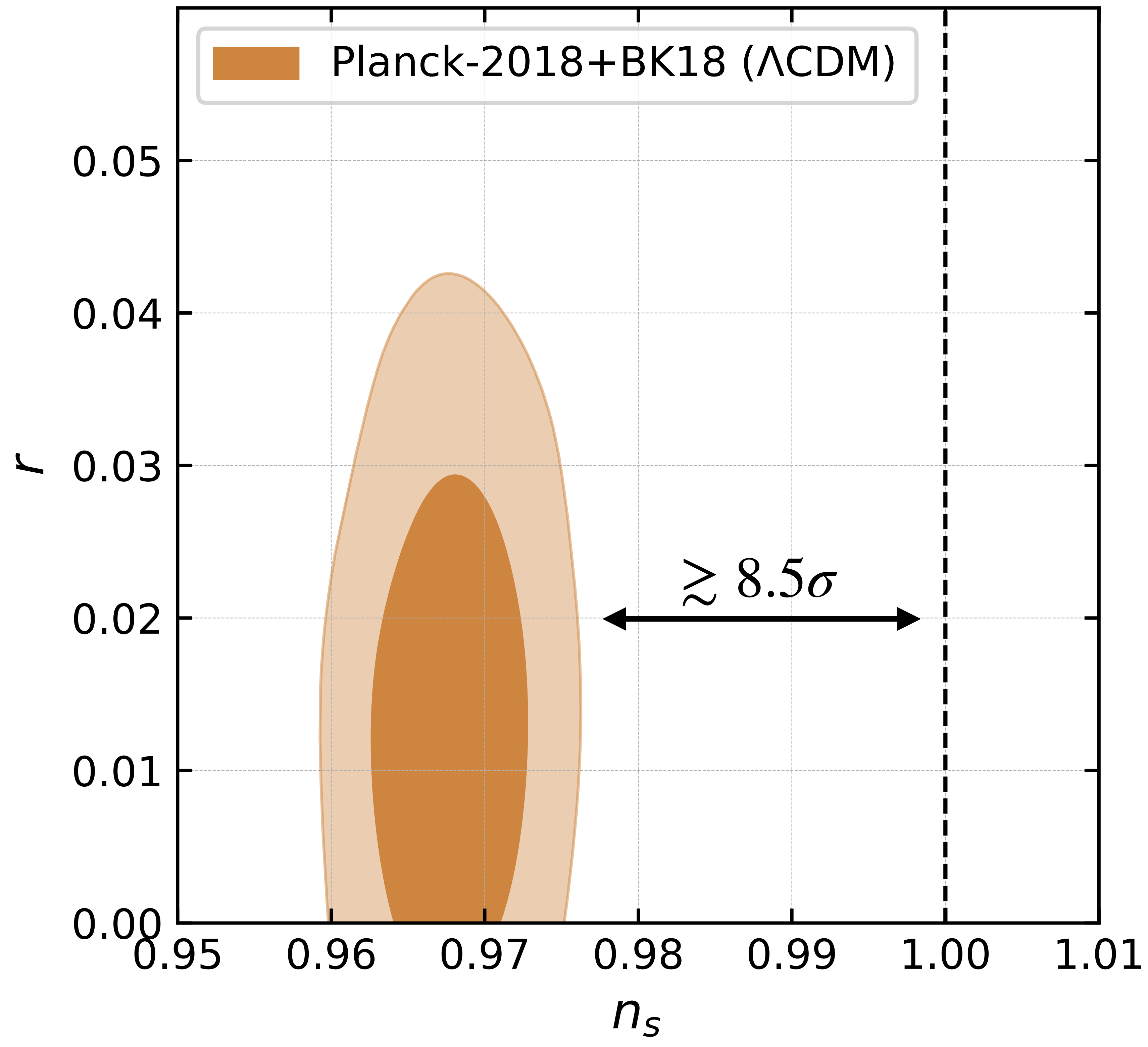
2) **No detection of tensor modes:** $r < 0.035$ (at 95%CL)

Slow-roll parameters:

1) η **measured to** $\eta = -0.0130^{+0.0024}_{-0.0029}$ (at 68% CL)

2) **upper limit** $\epsilon < 0.0022$ (at 95%CL)

3) Slow-roll hierarchy $1 \gg |\eta| \gg \epsilon$



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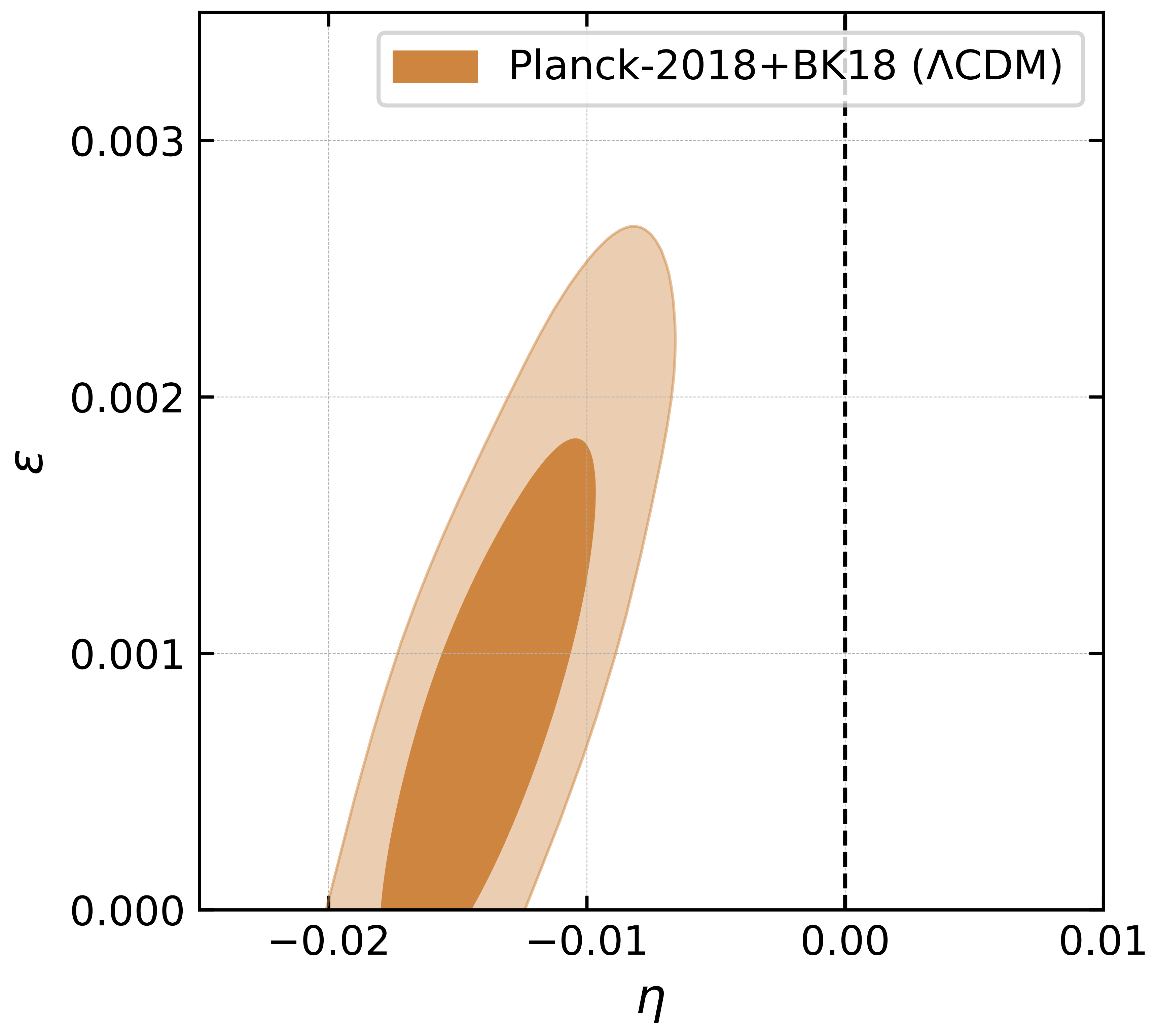

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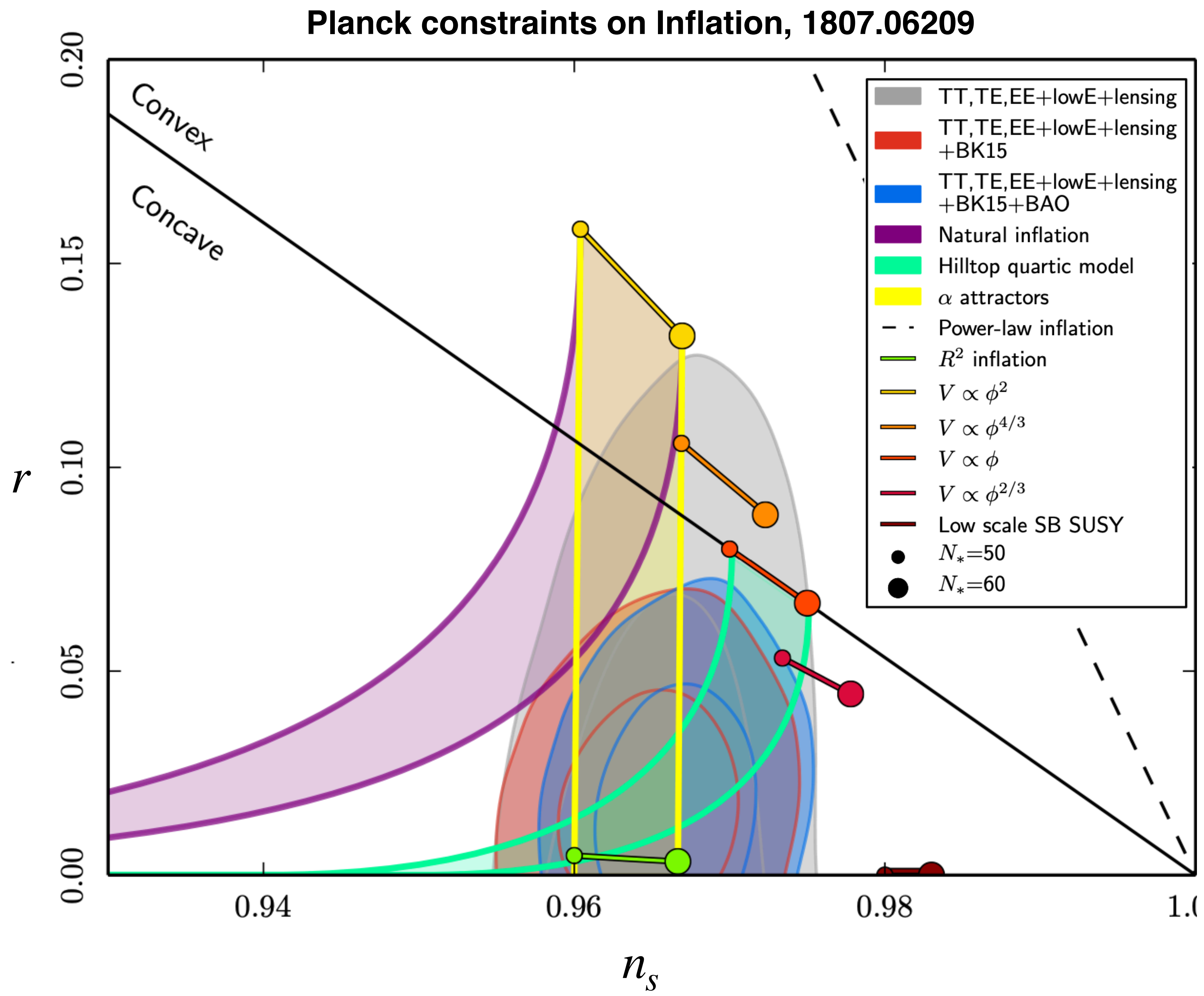

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“All models are equal, but some models are more equal than others”

Starobinsky Inflation

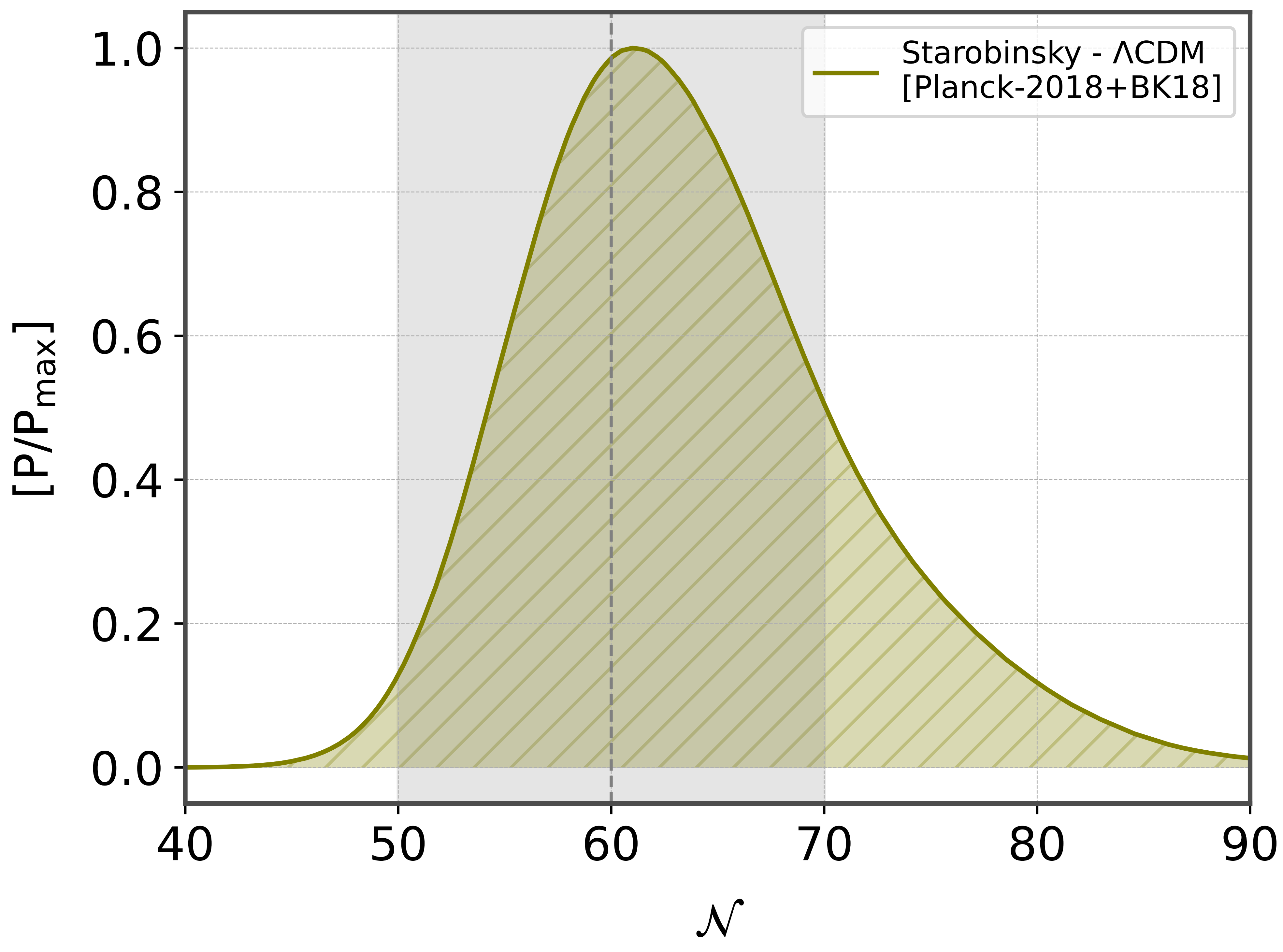
Inflation is controlled by the **squared Ricci scalar** in the effective action

$$S = \frac{1}{2M_{\rm Pl}^2} \int d^4x \sqrt{-g} \left(R + \frac{R^2}{6M^2} \right)$$

It gives ***predictions*** for n_s and r

$$n_s \simeq 1 - \frac{2}{\mathcal{N}} \qquad r \simeq \frac{12}{\mathcal{N}^2} \qquad 50 \lesssim \mathcal{N} \lesssim 70$$

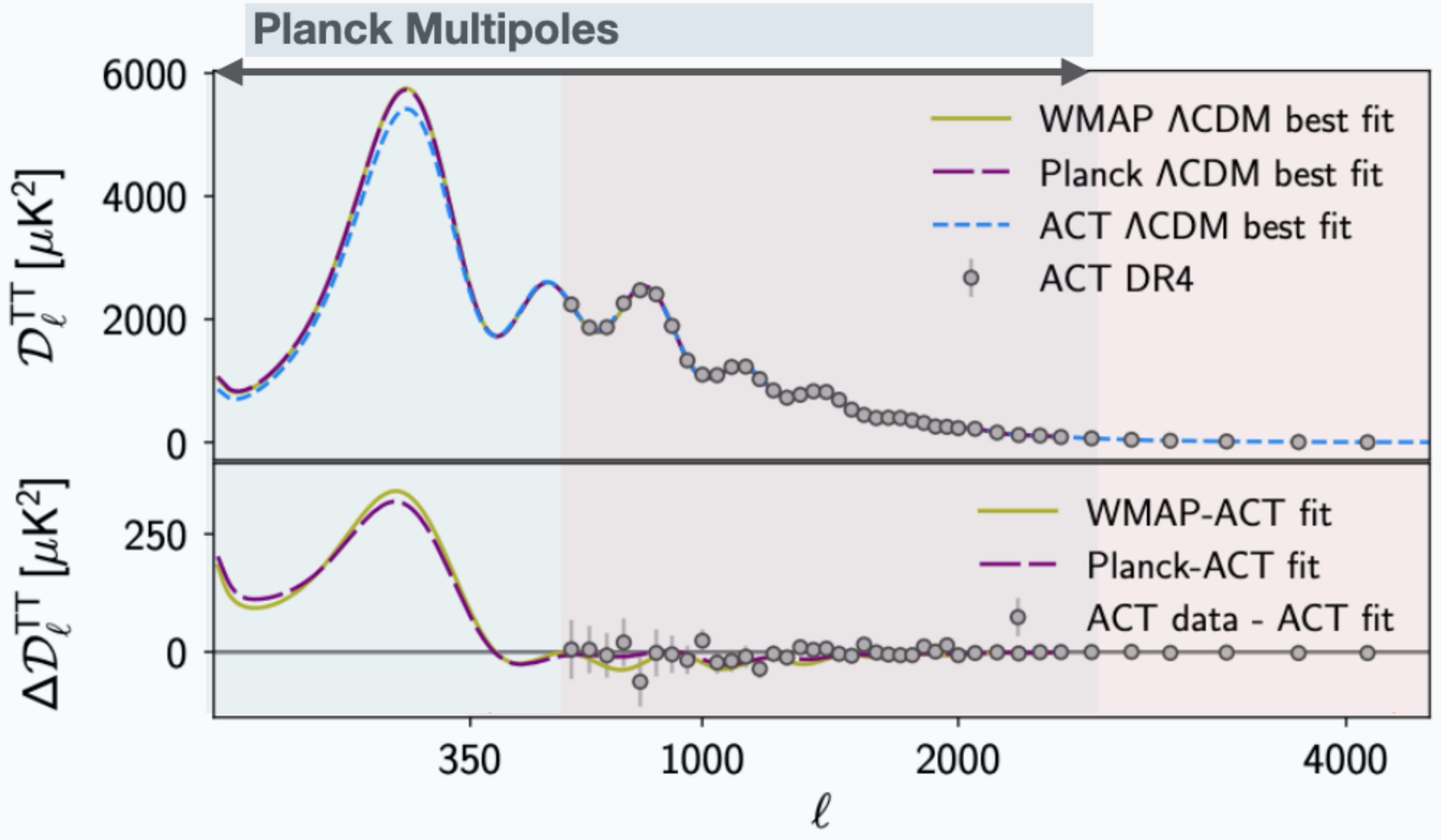
Model in perfect agreement with Planck and BICEP/KECK



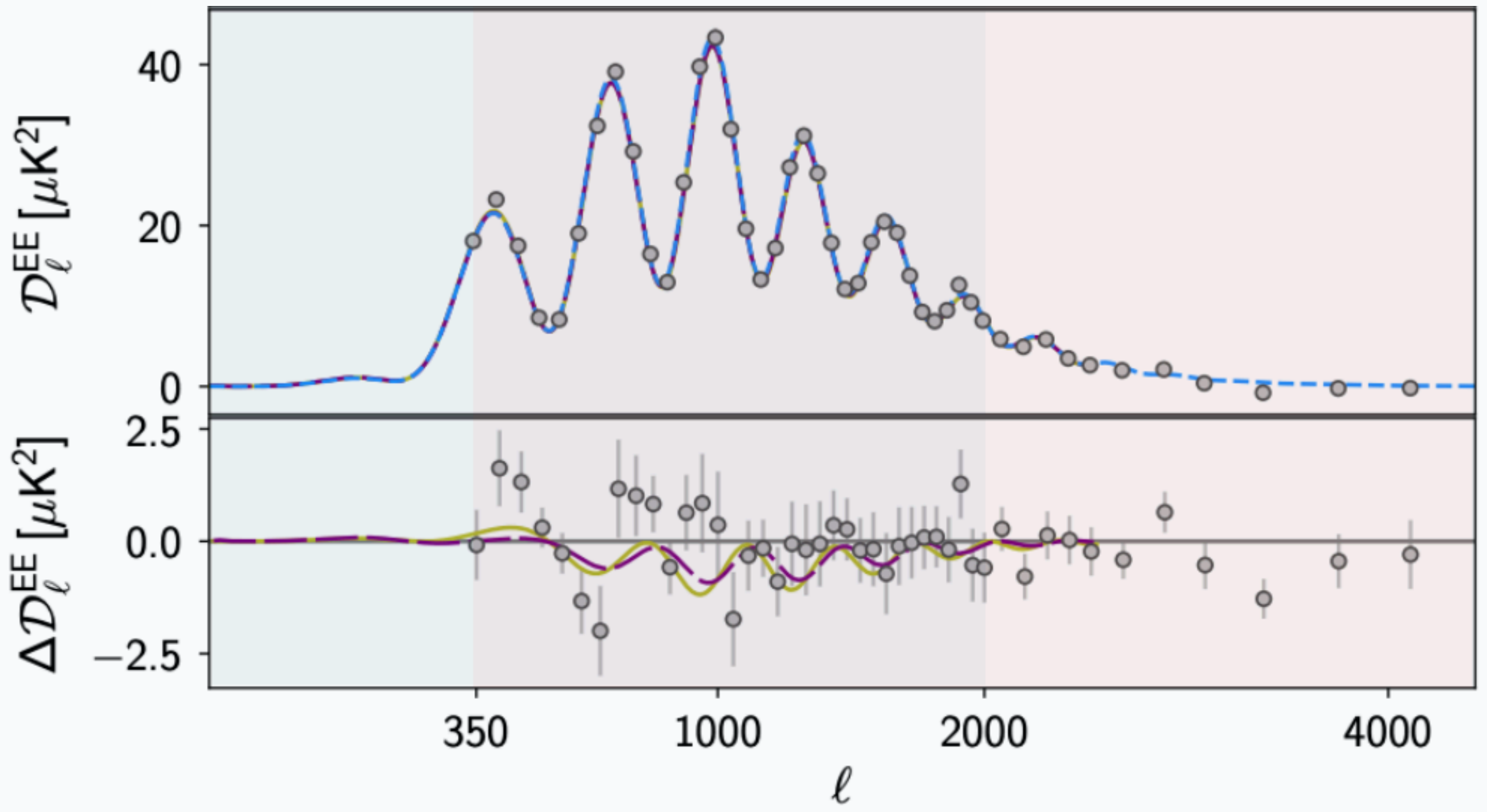
Atacama Cosmology Telescope

2007.07288

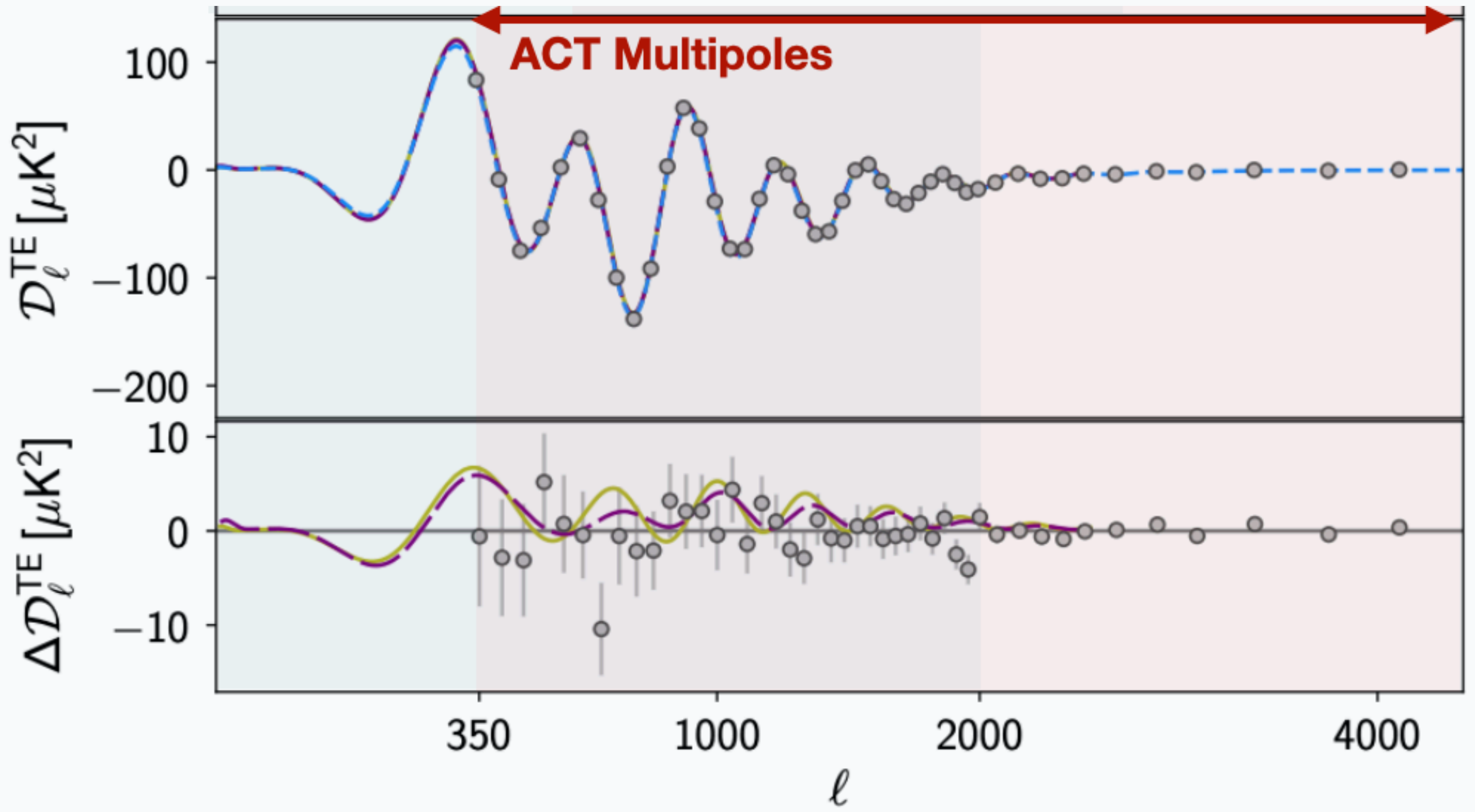
TT Spectrum



EE Spectrum



TE Cross-Spectrum



High-multipole temperature data

$600 < \ell \lesssim 4200$ in the TT Spectrum

High-multipole EE Polarization data

$350 < \ell \lesssim 4200$ in the EE Spectrum

High-multipole TE data

$350 < \ell \lesssim 42000$ in the TE Spectrum

Note:

Planck probes $\ell \in [2{,}2000]$

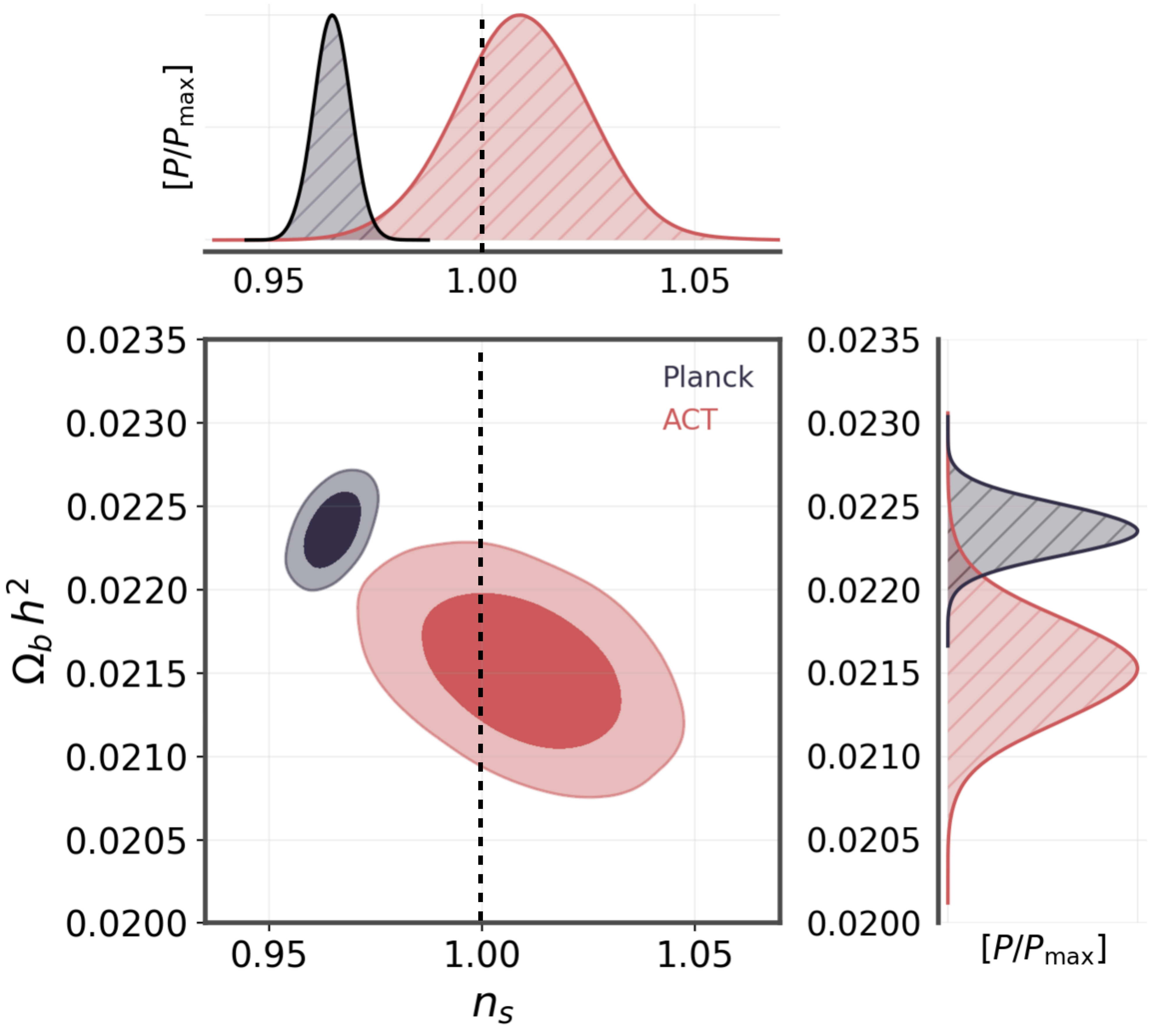
Atacama Cosmology Telescope

2007.07288

ACT shows a preference for $n_s \simeq 1$ (in 3σ disagreement with Planck)

Dataset	Scalar Spectral Index (n_s) ΛCDM
ACT	1.009 ± 0.015
ACT ($\tau = 0.0544 \pm 0.0070$)	1.007 ± 0.015
ACT + Planck low E	1.001 ± 0.011
ACT+BAO (DR12)	1.006 ± 0.013
ACT+BAO (DR16)	1.006 ± 0.014
ACT+DES	1.007 ± 0.013
ACT+SPT+BAO (DR16)	0.997 ± 0.013
ACT+SPT+BAO (DR12)	0.996 ± 0.012
Planck	0.9649 ± 0.0044
Planck+BAO (DR12)	0.9668 ± 0.0038
Planck+BAO (DR16)	0.9677 ± 0.0037
Planck+DES	0.9696 ± 0.0040
Planck ($2 \leq \ell \leq 650$)	0.9655 ± 0.0043
Planck ($\ell > 650$)	0.9634 ± 0.0085

Image taken from arXiv: 2210.09018



Inflation and Early Dark Energy

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Inflation and Early Dark Energy

1) EDE increases the expansion history before recombination $\propto f_{\mathrm{EDE}}$

2) EDE decreases the sound horizon, reducing the Hubble tension $\propto f_{\mathrm{EDE}}$

3) We move towards $n_s \to 1$, tho!

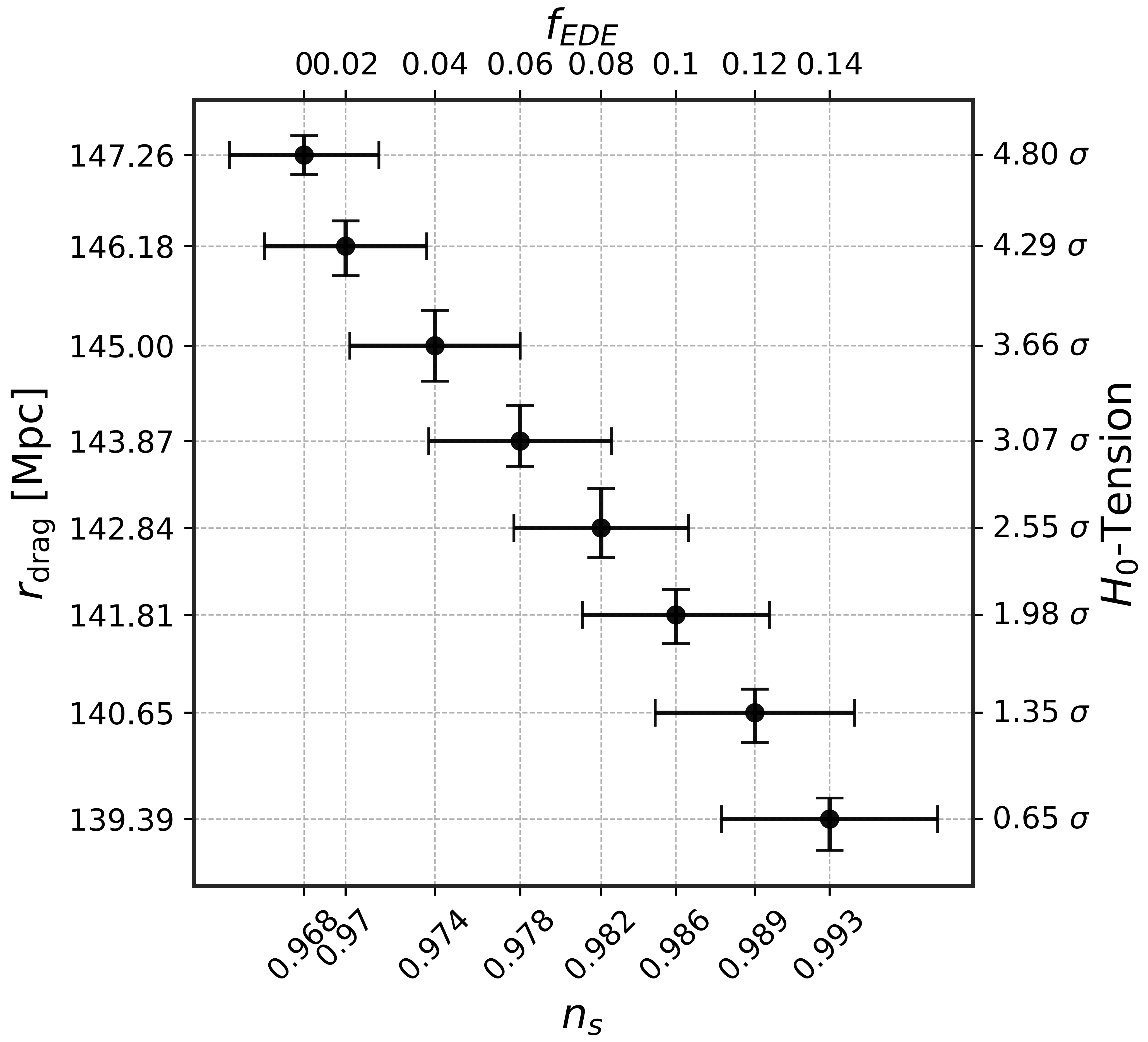

Image taken from arXiv:2404.12779

Inflation and Early Dark Energy

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Implications for Inflation

1) $|\eta| \gg \epsilon$ assuming ΛCDM

2) $|\eta| \gtrsim \epsilon$ for negligible $f_{\rm EDE}$

3) $|\eta| \sim \epsilon$ if EDE solves the H_0 tension

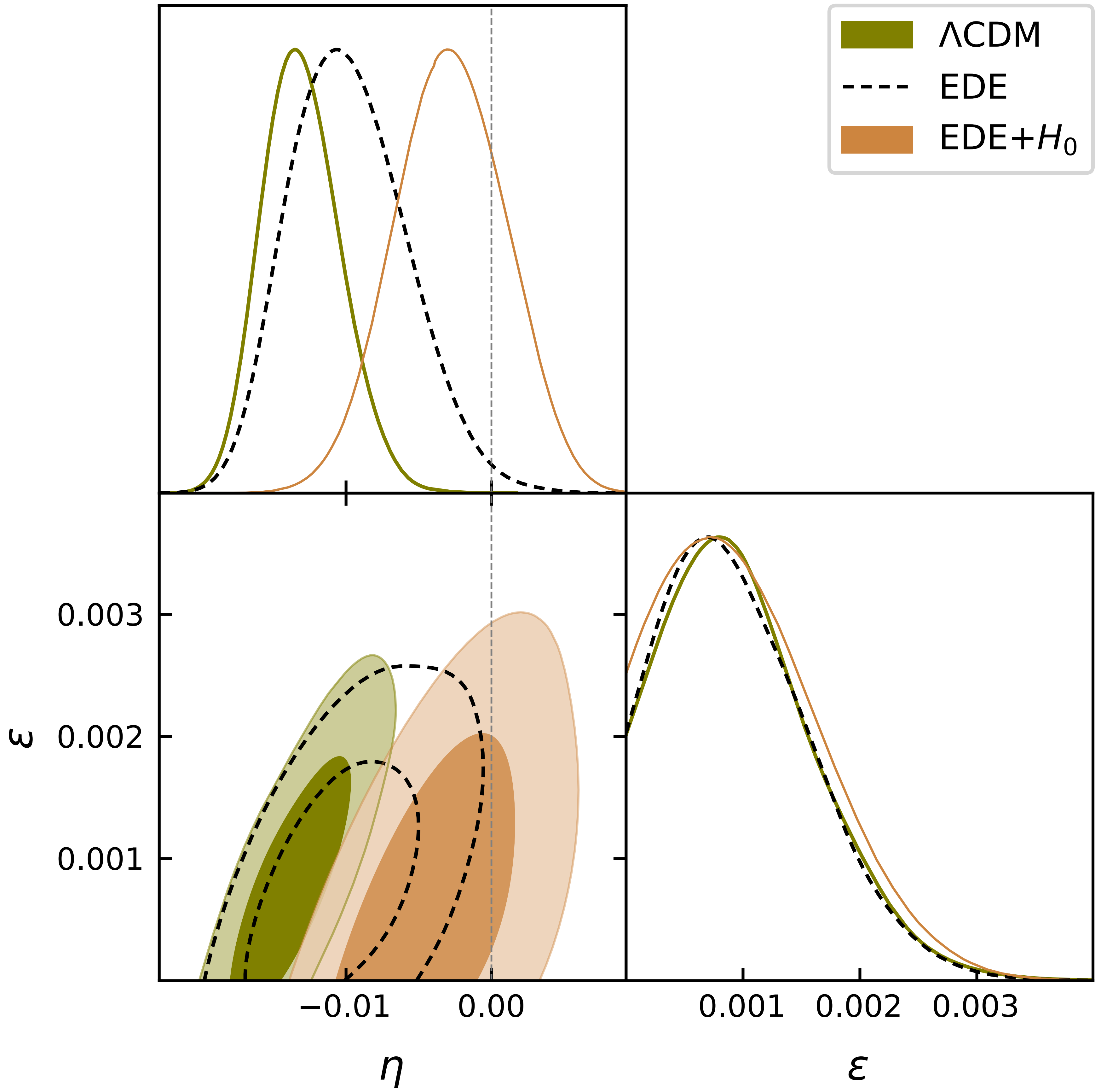

Image taken from arXiv:2404.12779

Inflation and the Neutrino Sector

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Inflation and The effective number of relativistic Particles

1) Increasing $\Delta N_{\rm eff}$ will increase the expansion history before recombination

2) The sound horizon will decrease, reducing the Hubble tension

3) The constraints on $n_s \to 1$

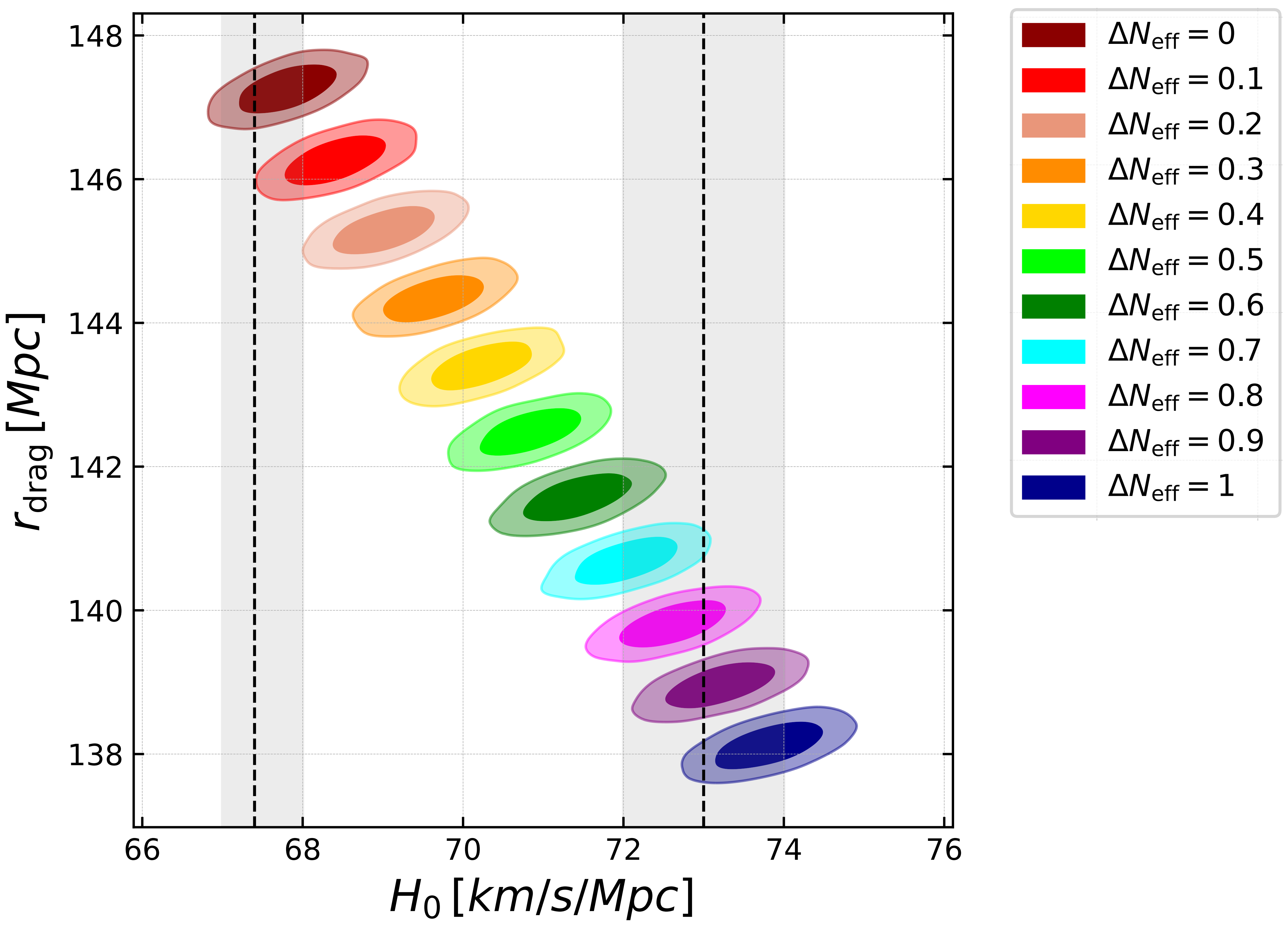

Image taken from arXiv:2404.12779

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Inflation and The effective number of relativistic Particles

1) Increasing $\Delta N_{\rm eff}$ will increase the expansion history before recombination

2) The sound horizon will decrease, reducing the Hubble tension

3) The constraints on $n_s \to 1$

4) Models with large $\Delta N_{\rm eff}$ are not favoured

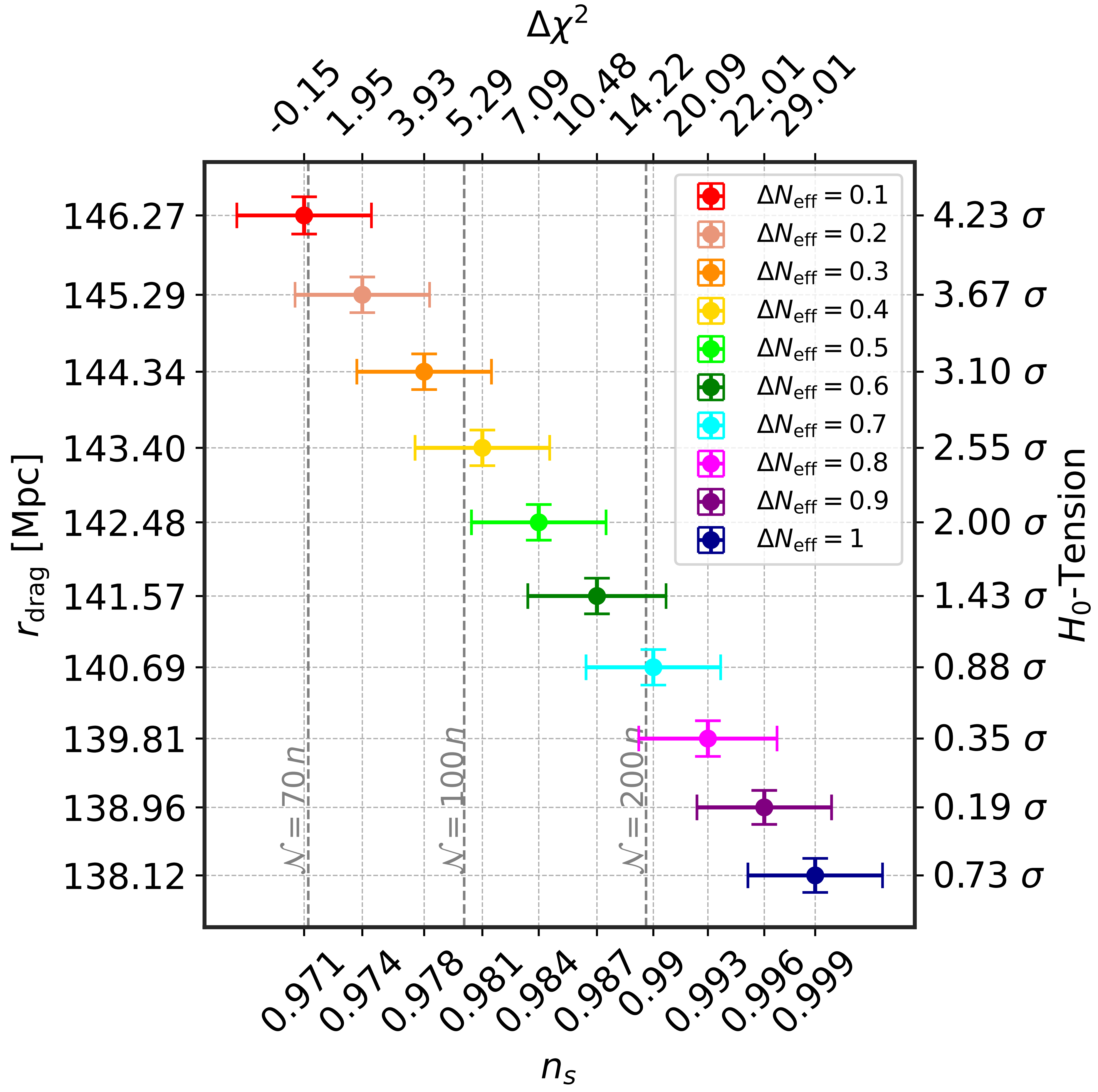

Image taken from arXiv:2404.12779

END of Lecture 2

Thank You!