From the Riemann curvature tensor, it is possible to compute the variation of the Ricci tensor

$$R^\rho{}_{\mu\sigma\nu} = \partial_\sigma \Gamma^\rho_{\nu\mu} - \partial_\nu \Gamma^\rho_{\sigma\mu} + \Gamma^\rho_{\sigma\lambda}\Gamma^\lambda_{\nu\mu} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\sigma\mu}$$

$$\Rightarrow R_{\mu\nu} = \partial_\rho \Gamma^\rho_{\nu\mu} - \partial_\nu \Gamma^\rho_{\rho\mu} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{\nu\mu} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\rho\mu}$$

$$\Rightarrow \delta R_{\mu\nu} = \partial_\rho \delta\Gamma^\rho_{\nu\mu} - \partial_\nu \delta\Gamma^\rho_{\rho\mu} + \delta\Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{\nu\mu} + \Gamma^\rho_{\rho\lambda}\delta\Gamma^\lambda_{\nu\mu}$$
$$- \delta\Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\rho\mu} - \Gamma^\rho_{\nu\lambda}\delta\Gamma^\lambda_{\rho\mu}$$

the above expression can be written in terms of covariant derivatives of $\delta\Gamma$

$$\nabla_\rho \delta\Gamma^\rho_{\nu\mu} = \partial_\rho \delta\Gamma^\rho_{\nu\mu} + \Gamma^\rho_{\rho\lambda}\delta\Gamma^\lambda_{\nu\mu} - \Gamma^\lambda_{\rho\nu}\delta\Gamma^\rho_{\lambda\mu} - \Gamma^\lambda_{\rho\mu}\delta\Gamma^\rho_{\nu\lambda}$$

$$\nabla_\nu \delta\Gamma^\rho_{\rho\mu} = \partial_\nu \delta\Gamma^\rho_{\rho\mu} + \Gamma^\rho_{\nu\lambda}\delta\Gamma^\lambda_{\rho\mu} - \Gamma^\lambda_{\nu\rho}\delta\Gamma^\rho_{\lambda\mu} - \Gamma^\lambda_{\nu\mu}\delta\Gamma^\rho_{\rho\lambda}$$

substracting the above two expression leads to

$$= \partial_\rho \delta\Gamma^\rho_{\nu\mu} + \Gamma^\rho_{\rho\lambda}\delta\Gamma^\lambda_{\nu\mu} - \Gamma^\lambda_{\rho\nu}\delta\Gamma^\rho_{\lambda\mu} - \Gamma^\lambda_{\rho\mu}\delta\Gamma^\rho_{\nu\lambda}$$
$$- \left[\partial_\nu \delta\Gamma^\rho_{\rho\mu} + \Gamma^\rho_{\nu\lambda}\delta\Gamma^\lambda_{\rho\mu} - \Gamma^\lambda_{\nu\rho}\delta\Gamma^\rho_{\lambda\mu} - \Gamma^\lambda_{\nu\mu}\delta\Gamma^\rho_{\rho\lambda}\right]$$

$$= \partial_\rho \delta\Gamma^\rho_{\nu\mu} - \partial_\nu \delta\Gamma^\rho_{\rho\mu} + \Gamma^\rho_{\rho\lambda}\delta\Gamma^\lambda_{\nu\mu} + \delta\Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{\nu\mu}$$
$$- \Gamma^\lambda_{\rho\mu}\delta\Gamma^\rho_{\nu\lambda} - \delta\Gamma^\lambda_{\rho\mu}\Gamma^\rho_{\nu\lambda} - \delta\Gamma^\rho_{\lambda\mu}\left(\Gamma^\lambda_{\rho\nu} - \Gamma^\lambda_{\nu\rho}\right)$$

$$= \delta R_{\mu\nu} - 2\,\delta\Gamma^{\rho}{}_{\lambda\mu}\, S^{\lambda}{}_{\rho\nu}$$

from which it is possible to deduce an expression for the variation of the Ricci tensor

$$\nabla_{\rho}\, \delta\Gamma^{\rho}{}_{\nu\mu} - \nabla_{\nu}\, \delta\Gamma^{\rho}{}_{\rho\mu} = \delta R_{\mu\nu} - 2\,\delta\Gamma^{\rho}{}_{\lambda\mu}\, S^{\lambda}{}_{\rho\nu}$$

$$\Rightarrow \quad \delta R_{\mu\nu} = \nabla_{\rho}\, \delta\Gamma^{\rho}{}_{\nu\mu} - \nabla_{\nu}\, \delta\Gamma^{\rho}{}_{\rho\mu} + 2\,\delta\Gamma^{\rho}{}_{\lambda\mu}\, S^{\lambda}{}_{\rho\nu}$$

where $S^{\lambda}{}_{\rho\nu}$ is the torsion tensor defined as

$$S^{\lambda}{}_{\rho\nu} = \frac{1}{2}\left\{ \Gamma^{\lambda}{}_{\rho\nu} - \Gamma^{\lambda}{}_{\nu\rho} \right\}$$