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POWER SYSTEMS CALCULATION

Josep Fanals

CITCEA

02/2021

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Basics

Unkowns are no longer numbers as such, but series with an arbitrary number of coefficients. Multiple ways to embed the equations for the power flow. For example:

$$\sum_{j=1}^{n} Y_{ij} V_j(s) = s \frac{S_i^*}{V_i^*(s)} \qquad \text{(Eq. 1)}$$

If we define $X_i(s) = 1/V_i(s)$ and expand the series:

$$\begin{aligned} &Y_{i1}(V_1[0] + sV_1[1] + ... + s^c V_1[c]) \\ &+...+Y_{ii}(V_i[0] + sV_i[1] + ... + s^c V_i[c]) \\ &+...+Y_{in}(V_n[0] + sV_n[1] + ... + s^c V_n[c]) = sS_i^*(X_i[0] + sX_i[1] + ... + s^{c-1} X_i[c-1]) \end{aligned} \qquad \text{(Eq. 2)}$$

This way we get to a linear system with a constant matrix. The coefficients to compute only depend on past terms!

Overview

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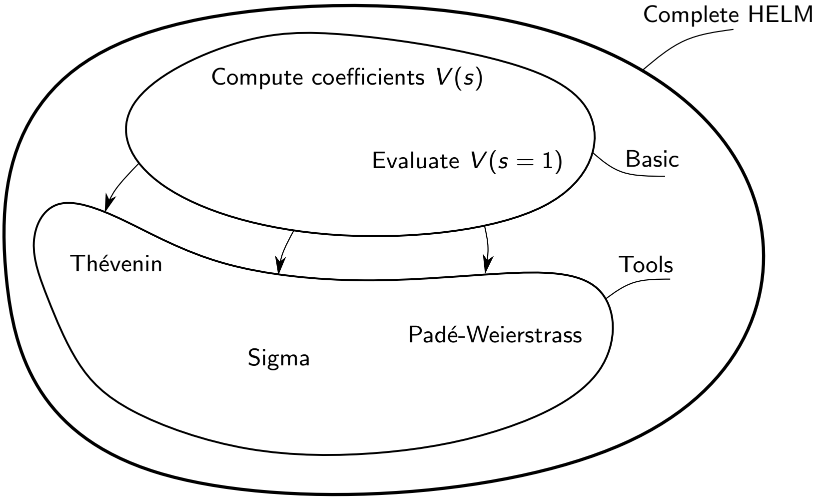

Figure 1. Perspective of the HELM. Basic reffers to obtaining the final solution whereas the tools complement the results.

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- Sigma plot of the IEEE30 system when active power changes.
- Sigma plot of the IEEE30 system when reactive power changes.

GridCal

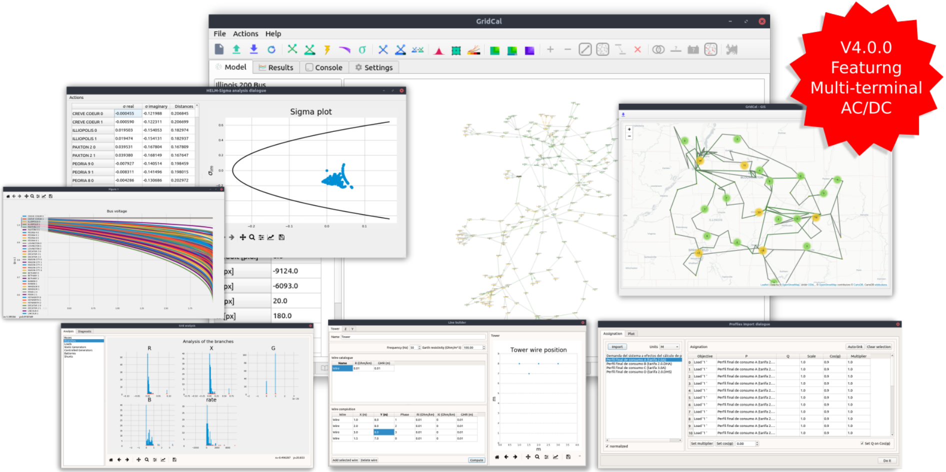

Figure 2. General view of GridCal with its GUI

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Modelling

We are interested in a generic model for power lines, transformers and converters.

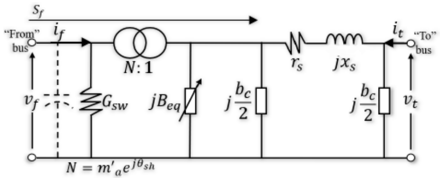

Figure 3. Flexible General Branch Model to model any element. Reference: Bustos, A. A. and Kazemtabrizi, B. (2018) 'Flexible general branch model unified power flow algorithm for future flexible AC/DC networks.' in 2018 IEEE International Conference on Environment and Electrical Engineering and 2018 IEEE Industrial and Commercial Power Systems Euroope (EEEIC / ICPS Europe): 12-15 June 2018, Palermo, Italy. Conference proceedings. Piscataway, NJ: IEEE.

Variable	Control
θ_{sh}	θ_{sh}
θ_{sh}	P_f
m_a	v_t
m_a	Q_t
B_{eq}	v_{dc}
B_{eq}	Zero Q constraint

Mode	Constraint 1	Constraint 2	VSC control
1	θ_{sh}	v_{ac}	I
2	P_f	Q_{ac}	I
3	P_f	v_{ac}	I
4	v_{dc}	Q_{ac}	II
5	v_{dc}	v_{ac}	II
6	v_{dc} droop	Q_{ac}	III
7	v_{dc} droop	v_{ac}	III

Table 1. VSC control models and relationship between the variables and the controlled magnitude

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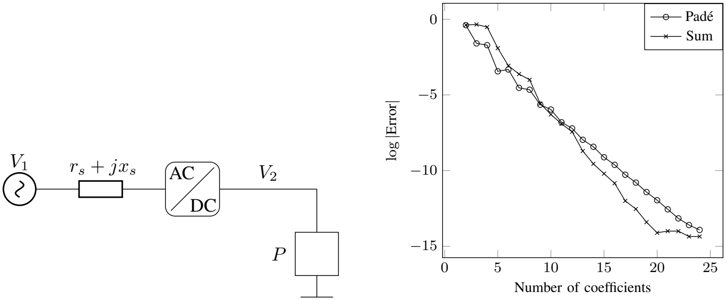

Figure 4. Left: simple system with a VSC converter. Right: maximum error depending on the number of coefficients in the series.

One embedded equation becomes for instance:

$$I_f(s) = y_s V_2(s) - y_s V_1(s) + sj\frac{b_c}{2} V_2(s) + sjB_{eq} V_2(s) + sGI_f^{re}(s) I_f^{re}(s) I_f^{im}(s) I_f^{im}(s) V_2(s) \quad \text{(Eq. 1)}$$

HELM is also suitable to solve rather complicated equations like this one.

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Formulation

We define the problem in two steps.

- Admittances side:

$$\begin{cases} I^{l+\frac{1}{2}} - I^{l} = \alpha(V^{l+\frac{1}{2}} - V^{l}), \\ YV^{l+\frac{1}{2}} = I_0 + I^{l+\frac{1}{2}}. \end{cases} \quad \text{(Eq. 1)}$$

- Load/generator side:

$$\begin{cases} I^{l+1} - I^{l+\frac{1}{2}} = \beta(V^{l+1} - V^{l+\frac{1}{2}}), \\ (V^{l+1})^{*} I^{l+1} = S^{*}. \end{cases} \quad \text{(Eq. 2)}$$

Matrices α and β can be arbitrarily defined, but for instance:

$$\begin{cases} \alpha = \text{diag}(S^{*}/|V|^{2}), \\ \beta = \text{diag}(Y + \alpha). \end{cases} \quad \text{(Eq. 3)}$$

These are constant matrices. Contrary to the typical NR, there are no inverses as such (expensive computation with $\mathcal{O}(n^3)$).

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Multidimensional power flow

- Dimensions: position (nodes), changes in power, time...
- Voltages expressed in the separated form: $V(x, q, t) = \sum_{m=1}^{M} V_m \otimes Q_m \otimes T_m$.

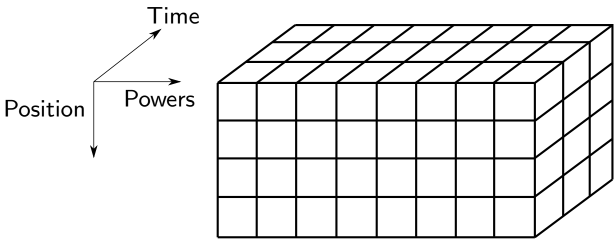

Figure 5. Representation of the cube of solutions

- Need to compute $M(n_{\text{buses}} + n_{\text{powers}} + n_{\text{time}})$ instead of $n_{\text{buses}} \cdot n_{\text{powers}} \cdot n_{\text{time}}$ cases.
- Can it be adapted for changes in the topology so that we can employ it for contingency analysis $(N-1,\ N-2...)$?

Algorithm

The outer loop follows the ASD procedure while the inner one is based on the alternating directions technique.

Algorithm 1 Pseudocode for the PGD combined with ASD

for $\gamma = 1$ to N_γ **do**
  Compute power side of the problem with PGD: $I = S^* \oslash V^{*[\gamma]}$
  for $m = 1$ to M **do**
    Define $I = \sum_{m=1}^{M-1} I_m \otimes Q_m \otimes T_m + I_M \otimes Q_M \otimes T_M$
    for $k = 1$ to N_k **do**
      Compute $I_M^{[k+1]}$ with $Q_M^{[k]}$ and $T_M^{[k]}$.
      Compute $Q_M^{[k+1]}$ with $I_M^{[k+1]}$ and $T_M^{[k]}$.
      Compute $T_M^{[k+1]}$ with $I_M^{[k+1]}$ and $Q_M^{[k+1]}$.
    end for
  end for
  Compute admittances side of the problem directly: $V^{[\gamma+1]} = Y^{-1}(I + I_0)$.
end for

- Show code and results.

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