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## 1. INTRODUCTION

It is a well-known fact that the power flow in power systems is represented by non-linear equations. Multiple solvers have been applied during these past decades. Some of them have been intentionally created for such purpose, while others are based on common numerical methods, being the Newton-Raphson the most popular. At its core, all these solvers are intended to be used with deterministic input data, that is, they leave no room for variation. In case an input changes, the power flow has to be recalculated.

With this conventional approach, given a power system where there are  $m$  uncertain parameters and each of them takes  $n$  values, the solver would have to be called  $n^m$  times in order to extract all solutions. Thus, an increase in the number of parameters involves a meteoric rise in the computational effort, which is most likely unjustified.

The probabilistic power flow addresses the uncertainty in the parameters' values. It aims at generating the probability distribution of a grid state (e.g., the voltage magnitude at a given bus) as a function of the input varying data (such as the active power provided by a generator). It was first formulated by Borkowska back in 1974 [1]. Nowadays it has gained attraction in the realm of renewable energy sources due to their relative unpredictability. In particular, it has been employed for photovoltaics and wind power, which rely on Gaussian and Weibull distributions respectively [2]–[4]. Moreover, the impact of the electric vehicle charging demand on the grid has been assessed with this technique [5]. However, using the probabilistic power flow does not allow to know with certainty a grid state considering deterministic input data.

The parametric power flow ideally provides a solution to the aforementioned challenge. Instead of dealing with distributions, it generates a closed-form expression that relates the input data (called parameters from now on) with the state of interest. It sacrifices a bit of precision in order to obtain a satisfactory approximation, much more computationally efficient than solving the  $n^m$  cases. It has been implemented in conjunction with an optimal power flow formulation [6], yet for the most part it has remained an unexplored methodology. García-Blanco et al. have formulated the so-called Proper Generalized Decomposition, which manages to obtain all  $n^m$  solutions reasonably fast and accurately [7], [8]. Nevertheless, the methodology is not scalable. It is tied to a traditional power system formed by PQ buses and a single slack bus. There are workarounds to add PV buses but they are prone to cause divergence. Thus, as power systems

tend to incorporate controllable power electronic devices, there is little relevance in such technique. A much more appealing approach, formulated by Shen et al., deals with expressing a state as a function of the parameters [9]. This is the central reference of this work.

This monograph presents a detailed formulation of the parametric power flow and justifies how the dimensions can be reduced. Then, a basic 5-bus system is tackled to exemplify the methodology. Some thoughts on whether or not this technique can be combined with power electronics are also verbalized. Finally, conclusions are extracted along with the conceptualization of potential research ideas.

## 2. FORMULATION

### 2.1. States, parameters and dimensions

States are denoted by  $\mathbf{x}$  and stand for the unknowns of the power flow, such as the voltage magnitude. The traditional power flow looks to obtain a solution for all states; in the parametric power flow, this is not necessarily the case, as there might be only one bus under study.

Parameters are defined as the input data which are meant to vary. Powers are part of this category, since loads change over time and most generation sources also experience variations. Considering a total of  $m$  parameters, grouped under  $\mathbf{p} = [p_1, p_2, \dots, p_m]$ , each parameter  $p_k \forall k \in \{1, \dots, m\}$  takes values inside the  $[a_k, b_k]$  range.

This way, the power flow problem can be written as:

$$\mathbf{f}(\mathbf{x}, \mathbf{p}) = 0, \quad (1)$$

where  $\mathbf{f}$  symbolizes all implicit functions involved in the solution of the problem. The formulation that follows is generic, in the sense that the methodology would be equally valid for other problems. Hence, this technique is not limited to the traditional power flow.

The parametric approach looks to express a certain state as a function of the parameters:

$$x = g(\mathbf{p}), \quad (2)$$

where  $g$  is a function pending to be found.

From (2) it becomes clear that all parameters will potentially affect a given state. Visually speaking, Fig. 1.a shows a representation of a state as a function of a single parameter, that is, assuming that only one input changes. Fig. 1.b presents a similar visualization with two parameters involved. This justifies why, when viewed as a spatial representation, parameters are also denoted as dimensions. Each one of them stands for a new axis independent axis, and thus, orthogonal to the rest.

More than two dimensions are expected to be present in a typical analysis of the parametric power flow. In a mid-size system, assuming all powers are treated as parameters, there could be hundreds of dimensions. If  $m \approx 100$ , opting for

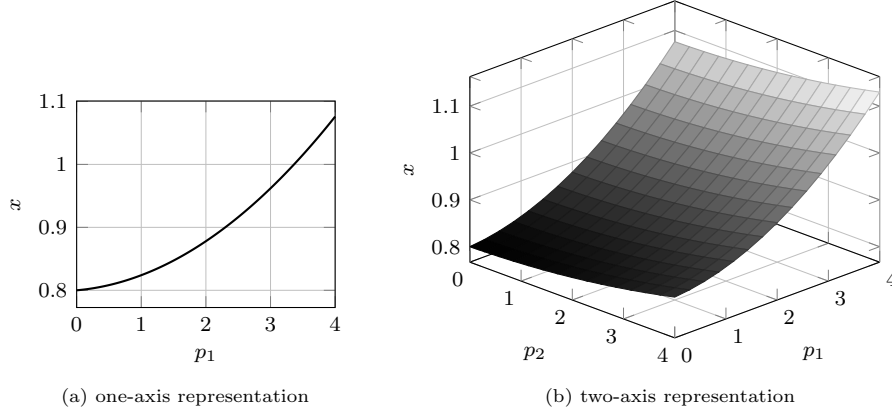


Figure 1. Representation of a state with one or two parameters

the naive approach with  $n^m$  cases would be painful to compute. This is often called the curse of dimensionality.

One fundamental step to circumvent this challenge is to reduce the dimensions. Note that in Fig. 1 the value of  $x$  is more or less the same in both Fig. 1.a and Fig. 1.b. That is,  $p_1$  has a larger influence than  $p_2$ . The essence of dimensionality reduction lies in compacting all dimensions into a few. A prerequisite is to determine the impact all parameters have on the state.

## 2.2. Principal Component Analysis

The Principal Component Analysis (PCA) weights the parameters according to their influence. To do so, the first step is concerned with computing the gradient:

$$\nabla_{\mathbf{p}} g = \left[ \frac{\partial x}{\partial p_1}, \dots, \frac{\partial x}{\partial p_k}, \dots, \frac{\partial x}{\partial p_m} \right]^T. \quad (3)$$

Calculating the gradient numerically is probably the most beneficial option because it allows to operate with implicit functions, just like the power flow is commonly presented. Hence, for all  $k$  in  $[1, \dots, m]$ :

$$\frac{\partial x}{\partial p_k} = \frac{g(p_1, \dots, p_k + \delta, \dots, p_m) - g(p_1, \dots, p_k, \dots, p_m)}{\delta}, \quad (4)$$

where  $\delta$  is an arbitrary small number, such as  $\delta = 1 \cdot 10^{-10}$  for instance. Computing the gradient with this approach means that the power flow solver has to be called  $m + 1$  times.

Let  $\mathbf{C} \in \mathbb{R}^{m \times m}$  be the covariance matrix. Its elements identify the influence of the parameters on the states. Large elements represent that the two involved

parameters have a significant impact on the state, whereas small elements that these two parameters are potentially irrelevant. Since a parameter  $k$  can take any value inside its corresponding range  $[a_k, b_k]$ , several samples have to be extracted to have a representative  $\mathbf{C}$  matrix:

$$\mathbf{C} \approx \frac{1}{M} \sum_{i=1}^M \nabla_{\mathbf{p}} g(\nabla_{\mathbf{p}} g)^T, \quad (5)$$

where  $M$  is the total number of samples and takes a relatively arbitrary value.

The parameters take random values in each sample. The power flow solver would have to be called  $M(m+1)$  times in total. Although it largely depends on the number of initial dimensions,  $M$  can take values in the order of a hundred. As a consequence, it can be foreseen that calculating  $M(m+1)$  power flows is generally way faster than computing  $n^m$  cases. The reason is that there is no dependence on the discretized  $n$  values and this methodology provides a continuous solution (in the form of a polynomial).

Matrix  $\mathbf{C}$  is symmetrical by definition, which means it can be diagonalized by orthogonal matrices:

$$\mathbf{C} = \mathbf{W} \mathbf{A} \mathbf{W}^T, \quad (6)$$

where  $\mathbf{A}$  is a diagonal  $m \times m$  matrix formed by eigenvalues  $[\lambda_1, \dots, \lambda_k, \dots, \lambda_m]$  sorted in order of relevance, and  $\mathbf{W}$  contains their respective eigenvectors, denoted as  $[\mathbf{w}_1, \dots, \mathbf{w}_k, \dots, \mathbf{w}_m]$ , in the form of columns.

The eigenvectors in  $\mathbf{W}$  can be understood as dimensions. The idea behind the PCA is to determine the impact of a given dimension, which ends up being proportional to its eigenvalue [9]. Since eigenvalues are sorted by order of importance,  $\mathbf{w}_1$  is more representative than  $\mathbf{w}_2$ ,  $\mathbf{w}_2$  is more important than  $\mathbf{w}_3$ , and so on. This process of determining the relevant dimensions is the core of the Proper Orthogonal Decomposition (POD), which has been widely employed in the field of fluid dynamics [10], [11].

If some dimensions are more influential than others, it makes sense to eliminate the insignificant ones. Although this is a subjective step, the truncation error can be defined as:

$$e_t = \frac{\lambda_{k+1} + \dots + \lambda_m}{\lambda_1 + \dots + \lambda_m}, \quad (7)$$

which implies that only eigenvalues  $[\lambda_1, \dots, \lambda_k]$  have been selected. The user ought to select a desired error (e.g.,  $e_t = 10\%$ ) and identify the eigenvalues that

allow obtaining a similar truncation error. The lower the truncation error, the more accurate and complex the model will be, and vice versa.

### 2.3. Dimension reduction

Up to this point, the most relevant dimensions have been identified. The next step is to reduce the dimensions:

$$\mathbf{y} = \mathbf{W}_{\mathbf{y}}^T \mathbf{p}, \quad (8)$$

where  $\mathbf{W}_{\mathbf{y}}^T$  is formed by the rows of eigenvectors  $[\mathbf{w}_1, \dots, \mathbf{w}_k]$ . As  $\mathbf{W}_{\mathbf{y}}^T \in \mathbb{R}^{k \times m}$ , it transforms all  $m$  directions into  $k$  impactful directions.

Consequently, the initial parametric power flow of the form  $x = g(\mathbf{p})$  is transformed into:

$$x \approx h(\mathbf{y}), \quad (9)$$

where  $h$  is a function which has yet to be found. In more detail, it will consist of a polynomial. Working with  $\mathbf{y}$ , which symbolizes  $k$  directions, instead of  $\mathbf{p}$  where there are  $m$  directions involved, is not mandatory. The state  $x$  could still be approximated by a polynomial function  $h(\mathbf{p})$ . However, if  $m$  tends to a large number, the method would suffer from the curse of dimensionality.

More details are provided in the following section, but for now it will be assumed that the number of terms  $N_t$  in the function  $h$  is given by [9], [12]:

$$N_t = \binom{l+m}{m} = \frac{(l+m)!}{m! (l+m-m)!} = \frac{(l+m)!}{m! l!}, \quad (10)$$

where  $l$  denotes the expansion order and it usually does not have to exceed 3. From (10), if  $l = 3$  and  $m = 5$ ,  $N_t = 56$ ; if  $m = 10$  while  $l = 3$ ,  $N_t = 286$ . The computational effort would have increased by a factor of approximately 5 even though the number of dimensions has only doubled. This numerical example indicates why it is appropriate to reduce the number of dimensions in spite of losing a bit of accuracy.

### 2.4. Polynomial function

Function  $h$  is a polynomial function of degree  $l$  at most. It is formed by a succession of terms such as:

$$x \approx h(\mathbf{y}) = \sum_{j=0}^l \mathbf{c}_j^T \phi_j(\mathbf{y}), \quad (11)$$

where  $\mathbf{c}_j$  is a vector of coefficients with the same length as  $\phi_j$ , which contains all basis functions of degree  $j$ . All coefficients have to be found in order to determine  $h$ . Once this is achieved,  $h$  will be fully defined and ready to be used to compute the state for whatever values the parameters take.

As (11) might be too compact, it is exemplified in the case where  $k = 3$  (that is, there are 3 meaningful directions) and  $l = 2$ . The three significant directions are denoted by  $y_1, y_2, y_3$ . The rule is that all three parameters have to be multiplied between them, and the degree of the resulting function has to be equal to  $j$ . Thus, when  $j = 0$ :

$$\begin{aligned} f_0(\mathbf{y}) &= [c_{0,1}] \cdot [y_1^0 y_2^0 y_3^0]^T \\ &= c_{0,1} \cdot 1, \end{aligned} \tag{12}$$

where  $c_{0,1}$  stands for the first (and only) coefficient of the basis function with degree 0, and it is unknown. The values of  $y_1, y_2, y_3$  are known since they are treated as inputs.

When  $j = 1$ :

$$\begin{aligned} f_1(\mathbf{y}) &= [c_{1,1}, c_{1,2}, c_{1,3}] \cdot [y_1^1 y_2^0 y_3^0, y_1^0 y_2^1 y_3^0, y_1^0 y_2^0 y_3^1]^T \\ &= c_{1,1} \cdot y_1 + c_{1,2} \cdot y_2 + c_{1,3} \cdot y_3. \end{aligned} \tag{13}$$

When  $j = 2$ :

$$\begin{aligned} f_2(\mathbf{y}) &= [c_{2,1}, c_{2,2}, c_{2,3}, c_{2,4}, c_{2,5}, c_{2,6}] \cdot [y_1^2 y_2^0 y_3^0, y_1^1 y_2^1 y_3^0, y_1^1 y_2^0 y_3^1, \\ &\quad y_1^0 y_2^2 y_3^0, y_1^0 y_2^1 y_3^1, y_1^0 y_2^0 y_3^2]^T \\ &= c_{2,1} \cdot y_1 y_2 + c_{2,2} \cdot y_1 y_3 + c_{2,3} \cdot y_2 y_3 + c_{2,4} \cdot y_1^2 + c_{2,5} \cdot y_2^2 + c_{2,6} \cdot y_3^2. \end{aligned} \tag{14}$$

Finally, all  $f$  are added:

$$x \approx h(\mathbf{y}) = f_0(\mathbf{y}) + f_1(\mathbf{y}) + f_2(\mathbf{y}). \tag{15}$$

Note that taking into account (12), (13) and (14), there are a total of 10 terms, just like (10) indicates when  $l = 2$  and  $m = 3$ .

Recall that a total of  $M$  samples, where the parameters take random values, are needed to build the covariance matrix. Now these samples have to be used in order to compute the coefficients  $c$ . Let  $\mathbf{Q} \in \mathbb{R}^{M \times N_t}$  denote the matrix where the terms that multiply the coefficients  $c$  are stored, and  $\mathbf{h}_x$  the vector that contains the value of the state  $x$  at each sample. Then, the goal is to minimize the error  $\epsilon$  in the generic expression:

$$\mathbf{h}_x = \mathbf{Q}\mathbf{c} + \epsilon, \tag{16}$$



where  $\mathbf{c}$  is constituted by the  $N_t$  coefficients.

Several ways to calculate the coefficients in  $\mathbf{c}$  exist [9]. If the least squares regression method is chosen [13], the best estimate of  $\mathbf{c}$  is:

$$\mathbf{c} = (\mathbf{Q}^T \mathbf{Q})^{-1} \mathbf{Q}^T \mathbf{h}_{\mathbf{x}}. \quad (17)$$

To apply this least squares regression, it is necessary to have  $M \geq N_t$ , and as a rule of thumb,  $M$  should be around 1.5 to 3 times  $N_t$  [13]. This concludes the calculation process, as  $h(\mathbf{y})$  is fully known.

## 2.5. Procedure

The steps to take in order to obtain the reduced dimension model are summarized below:

1. Compute the gradients for the  $M$  samples to calculate the covariance matrix  $\mathbf{C}$  with (5).
2. Perform the orthogonal decomposition of  $\mathbf{C}$  and select the most influential dimensions according to the desired truncation error.
3. With the random samples, transform  $\mathbf{p}$  into  $\mathbf{y}$  with (8).
4. Build the  $\mathbf{Q}$  matrix, the  $\mathbf{h}_{\mathbf{x}}$  vector, and find the coefficients with (17).

Once the coefficients  $\mathbf{c}$  are calculated, the procedure yields an explicit function which relates the state  $x$  with any value of the parameters.

### 3. CASE STUDY

The aim of this section is to show more in detail the calculation procedure. The methodology is applied to a basic power system such as the 5-bus system from the GridCal tutorial [14], shown in Fig. 2.

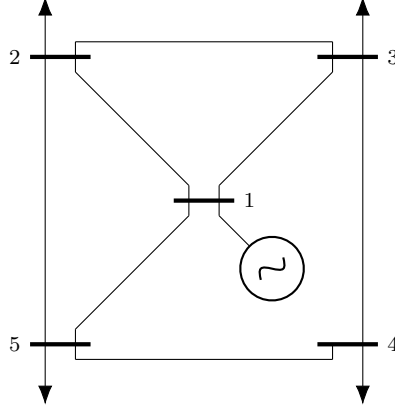


Figure 2. High-level representation of the 5-bus system

The system is formed by four PQ buses and one slack bus, corresponding to node 1. Lines are originally modeled with a  $\pi$  equivalent. Nevertheless, the parallel capacitances are removed in order to have voltages below the level imposed by the slack bus, which should help at obtaining a more intuitive result.

The state of interest will be the voltage magnitude at bus 5, thus,  $x = V_5$ . Choosing the parameters is rather a subjective choice which largely depends on the scope of the study. However, they are chosen to be the four active powers and the four reactive powers in this case:

$$[p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8] = [P_2, Q_2, P_3, Q_3, P_4, Q_4, P_5, Q_5], \quad (18)$$

where  $P$  and  $Q$  stand for the loads of active and reactive power.

These powers can take any value inside a predefined range of values. In a realistic scenario, such range could comprise a high confidence interval. Table 1 shows the chosen lower and upper bounds for all powers.

Limit	$P_2$	$Q_2$	$P_3$	$Q_3$	$P_4$	$Q_4$	$P_5$	$Q_5$
Lower	0	0	0	0	0	0	0	0
Upper	40	30	30	45	25	20	30	20

Table 1. Lower and upper limit for all parameters, in the order of MVA

The calculation process obeys the four steps summarized in Section 2.5. Each of them is numerically detailed below.

1. Compute the gradients for the  $M$  samples to calculate the covariance matrix  $\mathbf{C}$  with (5).

A total of  $M = 10$  samples is chosen. This becomes a resonable amount considering that the expansion order is set to  $l = 3$ . Moreover, as it will be shown in the next step, all eight parameters will be encapsulated in a single meaningful direction, so  $k = 1$ . Thus,  $N_t = 4$  in accordance with (10). The requirements are being met since  $M$  should be between 1.5 to 3 times  $N_t$ .

There is no complexity in generating the samples. All parameters take random values between the limits set in Table 1. The power flow is solved  $M(m + 1) = 10(8 + 1) = 90$  times. With it, a total of 10 gradients are obtained by following (4). The covariance matrix  $\mathbf{C}$  is built by adding the products of gradients, as shown in (5). In this particular case, it becomes:

$$\mathbf{C} = 10^{-8} \cdot \begin{pmatrix} 1.39 & 3.18 & 1.93 & 4.40 & 5.00 & 11.16 & 1.92 & 4.57 \\ 3.18 & 7.27 & 4.40 & 10.05 & 11.41 & 25.48 & 4.39 & 10.43 \\ 1.93 & 4.40 & 2.66 & 6.08 & 6.91 & 15.43 & 2.66 & 6.31 \\ 4.40 & 10.05 & 6.08 & 13.89 & 15.77 & 35.21 & 6.07 & 14.41 \\ 5.00 & 11.41 & 6.91 & 15.77 & 17.91 & 39.98 & 6.89 & 16.36 \\ 11.16 & 25.48 & 15.43 & 35.21 & 39.98 & 89.28 & 15.39 & 36.54 \\ 1.92 & 4.39 & 2.66 & 6.07 & 6.89 & 15.39 & 2.65 & 6.30 \\ 4.57 & 10.43 & 6.31 & 14.41 & 16.36 & 36.54 & 6.30 & 14.95 \end{pmatrix}. \quad (19)$$

It is symmetrical, with tiny values as the voltage does not change much with variations in the powers.

2. Perform the orthogonal decomposition of  $\mathbf{C}$  and select the most influential dimensions according to the desired truncation error.

The orthogonal decomposition of the covariance matrix follows (6). The eight eigenvalues are presented in Table 2.

	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$	$\lambda_5$	$\lambda_6$	$\lambda_7$	$\lambda_8$
Value ( $\cdot 10^{-12}$ )	$1.5 \cdot 10^6$	67	28	21	5	3	1	0

Table 2. Ordered eigenvalues obtained from the orthogonal decomposition

The first eigenvalue is significantly more relevant than the others. Choosing

$k = 1$  impactful dimensions translates into a minuscule truncation error. This shows that going from 8 to 1 dimension is possible without making a notorious mistake.

As  $k = 1$ ,  $\mathbf{W}_y$  is a single-column matrix containing the eigenvector  $\mathbf{w}_1$  associated with  $\lambda_1$ :

$$\mathbf{W}_y = \begin{pmatrix} 0.096509323157 \\ 0.220170675384 \\ 0.133325972120 \\ 0.304257354543 \\ 0.345498102071 \\ 0.771423934788 \\ 0.133032385496 \\ 0.315754699603 \end{pmatrix}. \quad (20)$$

This matrix weights all parameters. Notice how the sixth parameter, the one corresponding to  $P_4$ , is the largest by far. This could also had been observed in the  $\mathbf{C}$  matrix shown in (19), where the sixth column/row has significantly greater values than the rest. It makes sense that  $P_4$  has a notable impact on  $V_5$  because bus 4 is the only one not directly connected to the slack bus.

3. With the random samples, transform  $\mathbf{p}$  into  $\mathbf{y}$  with (8).

Then, the  $\mathbf{p}$  vector with the random samples and the matrix  $\mathbf{W}_y^T$  are multiplied for all samples. This is an straightforward step since the two involved objects are already known. The goal is to transform the 8 dimensions in  $\mathbf{p}$  to just one dimension stored in  $\mathbf{y}$ . In the end, the procedure looks for  $h(\mathbf{y})$ , so  $\mathbf{p}$  always has to be transformed.

Since there is only one dimension in  $\mathbf{y}$ , the basis function expansion turns out to be simplified. If  $l = 3$ :

$$h(y) = c_0 + c_1 \cdot y^1 + c_2 \cdot y^2 + c_3 \cdot y^3, \quad (21)$$

where  $y \in \mathbb{R}$  and the coefficients  $c$  have to be calculated.

4. Build the  $\mathbf{Q}$  matrix, the  $\mathbf{h}_x$  vector, and find the coefficients with (17).

The  $\mathbf{h}_x$  vector has been previously computed. It simply gathers the 10 solutions

(one for each sample) of  $V_5$ , and is equal to:

$$\mathbf{h}_x = \begin{pmatrix} 0.95989933482 \\ 0.956904257759 \\ 0.956758440075 \\ 0.964702869164 \\ 0.957956346033 \\ 0.965846162026 \\ 0.973066025427 \\ 0.961498057762 \\ 0.956583704834 \\ 0.966603851952 \end{pmatrix}. \quad (22)$$

The  $\mathbf{Q}$  matrix has dimensions  $10 \times 4$ , and it is formed by the four elements that are multiplied by the coefficients in (21), with each sample constituting a row. It is not shown here for simplicity.

Finally, the coefficients in  $\mathbf{c}$  are computed with (17). The resulting equation becomes:

$$V_5 \approx h(y) = 1.000044024546 - 1.147157669 \cdot 10^{-3}y - 1.101177 \cdot 10^{-6}y^2 - 3.244 \cdot 10^{-9}y^3. \quad (23)$$

To calculate the approximated voltage in every possible situation it is enough to convert the vector of parameters  $\mathbf{p}$  to  $y$  and input its value to (23). Fig. 3 shows the value  $V_5$  takes depending on  $y$ .

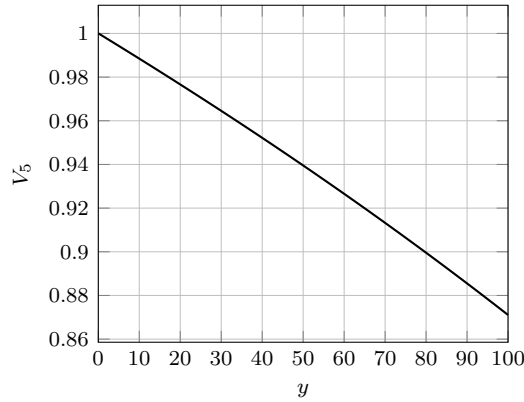


Figure 3. Graphical interpretation of  $V_5 \approx h(y)$

If the horizontal axis were to be extended, it would become clear how  $V_5(y)$  follows a parabolic profile, similar to the typical P-V curves. Also, it is relevant

to mention that the values of  $\mathbf{h}_{\mathbf{x}}$  are around 0.95 to 0.98 approximately. This is due to the lower and upper bounds defined in 1. Therefore, the computed value of  $y$  should be between 15 and 40 more or less.

The methodology has been tested for several cases with satisfactory results. Table 3 compares the results obtained with  $V_5 \approx h(y)$  and the results provided by the power flow solver in case  $\mathbf{p} = [25, 12, 8, 33, 21, 4, 17, 11]$  MVA.

Magnitude	Value
$V_5$ from (23)	0.9618089693761898
$V_5$ from the power flow solver	0.9618607699599497
Error	$5.180058375997554 \cdot 10^{-5}$

Table 3. Comparison of results when  $\mathbf{p} = [25, 12, 8, 33, 21, 4, 17, 11]$  MVA

It can be concluded that by only calling the power flow 90 times, a satisfactory model has been obtained. It can reproduce with great accuracy the actual  $V_5$  value as a function of the parameters.

In essence, the methodology is based on computing the gradients at different operating points. It may seem equally valid to calculate the gradient at a random operating point and estimate the state from this. While this basic approach could work for grids where the loads are at their lower levels, it is not suitable for higher load levels. Intuitively speaking, the operating points would move closer to the nose of the P-V curves, which are quadratic. As a consequence, a simple linearization at a random point becomes sub-optimal.

#### **4. POTENTIAL ADAPTATION TO POWER ELECTRONICS**

## 5. CONCLUSIONS



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