Positive and negative sequence currents to improve voltages during unbalanced faults

CITCEA

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1 Definition

We start by defining the grid code requirements as shown in Figure 1. The positive sequence plot corresponds to the Danish grid code as explained in [**mohseni2012review**], and matches with the one described by National Grid in [**nationalgrid**]. The positive sequence current to inject can be regarded as a function of the positive sequence voltage, and the same applies to the negative sequence current with respect to the negative sequence voltage. Note that the voltages are in reality the pre-correction ones, that is, we suppose they are not yet influenced by the injection of current coming from the VSC.

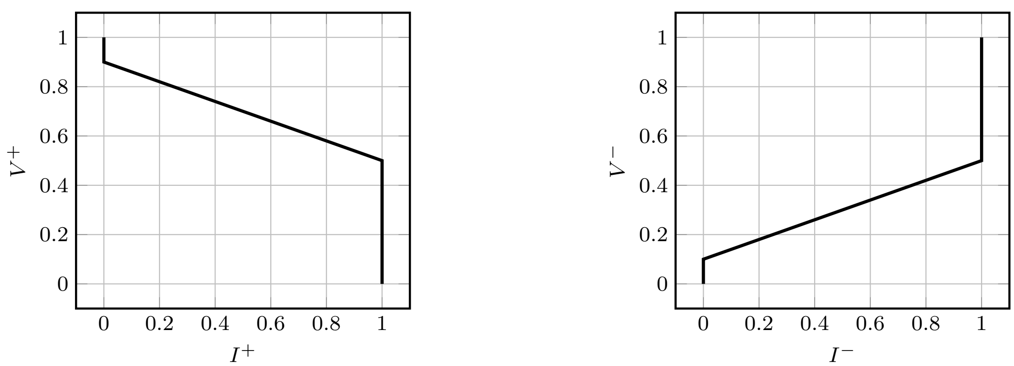

Figure 1: Grid code requirements for the positive and the negative sequence in case of voltage drops

This same profile is expressable in the form of a piecewise function for the positive sequence:

$$\begin{cases} I^+ = 0 & V^+ \geq 0.9 \\ I^+ = k_p(0.9 - V^+) & 0.5 \leq V^+ < 0.9 \\ I^+ = 1 & V^+ < 0.5 \end{cases} \tag{1}$$

where the k_p parameter is responsible for characterizing the slope of drop, and as can be deduced from Figure 1, it takes the value of 2.5. We have assumed the maximum current supported by the VSC to be 1.

Similarly, for the negative sequence:

$$\begin{cases} I^- = 0 & V^- \leq 0.1 \\ I^- = k_n(V^- - 0.1) & 0.1 \leq V^- < 0.5 \\ I^- = 1 & V- > 0.5 \end{cases} \tag{2}$$

where $k_n = 2.5$ as well.

To keep the expressions simple enough, we have not made any distinction between a positive or a negative voltage, nor a positive or negative current. As far as I understand it, we are concerned with improving the absolute value of the voltages, and hence, its angle is not specially relevant. However, it has to be considered to determine the direction of the current phasors. One approach would be to inject only reactive currents, as it is a commonality in grid codes [**mohseni2012review**, **haddadi2020negative**]. This is precisely the perspective taken in this study. Thus, the positive sequence current has to take a negative value to cause a positive voltage drop while the reactive negative sequence current has to become negative.

Besides, there is a potential incompatibility in the plots shown in Figure 1. For instance, the fault may turn out to be extremely severe and have both the positive and the negative sequence voltages approaching 0.5. In this case, the positive and the negative sequence current as well will tend to 1. Most likely some of the *abc* currents will surpass the limit. Therefore, some law is necessary to break from this conflicting situation and look for a trade-off.

In theory, the voltages V^+ and V^- are continously measured. The injected current depends on it, but it certainly has an effect on that as well. Thus, we expect to not follow a one-step procedure, but to update iteratively the injected current until we reach a converging state. If we denote the measurements as V_m^+, V_m^- and V^+, V^- the updated voltages due to the injection of current from the VSC, we ought to follow Algorithm 1.

Algorithm 1: Algorithm to determine the currents in steady-state

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Data: Measured voltages V_m^+ and V_m^-, $\max(\underline{I}_{abc})$ and tolerance tol (e.g. $tol = 1 \cdot 10^{-6}$)
Result: Injected currents I^+ and I^- in a converged situation
while $abs(abs(V_m^+)$ - $abs(V^+)) > tol$ *or* $abs(abs(V_m^-)$ - $abs(V^-)) > tol$ **do**
  Compute $I^+(V_m^+)$ with Equation 1;
  Compute $I^-(V_m^-)$ with Equation 2;
  Calculate $\underline{I}_{abc}$ from I^+ and I^-;
  if $max(\underline{I}_{abc}) > I_{max}$ **then**
    Reduce injected currents;
  else
    Calculate V^+ and V^-;
  end
  Calculate V^+ and V^-;
end

The critical step in Algorithm 1 has to do with the reduction of currents in case they surpass the maximum allowed by the VSC. Some laws have to be defined in order to minimize the exceeding current up to the point that it respects the limitations but also making sure it is a near-optimal solution. One approach would be to introduce a sort of relaxation factor f:

$$\begin{cases} I^+ = 0 & V^+ \geq 0.9 \\ I^+ = f \cdot k_p(0.9 - V^+) & 0.5 \leq V^+ < 0.9 \\ I^+ = f & V^+ < 0.5 \end{cases} \tag{3}$$

and the same applies to the negative sequence:

$$\begin{cases} I^- = 0 & V^- \leq 0.1 \\ I^- = f \cdot k_n(V^- - 0.1) & 0.1 \leq V^- < 0.5 \\ I^- = f & V- > 0.5 \end{cases} \tag{4}$$

This factor f has to be computed continously up to the point in which we do not exceed the current limits. For instance, it could follow the procedure described in Algorithm 2.

Algorithm 2: Algorithm to find the relaxation factor f

Data: Current $\max(\underline{I}_{abc})$, voltages V_m^+, V_m^- and tolerance tol_2
Result: Injected currents I^+ and I^- that satisfy the limits
$f = 1$;
while $max(\underline{I}_{abc}) > I_{max}$ *or* $\underline{I}_{abc} < (1 - tol_2)\ I_{max}$ **do**
  Calculate $\underline{I}_{abc}$ from I^+ and I^-;
  if $max(\underline{I}_{abc}) > I_{max}$ **then**
    $f = f - tol_2$;
  else
    $f = f + tol_2$;
  end
  Compute $I^+(V_m^+)$ with Equation 3;
  Compute $I^-(V_m^-)$ with Equation 4;
end

The tol_2 parameter will take an arbitrary value, such as $1 \cdot 10^{-4}$, for example. It is used to ensure that we operate close to the maximum current while not exceeding it. Figure 2 shows the evoluton of the positive and the negative sequence with the subsequent measurements of voltages and updates of currents. No dynamics have been considered, of course. The stabilization of both voltages is relatively straightforward. We have employed a fault impedance of 0.03, $\underline{Z}_a = 0.01{+}0.10j$ and $\underline{Z}_{th} = 0.01{+}0.05j$ for a line to ground fault.

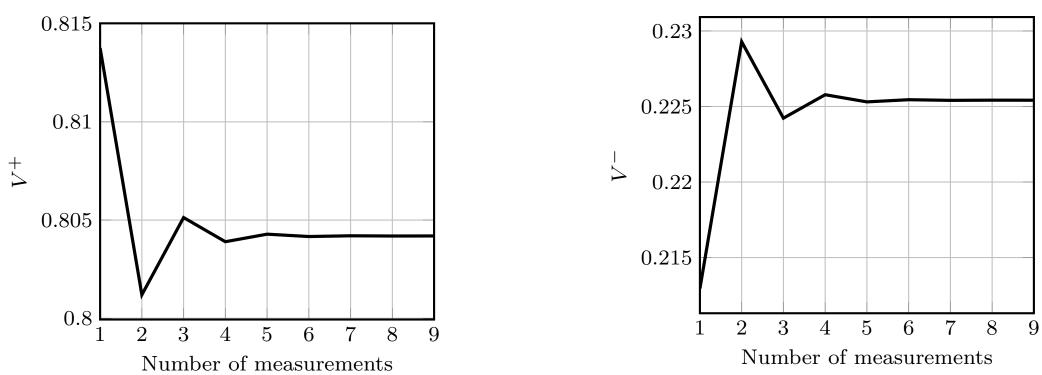

Figure 2: Positive and negative sequence evolution following the grid code approach

2 Variable resistance/inductance ratio

In this section we try to answer to the question of what happens when the R/X ratio varies. This way we experiment with cases where the resistive part is considerably larger than in the aforementioned analysis. The fault impedance $\underline{Z}_f$ has been forced to take real values, which seems realistic.

2.1 Balanced fault

Figure 3 depicts the optimal currents for the balanced fault.

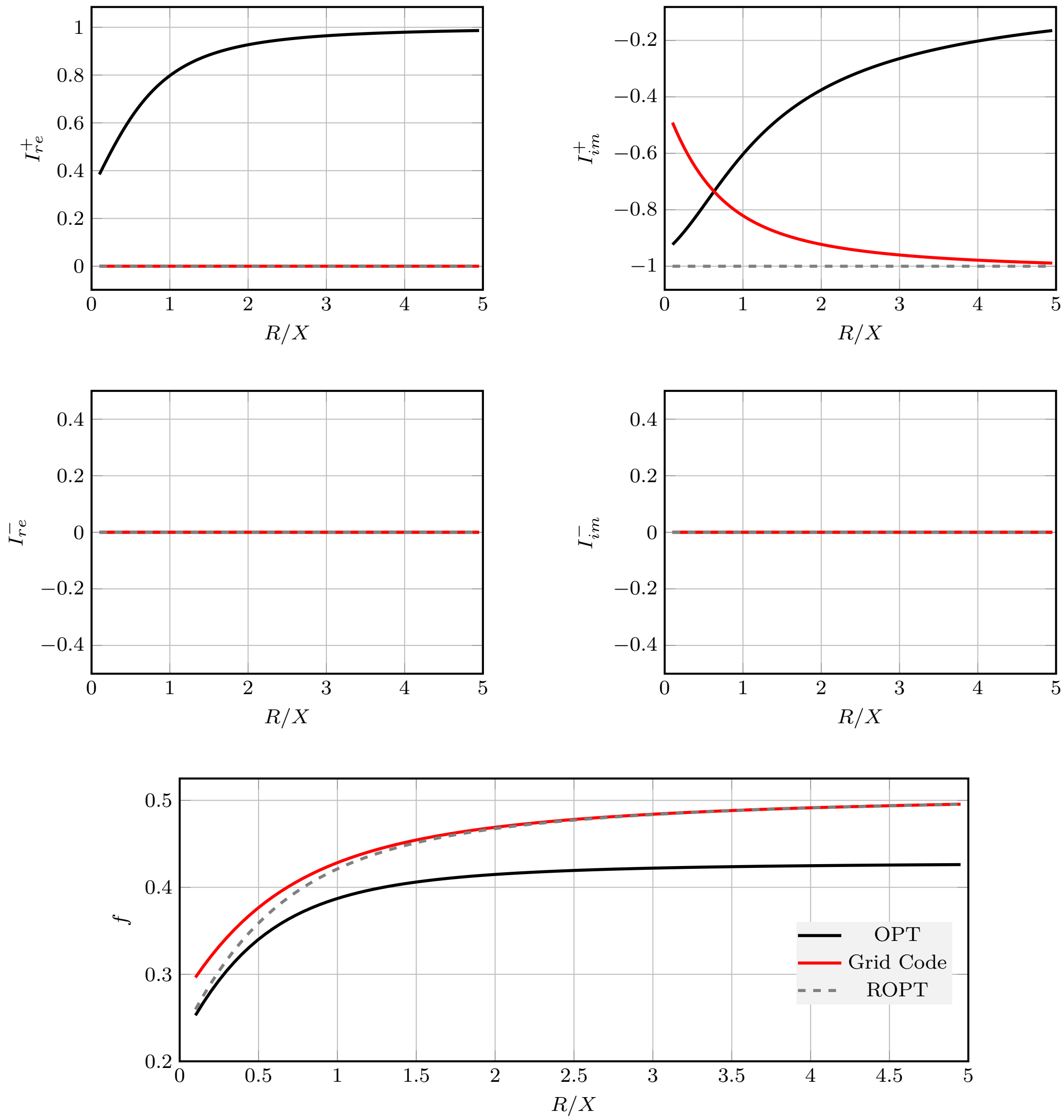

Figure 3: Influence of the currents on the objective function for the balanced fault with $\underline{Z}_f = 0.05$ and a changing R/X ratio. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

It becomes clear that the optimal current is highly dependent on the R/X ratio. That is, when $X > R$, the imaginary positive sequence is dominant, and vice versa for $R > X$. This phenomena makes complete sense when considering that we want to amplify the voltage drop in the positive sequence. However, even when there are serious differences between R and X in magnitude, neither the real nor the imaginary part go to zero. Thus, the objective function for the OPT case becomes slightly lower when compared to the ROPT case. Besides, since the negative sequence equivalent circuit is decoupled from the positive sequence one, the negative sequence currents are null in all cases.

The fault impedance has a noticeable effect on the results. When it is extremely small, the objective function almost does not vary. Opting for a too large fault impedance may cause the presence of multiple solutions. Hence, the real and imaginary currents may take unexpected values so it is hard to build intuition around the problem. We have checked that $\underline{Z}_f = 0.05$ for the balanced fault was a convenient value.

In addition, Figure 4 shows the absolute value of the voltages depending on the R/X ratio.

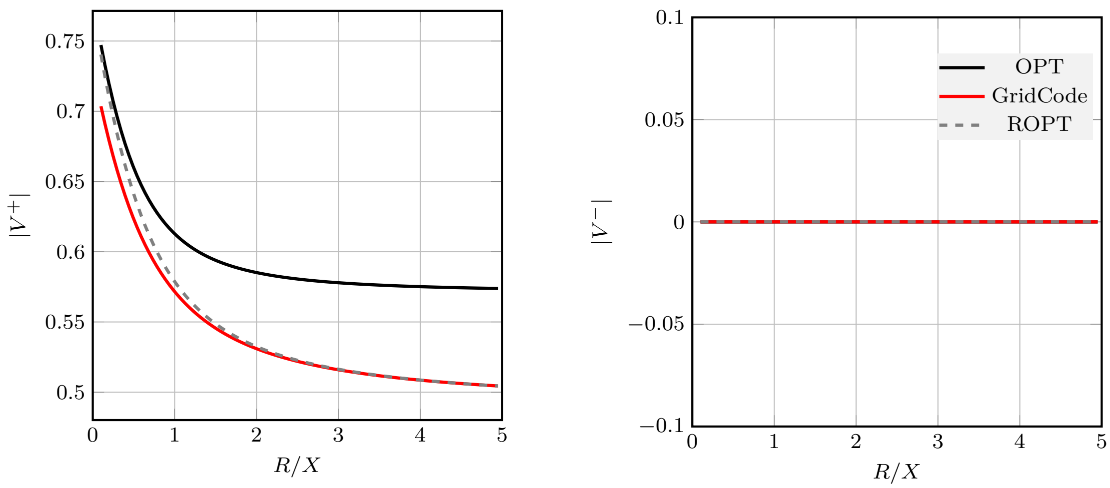

Figure 4: Influence of the R/X ratio on the voltages for the balanced fault with $\underline{Z}_f = 0.05$. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

As already noted in Figure 3, the objective function for the OPT case is inferior than the one for the ROPT case, and hence, closer to the ideal situation. Nevertheless, independently on having constraints on the active current, the negative sequence voltages remain at exactly zero for all the R/X range. They end up being superposed. The positive sequence voltages, instead, experience some variations depending on the R/X value. In some sense the positive sequence voltage trend is the contrary of the objective function pattern.

The best possible situation is having the reactive part larger than the real part of the impedance. This is when the positive sequence voltage tends to 0.75. Since the resistive characteristics of the impedance are so minimized, the ROPT case yields the same results as the OPT. When the resistance increases, the differences are exaggerated due to the fact that no active current can be injected in the ROPT situation.

2.2 Line to ground fault

Figure 6 depicts the optimal currents for the line to ground fault.

The line to ground fault has been analyzed for a fault impedance of 0.005 because otherwise the fault may not be severe enough. This is deduced from the presence of multiple optimal points for a same R/X ratio while the objective function improves substantially due to the injection of currents.

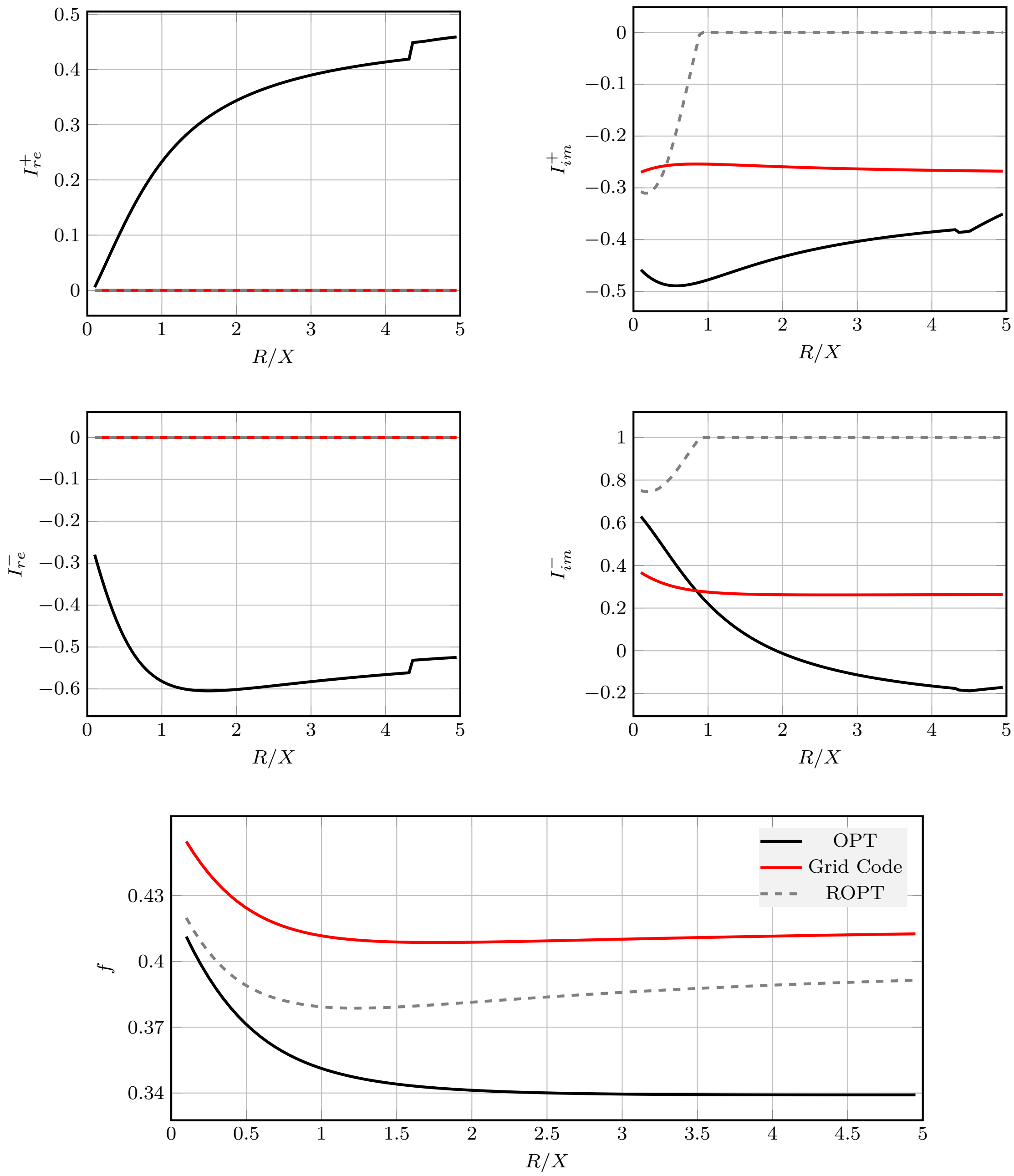

Figure 5: Influence of the currents on the objective function for the line to ground fault and a changing R/X ratio and $\underline{Z}_f = 0.03$. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

Instead, in this case the objective function does not vary much for all the R/X range. Notice also that it is not far apart from the objective function in the balanced fault case. Consequently, we are able to conclude that the severity of the fault is similar thanks to the convenient adjustment of the fault impedance.

On the other hand, the differences between the OPT and the ROPT are relevant. Despite that, there is not much room for improvement in the sense that real currents, for both the positive and negative sequences, take ideally values close to 0.5 for big enough R/X ratios. Imposing the constraint of not injecting any real current causes that all influence on the voltages is achieved by means of the imaginary currents, which are not enough to improve the voltages. For instance, for $R/X \approx 5$, the maximum absolute value of the *abc* currents is one order of magnitude lower than the maximum allowed current I_{max}. Therefore, this suggests that no combination of currents is able to reduce more the objective function.

Even though we can expect that the positive sequence voltage is far from the unit value and the negative sequence is also distant from zero, the evaluation of voltages becomes worth of a particular study. Figure **??** shows the objective function along with the positive and negative sequence for the OPT and the ROPT cases.

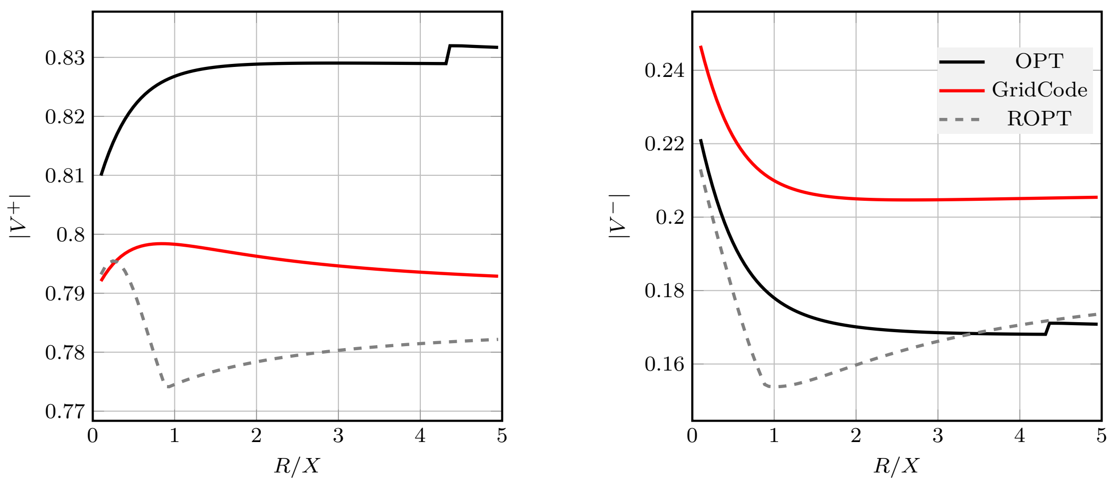

Figure 6: Influence of the R/X ratio on the voltages for the line to ground fault with $\underline{Z}_f = 0.03$. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

When looking at the bigger picture the objective function remains almost always the same, yet there exists a permanent difference between the OPT and the ROPT cases. The positive sequence voltages in the OPT situation are always above the ROPT ones for about 0.04 pu. For the negative sequence voltage the pattern is reversed. It is relevant to take into account that even if the voltages turn out to be practically constant, the currents experience large variations, as shown in Figure 6. Extracting conclusions regarding the fault by only observing the voltages may be misleading, as they can be maintained at the expense of injecting the specific optimal currents. Besides, just like it happened with the balanced fault, the shape of the objective function ressembles the shape of the voltages. This is logical when considering the proportionality between the objective function and the voltages.

2.3 Line to line fault

Figure 8 depicts the optimal currents for the line to line fault.

This time, even if the fault is equally or more severe than in the other cases, the currents take expectable values from an intuitive standpoint. The positive sequence real current is positive to cause a positive voltage drop and improve $\underline{V}_c^+$ while the positive sequence imaginary current becomes negative to provide a resulting positive voltage drop as well. The contrary applies to the negative sequence currents.

The real part of the sequence currents become substantial, even if the R/X takes small values (see the negative sequence real current). The increase in the R/X ratio implies an increment on the

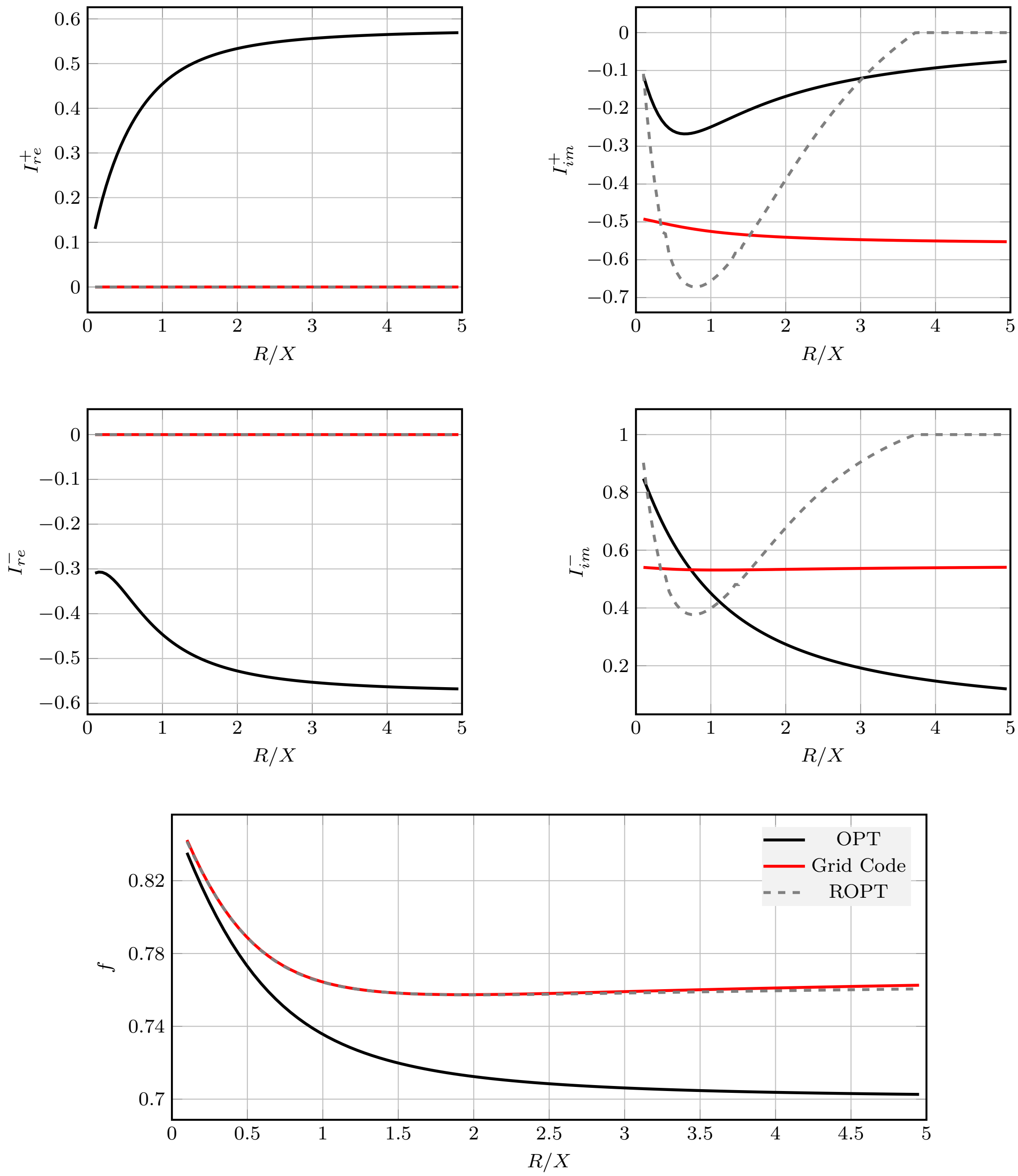

Figure 7: Influence of the currents on the objective function for the line to line fault and a changing R/X ratio with $\underline{Z}_f = 0.03$. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

real positive sequence current and specially a decrease in the negative sequence imaginary current. Contrarily, the ROPT case prioritizes the positive sequence imaginary current. Much of the current capability of the converter are invested in this component rather than in the negative sequence imaginary current. As visible in the objective function evolution, a consistent variation is present between the OPT and the ROPT case when R/X tends to grow. In further detail, the objective function for the ROPT increases a bit as a result of being incapable of injecting the much required real currents. This same phenomena happened in a more extreme way for the balanced fault.

Figure **??** displays the absolute value of the positive and negative sequence voltages, which change slightly for various R/X values. Consequently, the objective function does also vary as shown in Figure 8.

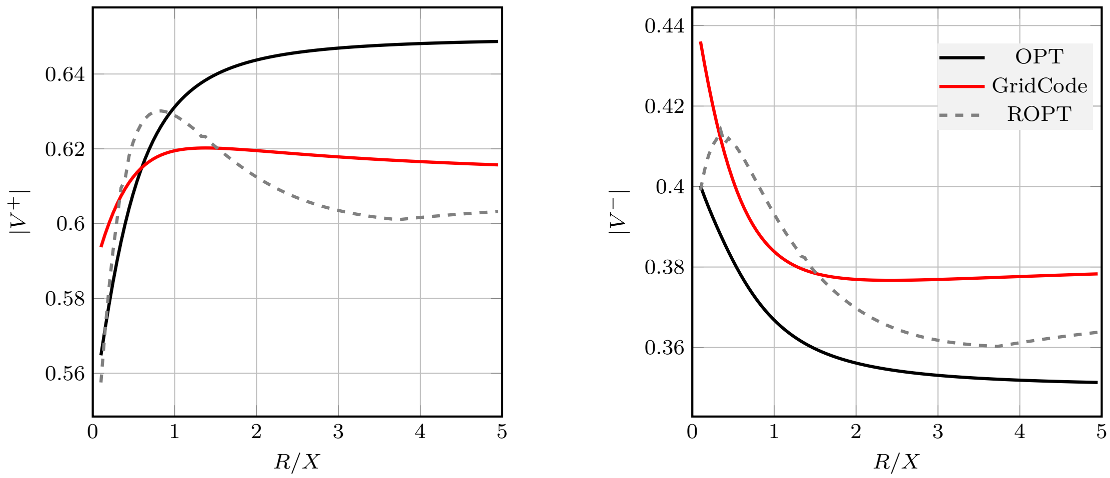

Figure 8: Influence of the R/X ratio on the voltages for the line to line fault with $\underline{Z}_f = 0.03$. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

Surprisingly, the positive sequence voltage for the OPT situation is not always higher than in the ROPT case. Thus, for small R/X ratios, which happen to be the most unfavorable, the large injection of positive sequence imaginary current yields higher positive sequence voltages. However, when considering the full objective function, the OPT is always better off than the ROPT. This can be explained with the negative sequence voltages. They are always smaller in the OPT situation compared to the ROPT case.

Also important, in this type of fault the result of the optimization has resulted in obtaining a positive sequence voltage that is approximately equally distant from the unit voltage than the zero voltage is from the negative sequence voltage. Although we were not looking for this balance, the optimization naturally produced it.

2.4 Double line to ground fault

Figure 9 depicts the optimal currents for the balanced fault.

The double line to ground fault is expected to become a more severe fault than all the previous cases considered since the connection between the two faulted phases is a solid one. This way, adjusting the fault impedance only affects the connection to ground but the fault remains worrying. As it can be anticipated, the converter has a limited influence on the objective function. Figure 9 shows indeed that such objective function is extremely close to one. The difference between the OPT and the ROPT becomes noticeable again. The converter is able to inject the optimal currents to keep a constant objective function whereas for the ROPT this does not happen.

The real positive and negative sequence currents take similar values (absolutely speaking) when compared to the imaginary parts. In some sense this ressembles the line to line fault findings. Since no real current can be injected in the ROPT situation, the optimal option consists of keeping the imaginary currents relatively constant across all R/X values.

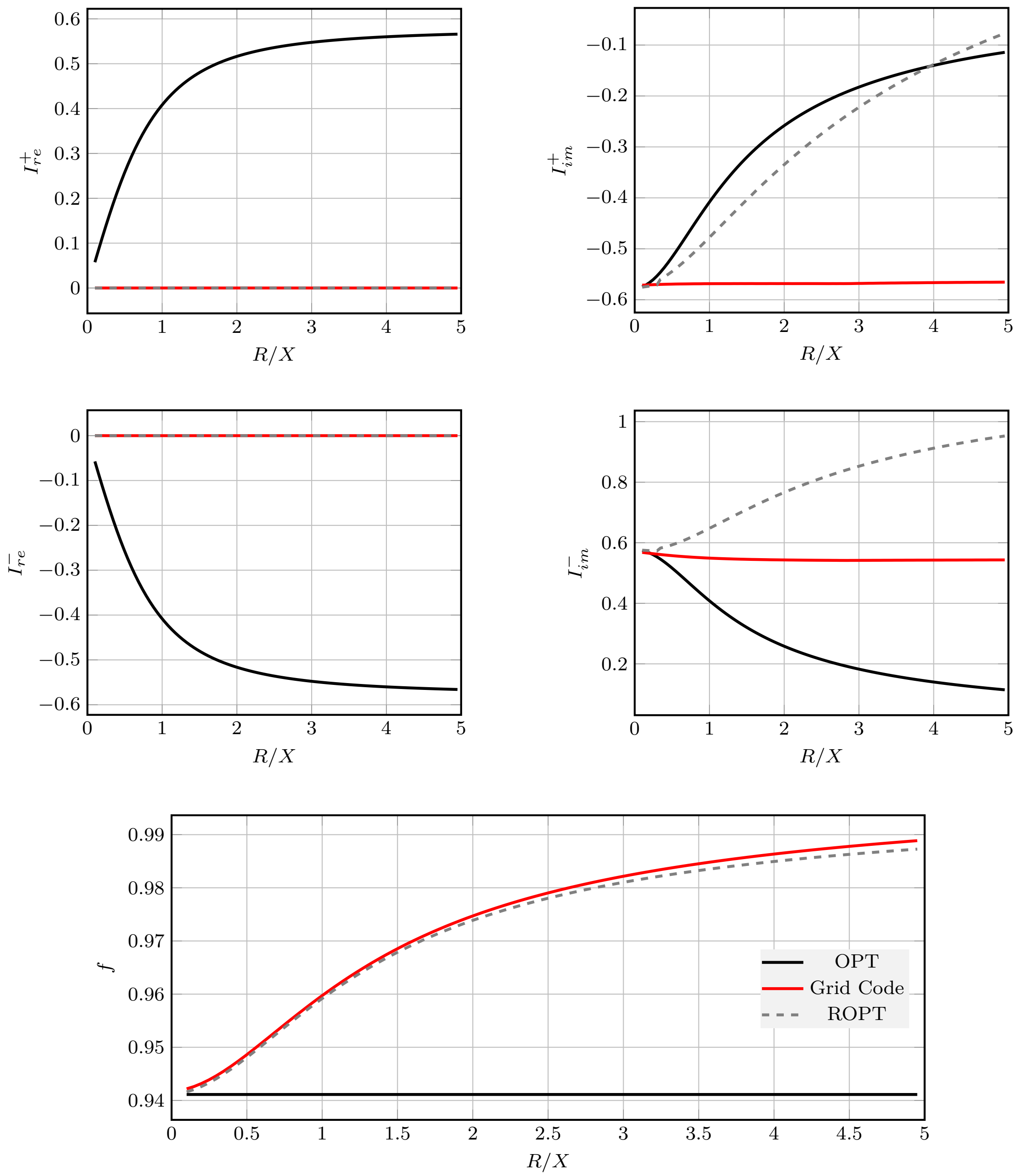

Figure 9: Influence of the currents on the objective function for the double line to ground fault and a changing R/X ratio with $\underline{Z}_f = 0.5$. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

Figure 10 shows the distribution of voltages for the multiple R/X ratios. Similar conclusions as before can be extracted.

When R/X is approximately null, the positive and negative sequence voltages match for the OPT and the ROPT cases. However, once the resistive part increases in comparison to the reactive part of the impedance, the impossibility of injecting active current causes a variation in voltages. For the

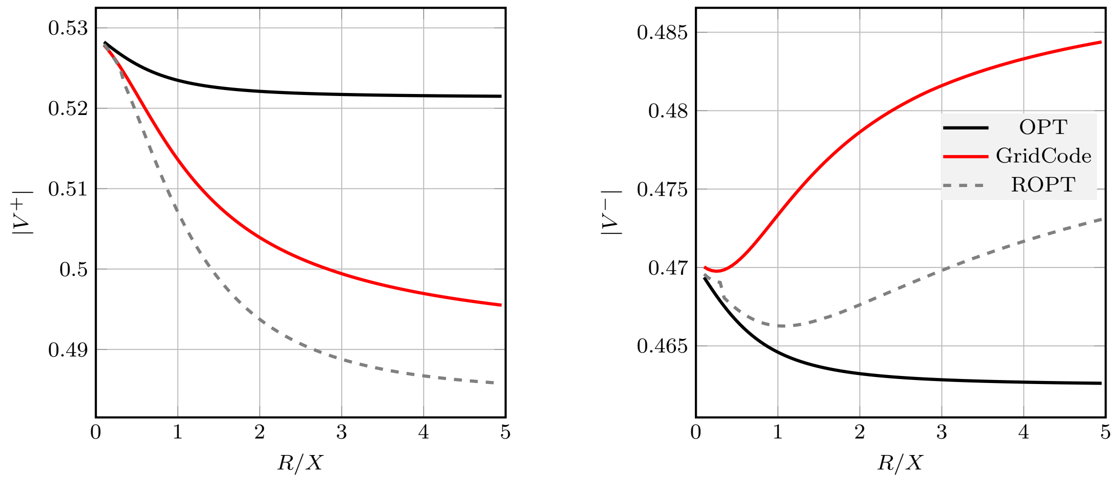

Figure 10: Influence of the R/X ratio on the voltages for the double line to ground fault with $\underline{Z}_f = 0.5$. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

positive sequence voltages, the OPT one is placed above the ROPT voltage; and the contrary happens to the negative sequence voltages.

3 Submarine cable

Having seen the influence of varying the ratio between the resistive and inductive part of the Thevenin impedance, we can now go a step further by including a hypothetical submarine cable between the grid and the point where the fault occurs. The systems takes the form shown in Figure 11.

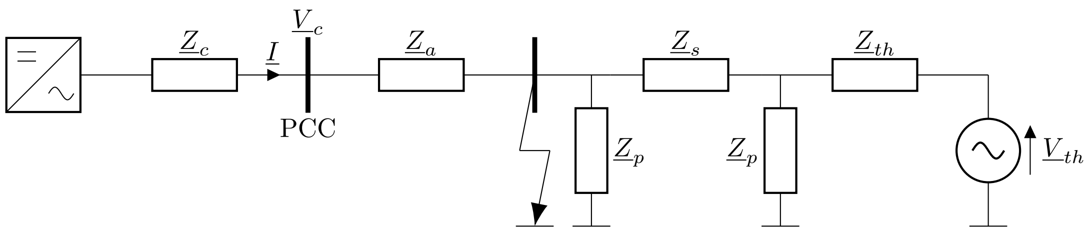

Figure 11: Single-phase representation of the simple system under a fault with a submarine cable

This system can be studied without added complexity once a Thevenin equivalent is formed on the right hand side of the fault. Opting for this will imply that we will be able to recycle the expressions developed before. The new Thevenin voltage and impedance, denoted by $\underline{V}'_{th}$ and $\underline{Z}'_{th}$, are given by:

$$\begin{cases} \underline{V}'_{th} = \dfrac{\underline{Z}_p\underline{Z}_p}{2\underline{Z}_{th}\underline{Z}_p + \underline{Z}_p\underline{Z}_s + \underline{Z}_p\underline{Z}_p + \underline{Z}_{th}\underline{Z}_s}\underline{V}_{th}, \\ \\ \underline{Z}'_{th} = \dfrac{\underline{Z}_p\underline{Z}_p\underline{Z}_{th} + \underline{Z}_s\underline{Z}_p\underline{Z}_p + \underline{Z}_{th}\underline{Z}_s\underline{Z}_p}{2\underline{Z}_{th}\underline{Z}_p + \underline{Z}_p\underline{Z}_s + \underline{Z}_p\underline{Z}_p + \underline{Z}_{th}\underline{Z}_s}. \end{cases} \tag{5}$$

The distance of the cable is going to take multiple values in order to weight its influence. Realistic data for the $\underline{Z}_s$ and $\underline{Z}_p$ impedances are taken from [**cheah2017offshore**] and adapted. They appear in Table 1.

Magnitude	Value	Units
$\underline{Z}_s$	$6.674 \cdot 10^{-5} + j2.597 \cdot 10^{-4}$	pu/km
$\underline{Z}_p$	$-j77.372$	pu·km

Table 1: Impedances values to be used in the submarine cable analysis

Figure 12 displays the resulting Thevenin voltage and impedance for a varying cable distance.

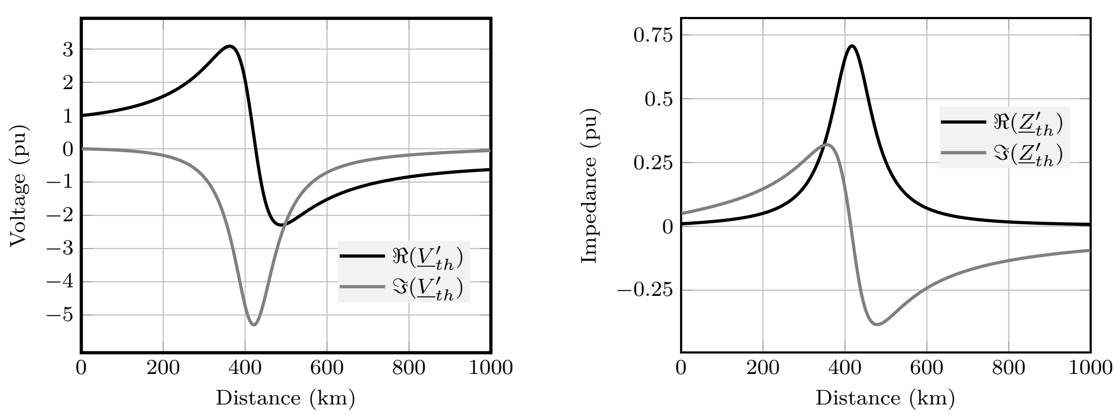

Figure 12: Influence of the cable distance on the Thevenin voltage and impedance

The real part of the impedance always remains above zero. However, the imaginary part starts being inductive and then becomes capacitive for large cable distances. The voltages can present extremely high values, but they coincide with a situation where the impedance is also high. Notice how abrupt changes take place around 400 km in both plots.

3.1 Balanced fault

Figure 13 depicts the optimal currents for the balanced fault.

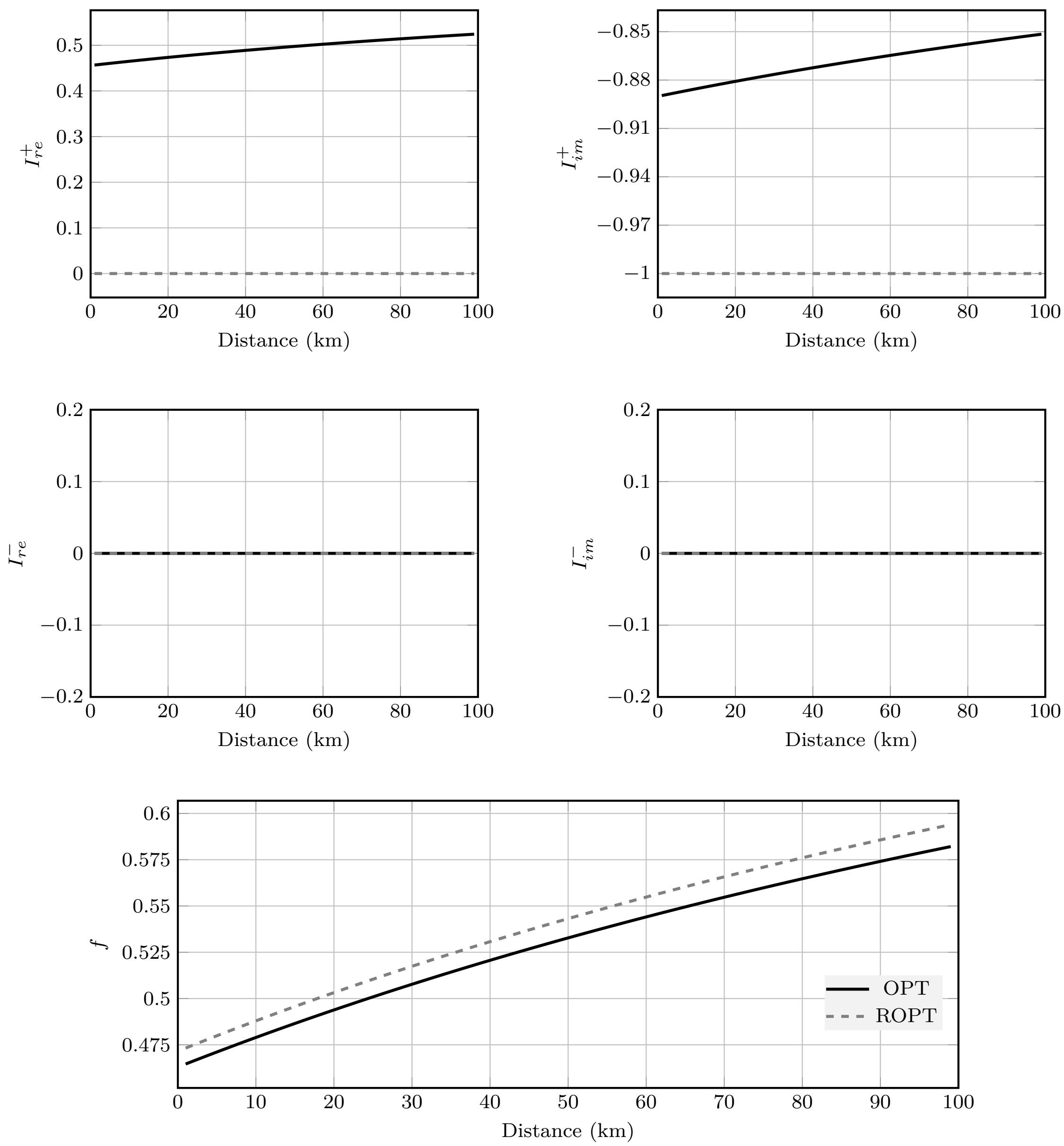

Figure 13: Influence of the currents on the objective function for the balanced fault with $\underline{Z}_f = 0.03$ and a submarine cable. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

As it can be deduced, the imaginary part of the positive sequence current is the one to be prioritized, specially for short distances. Notice that when the distance increases, as shown in Figure 12, both the real and imaginary part of the Thevenin impedances increase. However, in proportion, the real part becomes a bit more relevant. Therefore, in the OPT case some of the imaginary positive sequence current is traded for a bit more real current. The negative sequence currents are null for all range of distances because of the nature of the balanced fault. As it has been explained before, it makes no sense to inject negative sequence current for a balanced fault due to having initially an already null negative sequence voltage.

The ROPT turns out to be a more unfavorable case. This is fruit of not being able to inject any real current. Thus, the positive sequence current becomes totally reactive as it remains constant at -1, which corresponds to the maximum allowed current. Increasing the cable distance suggests that the longer the cable, the more severe the fault is. Nevertheless, the variations are relatively small. We can conclude that for this fault, injecting or not active current does not have a huge influence on the final results.

Figure 4 shows the voltages profile. The positive sequence absolute value of the voltage more or less mimics the reverse trend of objective function. It experiencex relatively small variations as well.

The negative sequence voltage is of course at 0 for the ROPT and the OPT cases as mentioned. The positive sequence voltages tend to decrease with longer distances, which makes sense because the impedance of the cable increases. For all distances there exists a rather constant difference between the voltages. This difference is the same as in the objective function. We have found out that the fault impedance is by far the most influential impedance in the system. If we had opted for higher values, the objective functions would have been reduced and the objective voltages would have improved. Sometimes, when $\underline{Z}_f$ is big enough, the objective function can become zero.

3.2 Line to ground fault

Figure 15 depicts the optimal currents for the line to ground fault.

The most noticeable aspect in this situation is that the longer the cable is, the more optimal the objective function becomes. There is about a 10% improvement if we compare the objective functions for an almost null distance with a distance close to 100 km. Again, there is a practically constant difference regarding the objective function between the OPT and the ROPT cases.

The currents vary slightly. For instance, the real positive sequence current grows a bit but is kept at around 0.1. The imaginary positive sequence currents follow a similar evolution. The currents for the OPT situation are usually further from the zero. The optimization suggests that the current limitations are not met in the ROPT case. This same phenomena happened for the line to ground fault when the R/X was a varying one. One hypothesis to explain that could be related to the presence of multiple optimal points. The imaginary part of the currents does not surpass the 0.3 mark in absolute value under any condition. About the negative sequence currents, the real one takes substantial values, which decrease with distance. In general one can spot that the trends of the positive and the negative sequence currents seem mirrored.

Figure 16 presents the absolute value of the positive and negative sequence voltages and the objective function for all considered distances.

When looking at the bigger picture, it is clear that the objective function remains quite constant for all distances. However, there is a noticeable difference between the OPT and the ROPT situations, as expressed before. Larger distances imply that the positive voltage sequence can be improved, yet the negative sequence voltage also increases. Fortunately, the increase in the first is superior to the increase in the latter.

This time injecting active currents turns out to be more beneficial than in the balanced fault, where differences were hard to notice. Besides, even though the fault impedances is an order of magnitude smaller in this line to ground fault, the objective functions fall into a similar range of values. As it can be concluded, the distance of the cable has a relatively weak influence on the final results.

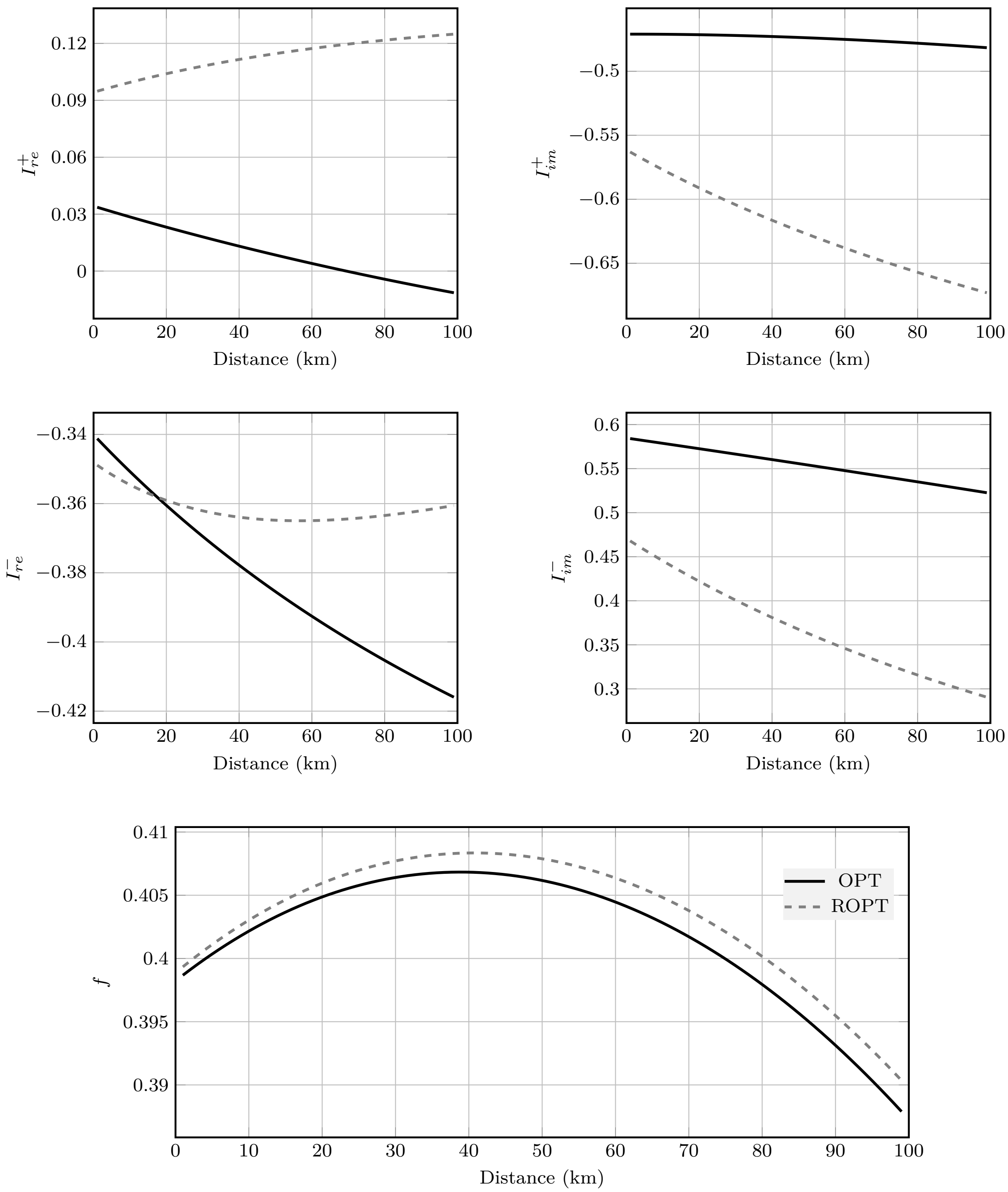

Figure 14: Influence of the currents on the objective function for the line to ground fault with $\underline{Z}_f = 0.03$ and a submarine cable. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

3.3 Line to line fault

Figure 17 depicts the optimal currents for the line to line fault.

The line to line fault has been found to become the one where the current values are more intuitive.

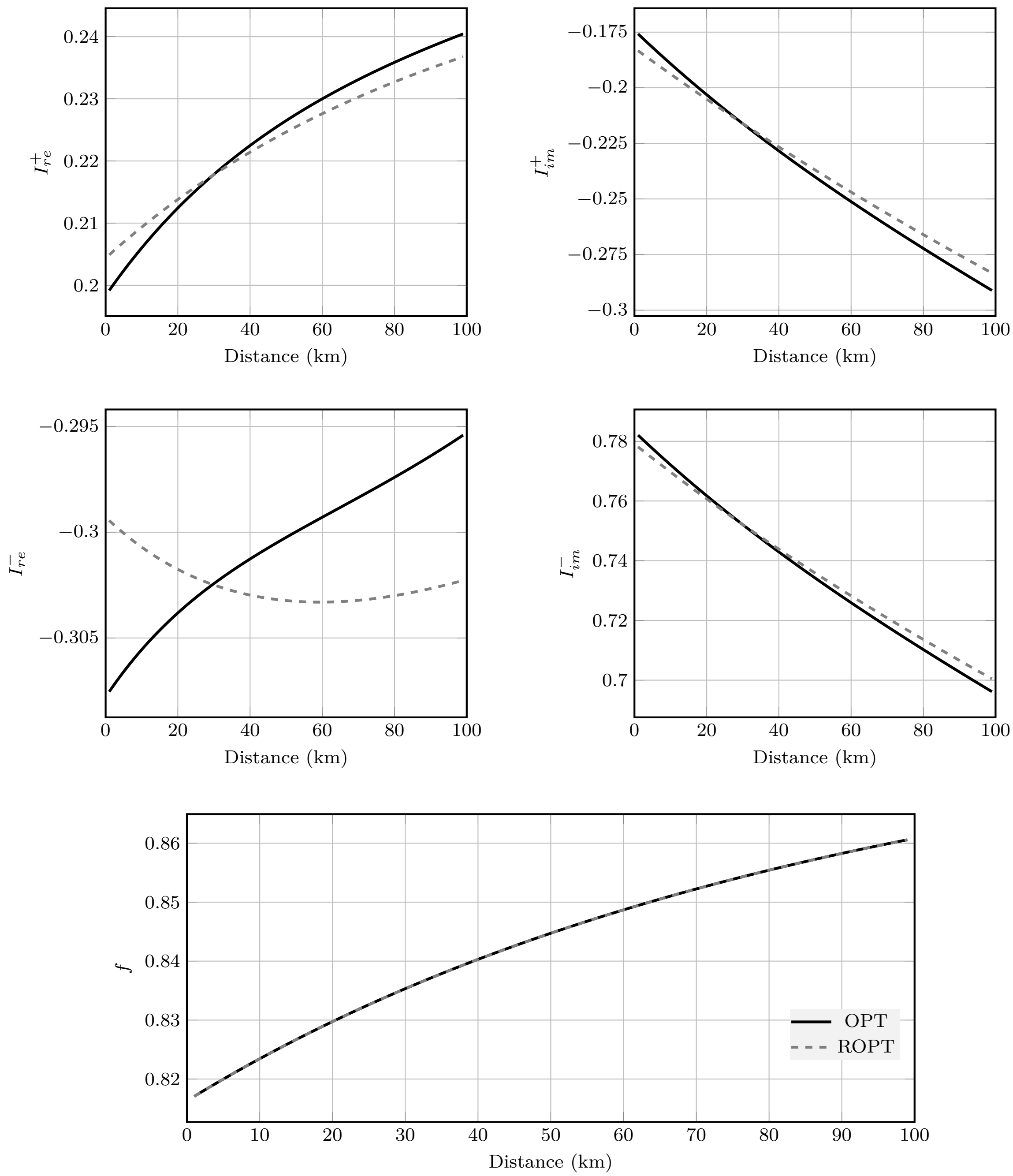

Figure 15: Influence of the currents on the objective function for the line to line fault with $\underline{Z}_f = 0.03$ and a submarine cable. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

That is, in the OPT situation, the real positive sequence current becomes positive while the imaginary positive sequence one is negative to provoke a positive voltage drop as well to maximize the positive sequence voltage at the PCC. The inverse reasoning applies to the negative sequence currents. When we limit the reactive current to zero, as in the ROPT case, not only the differences between the two active currents is acute, but they also are for the imaginary currents. The negative sequence real current is specially considerable, but despite that, the objective function is not largely affected if we constrain it.

The currents do not experience huge variations across all distances. However, the trend is to increase slightly the real currents in the OPT case. This allows to have always a more optimal objective function. Just like it happened for the balanced fault, shorter distances are better in order to keep the objective function to a value closer to zero. But in any case, generally speaking the differences in the objective function are small, around 1%.

Figure 18 displays the positive and negative sequence voltages for the considered range of distances. The objective functions are also depicted, although they are hardly differentiable.

Surprisingly, the positive sequence voltage is larger for the ROPT case than in the OPT situation. This could already be anticipated by looking at Figure 17, where the positive sequence OPT currents are inferior when compared to the imaginary positive sequence ROPT current. Therefore, the ROPT case makes a bigger effort to maximize the positive sequence voltage at the expense of causing an also larger negative sequence voltage. If we balance both, we get to the conclusion that the objective function is slightly better off in the OPT case, just like we could anticipate. All the absolute values of the voltages tend to increase for longer distances. Compared to the variation in the R/X ratio, the objective function now remains almost always the same. However, the fault is more severe and the differences between the OPT and the OPT scenarios have diminished.

3.4 Double line to ground fault

Figure 19 depicts the optimal currents for the double line to ground.

Due to the way it has been defined, the double line to ground is the most severe fault since there is solid connection between the two faulted phases. This means that the fault impedance, which represents the link to ground, becomes mostly irrelevant. The objective functions are close to one and practically constant across all lengths. The fault so severe that the influence of the cable is minimal.

The imaginary currents take values close to an order of magnitude higher than the real parts. From the analysis of all faults, it seems that the imaginary current is strongly linked to the severity of the fault. Consequently, when the fault happens to be a strong one, injecting or not active currents has a minor influence. As follows, the OPT and the ROPT cases yield very similar operation conditions. Varying the R/X ratio had a greater impact on the results than increasing the length of the cable, but generally speaking, the objective functions where higher than 0.9 as well.

Figure 10 displays the objective function values together with the positive and negative sequence voltages.

Again, differences are hardly noticeable between both cases. The lines are overlapped for all the range of distances. In spite of that, the voltages tend to increase. Increasing the positive sequence voltage means that we are improving the objective function, but in the meantime, the negative sequence voltage also grow. Balancing both contributions results in obtaining a constant objective function for all distances.

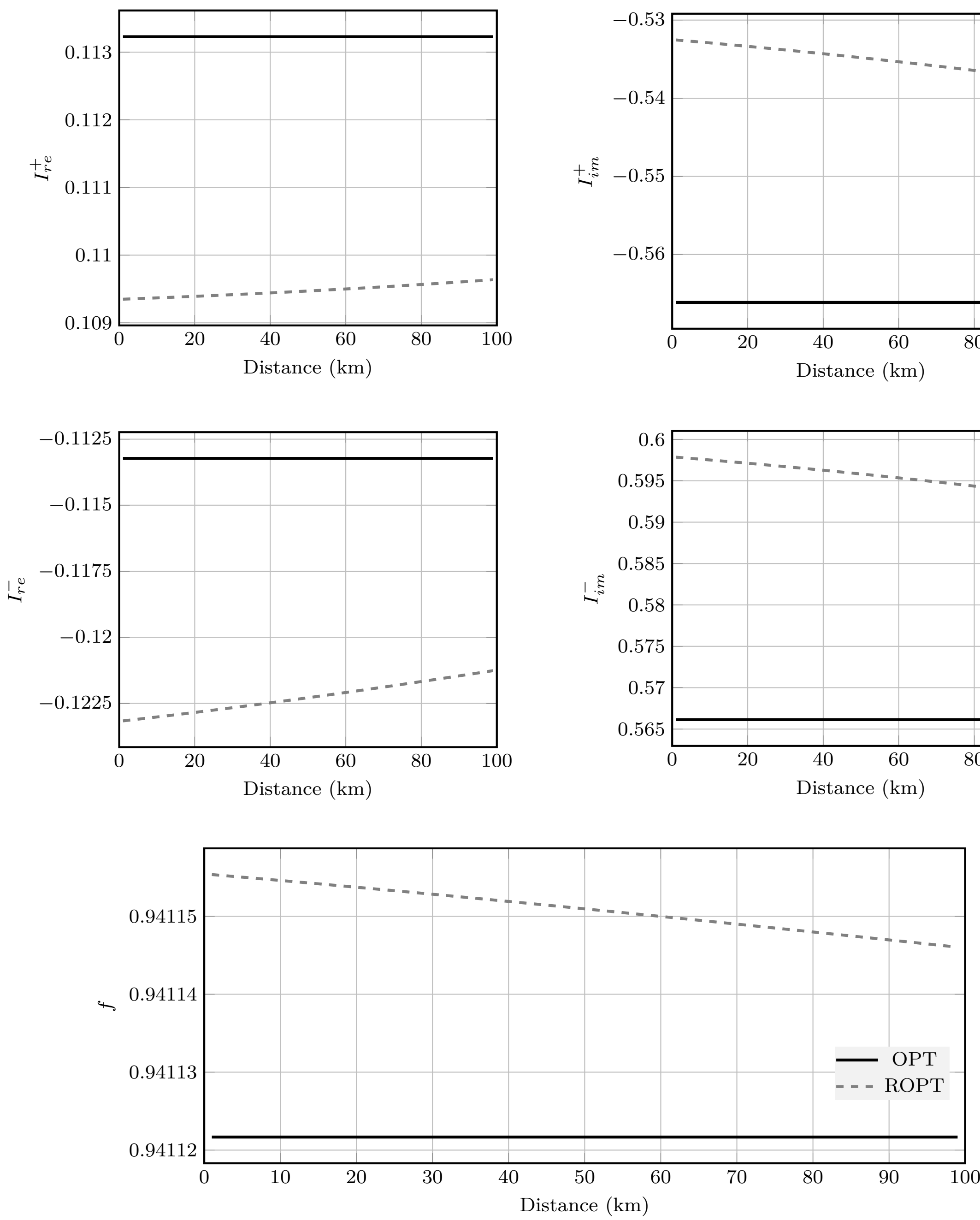

Figure 16: Influence of the currents on the objective function for the double line to ground fault with $\underline{Z}_f = 0.5$ and a submarine cable. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

4 Conclusion

This study has covered the analysis of a simple system to determine the optimal currents that ought to be injected to raise the positive sequence voltage and decrease the negative sequence voltage in case of a fault. Four types of faults have been considered, and in each case, two situations have been proposed: one in which the converter can inject real and reactive currents, and another where the converter can only inject reactive currents. Such currents are constrained to not surpass the current limitations of the converter. The results show that injecting only reactive powers, as could be imposed by the grid codes, is likely to be a near-optimal strategy. In all cases (constant impedance, changing R/X ratio and changing the cable distance) the objective function related to the ROPT situation has been close to the OPT one. However, it is mandatory to determine the type of fault in order to properly deduce the optimal values of the injected currents, which can vary significantly.

In addition to that, this work proposes two methodologies to arrive to the optimal solution. One has consisted of computing combinations where currents can take a wide range of values. When the intervals are small enough, such a computationally intensive approach matches with the solution coming from solving directly the optimization problem. The latter becomes the preferred option as it is faster and more precise. Generally speaking the double line to ground fault is the hardest in terms of minimizing the objective function.

Varying the R/X ratio shows that the optimal currents can be highly dependent on the ratio. However, the objective functions do not experience substantial differences, as the OPT case is always slightly better of than the ROPT one. The same conclusion can be extracted from the submarine cable under study. In both situations, it seems that the injection of real currents may dominate over the imaginary currents. We have also deduced that the fault impedance is usually the most determining factor. When it becomes small, the objective function has tiny room for improvement, while in case it takes large values, it remains close to zero.

A *abc* circuits

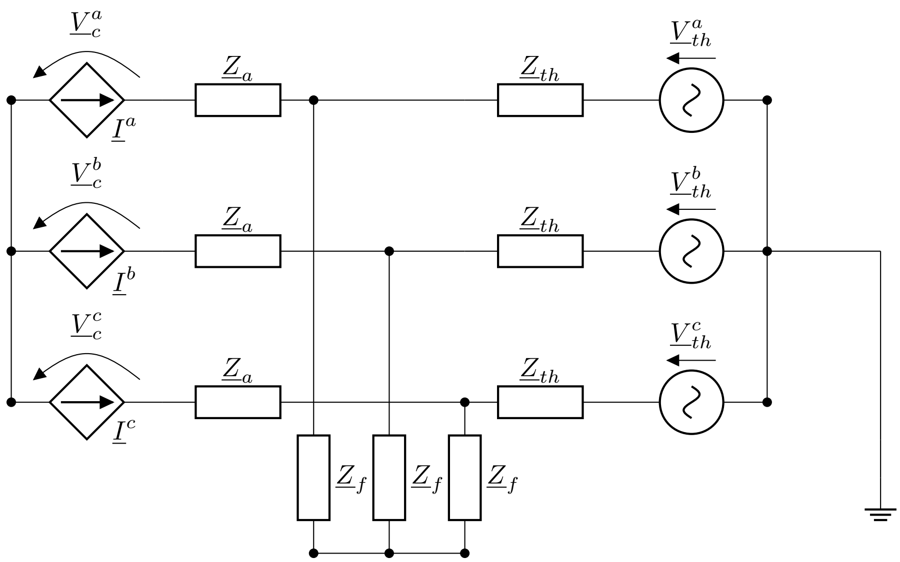

Figure 17: Balanced fault schematic

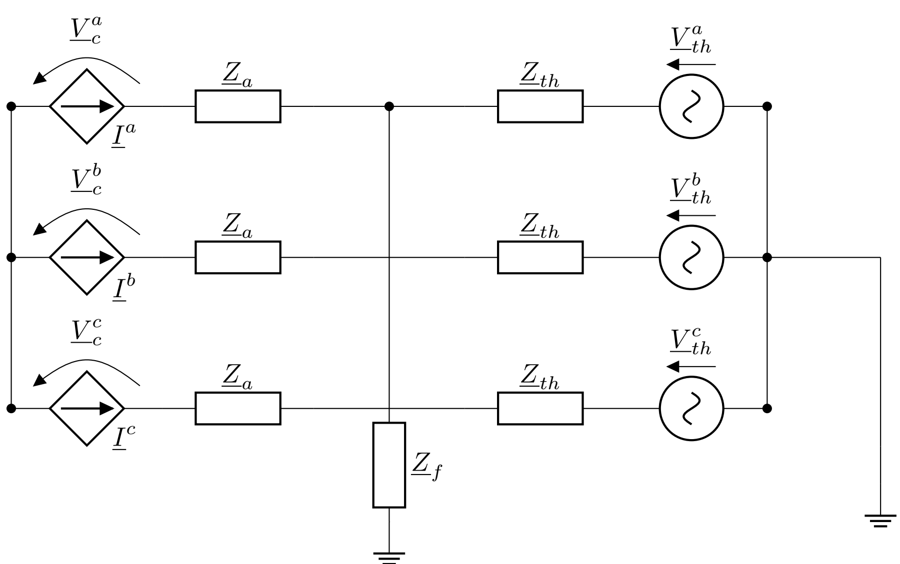

Figure 18: Line to ground fault schematic

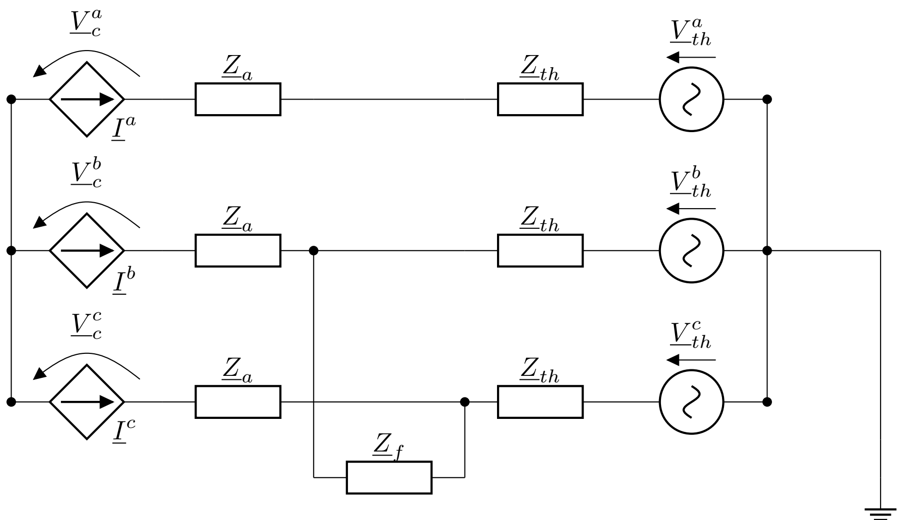

Figure 19: Line to line fault schematic

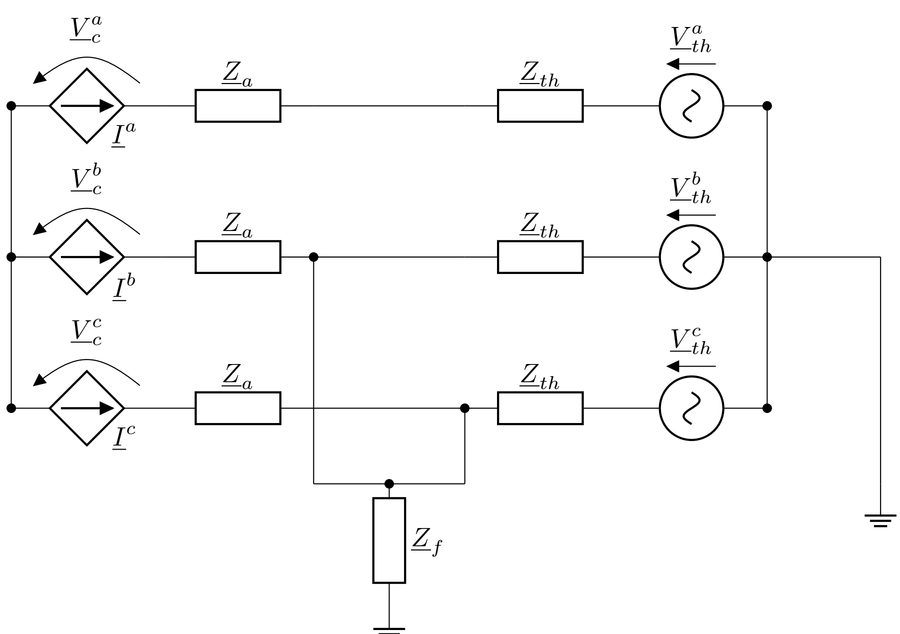

Figure 20: Double line to ground fault schematic

B $+-0$ schemes

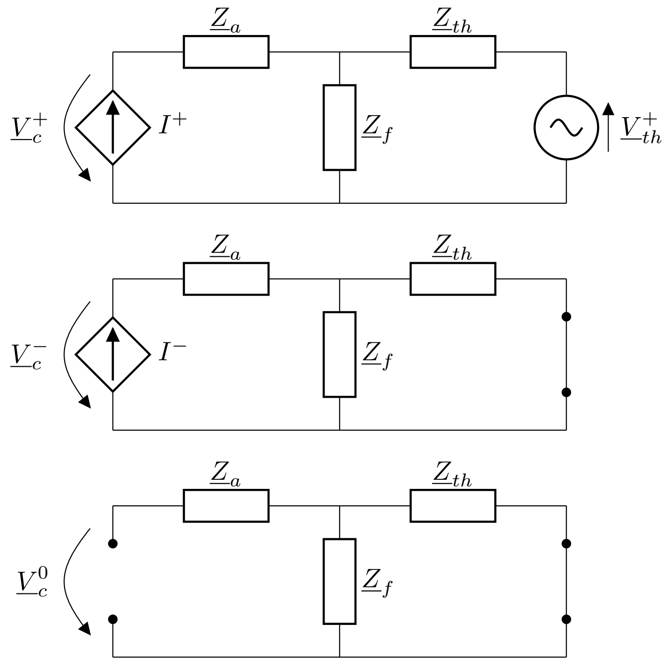

Figure 21: Equivalent circuit for the balanced fault analysis

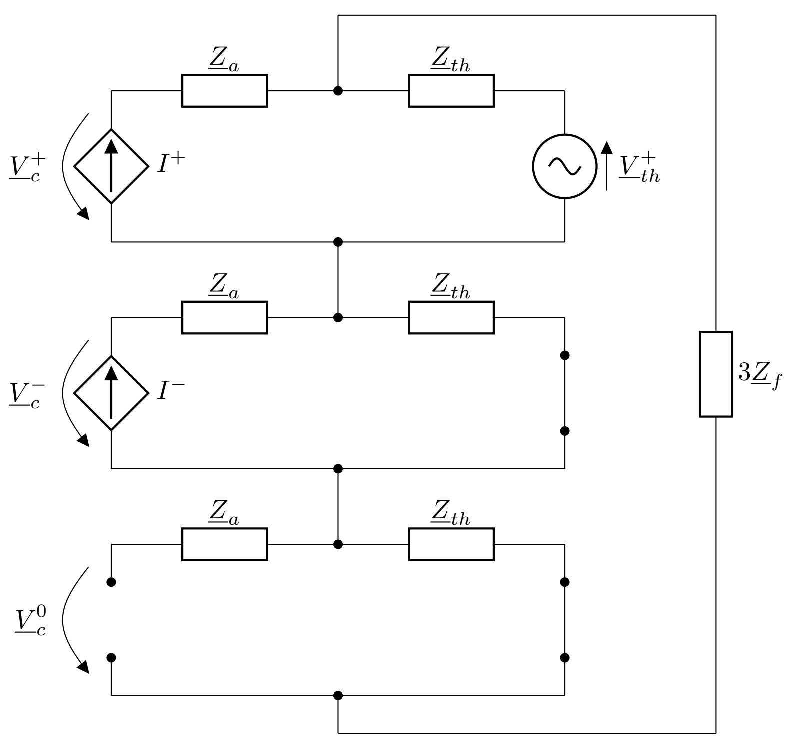

Figure 22: Equivalent circuit for the line to ground fault analysis

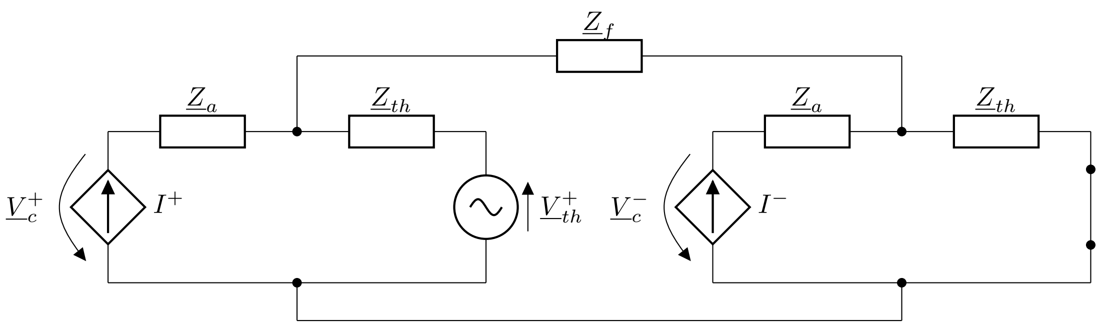

Figure 23: Equivalent circuit for the line to line fault analysis

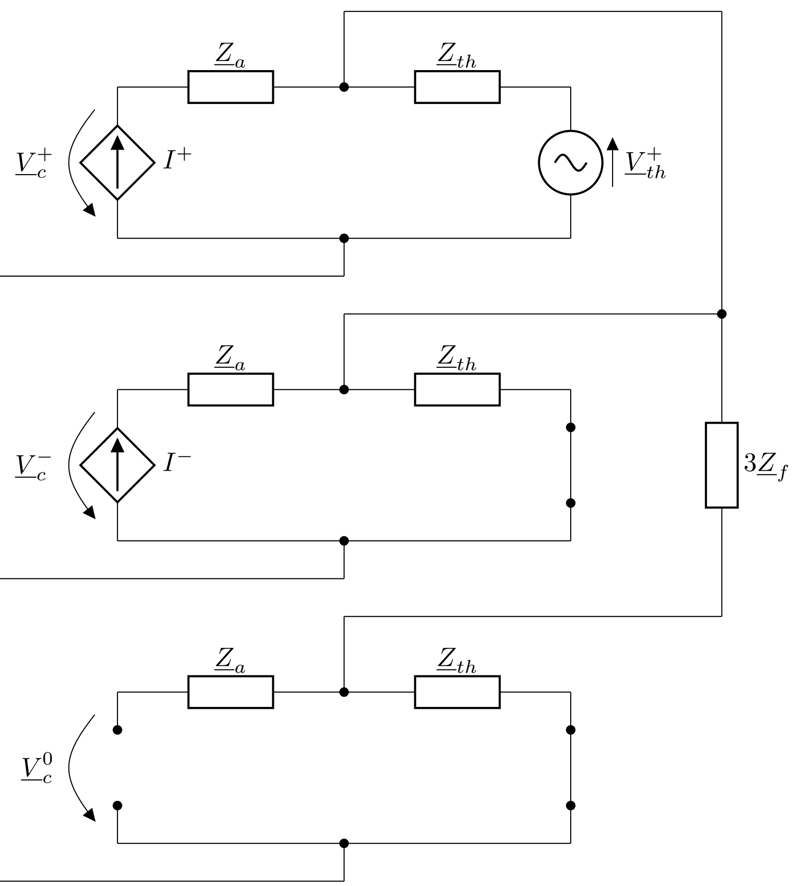

Figure 24: Equivalent circuit for the double line to ground fault analysis

C Expressions

C.1 Balanced fault

$$\begin{cases} \underline{V}_c^a = \frac{1}{\underline{Z}_f + \underline{Z}_{th}} [\underline{V}_{th}^a \underline{Z}_f + \underline{I}_a (\underline{Z}_a \underline{Z}_{th} + \underline{Z}_{th} \underline{Z}_f + \underline{Z}_f \underline{Z}_a)] \\ \underline{V}_c^b = \frac{1}{\underline{Z}_f + \underline{Z}_{th}} [\underline{V}_{th}^b \underline{Z}_f + \underline{I}_b (\underline{Z}_a \underline{Z}_{th} + \underline{Z}_{th} \underline{Z}_f + \underline{Z}_f \underline{Z}_a)] \\ \underline{V}_c^c = \frac{1}{\underline{Z}_f + \underline{Z}_{th}} [\underline{V}_{th}^c \underline{Z}_f + \underline{I}_c (\underline{Z}_a \underline{Z}_{th} + \underline{Z}_{th} \underline{Z}_f + \underline{Z}_f \underline{Z}_a)] \end{cases} \tag{6}$$

$$\begin{cases} \underline{V}_c^+ = \frac{1}{\underline{Z}_f + \underline{Z}_{th}} [\underline{V}_{th}^+ \underline{Z}_f + \underline{I}^+ (\underline{Z}_a \underline{Z}_f + \underline{Z}_a \underline{Z}_{th} + \underline{Z}_f \underline{Z}_{th})] \\ \underline{V}_c^- = \frac{1}{\underline{Z}_f + \underline{Z}_{th}} [\underline{I}^- (\underline{Z}_{th} \underline{Z}_f + \underline{Z}_a \underline{Z}_{th} + \underline{Z}_a \underline{Z}_f)] \\ \underline{V}_c^0 = 0 \end{cases} \tag{7}$$

C.2 Line to ground fault

$$\begin{cases} \underline{V}_c^a = \frac{1}{\underline{Z}_{th} + \underline{Z}_f} [\underline{I}_a (\underline{Z}_a \underline{Z}_{th} + \underline{Z}_a \underline{Z}_f + \underline{Z}_{th} \underline{Z}_f) + \underline{V}_{th}^a \underline{Z}_f] \\ \underline{V}_c^b = \underline{V}_{th}^b + \underline{I}_b (\underline{Z}_a + \underline{Z}_{th}) \\ \underline{V}_c^c = \underline{V}_{th}^c + \underline{I}_c (\underline{Z}_a + \underline{Z}_{th}) \end{cases} \tag{8}$$

$$\begin{cases} \underline{V}_c^+ = \underline{I}^+ (\underline{Z}_a + \underline{Z}_{th}) + \underline{V}_{th}^+ - \frac{\underline{Z}_{th}}{3\underline{Z}_f + 3\underline{Z}_{th}} [\underline{I}^+ \underline{Z}_{th} + \underline{I}^- \underline{Z}_{th} + \underline{V}_{th}^+] \\ \underline{V}_c^- = \underline{I}^- (\underline{Z}_a + \underline{Z}_{th}) - \frac{\underline{Z}_{th}}{3\underline{Z}_f + 3\underline{Z}_{th}} [\underline{I}^+ \underline{Z}_{th} + \underline{I}^- \underline{Z}_{th} + \underline{V}_{th}^+] \\ \underline{V}_c^0 = -\frac{\underline{Z}_{th}}{3\underline{Z}_f + 3\underline{Z}_{th}} [\underline{V}_{th}^+ + \underline{I}^+ \underline{Z}_{th} + \underline{I}^- \underline{Z}_{th}] \end{cases} \tag{9}$$

C.3 Line to line fault

$$\begin{cases} \underline{V}_c^a = \underline{I}_a (\underline{Z}_a + \underline{Z}_{th}) + \underline{V}_{th}^a \\ \underline{V}_c^b = \underline{I}_b \underline{Z}_a + \frac{1}{(\underline{Z}_f + \underline{Z}_{th})(\underline{Z}_f + 2\underline{Z}_{th})} [\underline{I}_b (\underline{Z}_{th} \underline{Z}_f \underline{Z}_f + 2\underline{Z}_{th} \underline{Z}_{th} \underline{Z}_f + \underline{Z}_{th} \underline{Z}_{th} \underline{Z}_{th}) \\ +\underline{I}_c (\underline{Z}_{th} \underline{Z}_{th} \underline{Z}_f + \underline{Z}_{th} \underline{Z}_{th} \underline{Z}_{th}) + \underline{V}_{th}^b (\underline{Z}_f \underline{Z}_f + 2\underline{Z}_f \underline{Z}_{th} + \underline{Z}_{th} \underline{Z}_{th}) + \underline{V}_{th}^c (\underline{Z}_{th} \underline{Z}_f + \underline{Z}_{th} \underline{Z}_{th})] \\ \underline{V}_c^c = \underline{I}_c \underline{Z}_a + \frac{1}{\underline{Z}_f + 2\underline{Z}_{th}} [\underline{I}_c (\underline{Z}_{th} (\underline{Z}_f + \underline{Z}_{th})) + \underline{V}_{th}^c (\underline{Z}_f + \underline{Z}_{th}) + \underline{I}_b \underline{Z}_{th} \underline{Z}_{th} + \underline{V}_{th}^b \underline{Z}_{th}] \end{cases} \tag{10}$$

$$\begin{cases} \underline{V}_c^+ = \underline{V}_{th}^+ + \underline{I}^+ (\underline{Z}_a + \underline{Z}_{th}) - \frac{\underline{Z}_{th}}{2\underline{Z}_{th} + \underline{Z}_f} [\underline{V}_{th}^+ + \underline{I}^+ \underline{Z}_{th} - \underline{I}^- \underline{Z}_{th}] \\ \underline{V}_c^- = \underline{V}_{th}^+ + \underline{I}^+ \underline{Z}_{th} + \underline{I}^- \underline{Z}_a - \frac{\underline{Z}_{th} + \underline{Z}_f}{2\underline{Z}_{th} + \underline{Z}_f} [\underline{V}_{th}^+ + \underline{I}^+ \underline{Z}_{th} - \underline{I}^- \underline{Z}_{th}] \\ \underline{V}_c^0 = 0 \end{cases} \tag{11}$$

C.4 Double line to ground fault

$$\begin{cases} \underline{V}_c^a = \underline{I}_a (\underline{Z}_a + \underline{Z}_{th}) + \underline{V}_{th}^a \\ \underline{V}_c^b = \underline{I}_b \underline{Z}_a + \frac{\underline{Z}_{th} \underline{Z}_f (\underline{I}_b + \underline{I}_c) + \underline{Z}_f (\underline{V}_{th}^b + \underline{V}_{th}^c)}{2\underline{Z}_f + \underline{Z}_{th}} \\ \underline{V}_c^c = \underline{I}_c \underline{Z}_a + \frac{\underline{Z}_{th} \underline{Z}_f (\underline{I}_b + \underline{I}_c) + \underline{Z}_f (\underline{V}_{th}^b + \underline{V}_{th}^c)}{2\underline{Z}_f + \underline{Z}_{th}} \end{cases} \tag{12}$$

$$
\begin{cases}
\underline{V}_c^+ = \underline{I}^+ \underline{Z}_a + \dfrac{\underline{Z}_{th} + 3\underline{Z}_f}{3\underline{Z}_{th} + 6\underline{Z}_f} [\underline{I}^+ \underline{Z}_{th} + \underline{I}^- \underline{Z}_{th} + \underline{V}_{th}^+] \\
\underline{V}_c^- = \underline{I}^- \underline{Z}_a + \dfrac{\underline{Z}_{th} + 3\underline{Z}_f}{3\underline{Z}_{th} + 6\underline{Z}_f} [\underline{I}^+ \underline{Z}_{th} + \underline{I}^- \underline{Z}_{th} + \underline{V}_{th}^+] \\
\underline{V}_c^0 = \dfrac{\underline{Z}_{th}}{3\underline{Z}_{th} + 6\underline{Z}_f} [\underline{I}^+ \underline{Z}_{th} + \underline{I}^- \underline{Z}_{th} + \underline{V}_{th}^+]
\end{cases}
\tag{13}
$$