Personal notes on gravitational waves

Jose Perdiguero Garate

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Notation and conventions

I am working with the following conventions, for the metric tensor

$$(-1, +1, +1, +1) \tag{1}$$

1 Linear general relativity

In this section I am presenting a general overview on how to build-up the Einstein's field equation, given a background metric tensor and a perturbation tensor.

1.1 General overview

Decomposed the metric tensor as the sum of a background metric, in this case the flat spacetime Minkowski metric $\eta_{\mu\nu}$,plus a perturbation $h_{\mu\nu}$ as follows

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{2}$$

Using the definition of the Kronecker delta object it is straightforward to obtain the inverse tensor of the perturbation

$$\delta^{\mu}_{\nu} = g^{\mu\sigma} g_{\sigma\nu}. \tag{3}$$

Replacing Eq.(2) in to Eq.(3), and neglecting second order terms in the perturbation field, leads to

$$h^{\mu\nu} = -\eta^{\mu\alpha}\eta^{\nu\beta} h_{\alpha\beta}, \tag{4}$$

notice that, in order to upper/lower the indices of the perturbation I am only using the background metric.[1] Schematically, second order terms are neglected

$$h_{\mu\nu} h_{\alpha\beta} \sim 0, \qquad h_{\mu\nu}\partial_\gamma h_{\alpha\beta} \sim 0, \qquad \partial_\delta h_{\mu\nu}\partial_\gamma h_{\alpha\beta} \sim 0. \tag{5}$$

This is the general overview of the fundamental field of general relativity, which at its core is the metric tensor. In the following subsection, I will be computing the Einstein's field equations for the metric tensor written in Eq.2.

[1] Including the perturbation tensor leads to second order terms, which I am ignoring.

2 Curved gravity