Let's compute $[\nabla_\mu, \nabla_\nu] V^\rho$ to obtain the Rieman curvature and the torsion tensor

$$= \nabla_\mu \nabla_\nu V^\rho - \nabla_\nu \nabla_\mu V^\rho$$

$$= \nabla_\mu \left(\partial_\nu V^\rho + \Gamma^\rho_{\beta\nu} V^\beta \right) - \{\mu \leftrightarrow \nu\}$$

$$= \nabla_\mu \left(g_\nu{}^\rho + f^\rho{}_\nu \right) - \{\mu \leftrightarrow \nu\}$$

$$= \nabla_\mu g^\rho{}_\nu + \nabla_\mu f^\rho{}_\nu$$

$$= \partial_\mu g^\rho{}_\nu + \Gamma^\rho_{\alpha\mu} g^\alpha{}_\nu - \Gamma^\alpha_{\nu\mu} g^\rho{}_\alpha + \partial_\mu f^\rho{}_\nu + \Gamma^\rho_{\alpha\mu} f^\alpha{}_\nu$$

$$- \Gamma^\alpha_{\nu\mu} f^\rho{}_\alpha - \{\mu \leftrightarrow \nu\}$$

$$= \partial_\mu \partial_\nu V^\rho + \Gamma^\rho_{\alpha\mu} \partial_\nu V^\alpha - \Gamma^\alpha_{\nu\mu} \partial_\alpha V^\rho +$$

$$\partial_\mu \left(\Gamma^\rho_{\beta\nu} V^\beta \right) + \Gamma^\rho_{\alpha\mu} \Gamma^\alpha_{\beta\nu} V^\beta - \Gamma^\alpha_{\nu\mu} \Gamma^\rho_{\beta\alpha} V^\beta - \{\mu \leftrightarrow \nu\}$$

$$= \partial_\mu \partial_\nu V^\rho + \Gamma^\rho_{\alpha\mu} \partial_\nu V^\alpha - \Gamma^\alpha_{\nu\mu} \partial_\alpha V^\rho + \partial_\mu \Gamma^\rho_{\beta\nu} V^\beta$$

$$+ \Gamma^\rho_{\beta\nu} \partial_\mu V^\beta + \Gamma^\rho_{\alpha\mu} \Gamma^\alpha_{\beta\nu} V^\beta - \Gamma^\alpha_{\nu\mu} \Gamma^\rho_{\beta\alpha} V^\beta -$$

$$\Big[\partial_\nu \partial_\mu V^\rho + \Gamma^\rho_{\alpha\nu} \partial_\mu V^\alpha - \Gamma^\alpha_{\mu\nu} \partial_\alpha V^\rho + \partial_\nu \Gamma^\rho_{\beta\mu} V^\beta$$

$$+ \Gamma^\rho_{\beta\mu} \partial_\nu V^\beta + \Gamma^\rho_{\alpha\nu} \Gamma^\alpha_{\beta\mu} V^\beta - \Gamma^\alpha_{\mu\nu} \Gamma^\rho_{\beta\alpha} V^\beta \Big]$$

$$= -\Gamma^{\alpha}{}_{\nu\mu}\,\partial_\alpha V^\rho + \partial_\mu \Gamma^{\rho}{}_{\beta\nu} V^\beta + \Gamma^{\rho}{}_{\alpha\mu}\Gamma^{\alpha}{}_{\beta\nu} V^\beta - \Gamma^{\alpha}{}_{\nu\mu}\Gamma^{\rho}{}_{\beta\alpha} V^\beta$$

$$- \left[-\Gamma^{\alpha}{}_{\mu\nu}\,\partial_\alpha V^\rho + \partial_\nu \Gamma^{\rho}{}_{\beta\mu} V^\beta + \Gamma^{\rho}{}_{\alpha\nu}\Gamma^{\alpha}{}_{\beta\mu} V^\beta - \Gamma^{\alpha}{}_{\mu\nu}\Gamma^{\rho}{}_{\beta\alpha} V^\beta \right]$$

$$\nabla_\alpha V^\rho = \partial_\alpha V^\rho + \Gamma^{\rho}{}_{\beta\alpha} V^\beta$$

$$= \partial_\mu \Gamma^{\rho}{}_{\beta\nu} V^\beta + \Gamma^{\rho}{}_{\alpha\mu}\Gamma^{\alpha}{}_{\beta\nu} V^\beta - \Gamma^{\alpha}{}_{\nu\mu}\left[\partial_\alpha V^\rho + \Gamma^{\rho}{}_{\beta\alpha} V^\beta\right]$$

$$- \left[\partial_\nu \Gamma^{\rho}{}_{\beta\mu} V^\beta + \Gamma^{\rho}{}_{\alpha\nu}\Gamma^{\alpha}{}_{\beta\mu} V^\beta - \Gamma^{\alpha}{}_{\mu\nu}\left[\partial_\alpha V^\rho + \Gamma^{\rho}{}_{\beta\alpha} V^\beta\right]\right]$$

$$= \partial_\mu \Gamma^{\rho}{}_{\beta\nu} V^\beta + \Gamma^{\rho}{}_{\alpha\mu}\Gamma^{\alpha}{}_{\beta\nu} V^\beta - \Gamma^{\alpha}{}_{\nu\mu}\nabla_\alpha V^\rho$$

$$- \partial_\nu \Gamma^{\rho}{}_{\beta\mu} V^\beta - \Gamma^{\rho}{}_{\alpha\nu}\Gamma^{\alpha}{}_{\beta\mu} V^\beta + \Gamma^{\alpha}{}_{\mu\nu}\nabla_\alpha V^\rho$$

$$= \left[\partial_\mu \Gamma^{\rho}{}_{\beta\nu} - \partial_\nu \Gamma^{\rho}{}_{\beta\mu} + \Gamma^{\rho}{}_{\alpha\mu}\Gamma^{\alpha}{}_{\beta\nu} - \Gamma^{\rho}{}_{\alpha\nu}\Gamma^{\alpha}{}_{\beta\mu}\right] V^\beta$$

$$+ \nabla_\alpha V^\rho \left[\Gamma^{\alpha}{}_{\mu\nu} - \Gamma^{\alpha}{}_{\nu\mu}\right]$$

$$= R^{\rho}{}_{\nu\mu\beta} V^\beta + 2\nabla_\alpha V^\rho\, T^{\alpha}{}_{\mu\nu}$$

where $R^{\alpha}{}_{\beta\gamma\delta}$ is the Rieman curvature tensor

and $2T^{\alpha}{}_{\mu\nu} = \Gamma^{\alpha}{}_{\mu\nu} - \Gamma^{\alpha}{}_{\nu\mu}$ is the torsion tensor.