Positive and negative sequence currents to improve voltages during unbalanced faults

CITCEA

Josep Fanals

03/2021

1 Introduction

Grid faults constitute a group of unfortunate events that cause severe perturbations in the grid. The voltages can take values below the established minimum, or on the contrary, exceed the maximum in non-faulted phases. The currents are also susceptible to vary considerably. Traditional power systems based on synchronous generators could encounter currents surpassing the nominal values, and therefore, the fault could be clearly detected. However, the increasing integration of renewables [**anees2012grid**] supposes a change of paradigm, in which currents can be controlled but are limited so as not to damage the Isolated-Gate Bipolar Transistors (IGBT) found in the Voltage Source Converter (VSC) [**abdou2013improving**].

Transmission System Operators (TSO) are responsible for imposing requirements related to the operation under voltage sags to generators and converters [**tsili2009review**, **iov2007mapping**]. Such requirements are gathered in the respective grid codes. There seems to be no clear consensus on how to restore the voltage. In this sense, even if for instance the Low Voltage Ride Through (LVRT) profiles present similarities [**conroy2007low**], analysis aimed at determining analytically the optimal injection of positive and negative sequence currents are not numerous. As far as we are aware, only Camacho et al. offer an optimal solution regarding the injection of active and reactive powers [**camacho2017positive**].

Consequently, this work focuses on finding the most convenient positive and negative sequence currents (and not powers) to improve the voltage at the point of common coupling. First, a simple model is presented, and then, the results are shown together with the corresponding discussion. We first analyze a basic case with brute force and finding the optimality with the SciPy as well. Then, we proceed to study what happens when the Thévenin impedance of the grid changes its R/X ratio and what occurs when we substitute the impedance linking the VSC to the grid by a cable with its π equivalent.

2 Optimization overview and constant impedance analysis

The system we are considering is formed by an ideal grid (only positive sequence voltage is present) coupled to a VSC. The model is depicted in Figure 1. The VSC will be controlled in a way that leads to an improvement in the voltage at the PCC.

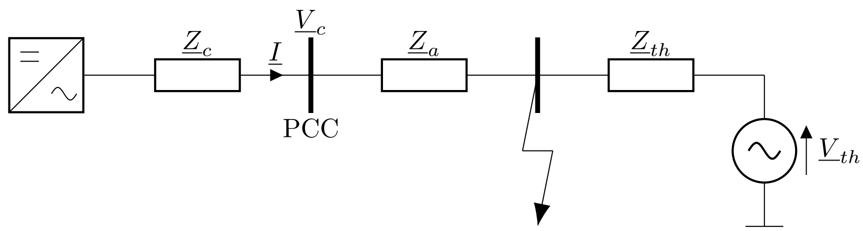

Figure 1: Single-phase representation of the simple system under a fault

This study considers the balanced fault type but also the unbalanced ones, which include the line to ground, the double line to ground and the line to line fault. The model in Figure 1 attempts to describe simple yet sufficient system to test the influence of the injected currents by the VSC on the voltage at the PCC. The system could be complicated by adding parallel capacitances on the right hand side of the Thevenin impedance, for instance, to model a hypothetical submarine cable. However, for now we prioritize keeping the system simple.

We are concerned with improving the voltage $\underline{V}_c$ as a function of the current $\underline{I}$ injected by the converter. Formally speaking there is no clear definition on what improving the voltage means as it depends on rather pre-stablished preferences. For instance, one could try to maximize the positive sequence voltage, minimize the negative sequence voltage or even maximize the difference between both. These three strategies have been covered in [**camacho2017positive**]. A more flexible approach is based on defining the objective function as

$$f(\underline{I}^+, \underline{I}^-) = \lambda^+ |(|\underline{V}_c^+(\underline{I}^+, \underline{I}^-)| - 1)| + \lambda^- |(|\underline{V}_c^-(\underline{I}^+, \underline{I}^-)| - 0)|, \tag{1}$$

where the weighting factors λ^+, $\lambda^- \in \mathbb{R}$. By adjusting these factors one can follow the three aforementioned strategies. Note that the goal is to obtain a positive sequence voltage as close as possible to one (in per unit) while simultaneously approaching zero in the negative sequence voltage. Even though the problem is described as a function of the positive and negative sequence currents, it could also be stated as a function of the original *abc* currents without loss of generality. The associated expressions are gathered in the appendix as well.

Minimizing f does not come with complete freedom. That is, the currents are constrained so as not to exceed the IGBT limits. It becomes more convenient to express the constraints in the *abc* frame:

$$g(\underline{I}_a, \underline{I}_b, \underline{I}_c) = \begin{cases} |\underline{I}_a| \leq I_{max}, \\ |\underline{I}_b| \leq I_{max}, \\ |\underline{I}_c| \leq I_{max}. \end{cases} \tag{2}$$

For now we are not concerned with the voltages imposed to the semiconductors due to the fact that the filter $\underline{Z}_c$ is most likely to take small values. In practical situations current limitations are the most serious constraint. Therefore, we define the optimization problem as follows:

$$\min_{\underline{I}^+, \underline{I}^-} \quad f(\underline{I}^+, \underline{I}^-) \tag{3a}$$

$$\text{subject to} \quad g(\underline{I}_a, \underline{I}_b, \underline{I}_c), \tag{3b}$$

where the currents in the *abc* frame can be related to the currents expressed in the symmetrical components form by means of Fortescue's transformation, and vice versa [**fortescue1918method**].

As a direct consequence of that, the analysis can be performed in the *abc* frame while expressing the voltages as a function of the positive, negative and homopolar components. Another valid procedure is to work with the symmetrical components and relate the currents to the *abc* ones. Both ways to confront the problem are equally valid. The results obtained in this study have been generated and validated with both paths.

Current grid codes define the low voltage ride through (LVRT) limit curve which indicates the relation between the duration of a fault and the voltage frontier at which for instance a wind turbine ought to disconnect. Such LVRT curves present slight variations from country to country [**tsili2009review**] but they all share the same pattern: in case of a severe fault, the disconnection should not take place if it has a low duration; on the contrary, less noticeable faults imply a larger disconnection time. Grid codes usually impose a curtailment of active power and enforce the generation of reactive power in order to collaborate on raising the voltage [**altin2010overview**, **serban2016voltage**].

Nevertheless, studies dealing with determining the optimal currents to inject are not precisely numerous. At most, some authors express the voltages in terms of active and reactive power but do not consider constraints [**camacho2012flexible**]. Others formulate the optimization problem and arrive to a closed-form expression, even though it becomes iterative [**camacho2017positive**]. The analysis is performed contemplating powers rather than currents. Because of that, this work focuses on computing the optimal currents to increase the positive sequence voltage as close to one as possible and achieving a negative sequence voltage that approaches zero. In essence, the results that follow come from solving Equations 3a and 3b.

Four types of faults are considered in this study. One is the balanced fault, which yields a simple yet valuable system to analyze the distribution of optimal currents. The remaining faults are unbalanced: the line to ground, the line to line and the double line to ground cases.

In all situations, we show plots representing the objective function depending on the real and imaginary parts of the positive and negative sequence currents. They are obtained in a brute force manner, that is, we generate multiple combinations of currents and store those points if they do not respect the constraints. The solution obtained by solving the optimization problem as such with the SciPy library is also displayed, together with optimal point supposing no active currents can be injected. The latter tries to emulate the outcome of following the grid codes to improve voltages. Note that grid codes also consider the injection of active power, but this serves the purpose of maintaining a stable system in terms of frequency, something not contemplated in this analysis.

For this and all the upcoming faults we have set the values shown in Table 1.

Magnitude	Value (pu)
$\underline{Z}_f$	$0.00 + 0.10j$
$\underline{Z}_a$	$0.01 + 0.10j$
$\underline{Z}_{th}$	$0.01 + 0.05j$
I_{max}	1.00
λ^+	1.00
λ^-	1.00
$\lvert\underline{V}_{th}^+\rvert$	1.00
$\lvert\underline{V}_{th}^-\rvert$	0.00

Table 1: Values for the system under study

We have considered the impedances to be mainly inductive. However, adding a resistive part not only makes them more realistic, but it also may shed some light on if the truly optimal decision is to inject only reactive currents. One could anticipate that the larger the resistive part becomes, the greater the active current should be to cause a considerable voltage drop (positive or negative) so that the voltage is improved. Thus, for the given impedances in Table 1, we foresee the fact that reactive currents will turn out to be substantially larger than active currents. Besides, the same weighting is arbitrarily attributed to the positive as well as to the negative sequence one.

2.1 Balanced fault

Balanced faults are commonly referred to as the most severe type of fault, as currents take the largest values [**kothari2003modern**]. Its representation in symmetrical components indicates a decoupling between sequences. As a result of that, a considerable positive sequence current should be injected to increase the positive sequence voltage. It should be mostly inductive. On the other hand, no negative sequence current has to be injected to keep a null negative sequence voltage (assuming that the grid presents no negative sequence voltage, as it is the case here). There are no homopolar currents in this and all the subsequent faults.

Figure 2 shows on the one hand the results with the brute force methodology. We have employed over 20^4 combinations to construct the plots. A more regular shape would be obtained in case we worked with smaller intervals. However, some points are suppressed because they exceed the current limitations. It is clear that the minimum is achieved when the negative sequence currents, both real and imaginary, tend to zero. The imaginary positive sequence current takes extreme negative values - close to I_{max} - while the real part remains small.

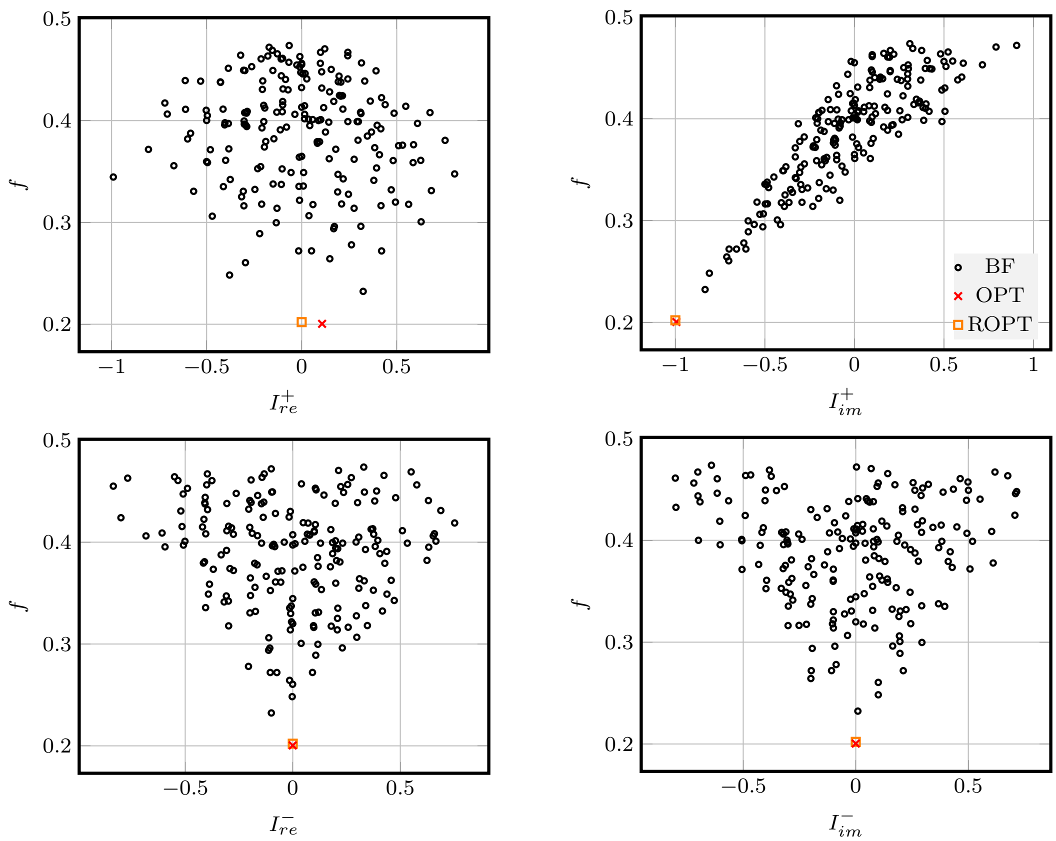

Figure 2: Influence of the currents on the objective function for the balanced fault when $\lambda^+ = 1$ and $\lambda^- = 1$. BF: brute force, OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

On the other hand, the point corresponding to the solution of the optimization problem illustrates what was already more or less deduced from the brute force computations. Nonetheless, solving the optimization problem yields a more favorable result. The objective function is slightly smaller and the optimal point can be located in a zone where not many points coming from the brute force are present. Such irregular distribution of points is due to the fact that around the optimal points many combinations of currents do not meet the constraints. In any case, injecting a negative imaginary positive sequence current causes a positive voltage drop so that the voltage we wish to improve can be far apart from the faulted one. Taking into account that in this case $X >> R$, the real positive sequence current becomes considerably smaller than the imaginary part.

The brute force calculation in Figure 2 was computed considering that $\lambda^+ = 1$ and $\lambda^- = 1$ as well. However, the optimality can also be studied for various λ values. This generic parameter would fit in the objective function as

$$f(\underline{I}^+, \underline{I}^-) = \lambda|(|\underline{V}_c^+(\underline{I}^+, \underline{I}^-)| - 1)| + (1-\lambda)|(|\underline{V}_c^-(\underline{I}^+, \underline{I}^-)| - 0)|, \tag{4}$$

where $\lambda = [0, 1]$. Therefore, bigger values of λ would imply that we prioritize the positive sequence voltage while small values will tend to give more importance to the negative sequence voltage. Figure 3 shows the voltage profiles for a sweep of λ values.

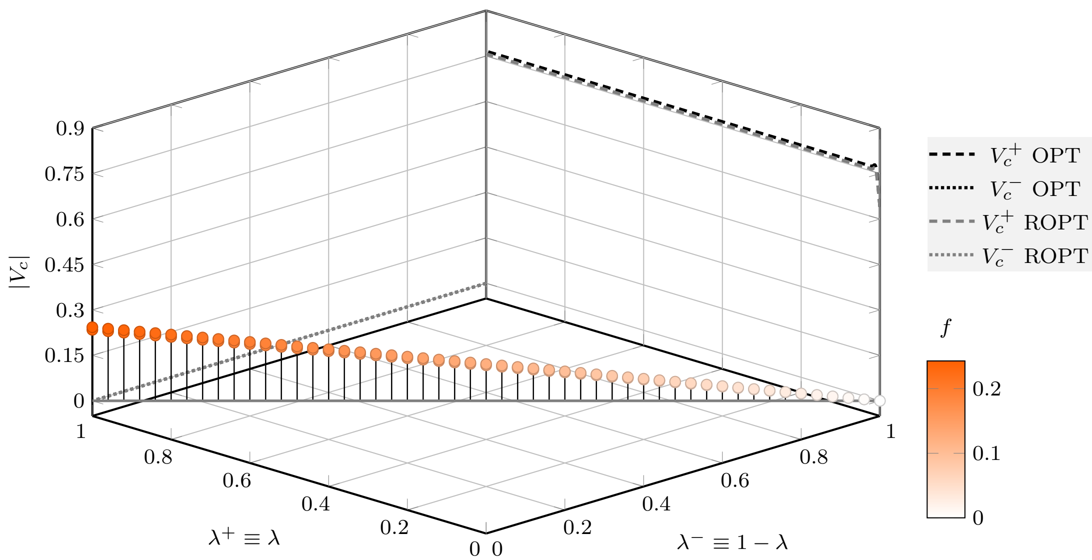

Figure 3: Sequence voltages together with the objective function for the balanced fault

2.2 Line to ground fault

The line to ground fault has the particularity of presenting a distribution of dots similar for both real and imaginary currents for both sequences. As shown in Figure 4, the minimum in the real currents plot takes place around the zero. Despite that, in reality, the real positive sequence current becomes slightly larger than zero, whilst for the negative sequence it takes a small but not negligible negative value. Observing Figure 33 it becomes clear that the real part of the positive sequence currents ought to be greater than zero whereas the negative sequence current should take negative values; these combinations cause the maximization of the positive sequence voltage and the minimization of the negative sequence voltage.

The imaginary part of the currents presents a similar distribution in both cases. Notice that contrarily to the balanced fault, where one vertex of the distribution coincided with the point at which f was at its minimum, there is a full edge in which the function becomes minimum. This phenomena suggests that maybe there is more than one minimum. For instance, carrying the analysis in the *abc* frame caused the optimal currents $[\underline{I}^+, \underline{I}^-]$ to become $[0.1715 - j0.4955, -0.0804 - j0.5003]$. Leaving aside the differences in current, the objective function became the same up to a precision of 10^{-10}.

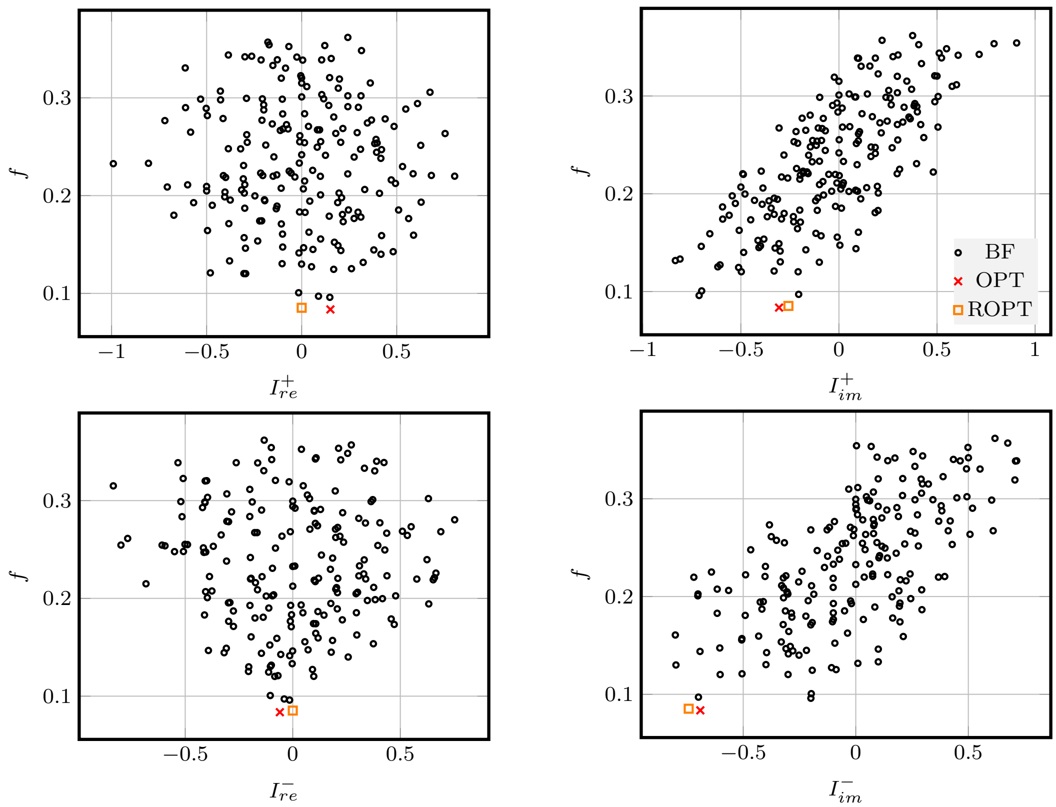

Figure 4: Influence of the currents on the objective function for the line to ground fault. BF: brute force, OPT: optimization problem, ROPT: optimization problem restricted to injecting reactive power.

The differences are due to initializing differently the currents passed to the SciPy `minimize()` function. Besides, Figure 5 shows the distribution of voltages and f across multiple values of λ.

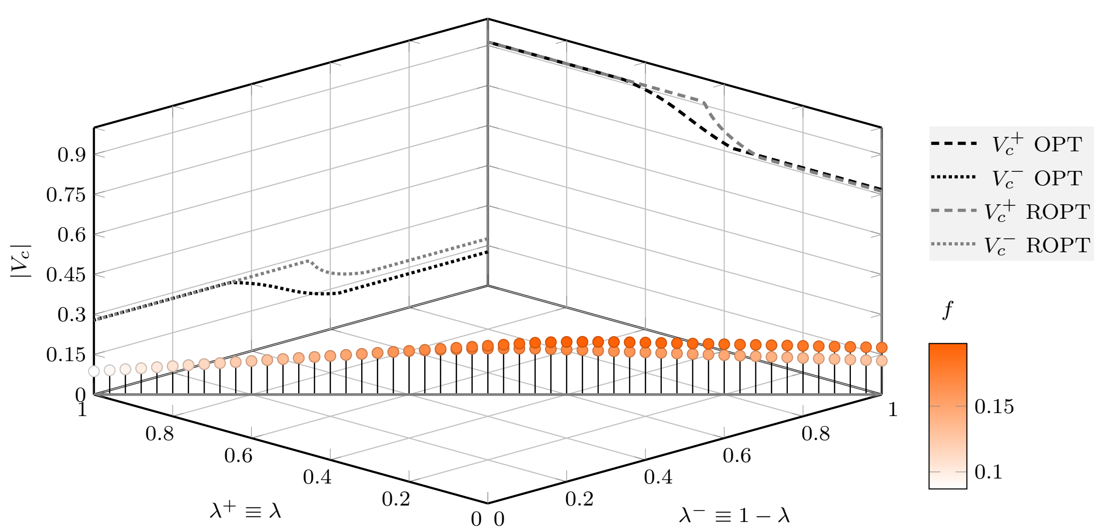

Figure 5: Sequence voltages together with the objective function for the line to ground fault

2.3 Line to line fault

The line to line fault can be considered to be a more severe fault compared to the line to ground case, since the function to minimize presents larger values. This can be understood when looking at the representations in Figures 29 and 30, where the fault impedance is between two phases and not phase and ground. Therefore, the voltage drop becomes larger. Figure 6 shows that even if the real part of the currents remains near the zero, visually speaking the imaginary parts take symmetrical values. The distribution of dots for the imaginary part of the negative sequence current is the main difference with respect to the line to ground fault.

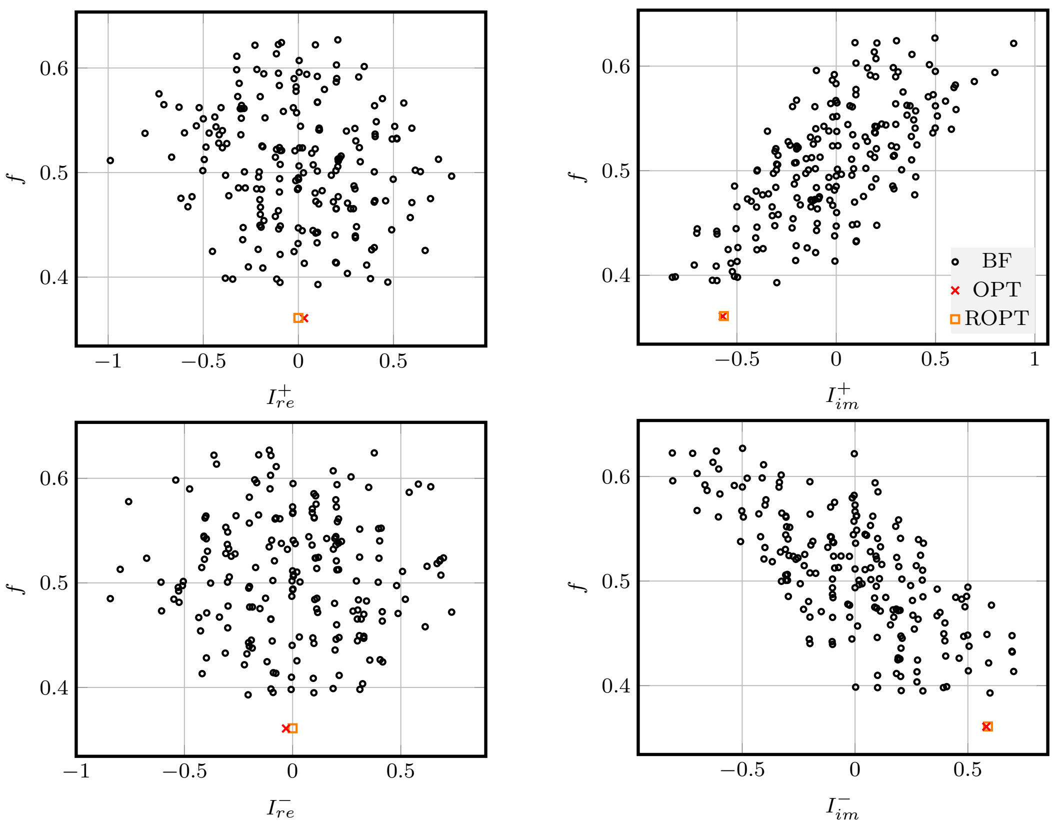

Figure 6: Influence of the currents on the objective function for the line to line fault. BF: brute force, OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

However, this time it has been checked that there is only one point that minimizes the function, independently of the initialization vector. Not less surprising, the imaginary positive and negative sequence currents are practically the same with a change of sign. We can intuitively make sense of it as the positive sequence voltage at the PCC tends to one while the negative one approaches zero. Thus, the current flowing towards the $\underline{Z}_f$ impedance is quite large and a big part of it is due to the injected positive sequence current. To achieve a small negative sequence voltage, as can be understood from Figure 34, the negative sequence current has to counteract the positive sequence current. Because of that, the optimal positive and negative sequence currents share almost the same magnitude and opposite signs.

Figure 7 depicts the plot where the positive and negative sequence voltages as well as the objective function are plotted.

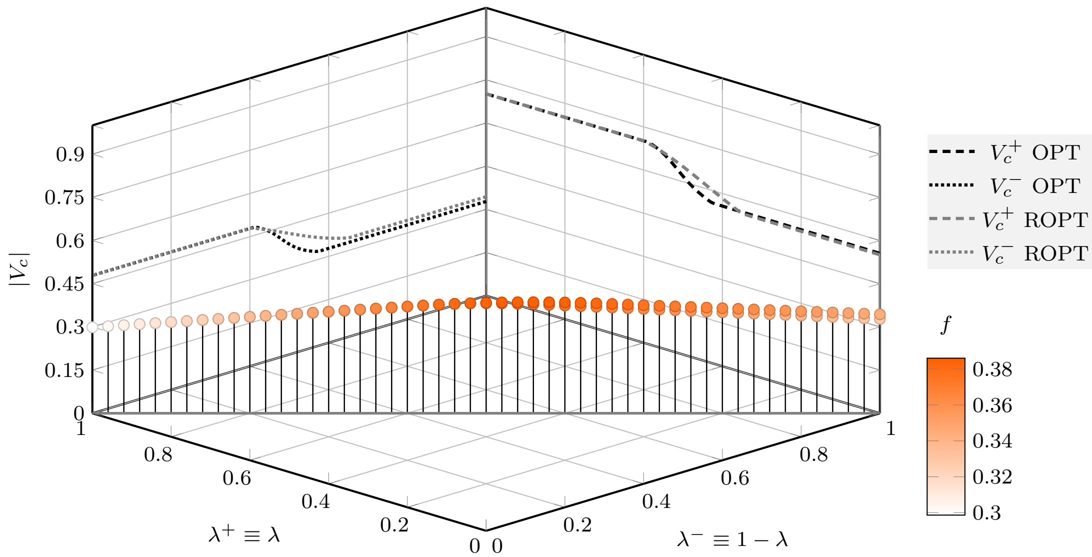

Figure 7: Sequence voltages together with the objective function for the line to line fault

2.4 Double line to ground fault

The double line to ground fault is likely to be the most severe in terms of voltage. It turns out to have the more distant voltages from the references, as shown in [**taul2020modeling**]. By nature, this fault is defined by a solid line to line connection and also a link between the two faulted phases and ground through an impedance. Figure 8 reveals that minimum of f falls considerably far from the ideal zero.

The double line to ground fault has many similarities with the line to line fault. First of all, the distribution of dots for the brute force computations is similar. Secondly, the optimal currents do not differ much from the ones obtained in Figure 6. All this could be deduced from its sequence representation. The same logic applies: the negative sequence current mirrors the positive sequence.

Again, only injecting reactive currents (ROPF case) has been experimentally proved to become a suboptimal yet highly convenient choice. The objective functions present tiny variations in this case. Even though the real currents in both sequence differ a bit, for the imaginary ones the OPT and the ROPT computations generate almost the same value. Therefore, the injection of real currents becomes secondary for highly inductive impedances.

Finally, figure 9 displays what the distribution of voltages and the subsequent objective function look like for variations in the λ parameter.

Table 2 gathers the numerical results for all the studied faults.

Fault	f	$\lvert V^+\rvert$	$\lvert V^-\rvert$	I^+_{re}	I^+_{im}	I^-_{re}	I^-_{im}
Balanced	0.200	0.800	0.000	0.107	-0.994	0.000	0.000
Line to ground	0.084	0.966	0.049	0.176	-0.535	-0.085	-0.461
Line to line	0.361	0.820	0.180	0.030	-0.571	-0.030	0.582
Double line to ground	0.884	0.525	0.408	0.064	-0.574	-0.064	0.574

Table 2: Main results for the balanced and the unbalanced faults

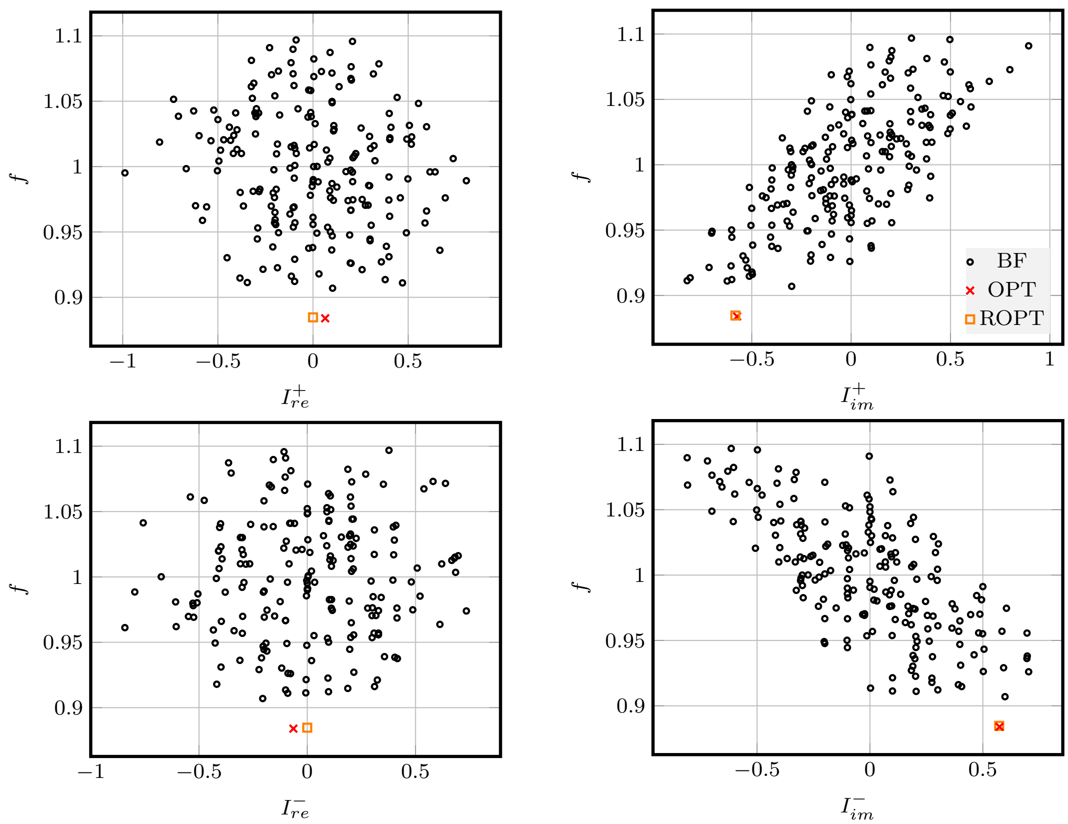

Figure 8: Influence of the currents on the objective function for the double line to ground fault. BF: brute force, OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

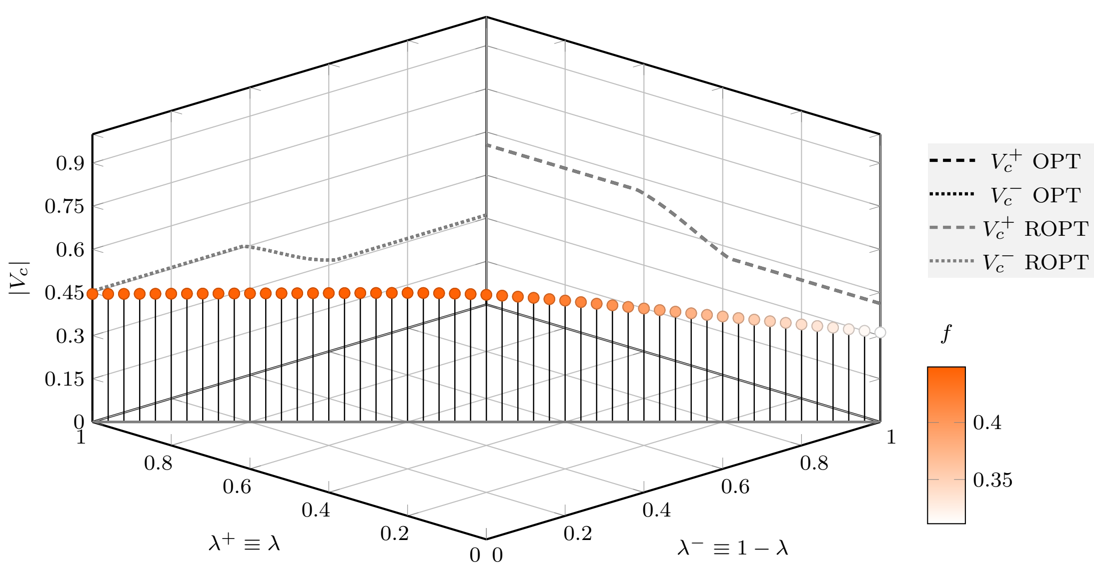

Figure 9: Sequence voltages together with the objective function for the double line to ground fault

3 Variable resistance/inductance ratio

In this section we try to answer to the question of what happens when the R/X ratio varies. This way we experiment with cases where the resistive part is considerably larger than in the aforementioned analysis. The fault impedance $\underline{Z}_f$ has been set at 0.1, which is probably more realistic than $0.0 + 0.1j$.

3.1 Balanced fault

Figure 10 depicts the optimal currents for the balanced fault.

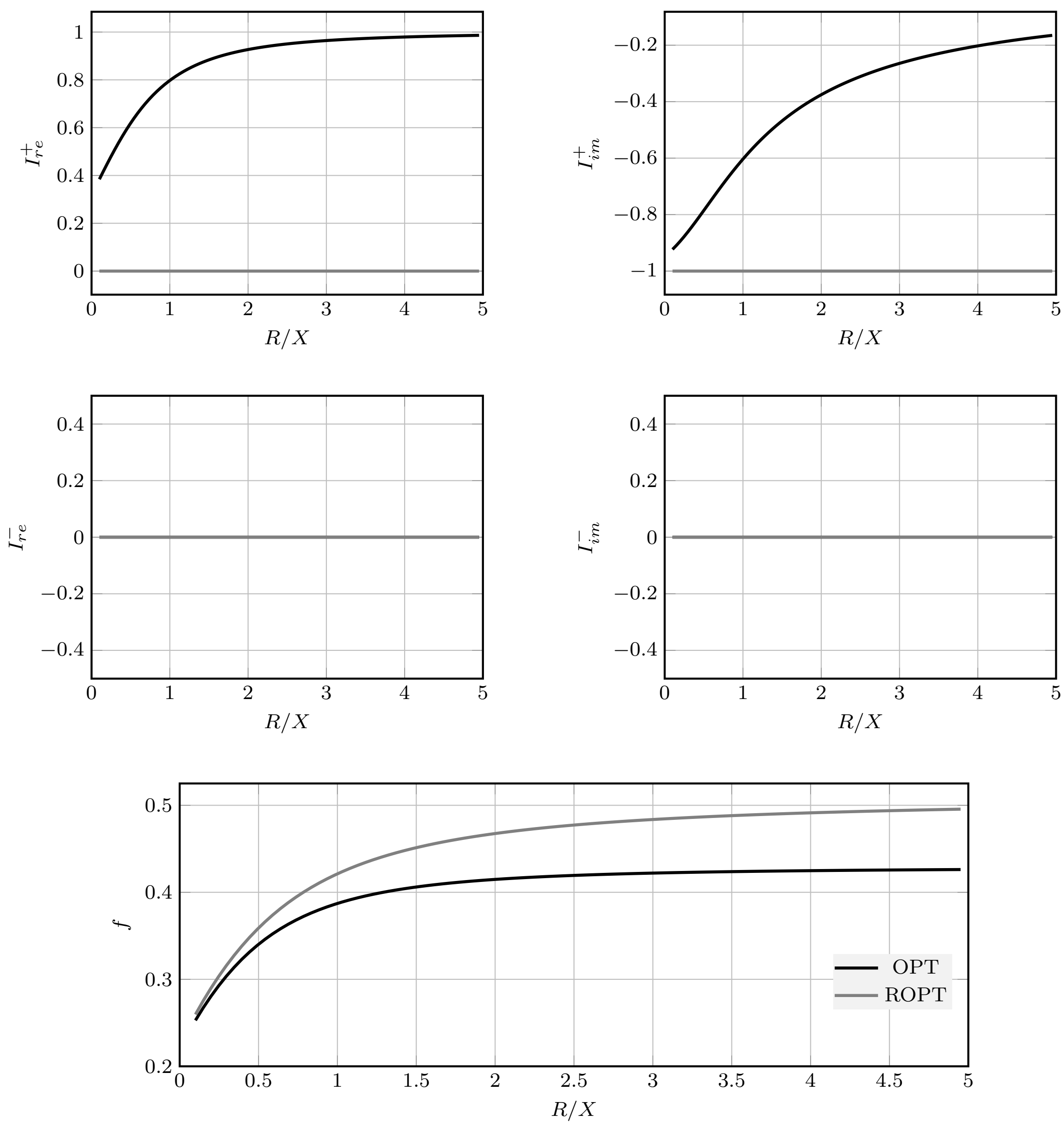

Figure 10: Influence of the currents on the objective function for the balanced fault with $\underline{Z}_f = 0.05$ and a changing R/X ratio. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

It becomes clear that the optimal current is highly dependent on the R/X ratio. That is, when $X > R$, the imaginary positive sequence is dominant, and vice versa for $R > X$. This phenomena makes complete sense when considering that we want to amplify the voltage drop in the positive sequence. However, even when there are serious differences between R and X in magnitude, neither the real nor the imaginary part go to zero. Thus, the objective function for the OPT case becomes slightly lower when compared to the ROPT case. Besides, since the negative sequence equivalent circuit is decoupled from the positive sequence one, the negative sequence currents are null in all cases.

The fault impedance has a noticeable effect on the results. When it is extremely small, the objective function almost does not vary. Opting for a too large fault impedance may cause the presence of multiple solutions. Hence, the real and imaginary currents may take unexpected values so it is hard to build intuition around the problem. We have checked that $\underline{Z}_f = 0.05$ for the balanced fault was a convenient value.

In addition, Figure 11 shows the absolute value of the voltages depending on the R/X ratio and together with the objective function.

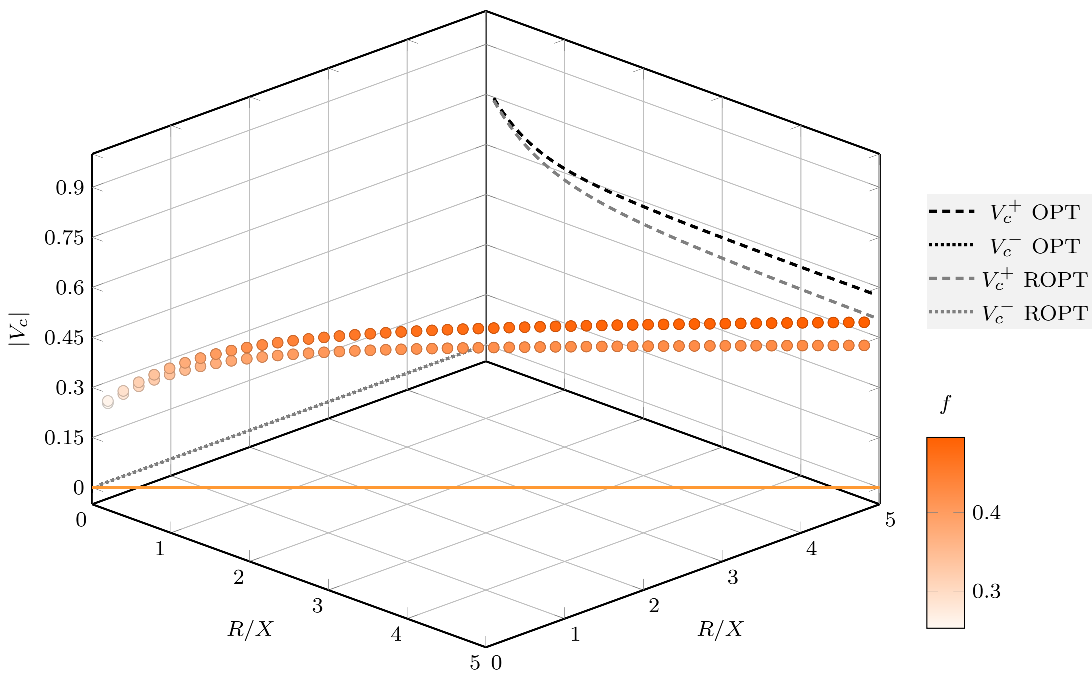

Figure 11: Sequence voltages together with the objective function for the balanced fault with $\underline{Z}_f = 0.05$ and a varying R/X ratio

As already noted in Figure 10, the objective function for the OPT case is inferior than the one for the ROPT case, and hence, closer to the ideal situation. Nevertheless, independently on having constraints on the active current, the negative sequence voltages remain at exactly zero for all the R/X range. They end up being superposed. The positive sequence voltages, instead, experience some variations depending on the R/X value. In some sense the positive sequence voltage trend is the contrary of the objective function pattern.

The best possible situation is having the reactive part larger than the real part of the impedance. This is when the positive sequence voltage tends to 0.75. Since the resistive characteristics of the impedance are so minimized, the ROPT case yields the same results as the OPT. When the resistance increases, the differences are exaggerated due to the fact that no active current can be injected in the ROPT situation.

3.2 Line to ground fault

Figure 12 depicts the optimal currents for the line to ground fault.

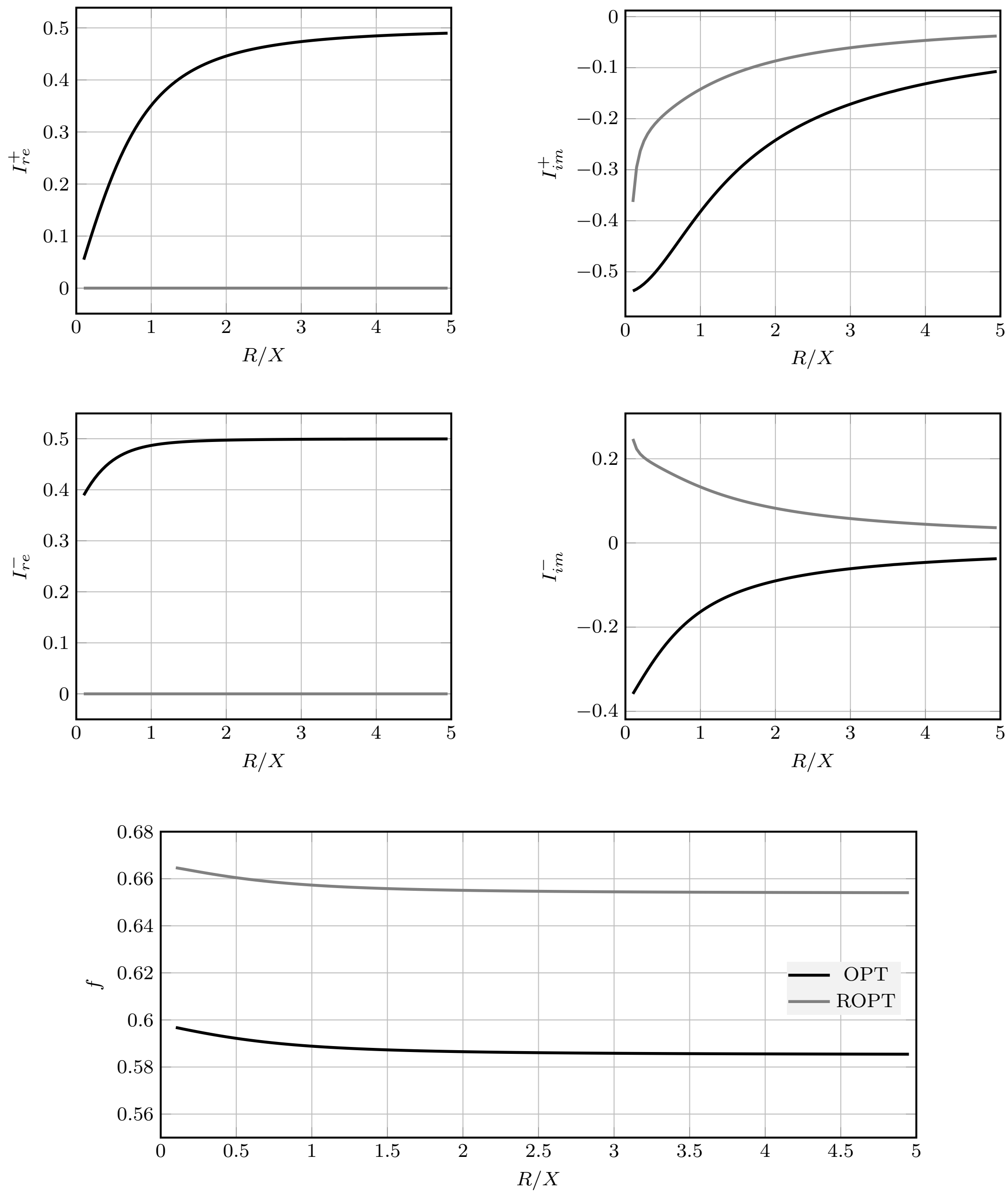

Figure 12: Influence of the currents on the objective function for the line to ground fault and a changing R/X ratio and $\underline{Z}_f = 0.001$. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

The line to ground fault has been analyzed for a fault impedance of 0.001 because otherwise the fault may not be severe enough. This is deduced from the presence of multiple optimal points for a same R/X ratio while the objective function improves substantially due to the injection of currents. Instead, in this case the objective function does not vary much for all the R/X range. Notice also that it is not far apart from the objective function in the balanced fault case. Consequently, we are able to conclude that the severity of the fault is similar thanks to the convenient adjustment of the fault impedance.

On the other hand, the differences between the OPT and the ROPT are relevant. Despite that, there is not much room for improvement in the sense that real currents, for both the positive and negative sequences, take ideally values close to 0.5 for big enough R/X ratios. Imposing the constraint of not injecting any real current causes that all influence on the voltages is achieved by means of the imaginary currents, which are not enough to improve the voltages. For instance, for $R/X \approx 5$, the maximum absolute value of the *abc* currents is one order of magnitude lower than the maximum allowed current I_{max}. Therefore, this suggests that no combination of currents is able to reduce more the objective function.

Even though we can expect that the positive sequence voltage is far from the unit value and the negative sequence is also distant from zero, the evaluation of voltages becomes worth of a particular study. Figure 13 shows the objective function along with the positive and negative sequence for the OPT and the ROPT cases.

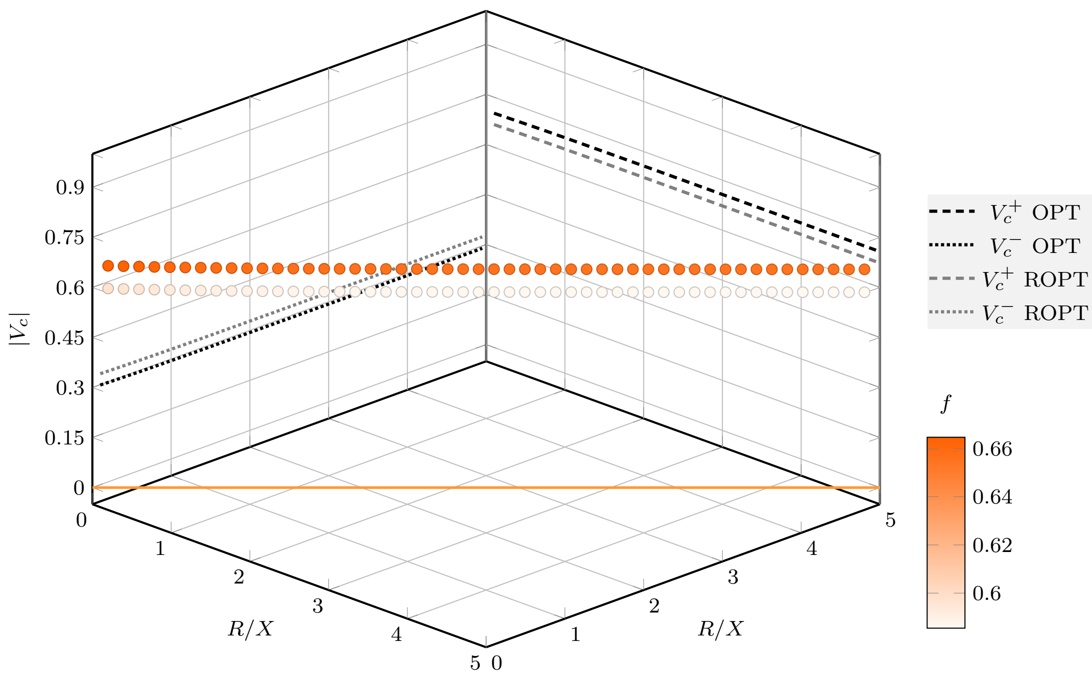

Figure 13: Sequence voltages together with the objective function for the line to ground fault with $\underline{Z}_f = 0.001$ and a varying R/X ratio

When looking at the bigger picture the objective function remains almost always the same, yet there exists a permanent difference between the OPT and the ROPT cases. The positive sequence voltages in the OPT situation are always above the ROPT ones for about 0.04 pu. For the negative sequence voltage the pattern is reversed. It is relevant to take into account that even if the voltages turn out to be practically constant, the currents experience large variations, as shown in Figure 12. Extracting conclusions regarding the fault by only observing the voltages may be misleading, as they can be maintained at the expense of injecting the specific optimal currents. Besides, just like it happened with the balanced fault, the shape of the objective function ressembles the shape of the voltages. This is logical when considering the proportionality between the objective function and the voltages.

3.3 Line to line fault

Figure 14 depicts the optimal currents for the line to line fault.

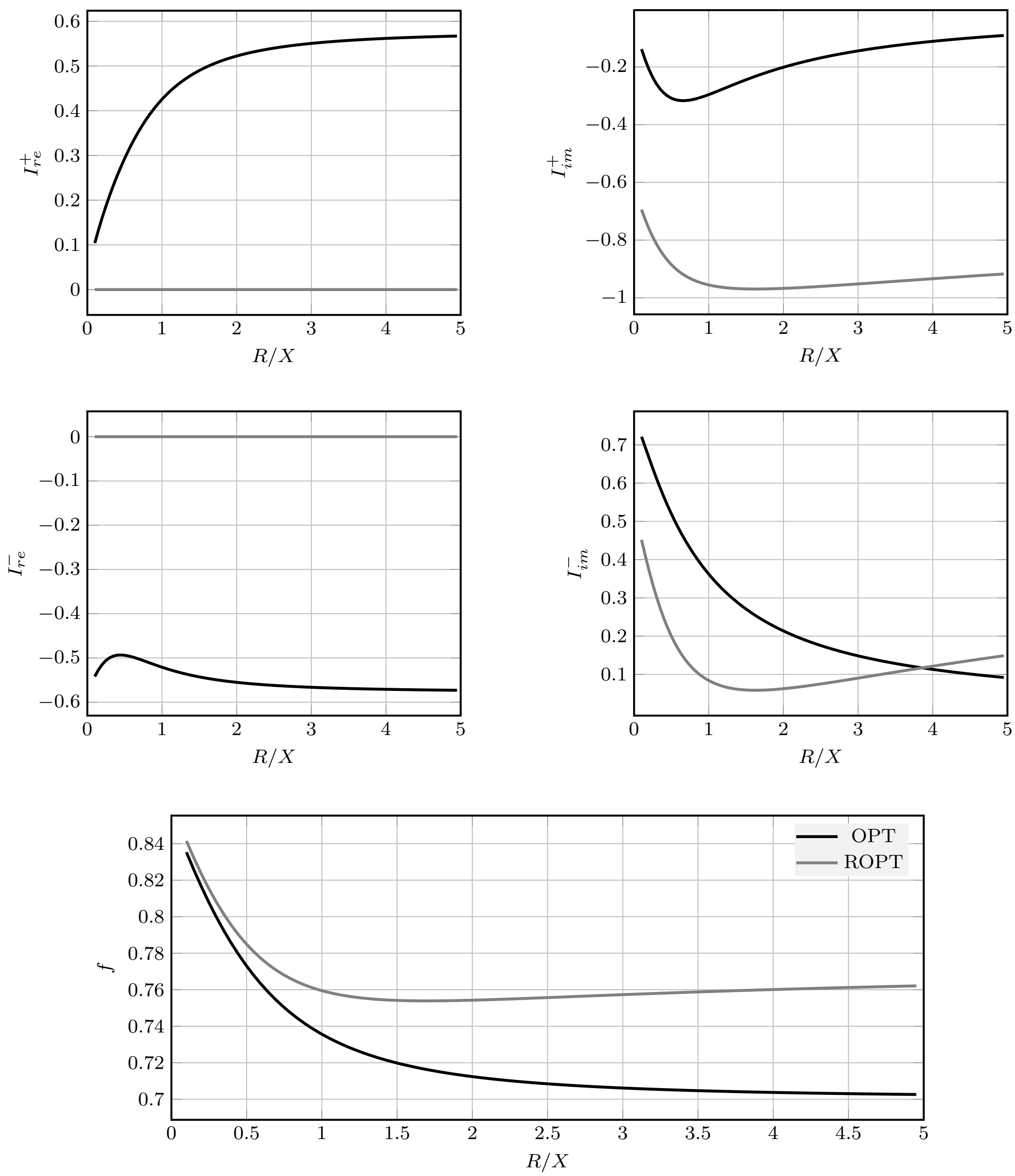

Figure 14: Influence of the currents on the objective function for the line to line fault and a changing R/X ratio with $\underline{Z}_f = 0.03$. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

This time, even if the fault is equally or more severe than in the other cases, the currents take expectable values from an intuitive standpoint. The positive sequence real current is positive to cause a positive voltage drop and improve $\underline{V}_c^+$ while the positive sequence imaginary current becomes negative to provide a resulting positive voltage drop as well. The contrary applies to the negative sequence currents.

The real part of the sequence currents become substantial, even if the R/X takes small values (see the negative sequence real current). The increase in the R/X ratio implies an increment on the real positive sequence current and specially a decrease in the negative sequence imaginary current. Contrarily, the ROPT case prioritizes the positive sequence imaginary current. Much of the current capability of the converter are invested in this component rather than in the negative sequence imaginary current. As visible in the objective function evolution, a consistent variation is present between the OPT and the ROPT case when R/X tends to grow. In further detail, the objective function for the ROPT increases a bit as a result of being incapable of injecting the much required real currents. This same phenomena happened in a more extreme way for the balanced fault.

Figure 15 displays the absolute value of the positive and negative sequence voltages, which change slightly for various R/X values. Consequently, the objective function does also vary as shown in Figure 14.

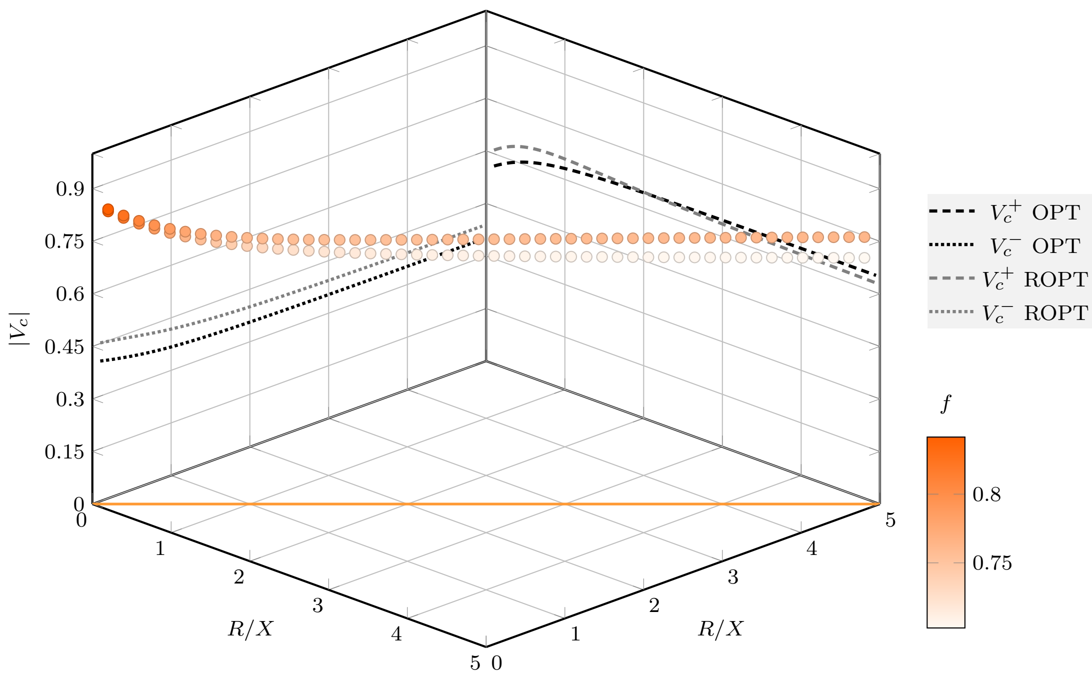

Figure 15: Sequence voltages together with the objective function for the line to line fault with $\underline{Z}_f = 0.03$ and a varying R/X ratio

Surprisingly, the positive sequence voltage for the OPT situation is not always higher than in the ROPT case. Thus, for small R/X ratios, which happen to be the most unfavorable, the large injection of positive sequence imaginary current yields higher positive sequence voltages. However, when considering the full objective function, the OPT is always better off than the ROPT. This can be explained with the negative sequence voltages. They are always smaller in the OPT situation compared to the ROPT case.

Also important, in this type of fault the result of the optimization has resulted in obtaining a positive sequence voltage that is approximately equally distant from the unit voltage than the zero voltage is from the negative sequence voltage. Although we were not looking for this balance, the optimization naturally produced it.

3.4 Double line to ground fault

Figure 16 depicts the optimal currents for the balanced fault.

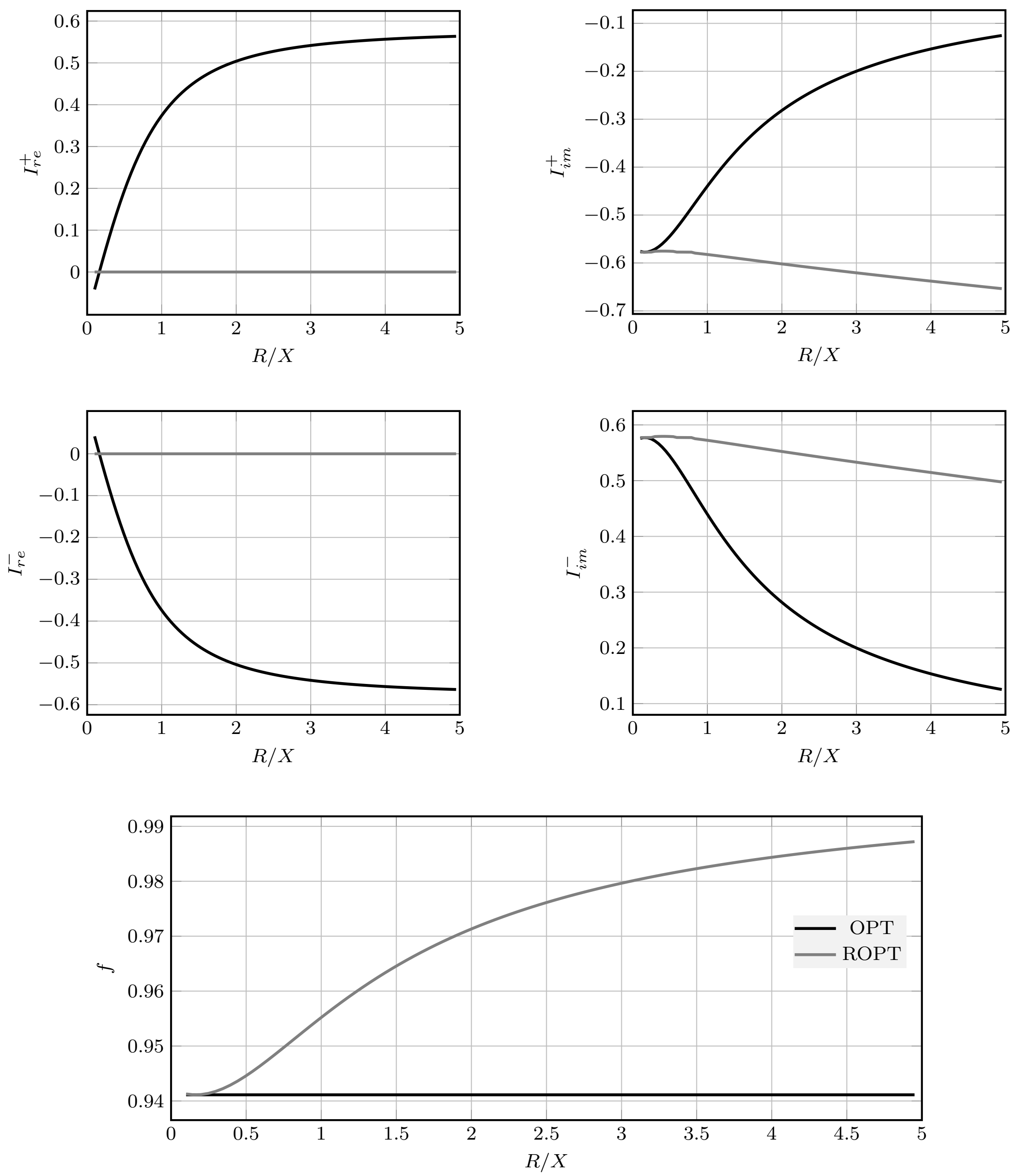

Figure 16: Influence of the currents on the objective function for the double line to ground fault and a changing R/X ratio with $\underline{Z}_f = 0.03$. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

The double line to ground fault is expected to become a more severe fault than all the previous cases considered since the connection between the two faulted phases is a solid one. This way, adjusting the fault impedance only affects the connection to ground but the fault remains worrying. As it can be anticipated, the converter has a limited influence on the objective function. Figure 16 shows indeed that such objective function is extremely close to one. The difference between the OPT and the ROPT becomes noticeable again. The converter is able to inject the optimal currents to keep a constant objective function whereas for the ROPT this does not happen.

The real positive and negative sequence currents take similar values (absolutely speaking) when compared to the imaginary parts. In some sense this ressembles the line to line fault findings. Since no real current can be injected in the ROPT situation, the optimal option consists of keeping the imaginary currents relatively constant across all R/X values.

Figure 17 shows the distribution of voltages for the multiple R/X ratios and the objective function as well. Similar conclusions as before can be extracted.

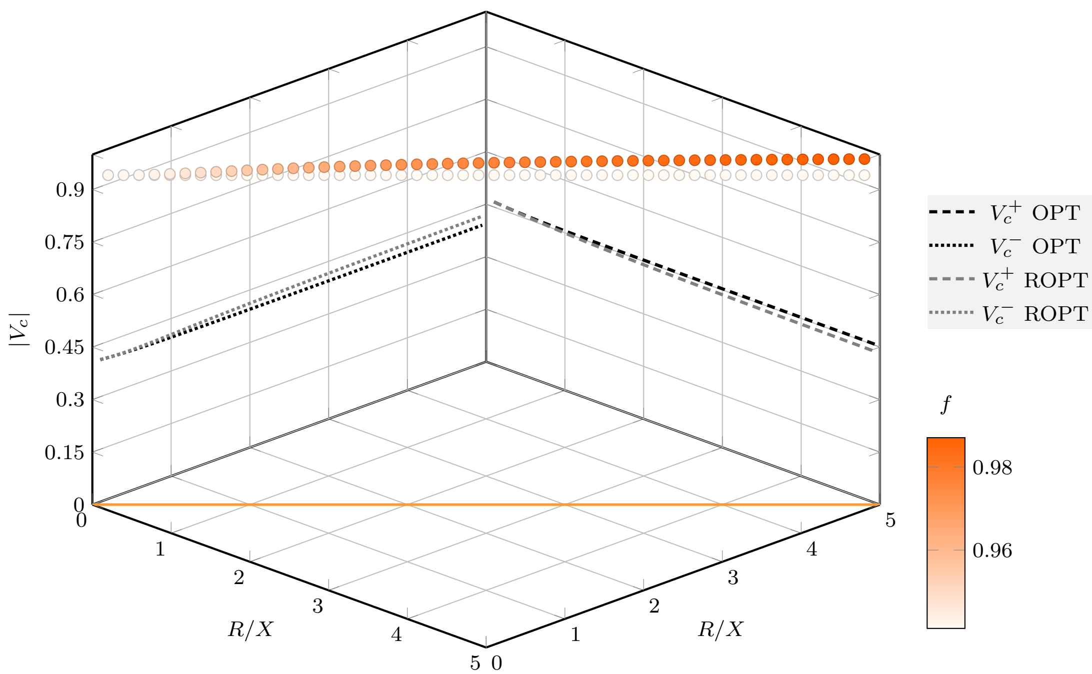

Figure 17: Sequence voltages together with the objective function for the double line to ground fault with $\underline{Z}_f = 0.03$ and a varying R/X ratio

When R/X is approximately null, the positive and negative sequence voltages match for the OPT and the ROPT cases. However, once the resistive part increases in comparison to the reactive part of the impedance, the impossibility of injecting active current causes a variation in voltages. For the positive sequence voltages, the OPT one is placed above the ROPT voltage; and the contrary happens to the negative sequence voltages.

4 Submarine cable

Having seen the influence of varying the ratio between the resistive and inductive part of the Thevenin impedance, we can now go a step further by including a hypothetical submarine cable between the grid and the point where the fault occurs. The systems takes the form shown in Figure 18.

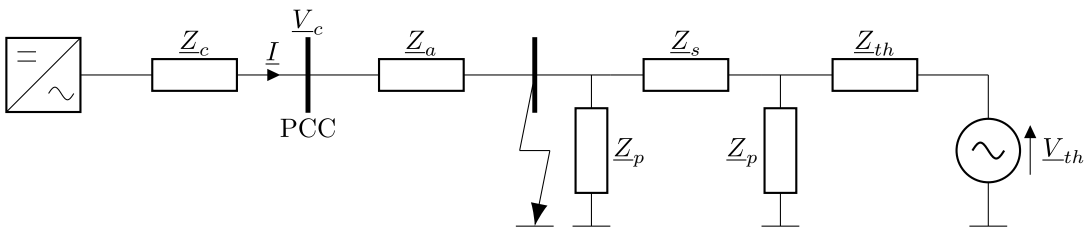

Figure 18: Single-phase representation of the simple system under a fault with a submarine cable

This system can be studied without added complexity once a Thevenin equivalent is formed on the right hand side of the fault. Opting for this will imply that we will be able to recycle the expressions developed before. The new Thevenin voltage and impedance, denoted by $\underline{V}'_{th}$ and $\underline{Z}'_{th}$, are given by:

$$\begin{cases} \underline{V}'_{th} = \dfrac{\underline{Z}_p\underline{Z}_p}{2\underline{Z}_{th}\underline{Z}_p + \underline{Z}_p\underline{Z}_s + \underline{Z}_p\underline{Z}_p + \underline{Z}_{th}\underline{Z}_s}\underline{V}_{th}, \\ \\ \underline{Z}'_{th} = \dfrac{\underline{Z}_p\underline{Z}_p\underline{Z}_{th} + \underline{Z}_s\underline{Z}_p\underline{Z}_p + \underline{Z}_{th}\underline{Z}_p + \underline{Z}_{th}\underline{Z}_s\underline{Z}_p}{2\underline{Z}_{th}\underline{Z}_p + \underline{Z}_p\underline{Z}_s + \underline{Z}_p\underline{Z}_p + \underline{Z}_{th}\underline{Z}_s}. \end{cases} \tag{5}$$

The distance of the cable is going to take multiple values in order to weight its influence. Realistic data for the $\underline{Z}_s$ and $\underline{Z}_p$ impedances are taken from [**cheah2017offshore**] and adapted to per unit values. They are shown in Table 3.

Magnitude	Value	Units
$\underline{Z}_s$	$6.674 \cdot 10^{-5} + j2.597 \cdot 10^{-4}$	pu/km
$\underline{Z}_p$	$-j77.372$	pu·km

Table 3: Impedances values to be used in the submarine cable analysis

Figure 19 displays the resulting Thevenin voltage and impedance for a varying cable distance.

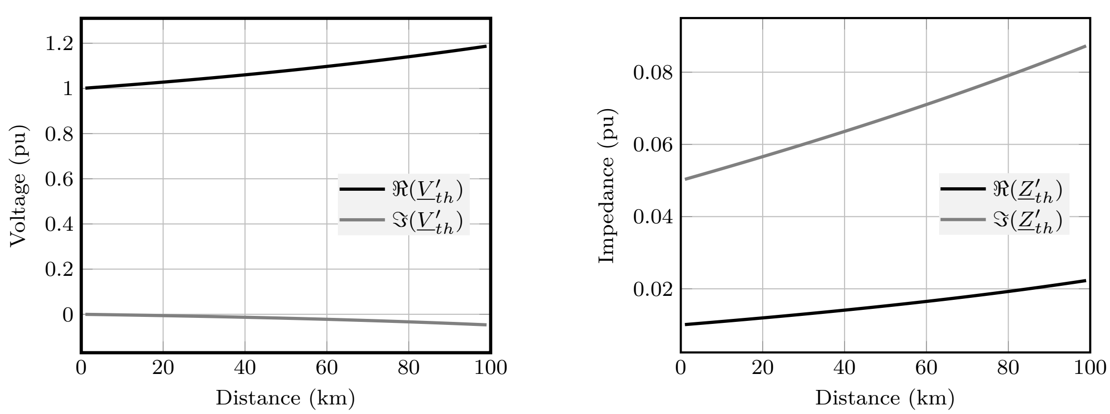

Figure 19: Influence of the cable distance on the Thevenin voltage and impedance

The real part of the voltage remains dominant across all distances, although it tends to increase with larger distances. The contrary pattern is experienced by the imaginary part. About the impedances, as one could expect, both of them grow with longer distances. The reactive part is considerably bigger than the real one for all distances.

4.1 Balanced fault

Figure 20 depicts the optimal currents for the balanced fault.

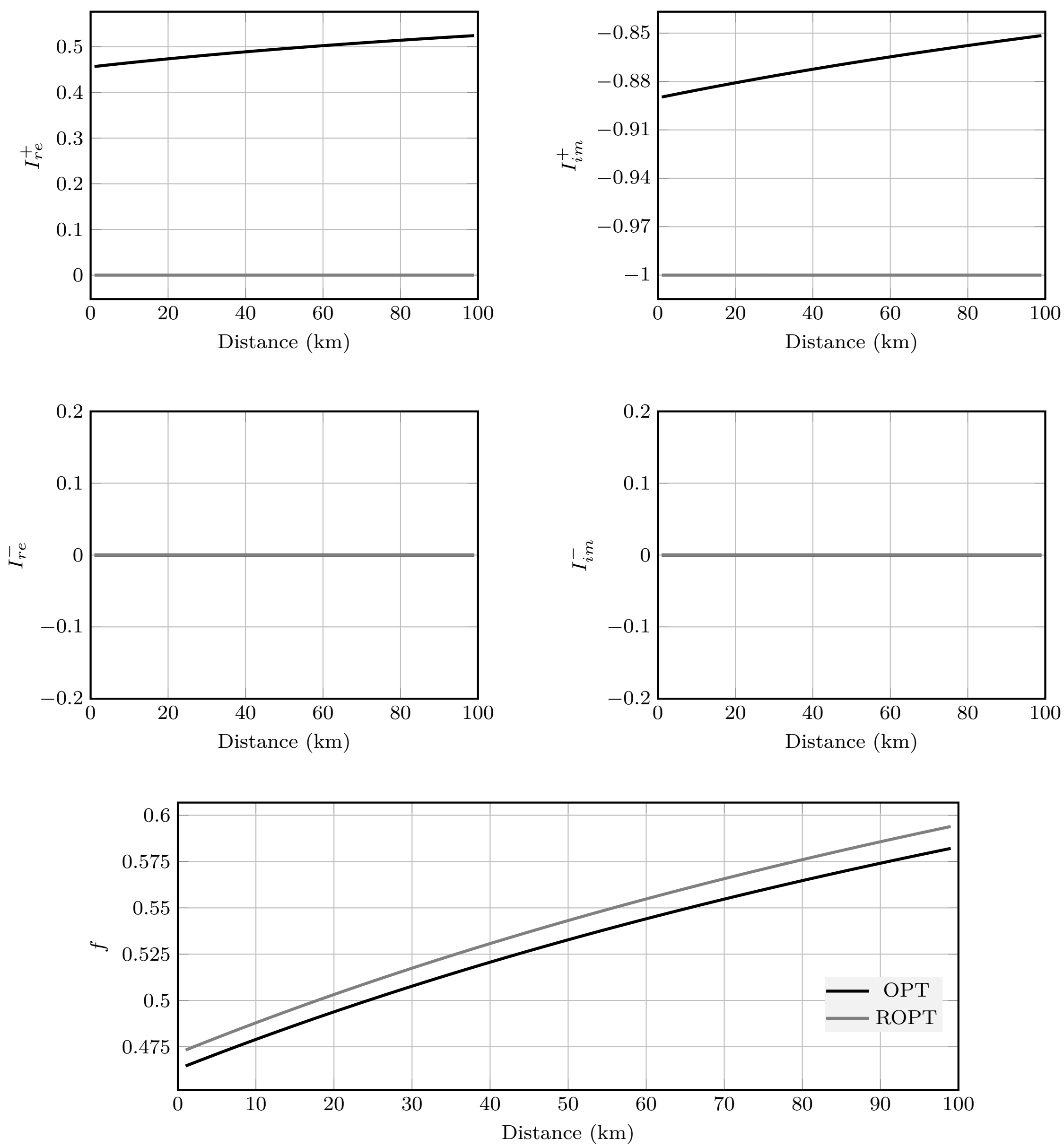

Figure 20: Influence of the currents on the objective function for the balanced fault with $\underline{Z}_f = 0.03$ and a submarine cable. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

As it can be deduced, the imaginary part of the positive sequence current is the one to be prioritized, specially for short distances. Notice that when the distance increases, as shown in Figure 19, both the real and imaginary part of the Thevenin impedances increase. However, in proportion, the real part becomes a bit more relevant. Therefore, in the OPT case some of the imaginary positive sequence current is traded for a bit more real current. The negative sequence currents are null for all range of distances because of the nature of the balanced fault. As it has been explained before, it makes no sense to inject negative sequence current for a balanced fault due to having initially an already null negative sequence voltage.

The ROPT turns out to be a more unfavorable case. This is fruit of not being able to inject any real current. Thus, the positive sequence current becomes totally reactive as it remains constant at -1, which corresponds to the maximum allowed current. Increasing the cable distance suggests that the longer the cable, the more severe the fault is. Nevertheless, the variations are relatively small. We can conclude that for this fault, injecting or not active current does not have a huge influence on the final results.

Figure 11 shows the voltages profile. The positive sequence absolute value of the voltage more or less mimics the reverse trend of objective function. It experiencex relatively small variations as well.

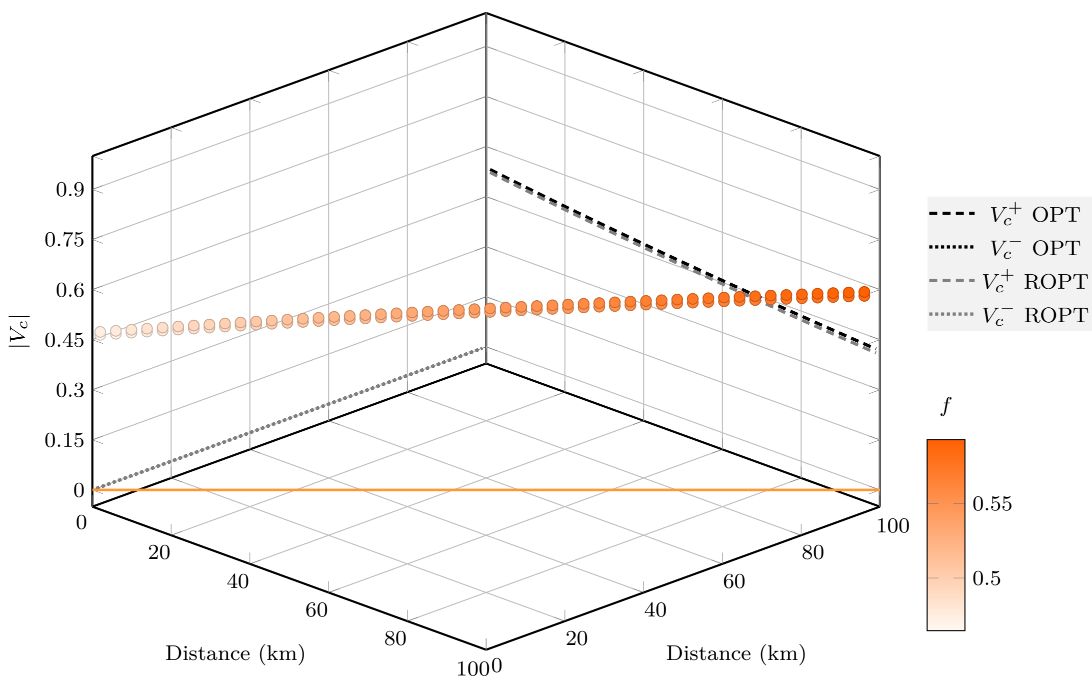

Figure 21: Sequence voltages together with the objective function for the balanced fault with $\underline{Z}_f = 0.03$ and a varying distance cable

The negative sequence voltage is of course at 0 for the ROPT and the OPT cases as mentioned. The positive sequence voltages tend to decrease with longer distances, which makes sense because the impedance of the cable increases. For all distances there exists a rather constant difference between the voltages. This difference is the same as in the objective function. We have found out that the fault impedance is by far the most influential impedance in the system. If we had opted for higher values, the objective functions would have been reduced and the objective voltages would have improved. Sometimes, when $\underline{Z}_f$ is big enough, the objective function can become zero.

4.2 Line to ground fault

Figure 22 depicts the optimal currents for the line to ground fault.

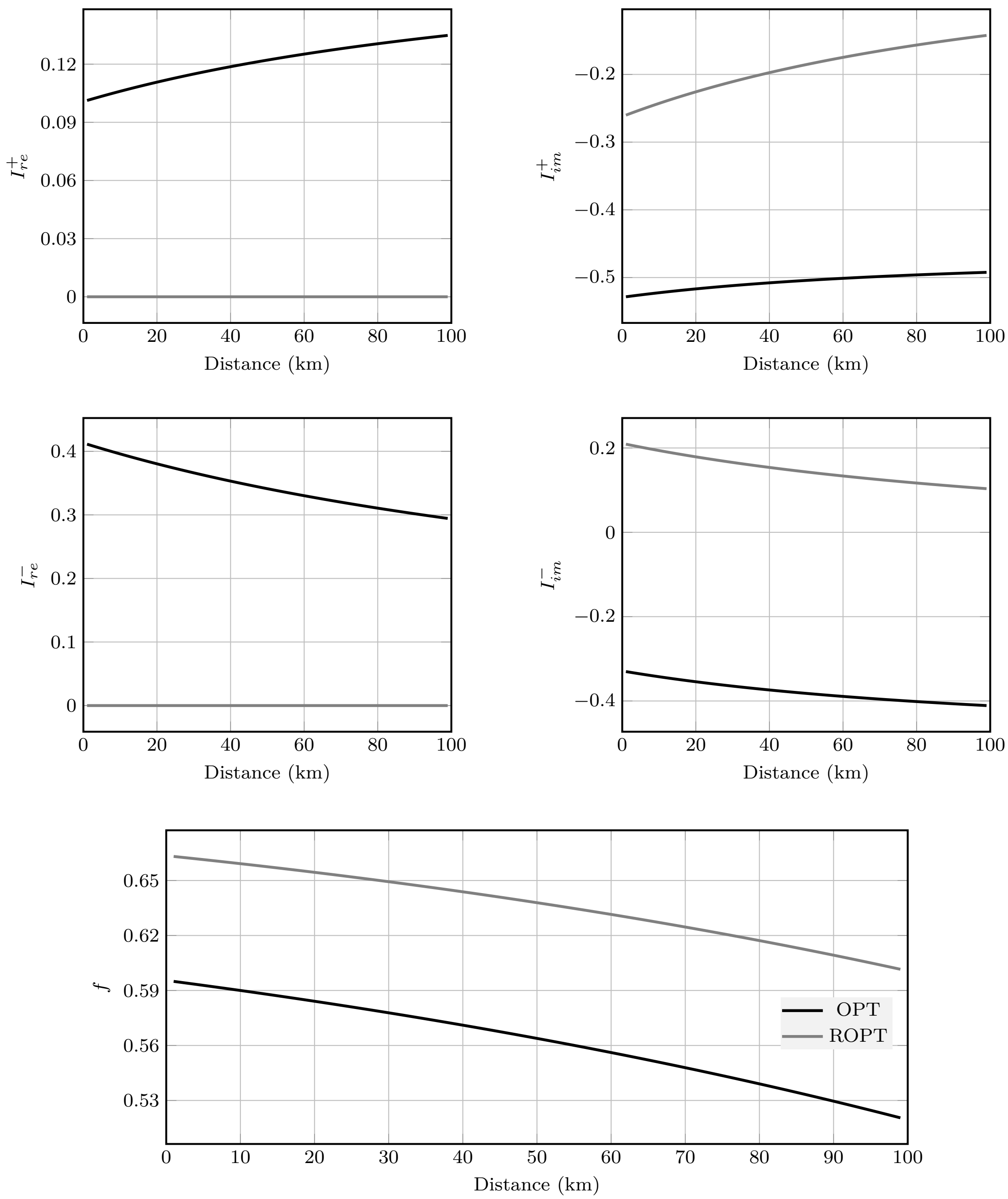

Figure 22: Influence of the currents on the objective function for the line to ground fault with $\underline{Z}_f = 0.001$ and a submarine cable. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

The most noticeable aspect in this situation is that the longer the cable is, the more optimal the objective function becomes. There is about a 10% improvement if we compare the objective functions for an almost null distance with a distance close to 100 km. Again, there is a practically constant difference regarding the objective function between the OPT and the ROPT cases.

The currents vary slightly. For instance, the real positive sequence current grows a bit but is kept at around 0.1. The imaginary positive sequence currents follow a similar evolution. The currents for the OPT situation are usually further from the zero. The optimization suggests that the current limitations are not met in the ROPT case. This same phenomena happened for the line to ground fault when the R/X was a varying one. One hypothesis to explain that could be related to the presence of multiple optimal points. The imaginary part of the currents does not surpass the 0.3 mark in absolute value under any condition. About the negative sequence currents, the real one takes substantial values, which decrease with distance. In general one can spot that the trends of the positive and the negative sequence currents seem mirrored.

Figure 23 presents the absolute value of the positive and negative sequence voltages and the objective function for all considered distances.

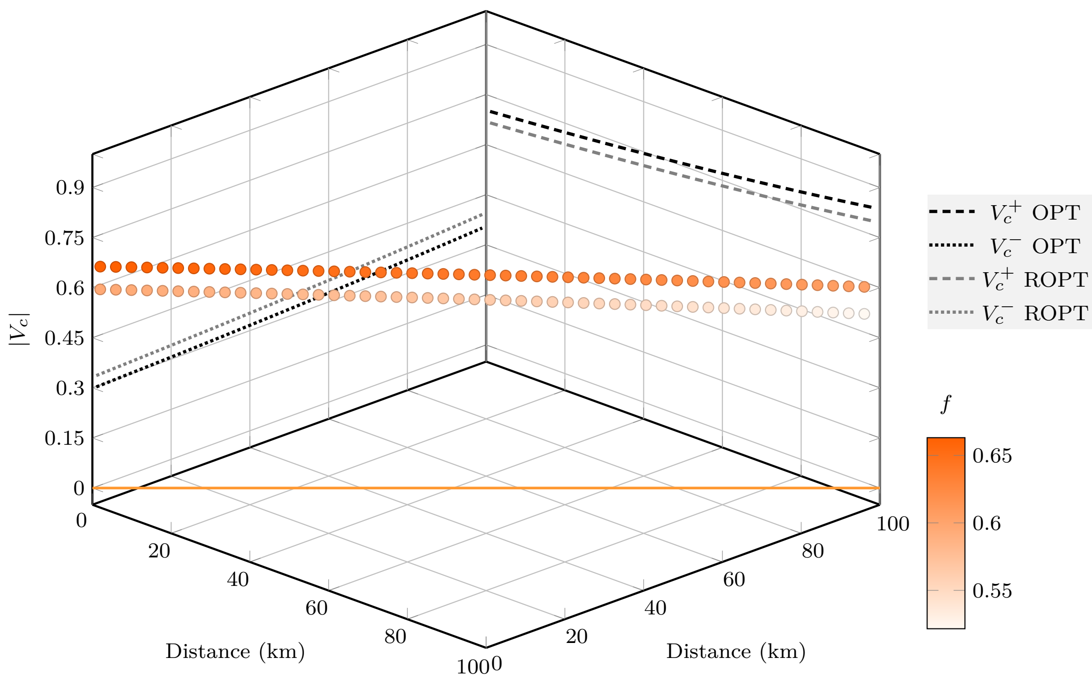

Figure 23: Sequence voltages together with the objective function for the line to ground fault with $\underline{Z}_f = 0.001$ and a varying distance cable

When looking at the bigger picture, it is clear that the objective function remains quite constant for all distances. However, there is a noticeable difference between the OPT and the ROPT situations, as expressed before. Larger distances imply that the positive voltage sequence can be improved, yet the negative sequence voltage also increases. Fortunately, the increase in the first is superior to the increase in the latter.

This time injecting active currents turns out to be more beneficial than in the balanced fault, where differences were hard to notice. Besides, even though the fault impedances is an order of magnitude smaller in this line to ground fault, the objective functions fall into a similar range of values. As it can be concluded, the distance of the cable has a relatively weak influence on the final results.

4.3 Line to line fault

Figure 24 depicts the optimal currents for the line to line fault.

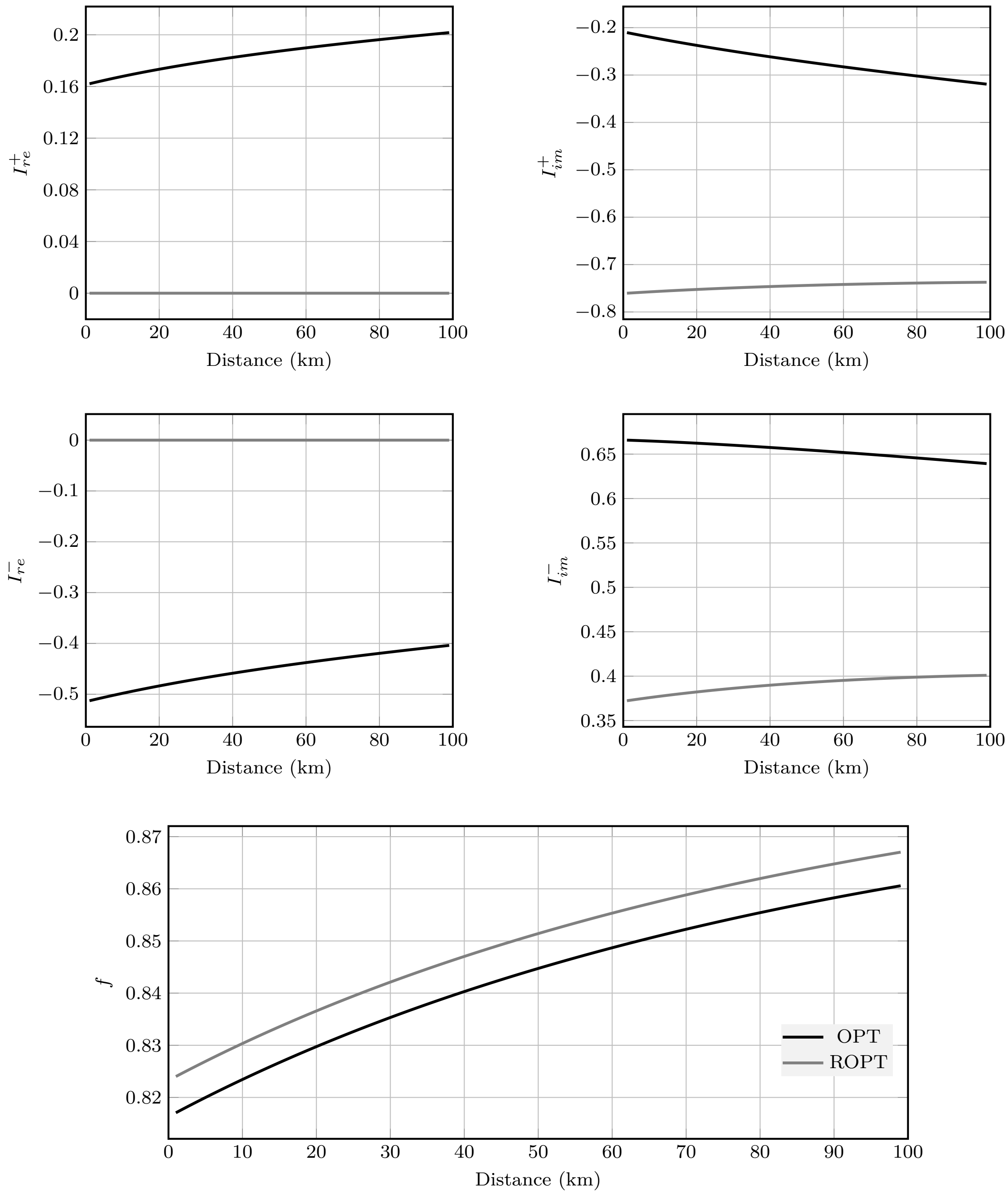

Figure 24: Influence of the currents on the objective function for the line to line fault with $\underline{Z}_f = 0.03$ and a submarine cable. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

The line to line fault has been found to become the one where the current values are more intuitive.

That is, in the OPT situation, the real positive sequence current becomes positive while the imaginary positive sequence one is negative to provoke a positive voltage drop as well to maximize the positive sequence voltage at the PCC. The inverse reasoning applies to the negative sequence currents. When we limit the reactive current to zero, as in the ROPT case, not only the differences between the two active currents is acute, but they also are for the imaginary currents. The negative sequence real current is specially considerable, but despite that, the objective function is not largely affected if we constrain it.

The currents do not experience huge variations across all distances. However, the trend is to increase slightly the real currents in the OPT case. This allows to have always a more optimal objective function. Just like it happened for the balanced fault, shorter distances are better in order to keep the objective function to a value closer to zero. But in any case, generally speaking the differences in the objective function are small, around 1%.

Figure 25 displays the positive and negative sequence voltages for the considered range of distances. The objective functions are also depicted, although they are hardly differentiable.

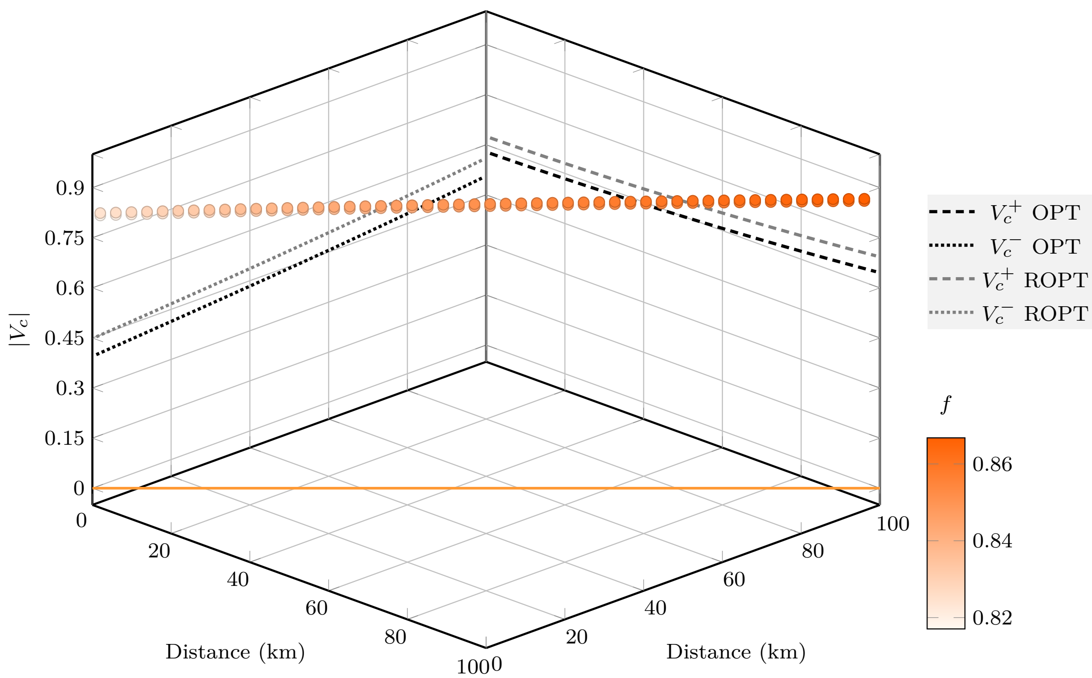

Figure 25: Sequence voltages together with the objective function for the line to line fault with $\underline{Z}_f = 0.03$ and a varying distance cable

Surprisingly, the positive sequence voltage is larger for the ROPT case than in the OPT situation. This could already be anticipated by looking at Figure 24, where the positive sequence OPT currents are inferior when compared to the imaginary positive sequence ROPT current. Therefore, the ROPT case makes a bigger effort to maximize the positive sequence voltage at the expense of causing an also larger negative sequence voltage. If we balance both, we get to the conclusion that the objective function is slightly better off in the OPT case, just like we could anticipate. All the absolute values of the voltages tend to increase for longer distances. Compared to the variation in the R/X ratio, the objective function now remains almost always the same. However, the fault is more severe and the differences between the OPT and the OPT scenarios have diminished.

4.4 Double line to ground fault

Figure 26 depicts the optimal currents for the double line to ground.

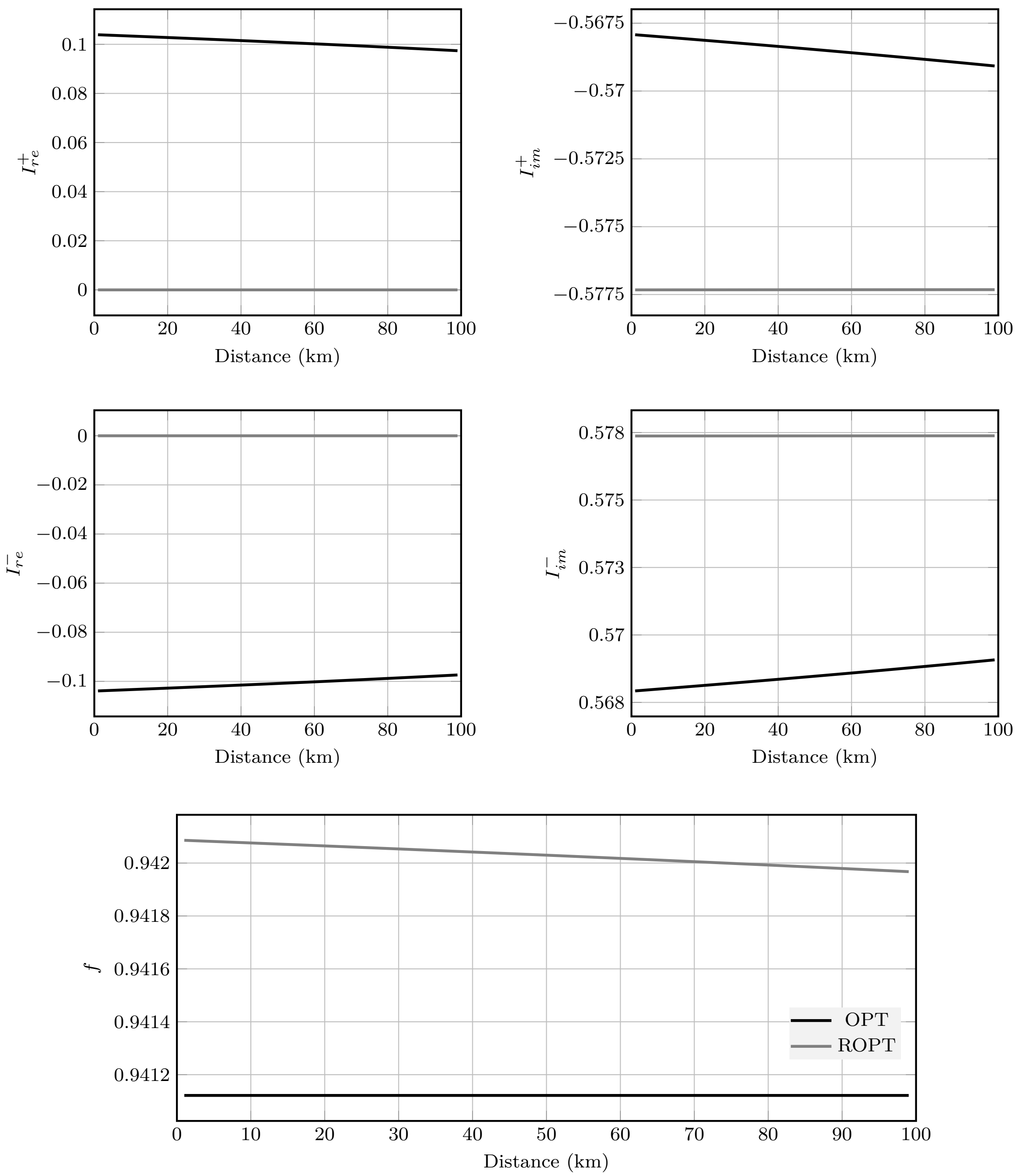

Figure 26: Influence of the currents on the objective function for the double line to ground fault with $\underline{Z}_f = 0.5$ and a submarine cable. OPT: solution to the optimization problem, ROPT: solution to the optimization problem restricted to only injecting reactive power.

Due to the way it has been defined, the double line to ground is the most severe fault since there is solid connection between the two faulted phases. This means that the fault impedance, which represents the link to ground, becomes mostly irrelevant. The objective functions are close to one and practically constant across all lengths. The fault so severe that the influence of the cable is minimal.

The imaginary currents take values close to an order of magnitude higher than the real parts. From the analysis of all faults, it seems that the imaginary current is strongly linked to the severity of the fault. Consequently, when the fault happens to be a strong one, injecting or not active currents has a minor influence. As follows, the OPT and the ROPT cases yield very similar operation conditions. Varying the R/X ratio had a greater impact on the results than increasing the length of the cable, but generally speaking, the objective functions where higher than 0.9 as well.

Figure 17 displays the objective function values together with the positive and negative sequence voltages.

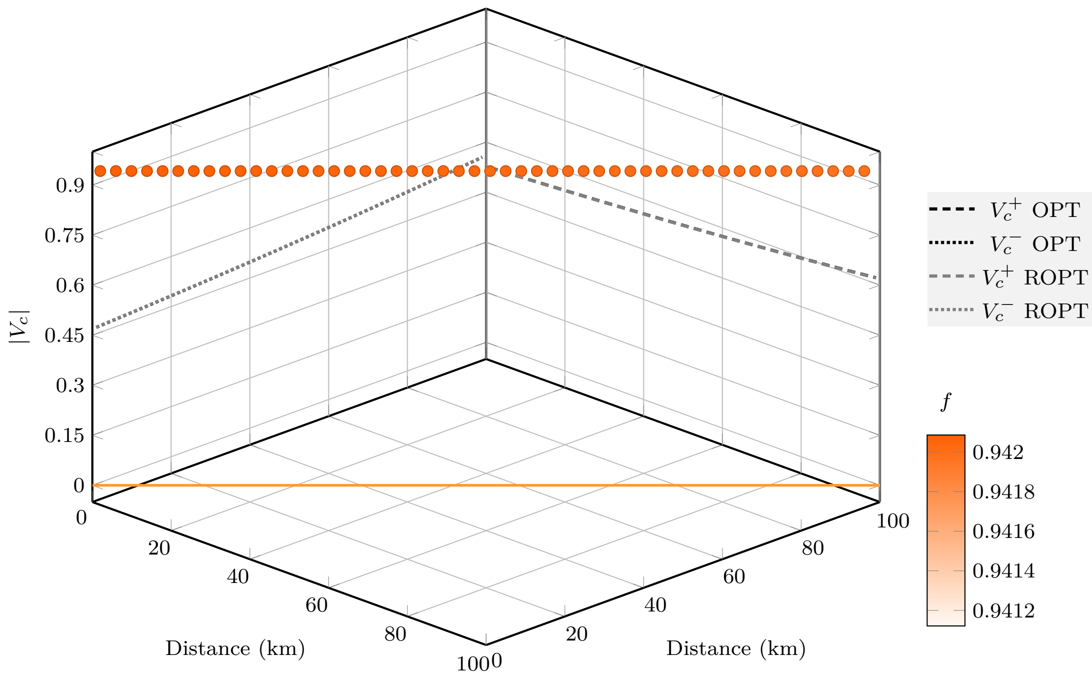

Figure 27: Sequence voltages together with the objective function for the double line to ground fault with $\underline{Z}_f = 0.5$ and a varying distance cable

Again, differences are hardly noticeable between both cases. The lines are overlapped for all the range of distances. In spite of that, the voltages tend to increase. Increasing the positive sequence voltage means that we are improving the objective function, but in the meantime, the negative sequence voltage also grow. Balancing both contributions results in obtaining a constant objective function for all distances.

5 Conclusion

This study has covered the analysis of a simple system to determine the optimal currents that ought to be injected to raise the positive sequence voltage and decrease the negative sequence voltage in case of a fault. Four types of faults have been considered. Despite the particularities, since the system is mainly inductive, the optimal active currents tend to be close to zero, while the optimal reactive currents become substantial. As a direct consequence, we conclude by saying that injecting only reactive powers, as imposed by the grid codes, is likely to be a convenient strategy for systems where lines are highly inductive. In all cases (constant impedance, changing R/X ratio and changing X_c) the objective function related to the ROPT situation has been close to the OPT one. However, it is mandatory to determine the type of fault in order to properly deduce the optimal values of the injected currents.

In addition to that, this work proposes two methodologies to arrive to the optimal solution. One has consisted of computing combinations where currents can take a wide range of values. When the intervals are small enough, such computationally intensive approach matches with the solution coming from solving directly the optimization problem. The results indicate that the double line to ground fault is the hardest in terms of minimizing the objective function, while the line to ground fault can present multiple minimums.

Varying the R/X ratio shows that the optimal currents can vary considerably. However, the objective functions do not experience substantial differences, as the OPT case is always slightly better of than the ROPT one. The same conclusion can be extracted from the submarine cable under study. In this last situation, it seems that the injection of real currents may dominate over the imaginary currents.

A *abc* circuits

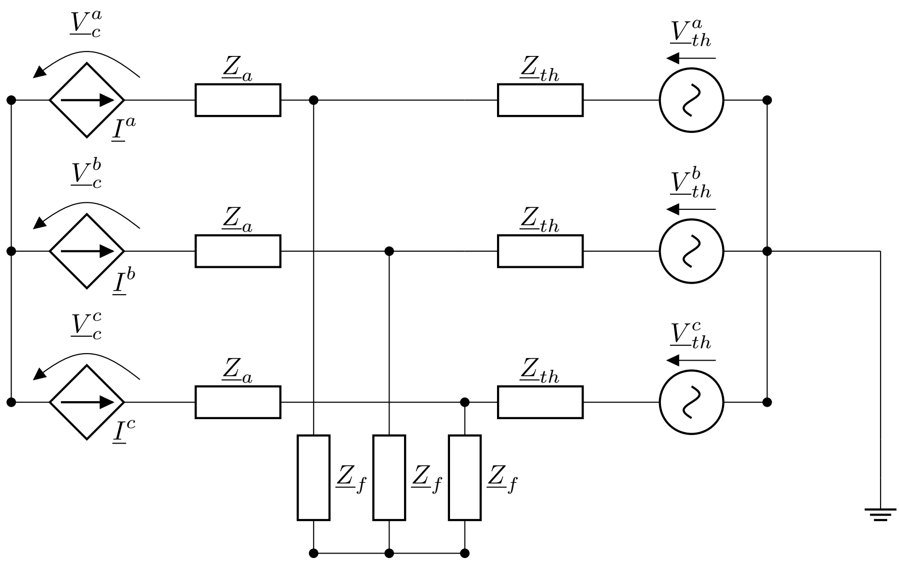

Figure 28: Balanced fault schematic

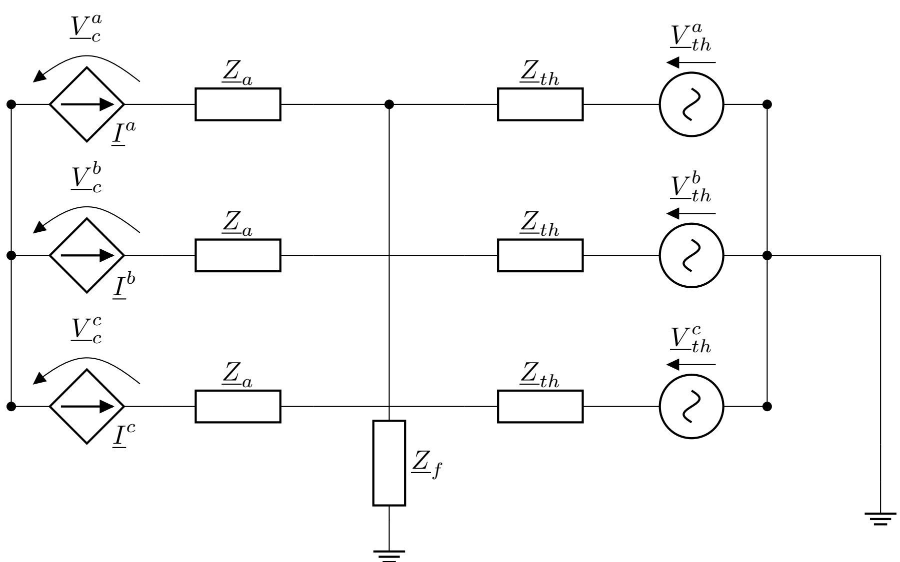

Figure 29: Line to ground fault schematic

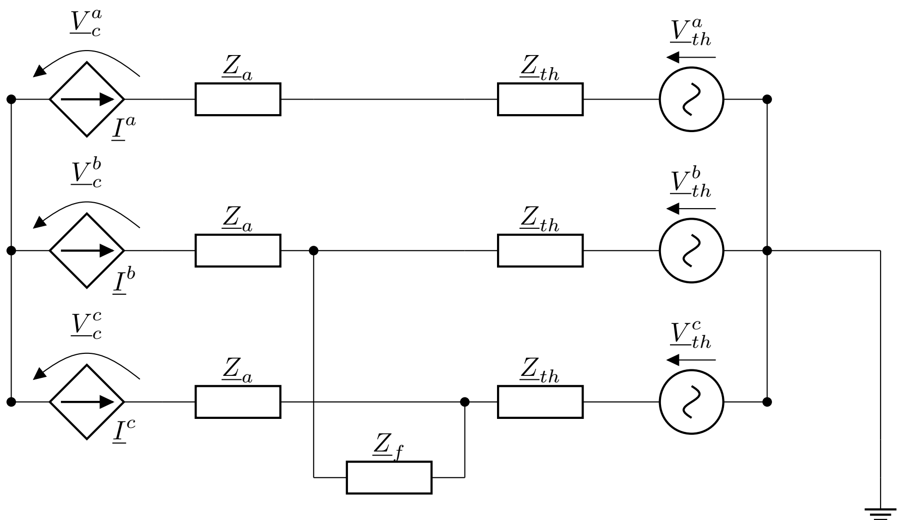

Figure 30: Line to line fault schematic

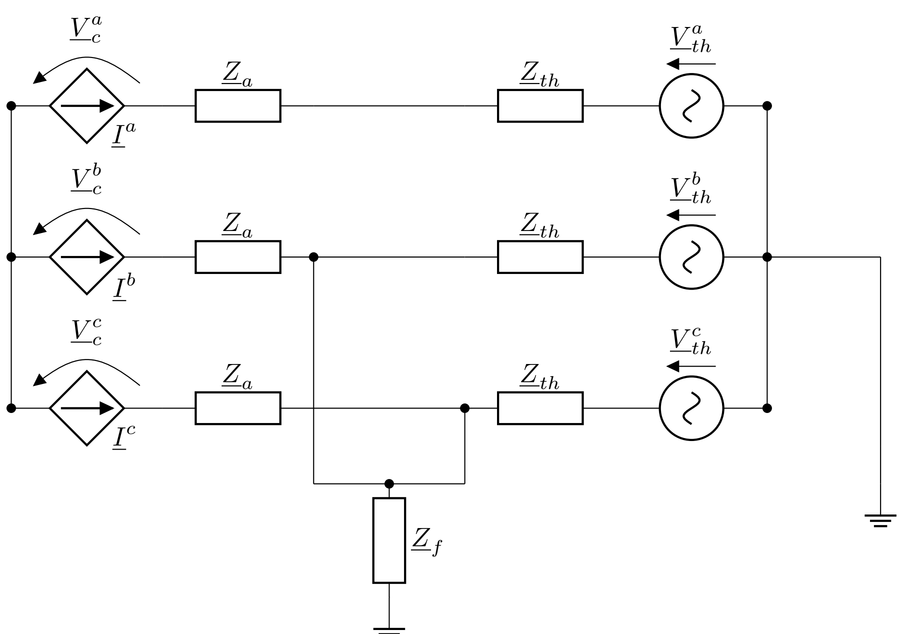

Figure 31: Double line to ground fault schematic

B $+-0$ schemes

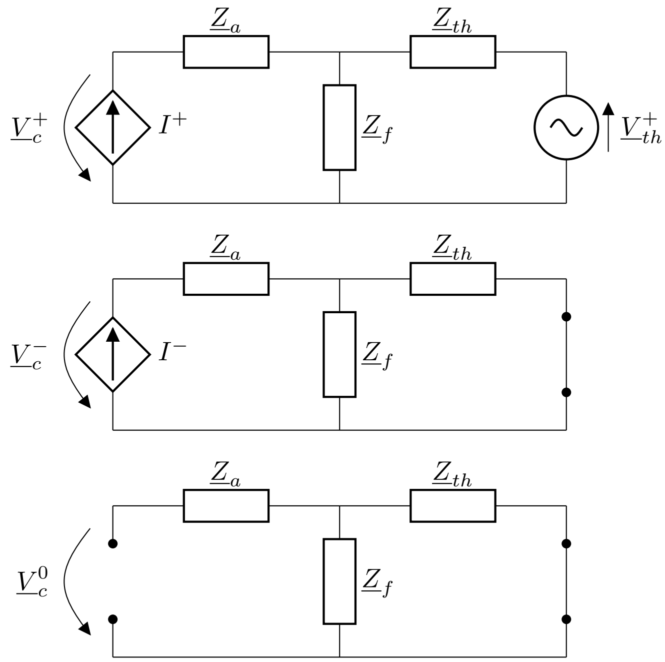

Figure 32: Equivalent circuit for the balanced fault analysis

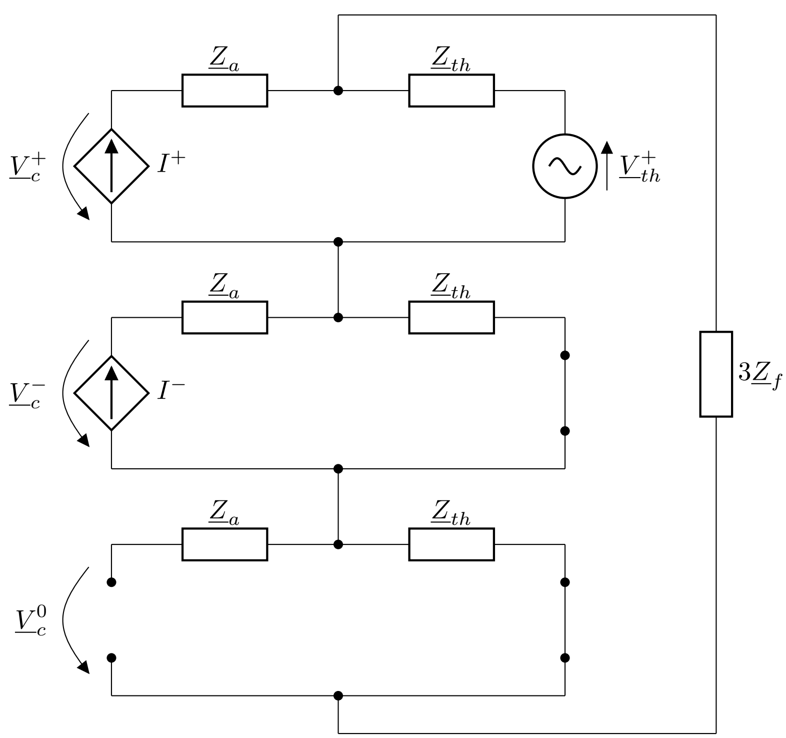

Figure 33: Equivalent circuit for the line to ground fault analysis

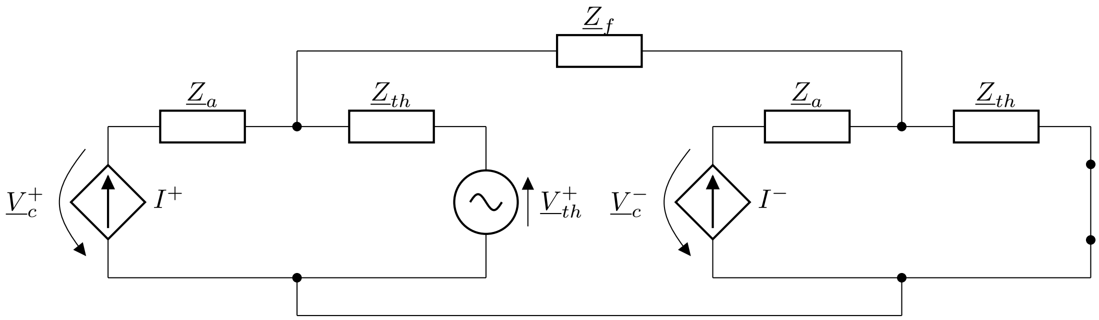

Figure 34: Equivalent circuit for the line to line fault analysis

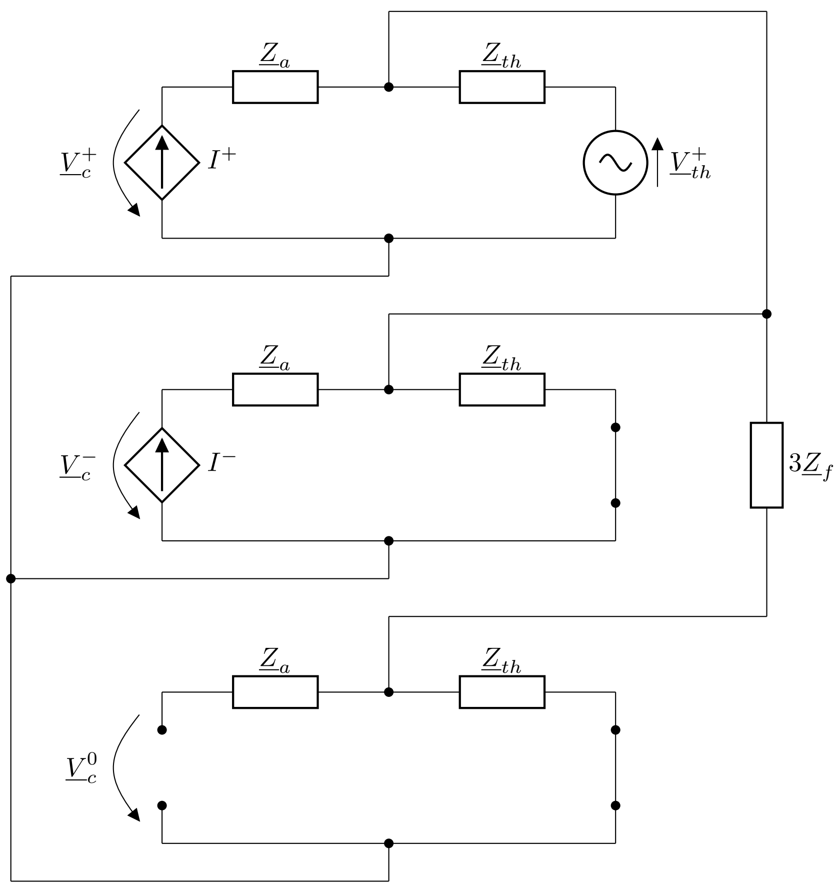

Figure 35: Equivalent circuit for the double line to ground fault analysis

C Expressions

C.1 Balanced fault

$$
\begin{cases}
\underline{V}_c^a = \dfrac{1}{\underline{Z}_f + \underline{Z}_{th}}[\underline{V}_{th}^a\underline{Z}_f + \underline{I}_a(\underline{Z}_a\underline{Z}_{th} + \underline{Z}_{th}\underline{Z}_f + \underline{Z}_f\underline{Z}_a)] \\
\underline{V}_c^b = \dfrac{1}{\underline{Z}_f + \underline{Z}_{th}}[\underline{V}_{th}^b\underline{Z}_f + \underline{I}_b(\underline{Z}_a\underline{Z}_{th} + \underline{Z}_{th}\underline{Z}_f + \underline{Z}_f\underline{Z}_a)] \\
\underline{V}_c^c = \dfrac{1}{\underline{Z}_f + \underline{Z}_{th}}[\underline{V}_{th}^c\underline{Z}_f + \underline{I}_c(\underline{Z}_a\underline{Z}_{th} + \underline{Z}_{th}\underline{Z}_f + \underline{Z}_f\underline{Z}_a)]
\end{cases}
\tag{6}
$$

$$
\begin{cases}
\underline{V}_c^+ = \dfrac{1}{\underline{Z}_f + \underline{Z}_{th}}[\underline{V}_{th}^+\underline{Z}_f + \underline{I}^+(\underline{Z}_a\underline{Z}_f + \underline{Z}_a\underline{Z}_{th} + \underline{Z}_f\underline{Z}_{th})] \\
\underline{V}_c^- = \dfrac{1}{\underline{Z}_f + \underline{Z}_{th}}[\underline{I}^-(\underline{Z}_{th}\underline{Z}_f + \underline{Z}_a\underline{Z}_{th} + \underline{Z}_a\underline{Z}_f)] \\
\underline{V}_c^0 = 0
\end{cases}
\tag{7}
$$

C.2 Line to ground fault

$$
\begin{cases}
\underline{V}_c^a = \dfrac{1}{\underline{Z}_{th} + \underline{Z}_f}[\underline{I}_a(\underline{Z}_a\underline{Z}_{th} + \underline{Z}_a\underline{Z}_f + \underline{Z}_{th}\underline{Z}_f) + \underline{V}_{th}^a\underline{Z}_f] \\
\underline{V}_c^b = \underline{V}_{th}^b + \underline{I}_b(\underline{Z}_a + \underline{Z}_{th}) \\
\underline{V}_c^c = \underline{V}_{th}^c + \underline{I}_c(\underline{Z}_a + \underline{Z}_{th})
\end{cases}
\tag{8}
$$

$$
\begin{cases}
\underline{V}_c^+ = \underline{I}^+(\underline{Z}_a + \underline{Z}_{th}) + \underline{V}_{th}^+ - \dfrac{\underline{Z}_{th}}{3\underline{Z}_f + 3\underline{Z}_{th}}[\underline{I}^+\underline{Z}_{th} + \underline{I}^-\underline{Z}_{th} + \underline{V}_{th}^+] \\
\underline{V}_c^- = \underline{I}^-(\underline{Z}_a + \underline{Z}_{th}) - \dfrac{\underline{Z}_{th}}{3\underline{Z}_f + 3\underline{Z}_{th}}[\underline{I}^+\underline{Z}_{th} + \underline{I}^-\underline{Z}_{th} + \underline{V}_{th}^+] \\
\underline{V}_c^0 = -\dfrac{\underline{Z}_{th}}{3\underline{Z}_f + 3\underline{Z}_{th}}[\underline{V}_{th}^+ + \underline{I}^+\underline{Z}_{th} + \underline{I}^-\underline{Z}_{th}]
\end{cases}
\tag{9}
$$

C.3 Line to line fault

$$
\begin{cases}
\underline{V}_c^a = \underline{I}_a(\underline{Z}_a + \underline{Z}_{th}) + \underline{V}_{th}^a \\
\underline{V}_c^b = \underline{I}_b\underline{Z}_a + \dfrac{1}{(\underline{Z}_f + \underline{Z}_{th})(\underline{Z}_f + 2\underline{Z}_{th})}[\underline{I}_b(\underline{Z}_{th}\underline{Z}_f\underline{Z}_f + 2\underline{Z}_{th}\underline{Z}_{th}\underline{Z}_f + \underline{Z}_{th}\underline{Z}_{th}\underline{Z}_{th}) \\
+\underline{I}_c(\underline{Z}_{th}\underline{Z}_{th}\underline{Z}_f + \underline{Z}_{th}\underline{Z}_{th}\underline{Z}_{th}) + \underline{V}_{th}^b(\underline{Z}_f\underline{Z}_f + 2\underline{Z}_f\underline{Z}_{th} + \underline{Z}_{th}\underline{Z}_{th}) + \underline{V}_{th}^c(\underline{Z}_{th}\underline{Z}_f + \underline{Z}_{th}\underline{Z}_{th})] \\
\underline{V}_c^c = \underline{I}_c\underline{Z}_a + \dfrac{1}{\underline{Z}_f + 2\underline{Z}_{th}}[\underline{I}_c(\underline{Z}_{th}(\underline{Z}_f + \underline{Z}_{th})) + \underline{V}_{th}^c(\underline{Z}_f + \underline{Z}_{th}) + \underline{I}_b\underline{Z}_{th}\underline{Z}_{th} + \underline{V}_{th}^b\underline{Z}_{th}]
\end{cases}
\tag{10}
$$

$$
\begin{cases}
\underline{V}_c^+ = \underline{V}_{th}^+ + \underline{I}^+(\underline{Z}_a + \underline{Z}_{th}) - \dfrac{\underline{Z}_{th}}{2\underline{Z}_{th} + \underline{Z}_f}[\underline{V}_{th}^+ + \underline{I}^+\underline{Z}_{th} - \underline{I}^-\underline{Z}_{th}] \\
\underline{V}_c^- = \underline{V}_{th}^+ + \underline{I}^+\underline{Z}_{th} + \underline{I}^-\underline{Z}_a - \dfrac{\underline{Z}_{th} + \underline{Z}_f}{2\underline{Z}_{th} + \underline{Z}_f}[\underline{V}_{th}^+ + \underline{I}^+\underline{Z}_{th} - \underline{I}^-\underline{Z}_{th}] \\
\underline{V}_c^0 = 0
\end{cases}
\tag{11}
$$

C.4 Double line to ground fault

$$
\begin{cases}
\underline{V}_c^a = \underline{I}_a(\underline{Z}_a + \underline{Z}_{th}) + \underline{V}_{th}^a \\
\underline{V}_c^b = \underline{I}_b\underline{Z}_a + \dfrac{\underline{Z}_{th}\underline{Z}_f(\underline{I}_b + \underline{I}_c) + \underline{Z}_f(\underline{V}_{th}^b + \underline{V}_{th}^c)}{2\underline{Z}_f + \underline{Z}_{th}} \\
\underline{V}_c^c = \underline{I}_c\underline{Z}_a + \dfrac{\underline{Z}_{th}\underline{Z}_f(\underline{I}_b + \underline{I}_c) + \underline{Z}_f(\underline{V}_{th}^b + \underline{V}_{th}^c)}{2\underline{Z}_f + \underline{Z}_{th}}
\end{cases}
\tag{12}
$$

$$
\begin{cases}
\underline{V}_c^+ = \underline{I}^+ \underline{Z}_a + \dfrac{\underline{Z}_{th} + 3\underline{Z}_f}{3\underline{Z}_{th} + 6\underline{Z}_f} [\underline{I}^+ \underline{Z}_{th} + \underline{I}^- \underline{Z}_{th} + \underline{V}_{th}^+] \\
\underline{V}_c^- = \underline{I}^- \underline{Z}_a + \dfrac{\underline{Z}_{th} + 3\underline{Z}_f}{3\underline{Z}_{th} + 6\underline{Z}_f} [\underline{I}^+ \underline{Z}_{th} + \underline{I}^- \underline{Z}_{th} + \underline{V}_{th}^+] \\
\underline{V}_c^0 = \dfrac{\underline{Z}_{th}}{3\underline{Z}_{th} + 6\underline{Z}_f} [\underline{I}^+ \underline{Z}_{th} + \underline{I}^- \underline{Z}_{th} + \underline{V}_{th}^+]
\end{cases}
\tag{13}
$$