

1 Holomorphic  
Embedding Load Flow  
Method (HELM)

1.1 Basics

1.2 Overview

1.3 Sigma approximants

1.4 GridCal

2 Flexible General  
Branch Model (FGBM)

2.1 Modelling

2.2 FGBM + HELM

3 Alternating Search  
Directions (ASD)

3.1 Formulation

4 Proper Generalised  
Decomposition (PGD)

4.1 Multidimensional  
power flow

4.2 Algorithm PGD +  
ASD

# POWER SYSTEMS CALCULATION

Josep Fanals

CITCEA

02/2021

## 1 Holomorphic Embedding Load Flow Method (HELM)

### 1.1 Basics

### 1.2 Overview

### 1.3 Sigma approximants

### 1.4 GridCal

## 2 Flexible General Branch Model (FGBM)

### 2.1 Modelling

### 2.2 FGBM + HELM

## 3 Alternating Search Directions (ASD)

### 3.1 Formulation

## 4 Proper Generalised Decomposition (PGD)

### 4.1 Multidimensional power flow

### 4.2 Algorithm PGD + ASD

## 1 Holomorphic Embedding Load Flow Method (HELM)

### 1.1 Basics

### 1.2 Overview

### 1.3 Sigma approximants

### 1.4 GridCal

## 2 Flexible General Branch Model (FGBM)

### 2.1 Modelling

### 2.2 FGBM + HELM

## 3 Alternating Search Directions (ASD)

### 3.1 Formulation

## 4 Proper Generalised Decomposition (PGD)

### 4.1 Multidimensional power flow

### 4.2 Algorithm PGD + ASD

## 1 Holomorphic Embedding Load Flow Method (HELM)

- 1.1 Basics
- 1.2 Overview
- 1.3 Sigma approximants
- 1.4 GridCal

## 2 Flexible General Branch Model (FGBM)

- 2.1 Modelling
- 2.2 FGBM + HELM

## 3 Alternating Search Directions (ASD)

- 3.1 Formulation

## 4 Proper Generalised Decomposition (PGD)

- 4.1 Multidimensional power flow
- 4.2 Algorithm PGD + ASD

## 1 Holomorphic Embedding Load Flow Method (HELM)

- 1.1 Basics
- 1.2 Overview
- 1.3 Sigma approximants
- 1.4 GridCal

## 2 Flexible General Branch Model (FGBM)

- 2.1 Modelling
- 2.2 FGBM + HELM

## 3 Alternating Search Directions (ASD)

- 3.1 Formulation

## 4 Proper Generalised Decomposition (PGD)

- 4.1 Multidimensional power flow
- 4.2 Algorithm PGD + ASD

Unknowns are no longer numbers as such, but series with an arbitrary number of coefficients. Multiple ways to embed the equations for the power flow. For example:

$$\sum_{j=1}^n Y_{ij} V_j(s) = s \frac{S_i^*}{V_i^*(s)} \quad (\text{Eq. 1})$$

If we define  $X_i(s) = 1/V_i(s)$  and expand the series:

$$\begin{aligned} & Y_{i1} (V_1[0] + sV_1[1] + \dots + s^c V_1[c]) \\ & + \dots + Y_{ii} (V_i[0] + sV_i[1] + \dots + s^c V_i[c]) \\ & + \dots + Y_{in} (V_n[0] + sV_n[1] + \dots + s^c V_n[c]) = sS_i^* (X_i[0] + sX_i[1] + \dots + s^{c-1} X_i[c-1]) \end{aligned} \quad (\text{Eq. 2})$$

This way we get to a linear system with a constant matrix. The coefficients to compute only depend on past terms!

## 1 Holomorphic Embedding Load Flow Method (HELM)

### 1.1 Basics

### 1.2 Overview

### 1.3 Sigma approximants

### 1.4 GridCal

## 2 Flexible General Branch Model (FGBM)

### 2.1 Modelling

### 2.2 FGBM + HELM

## 3 Alternating Search Directions (ASD)

### 3.1 Formulation

## 4 Proper Generalised Decomposition (PGD)

### 4.1 Multidimensional power flow

### 4.2 Algorithm PGD + ASD

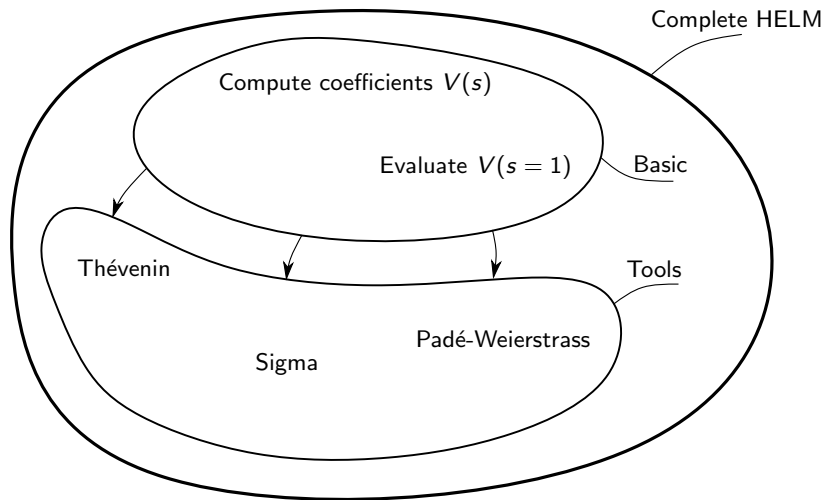


Figure 1. Perspective of the HELM. Basic refers to obtaining the final solution whereas the tools complement the results.

## 1 Holomorphic Embedding Load Flow Method (HELM)

### 1.1 Basics

### 1.2 Overview

### 1.3 Sigma approximants

### 1.4 GridCal

## 2 Flexible General Branch Model (FGBM)

### 2.1 Modelling

### 2.2 FGBM + HELM

## 3 Alternating Search Directions (ASD)

### 3.1 Formulation

## 4 Proper Generalised Decomposition (PGD)

### 4.1 Multidimensional power flow

### 4.2 Algorithm PGD + ASD

- Sigma plot of the IEEE30 system when active power changes.
- Sigma plot of the IEEE30 system when reactive power changes.

- 1 Holomorphic Embedding Load Flow Method (HELM)
  - 1.1 Basics
  - 1.2 Overview
  - 1.3 Sigma approximants
  - 1.4 GridCal
- 2 Flexible General Branch Model (FGBM)
  - 2.1 Modelling
  - 2.2 FGBM + HELM
- 3 Alternating Search Directions (ASD)
  - 3.1 Formulation
- 4 Proper Generalised Decomposition (PGD)
  - 4.1 Multidimensional power flow
  - 4.2 Algorithm PGD + ASD

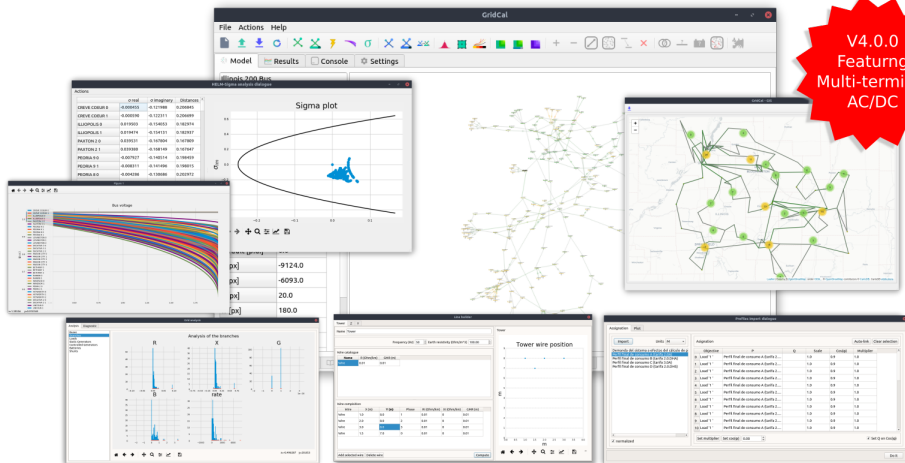


Figure 2. General view of GridCal with its GUI

## 1 Holomorphic Embedding Load Flow Method (HELM)

### 1.1 Basics

### 1.2 Overview

### 1.3 Sigma approximants

### 1.4 GridCal

## 2 Flexible General Branch Model (FGBM)

### 2.1 Modelling

### 2.2 FGBM + HELM

## 3 Alternating Search Directions (ASD)

### 3.1 Formulation

## 4 Proper Generalised Decomposition (PGD)

### 4.1 Multidimensional power flow

### 4.2 Algorithm PGD + ASD

## 1 Holomorphic Embedding Load Flow Method (HELM)

### 1.1 Basics

### 1.2 Overview

### 1.3 Sigma approximants

### 1.4 GridCal

## 2 Flexible General Branch Model (FGBM)

### 2.1 Modelling

### 2.2 FGBM + HELM

## 3 Alternating Search Directions (ASD)

### 3.1 Formulation

## 4 Proper Generalised Decomposition (PGD)

### 4.1 Multidimensional power flow

### 4.2 Algorithm PGD + ASD



We are interested in a generic model for power lines, transformers and converters.

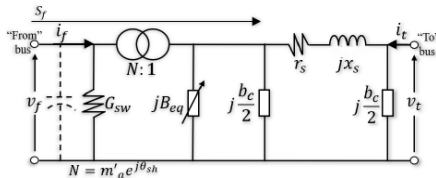


Figure 3. Flexible General Branch Model to model any element. Reference: Bustos, A. A. and Kazemtabrizi, B. (2018) 'Flexible general branch model unified power flow algorithm for future flexible AC/DC networks.' in 2018 IEEE International Conference on Environment and Electrical Engineering and 2018 IEEE Industrial and Commercial Power Systems Europe (IEEEIC / ICPS Europe): 12-15 June 2018, Palermo, Italy. Conference proceedings. Piscataway, NJ: IEEE.

Variable	Control
$\theta_{sh}$	$\theta_{sh}$
$\theta_{sh}$	$P_f$
$m_a$	$v_t$
$m_a$	$Q_t$
$B_{eq}$	$v_{dc}$
$B_{eq}$	Zero Q constraint

Mode	Constraint 1	Constraint 2	VSC control
1	$\theta_{sh}$	$v_{ac}$	I
2	$P_f$	$Q_{ac}$	I
3	$P_f$	$v_{ac}$	I
4	$v_{dc}$	$Q_{ac}$	II
5	$v_{dc}$	$v_{ac}$	II
6	$v_{dc}$ droop	$Q_{ac}$	III
7	$v_{dc}$ droop	$v_{ac}$	III

Table 1. VSC control models and relationship between the variables and the controlled magnitude

1 Holomorphic Embedding Load Flow Method (HELM)

1.1 Basics

1.2 Overview

1.3 Sigma approximants

1.4 GridCal

2 Flexible General Branch Model (FGBM)

2.1 Modelling

2.2 FGBM + HELM

3 Alternating Search Directions (ASD)

3.1 Formulation

4 Proper Generalised Decomposition (PGD)

4.1 Multidimensional power flow

4.2 Algorithm PGD + ASD

## 1 Holomorphic Embedding Load Flow Method (HELM)

### 1.1 Basics

### 1.2 Overview

### 1.3 Sigma approximants

### 1.4 GridCal

## 2 Flexible General Branch Model (FGBM)

### 2.1 Modelling

## 2.2 FGBM + HELM

## 3 Alternating Search Directions (ASD)

### 3.1 Formulation

## 4 Proper Generalised Decomposition (PGD)

### 4.1 Multidimensional power flow

### 4.2 Algorithm PGD + ASD

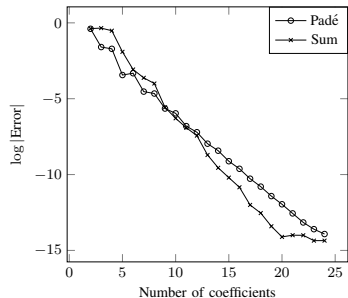
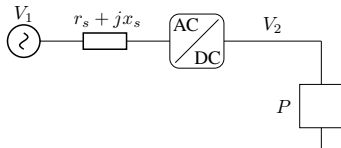


Figure 4. Left: simple system with a VSC converter. Right: maximum error depending on the number of coefficients in the series.

One embedded equation becomes for instance:

$$I_f(s) = y_s V_2(s) - y_s V_1(s) + sj \frac{b_c}{2} V_2(s) + sj B_{eq} V_2(s) + s G I_f^{re}(s) I_f^{re}(s) I_f^{im}(s) I_f^{im}(s) V_2(s) \quad (\text{Eq. 1})$$

HELM is also suitable to solve rather complicated equations like this one.

- 1 Holomorphic Embedding Load Flow Method (HELM)
  - 1.1 Basics
  - 1.2 Overview
  - 1.3 Sigma approximants
  - 1.4 GridCal
- 2 Flexible General Branch Model (FGBM)
  - 2.1 Modelling
  - 2.2 FGBM + HELM
- 3 Alternating Search Directions (ASD)
- 4 Proper Generalised Decomposition (PGD)
  - 4.1 Multidimensional power flow
  - 4.2 Algorithm PGD + ASD

## 1 Holomorphic Embedding Load Flow Method (HELM)

- 1.1 Basics
- 1.2 Overview
- 1.3 Sigma approximants
- 1.4 GridCal

## 2 Flexible General Branch Model (FGBM)

- 2.1 Modelling
- 2.2 FGBM + HELM

## 3 Alternating Search Directions (ASD)

- 3.1 Formulation

## 4 Proper Generalised Decomposition (PGD)

- 4.1 Multidimensional power flow
- 4.2 Algorithm PGD + ASD

We define the problem in two steps.

■ Admittances side:

$$\begin{cases} I^{l+\frac{1}{2}} - I^l = \alpha(V^{l+\frac{1}{2}} - V^l), \\ YV^{l+\frac{1}{2}} = I_0 + I^{l+\frac{1}{2}}. \end{cases} \quad (\text{Eq. 1})$$

■ Load/generator side:

$$\begin{cases} I^{l+1} - I^{l+\frac{1}{2}} = \beta(V^{l+1} - V^{l+\frac{1}{2}}), \\ (V^{l+1})^* I^{l+1} = S^*. \end{cases} \quad (\text{Eq. 2})$$

Matrices  $\alpha$  and  $\beta$  can be arbitrarily defined, but for instance:

$$\begin{cases} \alpha = \text{diag}(S^* / |V|^2), \\ \beta = \text{diag}(Y + \alpha). \end{cases} \quad (\text{Eq. 3})$$

These are constant matrices. Contrary to the typical NR, there are no inverses as such (expensive computation with  $\mathcal{O}(n^3)$ ).

## 1 Holomorphic Embedding Load Flow Method (HELM)

### 1.1 Basics

### 1.2 Overview

### 1.3 Sigma approximants

### 1.4 GridCal

## 2 Flexible General Branch Model (FGBM)

### 2.1 Modelling

### 2.2 FGBM + HELM

## 3 Alternating Search Directions (ASD)

### 3.1 Formulation

## 4 Proper Generalised Decomposition (PGD)

### 4.1 Multidimensional power flow

### 4.2 Algorithm PGD + ASD

## 1 Holomorphic Embedding Load Flow Method (HELM)

### 1.1 Basics

### 1.2 Overview

### 1.3 Sigma approximants

### 1.4 GridCal

## 2 Flexible General Branch Model (FGBM)

### 2.1 Modelling

### 2.2 FGBM + HELM

## 3 Alternating Search Directions (ASD)

### 3.1 Formulation

## 4 Proper Generalised Decomposition (PGD)

### 4.1 Multidimensional power flow

### 4.2 Algorithm PGD + ASD

- Dimensions: position (nodes), changes in power, time...
- Voltages expressed in the separated form:  $V(x, q, t) = \sum_{m=1}^M V_m \otimes Q_m \otimes T_m$ .

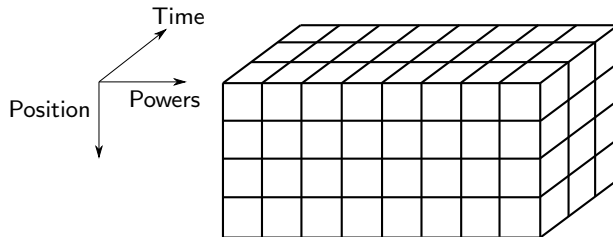


Figure 5. Representation of the cube of solutions

- Need to compute  $M(n_{\text{buses}} + n_{\text{powers}} + n_{\text{time}})$  instead of  $n_{\text{buses}} \cdot n_{\text{powers}} \cdot n_{\text{time}}$  unknowns.
- Can it be adapted for changes in the topology so that we can employ it for contingency analysis ( $N - 1$ ,  $N - 2$ ...)?

The outer loop follows the ASD procedure while the inner one is based on the alternating directions technique.

---

## Algorithm 1 Pseudocode for the PGD combined with ASD

---

```

1: for  $\gamma = 1$  to  $N_\gamma$  do
2:   Compute power side of the problem with PGD:  $I = S^* \oslash V^{*[\gamma]}$ 
3:   for  $m = 1$  to  $M$  do
4:     Define  $I = \sum_{m=1}^{M-1} I_m \otimes Q_m \otimes T_m + I_M \otimes Q_M \otimes T_M$ 
5:     for  $k = 1$  to  $N_k$  do
6:       Compute  $I_M^{[k+1]}$  with  $Q_M^{[k]}$  and  $T_M^{[k]}$ .
7:       Compute  $Q_M^{[k+1]}$  with  $I_M^{[k+1]}$  and  $T_M^{[k]}$ .
8:       Compute  $T_M^{[k+1]}$  with  $I_M^{[k+1]}$  and  $Q_M^{[k+1]}$ .
9:     end for
10:  end for
11:  Compute admittances side of the problem directly:  $V^{[\gamma+1]} = Y^{-1}(I + I_0)$ .
12: end for

```

---

■ Show code and results.

1 Holomorphic  
Embedding Load Flow  
Method (HELM)

1.1 Basics

1.2 Overview

1.3 Sigma approximants

1.4 GridCal

2 Flexible General  
Branch Model (FGBM)

2.1 Modelling

2.2 FGBM + HELM

3 Alternating Search  
Directions (ASD)

3.1 Formulation

4 Proper Generalised  
Decomposition (PGD)

4.1 Multidimensional  
power flow

4.2 Algorithm PGD +  
ASD

# POWER SYSTEMS CALCULATION

Josep Fanals

CITCEA

02/2021