MEASURES OF CENTRAL TENDENCY FOR UNGROUPED DATA

An Overview

MEAN (ARITHMETIC MEAN)

- The most common average.
- The sum of all values of the observations divided by the number of observations.

Formula:

$$\mu = \frac{\sum_{i=1}^{N} X_i}{N} \text{(population mean)}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} X_i}{n} \text{(sample mean)}$$

EXAMPLE 1:

Calculate the mean of the following:

2,4,6,8,10

Solution:

$$\bar{x} = \frac{\sum_{i=1}^{4} X_i}{5} = \frac{2+4+6+8+10}{5}$$

$$\bar{x} = \frac{30}{5} = 6$$

EXAMPLE 2:

Calculate the mean of the following:

47 350, 39 500, 38 000, 41 250, 44 000

Solution:

$$\bar{x} = \frac{\sum_{i=1}^{5} X_i}{5} = \frac{47\,350 + 39\,500 + 38\,000 + 41\,250 + 44\,000}{5}$$

$$\bar{x} = \frac{210\,100}{5} = 42\,020$$

EXAMPLE 3:

Six friends in a mathematics class of 20 students received test grades of 92, 84, 65, 76, 88 and 90. Find the mean of these test scores.

Solution:

$$\bar{x} = \frac{\sum_{i=1}^{6} X_i}{6} = \frac{92 + 84 + 85 + 76 + 88 + 90}{6}$$

$$\bar{x} = \frac{495}{6} = 82.5$$

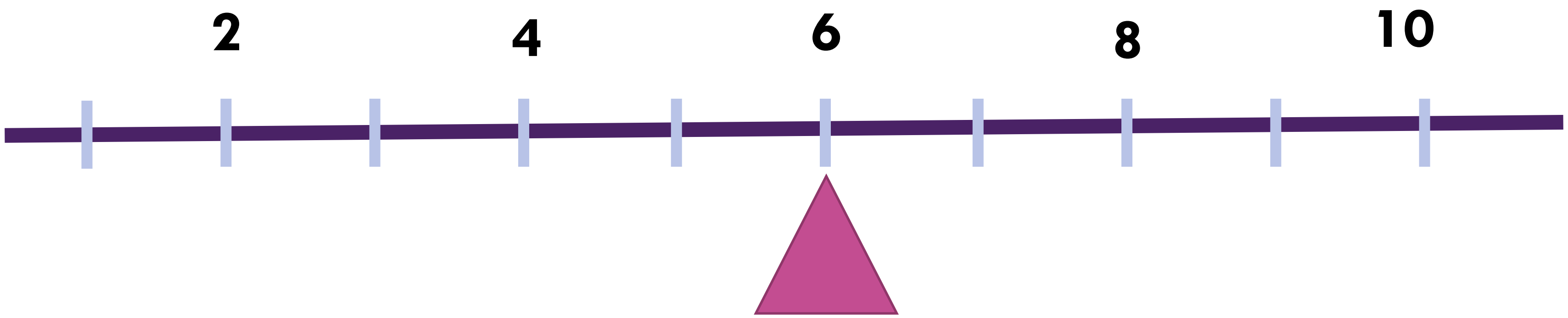
PROPERTIES OF THE MEAN

1. The mean is sensitive to the **exact value** of all the scores in the distribution.
2. The sum of the deviations about the mean equals to zero.

X_i	$X_i - \bar{x}$
2	$(2-6) = -4$
4	$(4-6) = -2$
6	$(6-6) = 0$
8	$(8-6) = +2$
10	$(10-6) = +4$
	$\sum(X_i - \bar{x}) = 0$

PROPERTIES OF THE MEAN

3. The mean is sensitive to extreme scores.



PROPERTIES OF THE MEAN

4. The sum of the squared of the deviations is the minimum.

X_i	$(X_i - 4)^2$	$(X_i - 6)^2$	$(X_i - 8)^2$
2	4	16	36
4	0	4	16
6	4	0	4
8	16	4	0
10	36	16	4
	68	40	60

5. Under most circumstances, of the measures used for central tendency, the mean is least subject to sampling variation

MEDIAN

- the positional middle of the arrayed data.
- in an array, one half of the values precede the median one half follow it.

Notation:

$\widetilde{\mu}$ (population median)

$\widetilde{x}$ (sample median)

EXAMPLE 4:

Calculate the median of the following:

$$8,10,4,3,1,15,9$$

Step 1: Arrange the data in an array.

$$1 \quad 3 \quad 4 \quad \boxed{8} \quad 9 \quad 10 \quad 15$$

Step 2: Determine the middle observation.

$$\tilde{x} = 8$$

EXAMPLE 5:

Calculate the median of the following:

46 23 92 89 77 108

Step 1: Arrange the data in an array.

$$23 \quad 46 \quad \boxed{77} \quad \boxed{89} \quad 92 \quad 108$$

Step 2: Determine the middle observation.

$$\tilde{x} = \frac{77 + 89}{2} = 83$$

PROPERTIES OF THE MEDIAN

1. The median is **less sensitive** than the mean to extreme scores.

Illustration:

Scores	Mean	Median
3,4,6,7,10	6	6
3,4,6,7,100	24	6
3,4,6,7,1000	204	6

PROPERTIES OF THE MEDIAN

2. Under usual circumstances, the median is more subject to sampling variability than the mean but less subject to sampling variability than the mode.

MODE

- the observed value that occurs most frequently
- locates the point where the observation values occur with the greatest density
- generally a less popular than the mean or the median.

Notation:

$\widehat{\mu}$ **(population mode)**

$\widehat{x}$ **(sample mode)**

MODE (continuation)

- In some data sets, the mode may not be unique.
  - Unimodal if the data set has a unique mode.
  - Bimodal if there are two modes.
  - Trimodal if there are three modes.
  - Multimodal if there are more than three modes.

EXAMPLE 6:

Find the mode of the following:

a. 18, 15, 21, 16, 15, 14, 15, 21 $\hat{x} = 15$ $unimodal$

b. 2,5,8,9,11,4,7,23 $no\ mode$

c. 6,2,0,5,25,20,6,25 $\hat{x} = 6, 25$ $bimodal$

d. S,L,M,M,L,XL,S,L,S,XL,M $\hat{x} = S, M, L$ $trimodal$

e. A,A,B,B,C,C,D,D,F $\hat{x} = A, B, C, D$ $multimodal$

f. A,A,AB,AB,O,O,B,B $no\ mode$

WEIGHTED MEAN

It is the modification of a usual mean that assigns weights (or measure of relative importance) to the observations to be averaged. If each observations X_i is assigned a weight W_i, where $i = 1, 2, \ldots, n$; the weighted mean is given by

$$\overline{X_w} = \frac{\sum_{i=1}^{n} W_i X_i}{\sum_{i=1}^{n} W_i}$$

EXAMPLE 7:

Suppose a teacher assigns the following weights to the various course requirements:

• Assignments	15%
• Project	25%
• Midterm Examination	20%
• Final Examination	40%

The maximum score a student may obtain for each component is 100. Jeffry obtains marks of 83 for assignments, 72 for the project, 41 for the midterm exam, and 47 for the final exam. Find his mean rank for the course.

SOLUTION:

Course Requirements	Percentage	Grade	W_iX_i
Assignments	15%	83	12.45
Project	25%	72	18
Midterm Examination	20%	41	8.2
Final Examination	40%	47	18.80
Total	100%	243	57.45

$$\overline{X_w} = \frac{\sum_{i=1}^{4} W_i X_i}{\sum_{i=1}^{4} W_i} = \frac{57.45}{100\%} = 57.45$$

EXAMPLE 8:

Table 4.1 shows A's first semester course grades. Use weighted mean formula to compute A's GWA for the semester.

Course	Course Grade	Course Unit
English	B	4
History	A	3
Chemistry	D	3
Algebra	C	4

In A's university, they use the 4-point grading system wherein A=4, B=3, C=2, D=1 and F=0.

SOLUTION:

Course	Course Grade	Course Unit	W_iX_i
English	B	4	12
History	A	3	12
Chemistry	D	3	3
Algebra	C	4	8
Total		14	35

In A's university, they use the 4-point grading system wherein A=4, B=3, C=2, D=1 and F=0.

$$\overline{X_w} = \frac{\sum_{i=1}^{4} W_iX_i}{\sum_{i=1}^{4} W_i} = \frac{35}{14} = 2.5$$