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1. INTRODUCTION

It is a well-known fact that the power flow in power systems is represented by non-linear equations. Multiple solvers have been applied during these past decades. Some of them have been intentionally created for such purpose, while others are based on common numerical methods, being the Newton-Raphson the most popular. At its core, all these solvers are intented to be used with deterministic input data, that is, they leave no room for variation. In case an input changes, the power flow has to be recalculated.

With this conventional approach, given a power system where there are m uncertain parameters and each of them takes n values, the solver would have to be called n^m times in order to extract all solutions. Thus, an increase in the number of parameters involves a meteoric rise in the computational effort, which is most likely unjustified.

The probabilistic power flow addresses the uncertainty in the parameters' values. It aims at generating the probability distribution of a grid state (e.g., the voltage magnitude at a given bus) as a function of the input varying data (such as the active power provided by a generator). It was first formulated by Borkowska back in 1974 [1]. Nowadays it has gained attraction in the realm of renewable energy sources due to their relative unpredictability. In particular, it has been employed for photovoltaics and wind power, which rely on Gaussian and Weibull distributions respectively [2]–[4]. Moreover, the impact of the electric vehicle charging demand on the grid has been assessed with this technique [5]. However, using the probabilistic power flow does not allow to know with certainty a grid state considering deterministic input data.

The parametric power flow ideally provides a solution to the aforementioned challenge. Instead of dealing with distributions, it generates a closed-form expression that relates the input data (called parameters from now on) with the state of interest. It sacrificies a bit of precision in order to obtain a satisfactory approximation, much more computationally efficient than solving the n^m cases. It has been implemented in conjunction with an optimal power flow formulation [6], yet for the most part it has remained an unexplored methodology. García-Blanco et al. have formulated the so-called Proper Generalized Decomposition, which manages to obtain all n^m solutions reasonably fast and accurately [7], [8]. Nevertheless, the methodology is not scalable. It is tied to a traditional power system formed by PQ buses and a single slack bus. There are workarounds to add PV buses but they are prone to cause divergence. Thus, as power systems tend to incorporate controllable power electronic devices, there is little relevance in such technique. A much more appealing approach, formulated by Shen et al., deals with expressing a state as a function of the parameters [9]. This is the central reference of this work.

This monograph presents a detailed formulation of the parametric power flow and justifies how the dimensions can be reduced. Then, a basic 5-bus system is tackled to exemplify the methodology. Some thoughts on whether or not this technique can be combined with power electronics are also verbalized. Finally, conclusions are extracted along with the conceptualization of potential research ideas.

2. FORMULATION

2.1. What are states, parameters and dimensions?

States are denoted by $\mathbf{x}$ and stand for the unknowns of the power flow, such as the voltage magnitude. The traditional power flow looks to obtain a solution for all states; in the parametric power flow, this is not necessarily the case, as there might be only one bus under study.

Parameters are defined as the input data which are meant to vary. Powers are part of this category, since loads change over time and most generation sources also experience variations. Considering a total of m parameters, grouped under $\mathbf{p} = [p_1, p_2, ..., p_m]$, each parameter p_k $\forall k \in \{1, ..., m\}$ takes values inside the $[a_k, b_k]$ range.

This way, the power flow problem can be written as:

$$\mathbf{f}(\mathbf{x}, \mathbf{p}) = 0, \tag{2.1}$$

where $\mathbf{f}$ symbolizes all implicit functions involved in the solution of the problem. The formulation that follows is generic, in the sense that the methodology would be equally valid for other problems. Hence, this technique is not limited to the traditional power flow.

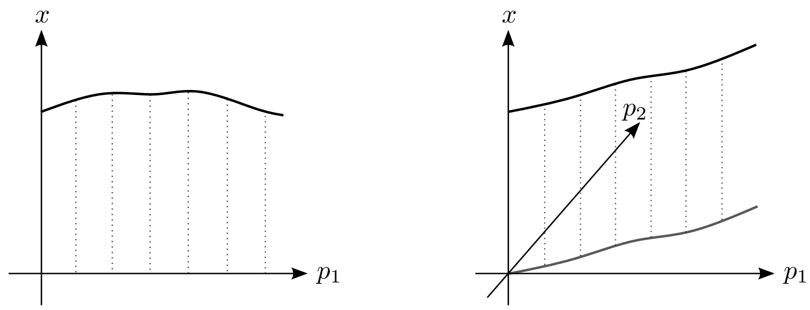
The parametric approach looks to express a certain state as a function of the parameters:

$$x = g(\mathbf{p}), \tag{2.2}$$

where g is a function pending to be found.

From (2.2) it becomes clear that all parameters will potentially affect a given state. Visually speaking, Fig. 1.a shows a representation of a state as a function of a single parameter, that is, assuming that only one input changes. Fig. 1.b presents a similar visualization with two parameters involved. This justifies why, when viewed as a spatial representation, parameters are also denoted as dimensions. Each one of them stands for a new axis independent axis, and thus, orthogonal to the rest.

More than two dimensions are expected to be present in a typical analysis of the parametric power flow. In a mid-size system, assuming all powers are treated as parameters, there could be hundreds of dimensions. If $m \approx 100$, opting for



(a) one-axis representation (b) two-axis representation

Figure 1. Representation of a state with one or two parameters

the naive approach with m^n cases would be painful to compute. This is often called the curse of dimensionality.

One fundamental step to circumvent this challenge is to reduce the dimensions. Note that in Fig. 1 the value of x is more or less the same in both Fig. 1.a and Fig. 1.b. That is, p_1 has a larger influence than p_2. The essence of dimensionality reduction lies in weighting each parameter according to its impact on the state and compacting them as a single parameter.

3. POTENTIAL ADAPTATION TO POWER ELECTRONICS

4. CONCLUSIONS

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