The irreducible fields of the affine connection are

$$\tilde{\Gamma}_{\alpha}{}^{\gamma}{}_{\beta} = \Gamma_{(\alpha}{}^{\gamma}{}_{\beta)} + \Gamma_{[\alpha}{}^{\gamma}{}_{\beta]}$$

$$= \Gamma_{\alpha}{}^{\gamma}{}_{\beta} + B_{\alpha}{}^{\gamma}{}_{\beta} + \delta^{\gamma}_{[\alpha} A_{\beta]}$$

To preserve the invariance under diffeomorphism $\Gamma_{\alpha}{}^{\gamma}{}_{\beta} \to \nabla_{\mu}$, and the volume element is defined by the wedge product

$$dV^{\alpha\beta\gamma\delta} = dx^{\alpha} \wedge dx^{\beta} \wedge dx^{\gamma} \wedge dx^{\delta} \qquad 4D$$

$$dV^{\alpha\beta\gamma} = dx^{\alpha} \wedge dx^{\beta} \wedge dx^{\gamma} \qquad 3D$$

$$dV^{\alpha\beta} = dx^{\alpha} \wedge dx^{\beta} \qquad 2D$$

Let's define the operators $\hat{N}$ and $\hat{W}$ that counts the number of free indices and the weight of the object respectively:

$\hat{N}(A) = -1 \longleftrightarrow \hat{W}(A) = 0$

$\hat{N}(B) = -1 \longleftrightarrow \hat{W}(B) = 0$

$\hat{N}(D) = -1 \longleftrightarrow \hat{W}(D) = 0$

$\hat{N}(dU) = D \longleftrightarrow \hat{W}(dU) = 1$

dimension

An arbitrary term will have power of each field χ

$$\chi = A^m B^n D^p dU^f$$

the most general scalar density requires that

$$\hat{N}(\chi) = -m - n - p + Df = 0$$

$$\hat{W}(\chi) = f = 1$$

The geometrical constraint equation

$$m + n + p = D$$

2D - action : $m+n+p=2$

	m	n	p	term	configuration
1)	2	0	0	AA	trivial
2)	0	2	0	BB	trivial
3)	0	0	2	DD	2 - terms
4)	1	0	1	AD	trivial
5)	1	1	0	AB	1 - term
6)	0	1	1	BD	1 - term

analysis of each configuration

1) AA $\Rightarrow$ $\int dV^{\alpha\beta} A_\alpha A_\beta = 0$

this produce a trivial term because of the symmetric product $A_\alpha A_\beta$ with the anti-symmetric $dV^{\alpha\beta}$

2) BB $\Rightarrow$ $\int dV^{\alpha\beta} B_\sigma{}^\theta{}_\alpha B_\beta{}^\sigma{}_\theta = 0$

the contraction $B_\sigma{}^\theta{}_\alpha B_\beta{}^\sigma{}_\theta$ is symmetric, just like ①, the term is trivial.

3) $\nabla\nabla \Rightarrow \int dV^{\alpha\beta} \nabla_\alpha \nabla_\beta$

$= \int dV^{\alpha\beta} \{ R_{\alpha\beta}{}^{\sigma}{}_{\sigma} + R_{\alpha\sigma}{}^{\sigma}{}_{\beta} \}$

the anti-symmetrization of the covariants derivatives produce the Riemann curvature tensor.

4) $A\nabla \Rightarrow \int dV^{\alpha\beta} \nabla_\alpha A_\beta$

$= \int dV^{\alpha\beta} F_{\alpha\beta}$ where $F_{\alpha\beta} = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha$

5) $\nabla B \Rightarrow \int dV^{\alpha\beta} \{ \nabla_\sigma B^{\sigma}_{\alpha\beta} \}$

6) $AB \Rightarrow \int dV^{\alpha\beta} \{ A_\sigma B_{\alpha}{}^{\sigma}{}_{\beta} \}$

the action in two dimensions

$S[\nabla, A, B] = \int dV^{\alpha\beta} \{ a_1 R_{\alpha\beta}{}^{\sigma}{}_{\sigma} + a_2 R_{\alpha\sigma}{}^{\sigma}{}_{\beta}$
$+ b_1 F_{\alpha\beta} + c_1 \nabla_\sigma B_{\alpha}{}^{\sigma}{}_{\beta} + d_1 A_\sigma B_{\alpha}{}^{\sigma}{}_{\beta} \}$

There are only five possible non-trivial

contributions in 2-dimensions