SET readout using RF reflectometry and kinetic inductance nonlinearity

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Meaning of the title

- **SET:** Single electron transistor
- **RF reflectometry:** Measuring method for the SET
- **Kinetic inductance:** Inductance using kinetic energy

Outline of defense

Why silicon-based quantum computing?

Other forms of quantum computing

- Decent scalability ($\geq$ 100 qubits)
- Excellent fidelity ($\geq$ 0.9999)

Silicon-based quantum computing

- Excellent scalability ($\geq$ 6000000+ qubits)
- Decent fidelity ($\geq$ 0.99)

Objectives

1. Learn basics of: SET functions, RF reflectometry, kinetic inductance non-linearity.
2. Analyze conventional measuring circuit & optimize
3. Explore using a kinetic inductor in measuring circuit

3 Theoretical background

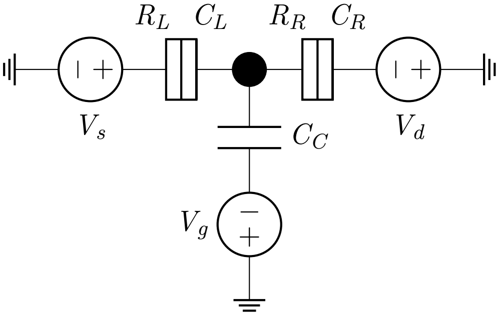
- The SET for spin sensing
- RF reflectometry
- Kinetic inductance and his nonlinearity

Coulomb blockade

$$E = \frac{Q^2}{2C} = \frac{e^2}{2C} N^2 = E_C N^2$$

$$E_N = E_C N^2 - E_C (N-1)^2 = E_C (2N - 1)$$

SET Circuit



Conditions for SET operation

- $R \gg 50\mathrm{k}\Omega$
- $2E_C/e > |V|$
- $E_C > k_B T$

Spin to charge conversion for SET

- **Elzerman readout:** Hopping to charge reservoir depends on spin
- **Pauli Spin Blockade:** Hopping to QD with spin inside depends on spin

3 Theoretical background

- The SET for spin sensing
- RF reflectometry
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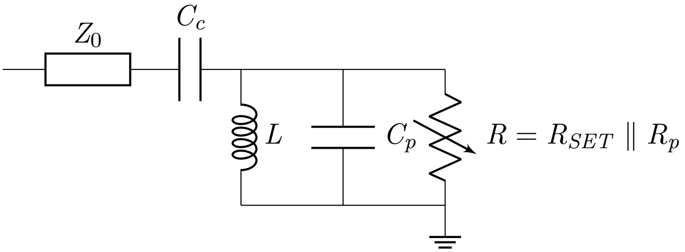
Reflection coefficient and SNR

$$\Gamma = \frac{Z(\omega) - Z_0}{Z(\omega) + Z_0} \qquad \mathrm{SNR} = |\Delta\Gamma|^2 \frac{P_0}{P_N}$$

Kinetic inductance nonlinearity

$$L_k = L_{k0}\left(1 + \frac{j^2}{j_*^2} + \dots\right)$$

Measuring circuit



4 Results

- The parallel RLC resonator
- The parallel kinetic RLC resonator

Effective frequency and impedance

$$Z(\omega) = \frac{1}{j\omega C_p + \frac{1}{j\omega L} + \frac{1}{R}} + \frac{1}{j\omega C_c}$$

$$\Downarrow$$

$$\omega_r = \frac{1}{\sqrt{L(C_c + C_p)}} \text{ with } \frac{C_c}{C_p}, \frac{L}{R^2 C_p} \ll 1$$

$$Z_{\mathit{eff}} = \frac{L(C_c + C_p)}{R C_c^2} \text{ with } \frac{L}{R^2 C_p} \ll \left(\frac{C_c}{C_p}\right)^2$$

- The parallel RLC resonator
  - Contrast and optimization

What variable to optimize for?

$$\omega = \frac{1}{\sqrt{L(C_c + C_p)}}$$

L to select a suitable frequency, C_c as the optimization variable

Optimum C_c

$$\omega = \frac{1}{\sqrt{L(C_c + C_p)}}$$

$$\begin{aligned} Z(\omega) &= \frac{1}{j\omega C_p + \frac{1}{j\omega L} + \frac{1}{R}} + \frac{1}{j\omega C_c} \\ &= \frac{1}{RS^2 + jS} \text{ with } S = \frac{C_c}{\sqrt{L(C_c + C_p)}} = \omega C_c \end{aligned}$$

Optimum C_c

$$
\begin{aligned}
\Gamma &= \frac{Z(\omega) - Z_0}{Z(\omega) + Z_0} \\
&\approx \frac{2\,Y_0}{RS^2 + Y_0} - 1 \text{ with } \omega \approx \omega_r \text{ and } (RS^2 + Y_0)^2 \gg S^2
\end{aligned}
$$

$$
\begin{aligned}
|\Delta\Gamma| &= |\Gamma(R = R_{\mathrm{Off}}) - \Gamma(R = R_{\mathrm{On}})| \\
&\approx 2\,Y_0 \left| \frac{1}{R_{\mathrm{Off}} S^2 + Y_0} - \frac{1}{R_{\mathrm{On}} S^2 + Y_0} \right|
\end{aligned}
$$

Optimum C_c

$$\partial_{C_c}|\Delta\Gamma| = 0$$

$$\Downarrow$$

$$S^2 = \frac{Y_0}{\sqrt{R_{\mathrm{Off}} R_{\mathrm{On}}}}$$

$$\Downarrow$$

$$C_{c\mathrm{Max}} = \frac{L Y_0}{2\sqrt{R_{\mathrm{Off}} R_{\mathrm{On}}}} \left(1 + \sqrt{1 + 4\, C_p \frac{\sqrt{R_{\mathrm{Off}} R_{\mathrm{On}}}}{L Y_0}} \right)$$

Introduction of ρ

$$\rho = \frac{R_{\mathrm{Off}}}{R_{\mathrm{On}}} \geq 1$$

$$S^2 = \frac{Y_0}{\sqrt{\rho} R_{\mathrm{On}}}$$

$$C_{c\mathrm{Max}} = \frac{L Y_0}{2\sqrt{\rho} R_{\mathrm{On}}} \left(1 + \sqrt{1 + 4\, C_p \frac{\sqrt{\rho} R_{\mathrm{On}}}{L Y_0}} \right)$$

Optimized contrast

$$Z_0 = \frac{L(C_c + C_p)}{\sqrt{\rho} R_{\mathrm{On}} C_c^2}$$

$$\begin{aligned} |\Delta\Gamma| &\approx 2\, Y_0 \left| \frac{1}{R_{\mathrm{Off}} S^2 + Y_0} - \frac{1}{R_{\mathrm{On}} S^2 + Y_0} \right| \\ &= 2 \left| \frac{1}{\sqrt{\rho}+1} - \frac{\sqrt{\rho}}{1+\sqrt{\rho}} \right| = 2 \left| \frac{1-\sqrt{\rho}}{1+\sqrt{\rho}} \right| \end{aligned}$$

Introducing the parasitic resistance

$$\pi = \frac{R_p}{R_{\mathrm{On}}^{SET}} \text{ with } 0 < \pi < \infty$$

$$R_{\mathrm{On}} = \frac{\pi}{1+\pi} R_{\mathrm{On}}^{SET}$$

$$R_{\mathrm{Off}} = \frac{\rho_{SET}\pi}{\rho_{SET}+\pi} R_{\mathrm{On}}^{SET}$$

$$\rho = \frac{\rho_{SET}(1+\pi)}{\rho_{SET}+\pi}$$

Introducing the parasitic resistance

With $\rho_{SET} \approx \infty$

$$R_{\mathrm{On}} = \frac{\pi}{1+\pi} R_{\mathrm{On}}^{SET}$$

$$R_{\mathrm{Off}} = \pi R_{\mathrm{On}}^{SET}$$

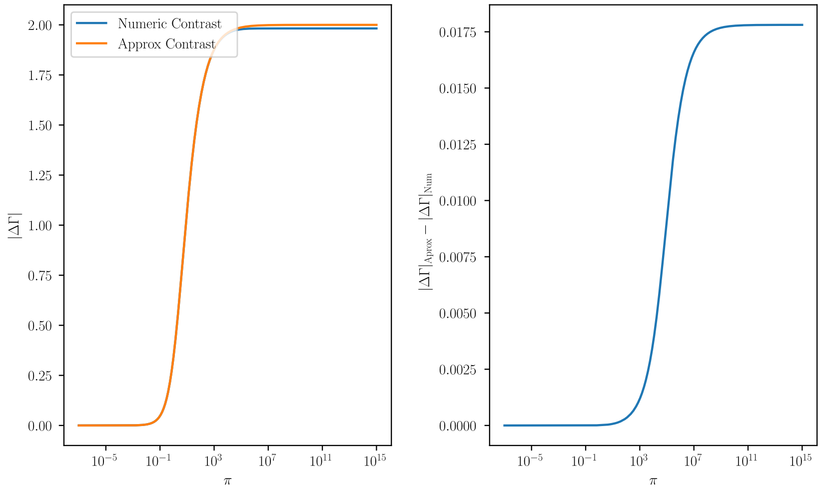
$$\rho = 1 + \pi$$

$$|\Delta\Gamma| \approx 2 \left| \frac{1 - \sqrt{1+\pi}}{1 + \sqrt{1+\pi}} \right|$$

Checking with a simulation

Z_0	R_{On}^{SET}	ρ_{SET}	C_p	L
50Ω	$50\text{k}\Omega$	$2 \cdot 10^6$	500fF	180nH

Checking with simulations



4 Results

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- The parallel kinetic RLC resonator

Introducing a variable inductance

$$\lambda = \frac{L_{\mathrm{Off}}}{L_{\mathrm{On}}} \leq 1 \text{ with } 0 < \lambda \leq 1$$

$$\omega(\lambda_t) = \frac{1}{\sqrt{\lambda_t L_{\mathrm{On}}(C_c + C_p)}} \text{ with } \lambda \leq \lambda_t \leq 1$$

Introducing a variable inductance

- L_{On} tuning ($\lambda_t = 1$)
- L_{Off} tuning ($\lambda_t = \lambda$)
- Middle tuning ($\lambda < \lambda_t < 1$)

Why no analytical expression?

$$
\begin{aligned}
Z &= \frac{1}{j\omega(\lambda_t)C_p + \frac{1}{j\omega(\lambda_t)\lambda L_{\mathrm{On}}} + \frac{1}{R}} + \frac{1}{j\omega(\lambda_t)C_c} \\
&= \frac{jR}{R\omega(\lambda_t)\left(\frac{\lambda_t}{\lambda}(C_c + C_p) - C_p\right) + j} + \frac{1}{j\omega(\lambda_t)C_c}
\end{aligned}
$$

- The parallel kinetic RLC resonator
  - Simulation of the effect on kinetic inductance on the contrast

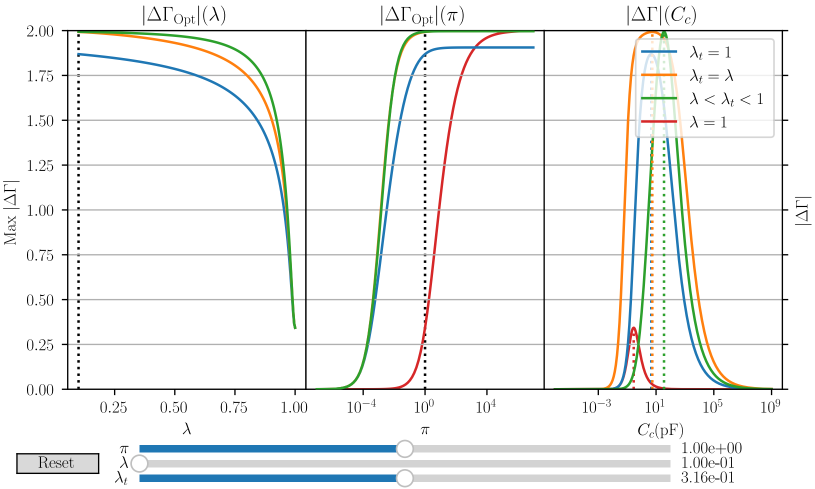
What to simulate

$$\Delta\Gamma(Z_0, \rho_{SET}, R_{\mathrm{On}}^{SET}, C_p, L_{\mathrm{On}}, \lambda_t, \pi, C_c, \lambda)$$

Z_0	ρ_{SET}	R_{On}^{SET}	C_p	L_{On}
50Ω	$2 \cdot 10^6$	$50\mathrm{k}\Omega$	500fF	180nH

$$\Delta\Gamma(\lambda_t, \pi, C_c, \lambda)$$

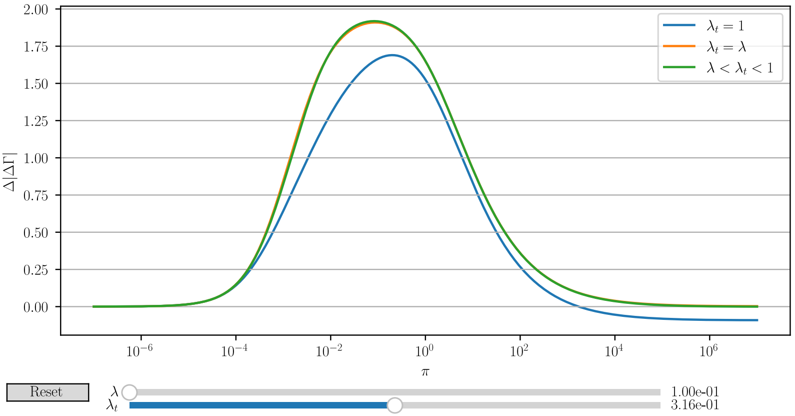
Simulations



Simulations

- A smaller λ always better
- $\lambda_t = \lambda$ and $\lambda < \lambda_t < 1$ objectively better than non-kinetic case, with intermediate λ_t having higher $C_{c\mathrm{Opt}}$
- $\lambda_t = 1$ has a ceiling. It depends on λ, but also R_{On}^{SET}, C_p and L_{On}

Improvement



Conclusions

- **Analytical** optimization of the non-kinetic measuring circuit
  - Tuning to geometric mean of resistances
  - Optimum contrast only depends on ρ/π
- Showed kinetic inductance objectively improves SNR with appropriate configuration
  - L_{On} tuning: Low w + some losses
  - Middle tuning: Greatest C_c + virtually max performance
  - L_{Off} tuning: Max performance at greater π
- Most improvement at around $\pi = 1$, so best for giving freedom in circuit design

Next steps

- Building the actual sensor and measuring it and with it
- Developing the analytical analysis of the kinetic circuit
- Investigating the use of a kinetic inductor with breaking inductance (credit to my supervisor Fernando Gonzalez Zalba for the idea)

Closing remarks

I would like to thank my supervisor Fernando Gonzalez Zalba for all the guidance and help he gave me, my family and significant other for all the love and support, and to all attendees to this defense for your time and attention.

This presentation, the full thesis and the simulations used in this thesis are available in the Github repository of the project github.com/JoseluMontoya/TFM.