Spherically solutions of Polynomial Affine Gravity in the torsion-free sector

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1 Introduction

2 Polynomial Affine Gravity

The Polynomial Affine Gravity uses the affine connection to mediated gravitational interactions, instead of the metric tensor. To build the action, we decompose the affine connection into its irreducible fields

$$\begin{aligned} \hat{\Gamma}_{\alpha}{}^{\beta}{}_{\gamma} &= \hat{\Gamma}_{(\alpha}{}^{\beta}{}_{\gamma)} + \hat{\Gamma}_{[\alpha}{}^{\beta}{}_{\gamma]}, \\ &= \Gamma_{\alpha}{}^{\beta}{}_{\gamma} + \mathcal{B}_{\alpha}{}^{\beta}{}_{\gamma} + \delta^{\beta}_{[\gamma}\mathcal{A}_{\alpha]}, \end{aligned} \tag{1}$$

where the first term Γ stands for the symmetric part and $\mathcal{A}$, $\mathcal{B}$ fields are related to the skew symmetric part of the affine connection.

Additionally, is necessary to introduce a volume form without the use of the metric tensor. In order to achieve this, we used the volume element defined as $\mathrm{d}V^{\alpha\beta\gamma\delta} = \mathrm{d}x^{\alpha} \wedge \mathrm{d}x^{\beta} \wedge \mathrm{d}x^{\gamma} \wedge \mathrm{d}x^{\delta}$, which is completely antisymmetric.

Moreover, we want to preserve the invariance under diffeomorphism, which is why the symmetric part of the affine connection, goes indirectly through the covariant derivative ∇^{Γ}.

The action is built up using a sort of *dimensional analysis* technique. This strategy allows us to generate every scalar density composed by powers of the irreducible fields of the affine connection.

The most general action (up to topological invariants and boundary terms) in four dimensions is given by

$$
\begin{aligned}
S = \int \mathrm{d}V^{\alpha\beta\gamma\delta}\Big[& B_1 \mathcal{R}_{\mu\nu}{}^{\mu}{}_{\rho} \mathcal{B}_{\alpha}{}^{\nu}{}_{\beta} \mathcal{B}_{\gamma}{}^{\rho}{}_{\delta} + B_2 \mathcal{R}_{\alpha\beta}{}^{\mu}{}_{\rho} \mathcal{B}_{\gamma}{}^{\nu}{}_{\delta} \mathcal{B}_{\mu}{}^{\rho}{}_{\nu} + B_3 \mathcal{R}_{\mu\nu}{}^{\mu}{}_{\alpha} \mathcal{B}_{\beta}{}^{\nu}{}_{\gamma} \mathcal{A}_{\delta} \\
& + B_4 \mathcal{R}_{\alpha\beta}{}^{\sigma}{}_{\rho} \mathcal{B}_{\gamma}{}^{\rho}{}_{\delta} \mathcal{A}_{\sigma} + B_5 \mathcal{R}_{\alpha\beta}{}^{\rho}{}_{\rho} \mathcal{B}_{\gamma}{}^{\sigma}{}_{\delta} \mathcal{A}_{\sigma} + C_1 \mathcal{R}_{\mu\alpha}{}^{\mu}{}_{\nu} \nabla_{\beta} \mathcal{B}_{\gamma}{}^{\nu}{}_{\delta} \\
& + C_2 \mathcal{R}_{\alpha\beta}{}^{\rho}{}_{\rho} \nabla_{\sigma} \mathcal{B}_{\gamma}{}^{\sigma}{}_{\delta} + D_1 \mathcal{B}_{\nu}{}^{\mu}{}_{\lambda} \mathcal{B}_{\mu}{}^{\nu}{}_{\alpha} \nabla_{\beta} \mathcal{R}_{\gamma}{}^{\lambda}{}_{\delta} + D_2 \mathcal{B}_{\alpha}{}^{\mu}{}_{\beta} \mathcal{B}_{\mu}{}^{\lambda}{}_{\nu} \nabla_{\lambda} \mathcal{B}_{\gamma}{}^{\nu}{}_{\delta} \\
& + D_3 \mathcal{B}_{\alpha}{}^{\mu}{}_{\nu} \mathcal{B}_{\beta}{}^{\lambda}{}_{\gamma} \nabla_{\lambda} \mathcal{B}_{\mu}{}^{\nu}{}_{\delta} + D_4 \mathcal{B}_{\alpha}{}^{\lambda}{}_{\beta} \mathcal{B}_{\gamma}{}^{\sigma}{}_{\delta} \nabla_{\lambda} \mathcal{A}_{\sigma} + D_5 \mathcal{B}_{\alpha}{}^{\lambda}{}_{\beta} \mathcal{A}_{\sigma} \nabla_{\lambda} \mathcal{B}_{\gamma}{}^{\sigma}{}_{\delta} \\
& + D_6 \mathcal{B}_{\alpha}{}^{\lambda}{}_{\beta} \mathcal{A}_{\gamma} \nabla_{\lambda} A_{\delta} + D_7 \mathcal{B}_{\alpha}{}^{\lambda}{}_{\beta} \mathcal{A}_{\lambda} \nabla_{\gamma} A_{\delta} + E_1 \nabla_{\rho} \mathcal{B}_{\alpha}{}^{\rho}{}_{\beta} \nabla_{\sigma} \mathcal{B}_{\gamma}{}^{\sigma}{}_{\delta} \\
& + E_2 \nabla_{\rho} \mathcal{B}_{\alpha}{}^{\rho}{}_{\beta} \nabla_{\gamma} \mathcal{A}_{\delta} + F_1 \mathcal{B}_{\alpha}{}^{\mu}{}_{\beta} \mathcal{B}_{\gamma}{}^{\sigma}{}_{\delta} \mathcal{B}_{\mu}{}^{\lambda}{}_{\rho} \mathcal{B}_{\sigma}{}^{\rho}{}_{\lambda} + F_2 \mathcal{B}_{\alpha}{}^{\mu}{}_{\beta} \mathcal{B}_{\gamma}{}^{\nu}{}_{\lambda} \mathcal{B}_{\delta}{}^{\lambda}{}_{\rho} \mathcal{B}_{\mu}{}^{\rho}{}_{\nu} \\
& + F_3 \mathcal{B}_{\nu}{}^{\mu}{}_{\lambda} \mathcal{B}_{\mu}{}^{\nu}{}_{\alpha} \mathcal{B}_{\beta}{}^{\lambda}{}_{\gamma} \mathcal{A}_{\delta} + F_4 \mathcal{B}_{\alpha}{}^{\mu}{}_{\beta} \mathcal{B}_{\gamma}{}^{\nu}{}_{\delta} \mathcal{A}_{\mu} \mathcal{A}_{\nu} \Big].
\end{aligned}
\tag{2}
$$

3 Solutions

4 Final remarks