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# POWER SYSTEMS CALCULATION

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CITCEA

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We are interested in a generic model for power lines, transformers and converters.

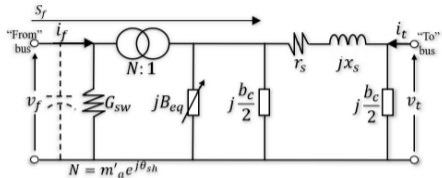


Figure 1. Flexible General Branch Model to model any element. Reference: Bustos, A. A. and Kazemtabrizi, B. (2018) 'Flexible general branch model unified power flow algorithm for future flexible AC/DC networks.' in 2018 IEEE International Conference on Environment and Electrical Engineering and 2018 IEEE Industrial and Commercial Power Systems Europe (EEEIC / ICPS Europe): 12-15 June 2018, Palermo, Italy. Conference proceedings. Piscataway, NJ: IEEE.

Variable	Control
$\theta_{sh}$	$\theta_{sh}$
$\theta_{sh}$	$P_f$
$m_a$	$v_t$
$m_a$	$Q_t$
$B_{eq}$	$v_{dc}$
$B_{eq}$	Zero $Q$ constraint

Mode	Constraint 1	Constraint 2	VSC control
1	$\theta_{sh}$	$v_{ac}$	I
2	$P_f$	$Q_{ac}$	I
3	$P_f$	$v_{ac}$	I
4	$v_{dc}$	$Q_{ac}$	II
5	$v_{dc}$	$v_{ac}$	II
6	$v_{dc}$ droop	$Q_{ac}$	III
7	$v_{dc}$ droop	$v_{ac}$	III

Table 1. VSC control models and relationship between the variables and the controlled magnitude

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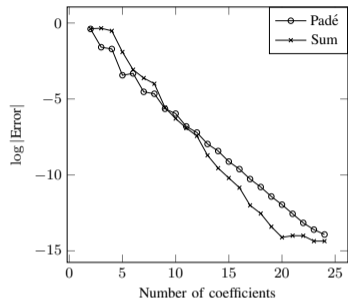
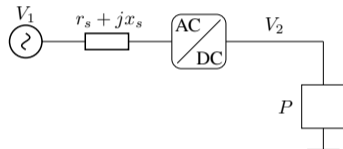


Figure 2. Left: simple system with a VSC converter. Right: maximum error depending on the number of coefficients in the series.

One embedded equation becomes for instance:

$$I_f(s) = y_s V_2(s) - y_s V_1(s) + sj \frac{b_c}{2} V_2(s) + sj B_{eq} V_2(s) + s G I_f^{re}(s) I_f^{re}(s) I_f^{im}(s) I_f^{im}(s) V_2(s) \quad (\text{Eq. 1})$$

HELM is also suitable to solve rather complicated equations like this one.

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We define the problem in two steps.

■ Admittances side:

$$\begin{cases} I^{l+\frac{1}{2}} - I^l = \alpha(V^{l+\frac{1}{2}} - V^l), \\ YV^{l+\frac{1}{2}} = I_0 + I^{l+\frac{1}{2}}. \end{cases} \quad (\text{Eq. 1})$$

■ Load/generator side:

$$\begin{cases} I^{l+1} - I^{l+\frac{1}{2}} = \beta(V^{l+1} - V^{l+\frac{1}{2}}), \\ (V^{l+1})^* I^{l+1} = S^*. \end{cases} \quad (\text{Eq. 2})$$

Matrices  $\alpha$  and  $\beta$  can be arbitrarily defined, but for instance:

$$\begin{cases} \alpha = \text{diag}(S^* / |V|^2), \\ \beta = \text{diag}(Y + \alpha). \end{cases} \quad (\text{Eq. 3})$$

These are constant matrices. Contrary to the typical NR, there are no inverses as such (expensive computation with  $\mathcal{O}(n^3)$ ).

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- Dimensions: position (nodes), changes in power, time...
- Voltages expressed in the separated form:  $V(x, q, t) = \sum_{m=1}^M V_m \otimes Q_m \otimes T_m$ .

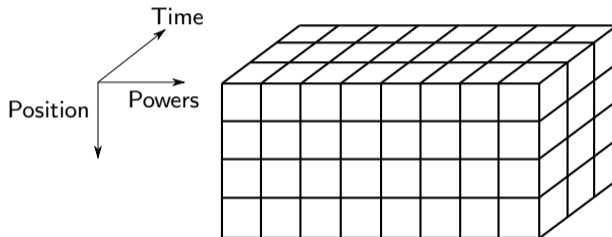


Figure 3. Representation of the cube of solutions

- Need to compute  $M(n_{\text{buses}} + n_{\text{powers}} + n_{\text{time}})$  instead of  $n_{\text{buses}} \cdot n_{\text{powers}} \cdot n_{\text{time}}$  cases.
- Can it be adapted for changes in the topology so that we can employ it for contingency analysis ( $N - 1$ ,  $N - 2$ ...)?

The outer loop follows the ASD procedure while the inner one is based on the alternating directions technique.

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## Algorithm 1 Pseudocode for the PGD combined with ASD

---

```

1: for  $\gamma = 1$  to  $N_\gamma$  do
2:   Compute power side of the problem with PGD:  $I = S^* \oslash V^{*[\gamma]}$ 
3:   for  $m = 1$  to  $M$  do
4:     Define  $I = \sum_{m=1}^{M-1} I_m \otimes Q_m \otimes T_m + I_M \otimes Q_M \otimes T_M$ 
5:     for  $k = 1$  to  $N_k$  do
6:       Compute  $I_M^{[k+1]}$  with  $Q_M^{[k]}$  and  $T_M^{[k]}$ .
7:       Compute  $Q_M^{[k+1]}$  with  $I_M^{[k+1]}$  and  $T_M^{[k]}$ .
8:       Compute  $T_M^{[k+1]}$  with  $I_M^{[k+1]}$  and  $Q_M^{[k+1]}$ .
9:     end for
10:   end for
11:   Compute admittances side of the problem directly:  $V^{[\gamma+1]} = Y^{-1}(I + I_0)$ .
12: end for
  
```

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■ Show code and results.

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