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Application of asymptotic numerical method with homotopy techniques to power flow problems

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ABSTRACT

Traditional power flow methods such as the Newton-like methods are locally convergent and may be ineffective in some circumstances. In this paper, we propose a novel computational approach which associates a Newton homotopy with an asymptotic numerical method (ANM) to solve the nonlinear power flow equations. ANM is a family of algorithms based on the computation of Taylor series expansion per step. With ANM, as the homotopy path has been expressed into a closed analytical form section by section, the multiple power flow solutions on the path are computed by solving a simple polynomial equation. The proposed method provides a reliable way to study the ill-conditioned power flow problems thanks to the use of homotopy transformation and higher-order predictor of ANM. Numerical examples of several power systems are presented to validate the effectiveness of the method.

1. Introduction

Power flow computations play an important role in power system analysis. The power flow equations are modeled by a set of nonlinear algebraic equations. Traditionally, the power flow equations are solved by use of the Newton's method and its variants (e.g., fast P–Q decoupled method, optimal multiplier method, etc.) [1–6].

The power flow problem may be divided into the well-conditioned and the ill-conditioned cases. A brief review of the contributions in this area is given by Milano [7]. For well-conditioned case, the conventional Newton-like methods will converge quickly if the initial guess is in the neighborhood of a solution. As for the ill-conditioned case (e.g., heavily stressed system, high branch r/x ratios, etc.), the power flow Jacobian matrix will become singular or ill-conditioned when approaching the solution, and may make the iteration sequences fail to converge or converge very slowly. More recently, some robust power flow methods have been proposed to solve the issue, e.g., continuous Newton's method [7], tensor methods based approach [8], etc.

It is well known that the homotopy methods have been developed to overcome the local convergence of the Newton-like methods and may provide a reliable way to solve the nonlinear equations [9,10]. The homotopy methods have been applied to power system in different aspects [11–14]. In the last two decades, the idea of homotopy was combined with perturbation [15–18]. Using the perturbation technique, the nonlinear problem to be solved is transformed into a sequence of linear ones with the same operator. This fundamental work was done by Liao and He. Liao proposed the homotopy analysis method (HAM) [15,16], and He introduced the homotopy perturbation method (HPM) [17,18]. The above homotopy methods are constantly being developed and applied to solve various nonlinear problems in science and engineering.

In this paper, by using the homotopy technique, we present a new numerical method for the solutions of the power flow equations. The method is a combination of the homotopy transformation with the asymptotic numerical method (ANM), which is a family of algorithms based on the computation of Taylor series expansion per step (in this way, the ANM may also belong to a kind of perturbation method) [19–21]. The advantage of ANM over the classical incremental iterative approach is that it yields an analytical continuous representation of the path, which provides significant benefits to the computation. ANM has been widely applied to solve the nonlinear equations with a changeable parameter mainly in structural and fluid mechanics. Many applications of ANM to these areas show the performance of this technique with respect to the computation time and adaptive of the step-length. In [22], ANM is also used to solve the continuation power flow (CPF) in power systems.

The basic idea of the proposed algorithm is to embed the given problem into one parameter-dependent system and to compute the homotopy path with ANM. The power flow solutions on the

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path are computed by solving a simple polynomial equation. The main contributions of this paper are twofold: (1) to provide a robust power flow method for the ill-conditioned power flow, thanks to the use of the homotopy transformation and higher-order predictor of ANM; (2) to propose an efficient method to study the multiple solutions of the power flow equations.

This paper is organized as follows: Section 2 presents the quadratic form of the power flow equations. Section 3 shows the principle of the techniques applying homotopy transformation with ANM to solve the power flow equations. Numerical examples are given in Section 4 to test the algorithm. Finally, the conclusions are drawn in Section 5.

2. The nonlinear power flow equations

In rectangular coordinates, the power flow equations of a power system are represented by a set of polynomial equations

$$P_{Gi} - P_{Li} - e_i\sum_{j=1}^{N}(G_{ij}e_j - B_{ij}f_j) - f_i\sum_{j=1}^{N}(G_{ij}f_j + B_{ij}e_j) = 0 \tag{1}$$

$$Q_{Gi} - Q_{Li} - f_i\sum_{j=1}^{N}(G_{ij}e_j - B_{ij}f_j) + e_i\sum_{j=1}^{N}(G_{ij}f_j + B_{ij}e_j) = 0 \tag{2}$$

As for a PV bus, Eq. (2) is replaced by

$$V_{is}^2 - e_i^2 - f_i^2 = 0 \tag{3}$$

where N is the bus number; P_{Gi}, Q_{Gi}, P_{Li}, Q_{Li} are the active and reactive generation and load power at bus i; G_{ij}, B_{ij} are the real and imaginary part of (i, j) entry of the bus admittance matrix; e_i, f_i are the real and imaginary part of voltage at bus i; V_{is} is the specified known voltage at bus i.

For nonlinear loads or other devices in power systems (e.g., FACTS, HVDC, etc.), by introducing auxiliary variables and different relations between the variables, we may transform the power flow equations into the quadratic form [22,23].

The power flow problem of above (1)–(3) can be reformulated as finding a solution of nonlinear equation such that

$$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0} \tag{4}$$

where $\boldsymbol{f}:\boldsymbol{R}^n \to \boldsymbol{R}^n$ is the power flow equation, and $\boldsymbol{x} \in \boldsymbol{R}^n$ is the bus voltage vector. Eq. (4) may be solved by the following proposed methodology, which associates a homotopy transformation with an asymptotic numerical method together.

3. Proposed methodology

3.1. Homotopy transformation

The simplest way of globalization of local Newton's method is to embed the given problem (4) into one-parameter family of problems [9,10]:

$$\boldsymbol{H}(\boldsymbol{x}, \lambda) = \boldsymbol{0} \tag{5}$$

where $\boldsymbol{H}:\boldsymbol{R}^n \times \boldsymbol{R} \to \boldsymbol{R}^n$ is a homotopy transformation, and $\lambda \in [0, 1]$ is a scalar embedding parameter. The Newton homotopy is a popular homotopy. It takes the form

$$\boldsymbol{H}(\boldsymbol{x}, \lambda) = \boldsymbol{f}(\boldsymbol{x}) - (1 - \lambda)\boldsymbol{f}(\boldsymbol{x}_0) \tag{6}$$

where $\boldsymbol{x}_0$ is the starting point. Define $\boldsymbol{H}^{-1}(\boldsymbol{0}) = \{(\boldsymbol{x}, \lambda)|\boldsymbol{H}(\boldsymbol{x}, \lambda) = \boldsymbol{0}\}$ as the set of solutions to the system (5). Fig. 1 shows the principle of the homotopy continuation. Firstly we set $\lambda = 0$ and the starting point $\boldsymbol{x} = \boldsymbol{x}_0$, then increase λ in small increments. For each value of λ, we compute the solution of the system (5). The final value $\boldsymbol{x}^*$ corresponding to $\lambda = 1$ will solve the original problem $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0}$. Continue the process when λ is outside the interval $[0, 1]$, and another power flow solution $\boldsymbol{x}^{**}$ may be obtained (Case I in Fig. 1). A special case is that the two points $\boldsymbol{x}^*$ and $\boldsymbol{x}^{**}$ coalesce at $\lambda = 1$. In this respect, there exists only one power flow solution, and $\boldsymbol{x}^*(\boldsymbol{x}^{**})$ is actually the saddle node bifurcation point (SNBP) (or called fold point) of the power flow equations. When there is no intersections between $\lambda = 1$ and $\boldsymbol{H}^{-1}(\boldsymbol{0})$, the power flow equations have no real solutions (Case II in Fig. 1).

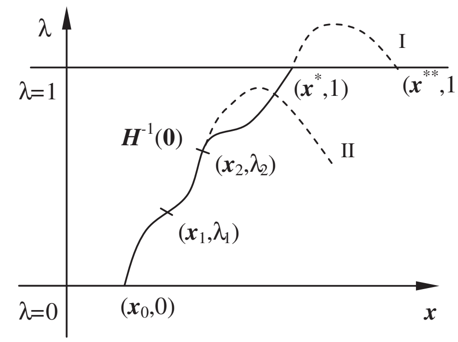

Fig. 1. Principle of the homotopy continuation (Case I: multiple solutions; Case II: no solution).

Usually, the homotopy path is computed by the predictor-corrector path following algorithms [9,24], which requires a high computational cost. To solve (5) efficiently, questions like the adaptive choices of step-lengths are of interest. The ANM described in the next subsection will greatly enhance the efficiency of the homotopy continuation.

3.2. Solve $\boldsymbol{H}^{-1}(\boldsymbol{0})$ with ANM

Eq. (6) may be decomposed into the following parts:

$$\boldsymbol{H}(\boldsymbol{x}, \lambda) := \boldsymbol{L}(\boldsymbol{x}) + \boldsymbol{Q}(\boldsymbol{x}, \boldsymbol{x}) + \lambda\boldsymbol{F} = \boldsymbol{0} \tag{7}$$

where $\boldsymbol{L}$ and $\boldsymbol{Q}$ represent linear and bilinear operators respectively, and $\boldsymbol{F}$ is a constant vector. Notice that $\boldsymbol{F}$ is chosen as $\boldsymbol{f}(\boldsymbol{x}_0)$ at the initial step, and $\boldsymbol{x}_0$ may be a flat start in the computation as general power flow methods [1–6].

The basic idea of ANM consists in searching the solution path under an asymptotic expansion form with respect to a path parameter (e.g., the arclength s). The algorithms have been described in previous literatures [19–22] and we limit ourselves to a brief account.

Suppose that $(\boldsymbol{x}_j, \lambda_j)$ is the jth $(j = 0, 1, 2, \cdots)$ computed point on $\boldsymbol{H}^{-1}(\boldsymbol{0})$, any points between jth and $(j + 1)$th step can be expressed as the series like

$$\boldsymbol{x}(s) = \boldsymbol{x}_j + \sum_{p=1}^{K}\boldsymbol{x}_j^{(p)}(\Delta s)^p, \lambda(s) = \lambda_j + \sum_{p=1}^{K}\lambda_j^{(p)}(\Delta s)^p \tag{8}$$

where K is the truncation order of the series, $(\boldsymbol{x}_j^{(p)}, \lambda_j^{(p)})$ is the Taylor coefficients, and the arclength $\Delta s = s - s_j$ satisfies

$$\Delta s = \langle \boldsymbol{x}_j^{(1)}, \boldsymbol{x} - \boldsymbol{x}_j\rangle + \lambda_j^{(1)}(\lambda - \lambda_j) \tag{9}$$

where $\langle\cdot,\cdot\rangle$ indicates the inner product. The Taylor coefficients $(\boldsymbol{x}_j^{(p)}, \lambda_j^{(p)})$ at different orders are determined by a series of equations:

Order $p = 1$:

$$\begin{aligned} &\boldsymbol{f}_x^j\boldsymbol{x}_j^{(1)} + \lambda_j^{(1)}\boldsymbol{F} = \boldsymbol{0} \\ &\|\boldsymbol{x}_j^{(1)}\|_2^2 + |\lambda_j^{(1)}|^2 = 1 \end{aligned} \tag{10}$$

Order $p \geqslant 2$:

$$
\begin{aligned}
&\boldsymbol{f}_x^j \boldsymbol{x}_j^{(p)} + \lambda_j^{(p)} \boldsymbol{F} = -\sum_{r=1}^{p-1} \boldsymbol{Q}(\boldsymbol{x}_j^{(r)}, \boldsymbol{x}_j^{(p-r)}) \\
&< \boldsymbol{x}_j^{(p)}, \boldsymbol{x}_j^{(1)} > + \lambda_j^{(p)} \lambda_j^{(1)} = 0
\end{aligned}
\tag{11}
$$

where $\boldsymbol{f}_x^j$ denotes the power flow Jacobian matrix. The above problem is solved with Keller's bordering algorithm [24] recursively:

Order $p = 1$:

$$
\begin{aligned}
&\boldsymbol{f}_x^j \boldsymbol{v} = -\boldsymbol{F} \\
&(\boldsymbol{x}_j^{(1)}, \lambda_j^{(1)}) = \pm(\boldsymbol{v}, 1) / \sqrt{(1 + \|\boldsymbol{v}\|_2^2)}
\end{aligned}
\tag{12}
$$

Order $p \geqslant 2$:

$$
\begin{aligned}
&\boldsymbol{f}_x^j \boldsymbol{v}_p = -\sum_{r=1}^{p-1} \boldsymbol{Q}(\boldsymbol{x}_j^{(r)}, \boldsymbol{x}_j^{(p-r)}) \\
&\lambda_j^{(p)} = -\lambda_j^{(1)} \langle \boldsymbol{v}_p, \boldsymbol{x}_j^{(1)} \rangle \\
&\boldsymbol{x}_j^{(p)} = \boldsymbol{v}_p + \lambda_j^{(p)} \boldsymbol{v}
\end{aligned}
\tag{13}
$$

The sign '±' in (12) depends on the chosen orientation. It satisfies $\langle \boldsymbol{x}_{j-1}^{(1)}, \boldsymbol{x}_j^{(1)} \rangle + \lambda_{j-1}^{(1)} \lambda_j^{(1)} > 0$, where $(\boldsymbol{x}_{j-1}^{(1)}, \lambda_{j-1}^{(1)})$ is the preceding tangent vector. We choose '+' at the start. Note that for all these linear systems only the right-hand-side terms change, and only one sparse Jacobian matrix factorization is required at each step. The range of validity of the series (8) is computed *a posteriori* with a formula [19–22]

$$
\Delta s_{\max} = (\varepsilon \|\boldsymbol{x}_j^{(1)}\|_\infty / \|\boldsymbol{x}_j^{(K)}\|_\infty)^{1/(K-1)} \tag{14}
$$

where ε is the accuracy parameter chosen by the user. As the range of validity of the series (8) is limited, the solution path is computed in a step-by-step manner. With proper choices of ANM parameters, the solution path can be very precise, and no corrections are needed for ANM.

3.3. Compute the power flow solutions on $\boldsymbol{H}^{-1}(\boldsymbol{0})$ by solving a simple polynomial equation

Since $\boldsymbol{H}^{-1}(\boldsymbol{0})$ is represented by a closed-form expression section by section, the power flow solutions on $\boldsymbol{H}^{-1}(\boldsymbol{0})$ can be easily computed with no extra computational costs. The flowchart of the algorithm is depicted in Fig. 2. The dotted portion in Fig. 2 corresponds to finding a power flow solution on the path. The procedure is illustrated in Fig. 3.

A power flow solution is computed as follows: at two consecutive ANM steps, find a value of the step-length for which $\lambda(s) = 1$ (e.g., $\lambda_j < 1 < \lambda_{j+1}$ or $\lambda_j > 1 > \lambda_{j+1}$). The parameter λ changes monotonically on the interval $(\lambda_j, \lambda_{j+1})$ (Fig. 3(a)). The step-length for the power flow solution is determined by solving the following simple polynomial equation

$$
1 = \lambda_j + \sum_{p=1}^{K} \lambda_j^{(p)} (\Delta s^*)^p \tag{15}
$$

where Δs^* is the step-length for determining the power flow solution at step j. The bisection method can be used here. Then, the power flow solution $\boldsymbol{x}^*$ is computed by

$$
\boldsymbol{x}^* = \boldsymbol{x}_j + \sum_{p=1}^{K} \boldsymbol{x}_j^{(p)} (\Delta s^*)^p \tag{16}
$$

An important case should be considered when the power flow solutions are very close to the SNBP of $\boldsymbol{H}^{-1}(\boldsymbol{0})$. It usually occurs when the power system is under heavy load condition. Assume that there exists a SNBP on the interval $(\lambda_j, \lambda_{j+1})$ (Fig. 3(b)), and it satisfies $\lambda_j, \lambda_{j+1} < 1$ and $\lambda_{SNBP} > 1$. It is easier to locate the SNBP on $\boldsymbol{H}^{-1}(\boldsymbol{0})$ with ANM. As the first-order derivative of λ with respect to the arclength changes sign at the SNBP, the step-length for the SNBP is determined by the following condition:

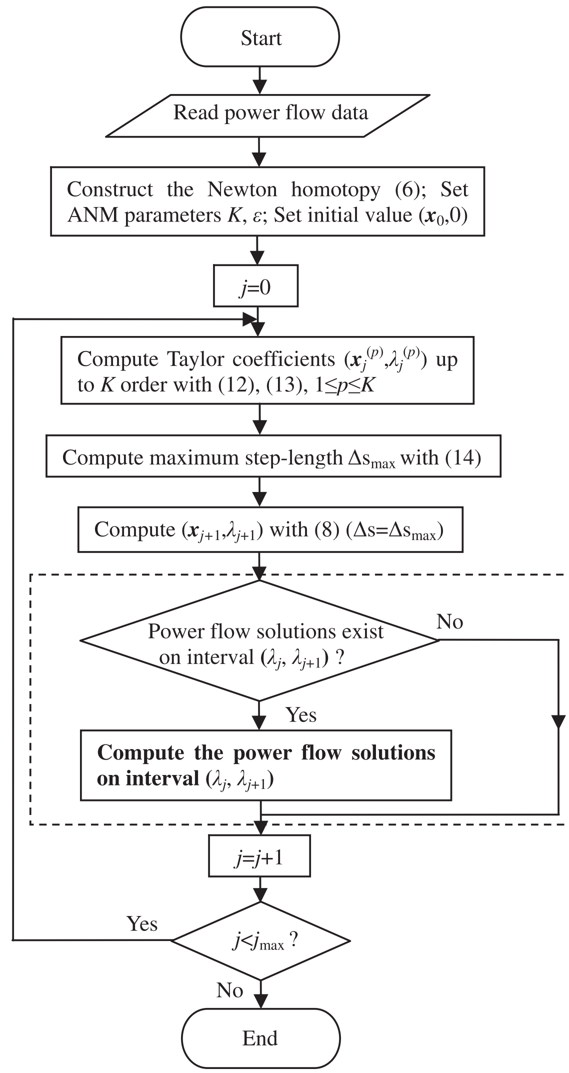

Fig. 2. Flowchart of the algorithm.

$$
\lambda_{SNBP}^{(1)} = \sum_{p=1}^{K} p \lambda_j^{(p)} (\Delta s_{SNBP})^{p-1} = 0 \tag{17}
$$

where Δs_{SNBP} is the step-length for locating the SNBP at step j. Also, the bisection method can be used here. After the SNBP has been determined, applying the same procedure on the interval $(\lambda_j, \lambda_{SNBP})$ as the general case, the power flow solution $\boldsymbol{x}^*$ can be obtained. Similar procedure may be used to determine another solution $\boldsymbol{x}^{**}$ on the interval $(\lambda_{SNBP}, \lambda_{j+1})$.

When $\lambda_{SNBP} < 1$, it means that the power flow equations have no real solutions. In this case, $|\lambda_{SNBP} - 1|$ may provide a measure for the unsolvability.

The proposed method is more reliable to compute the power flow solutions when the system is at stressed load condition because it uses a higher-order Taylor series based predictor near the singular point of the power flow equations. A similar idea by using the extrapolation technique to approximate the singularities of general nonlinear equations has been proposed by Griewank

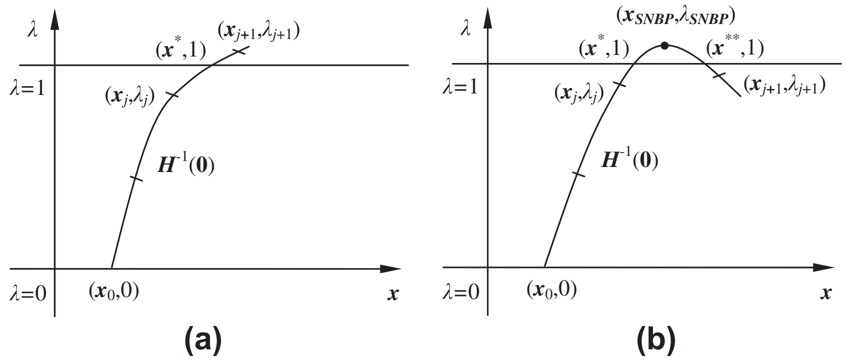

Fig. 3. Find a power flow solution on the interval $(\lambda_j, \lambda_{j+1})$. (a) Normal case. (b) Solution near the SNBP.

[25]. With the high-order predictor, many factorizations of the Jacobian matrix near the singular point will be avoided, and it allows to achieve a more rapid and reliable method to obtain the solution.

3.4. Consider generator reactive power limits violations

For practical power flow computations in power systems, it is important to take the generator reactive power limits into consideration. The procedure may be similar to that of the Q-limit point determination described in [22]. As $\boldsymbol{H}^{-1}(\mathbf{0})$ has been expressed into a closed analytical form, the break point where a generator reaches its limit may be precisely located. Suppose $(\boldsymbol{x}_q, \lambda_q)$ is the break point. At $(\boldsymbol{x}_q, \lambda_q)$, a PV bus is switched to a PQ bus. The function type or dimension of Eq. (7) will be changed. To keep the continuity of the solution path at $(\boldsymbol{x}_q, \lambda_q)$, the constant vector $\boldsymbol{F}$ in Eq. (7) should be modified to $\boldsymbol{f}(\boldsymbol{x}_q)/(1-\lambda_q)$.

We would like to emphasize that if there exist a number of generators which violate the reactive limits in the computation, ANM must restart at the break points, which may lower the efficiency of the proposed method (*Note:* this is quite common in actual power flow computations because we do not know the results before computing; while for CPF computation (e.g., Ref. [22]), as the reactive limits are satisfied at the initial power flow computation, chances of the reactive power violations in the consequent continuation procedure will be greatly reduced.). For this reason, we take the following strategy, which has been adopted by MATPOWER [26] and many other power flow programs. First, the power flow is solved with no limits consideration. Then the limits are checked after computation. Switch the PV buses to the PQ buses whose reactive generation violates the limits. The procedure is repeated until there are no more reactive limits violations. This requires several trial computations before all the reactive limits are satisfied.

4. Numerical examples

In this section, we present some examples to show the performance of the proposed method. The computational program was implemented in C++ codes, using double precision arithmetic. All tests were performed on a Pentium (R) IV microcomputer with 2.2 GHz microprocessor and 512M RAM.

The bus voltage unknowns are set as $e_{i,0} + jf_{i,0} = 1 + j0$ p.u. at the start. The ANM parameters are: $K = 10$ and $\varepsilon = 10^{-5}$ (Note that in our previous ANM program [22], the accuracy control parameter ε in (14) is actually outside of the parentheses. We have modified this in the newly developed program. So, ε taken as 0.1 in [22] corresponds to $10^{-(K-1)}$ in the present paper.).

We compare the results of the proposed method with those of the conventional power flow methods such as Newton's method and optimal multiplier method (we call these methods the Newton-like methods later). The convergence tolerances of the Newton-like methods are set to TOL = 10^{-5} p.u., and the stopping criteria is $\|\boldsymbol{f}(\boldsymbol{x})\|_\infty <$ TOL and $\|\Delta\boldsymbol{x}\|_\infty <$ TOL.

4.1. Example 1: a didactic two-bus example

The goal of this example is used to show the ability of proposed method to precisely locate the singular point (e.g., SNBP) of the power flow equations. Consider a simple two-bus power system [27] as shown in Fig. 4.

For the two-bus power system, the power flow equations are

$$\begin{aligned} &10f_2 + P = 0 \\ &10e_2^2 + 10f_2^2 - 10e_2 + Q = 0 \end{aligned} \tag{18}$$

The power flow Jacobian matrix will be singular when P + jQ at bus 2 becomes 5 + j0 p.u., and the voltage is $V_2 = 0.5 - j0.5$ p.u.

If Eq. (18) is solved by use of the Newton's method, it converges very slowly to reach the required accuracy within 17 iterations because of the singularity of the Jacobian matrix. When the proposed method is used, however, it takes only 6 ANM steps to obtain the result with the same order of accuracy. Fig. 5 shows the computational result. In Fig. 5(a), λ reaches its maximum value at $\lambda = 1$. The SNBP of $\boldsymbol{H}^{-1}(\mathbf{0})$ coincides with the singularity of the power flow equations. The power flow solution is $e_2 + jf_2 = 0.499999999 - j$ 0.500002022 p.u. The proposed method is more efficient to compute the singular point than the Newton-like methods because it uses a higher-order Taylor series based predictor near the singular point (between 5 ~ 6th ANM steps, using Eq. (17)). Similar to ANM applications in fluid and mechanics [19–21], the step-lengths of ANM are naturally adaptive in the computation procedure as shown in Fig. 5(b), and the residue of $\boldsymbol{H}^{-1}(\mathbf{0})$ is 10^{-5} order (Fig. 5(c)).

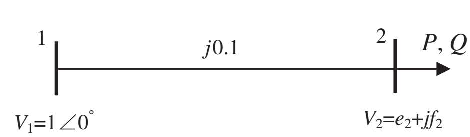

Fig. 4. Two-bus power system [27].

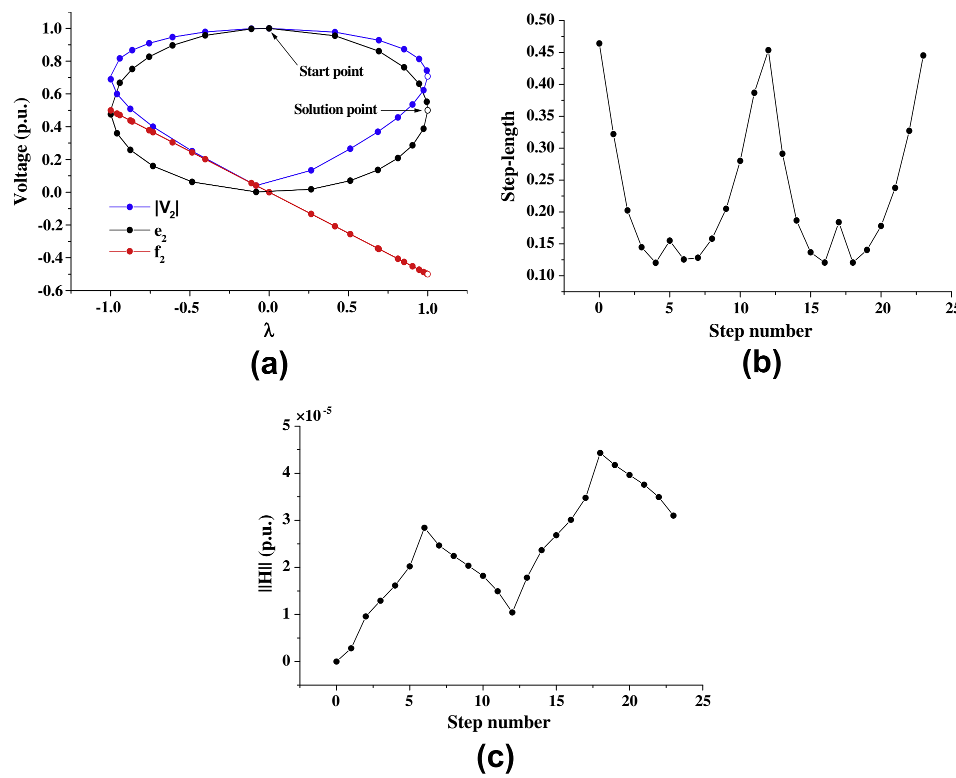

Fig. 5. Computing $\boldsymbol{H}^{-1}(\mathbf{0})$ with ANM for the two-bus power system. (a) e_2, f_2, $|V_2|$ vs. λ. (b) Step-length curve. (c) Residue curve.

Remark. We see in Fig. 5(a) that the homotopy path $|V_2|-\lambda$ is a closed curve because the Newton homotopy of (18) is

$$\begin{aligned} &10f_2 + \lambda P = 0 \\ &10e_2^2 + 10f_2^2 - 10e_2 + \lambda Q = 0 \end{aligned} \tag{19}$$

It is easier to know from (19) that $e_2-\lambda$ is an elliptic curve, while $f_2-\lambda$ is a degenerated line ($f_2-\lambda$ will also be an elliptic curve when the phase angle of bus 1 is not zero). Thus the homotopy path $|V_2|-\lambda$ will be closed. In fact, this observation has been verified by different sizes of power systems in our numerical tests besides the two-bus example.

4.2. Example 2: 3-bus power system

This example is used to show the ability of proposed method to compute the multiple power flow solutions of power flow equations. The one-line diagram of the 3-bus power system [11] is shown in Fig. 6. Assume that the load power at bus 1 may be altered. At first, we set $P_1 + jQ_1 = 0.15 + j0.1$ p.u.

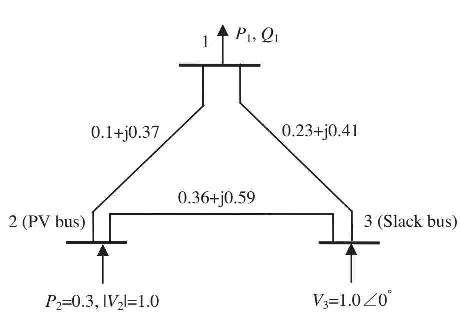

Fig. 6. 3-Bus power system [11].

Fig. 7 shows the computational result. The power flow solutions are the intersection of $\boldsymbol{H}^{-1}(\mathbf{0})$ with $\lambda = 1$. There exist four intersection points in this example. The four intersections occur between $0 \sim 1$th, $21 \sim 22$th, $32 \sim 33$th, and $41 \sim 42$th ANM steps, respectively. The results are listed in Table 1. Solution 1 is just the one computed by the general power flow methods.

Suppose that the load power at bus 1 increases with constant power factor. The power at bus 1 changes to $P_1 + jQ_1 = 0.3 + j0.2$, $0.75 + j0.5$, and $1.2 + j0.8$ p.u., respectively. Fig. 8(a)–(c) shows the variations of the process. As the load power at bus 1 increases, the homotopy path gradually moves to the left. The corresponding SNBP of $\boldsymbol{H}^{-1}(\mathbf{0})$ in Fig. 8(a)–(c) are $\lambda_{SNBP} = 3.63103$, 1.39032, and 0.84904, respectively. The difference between the SNBP of $\boldsymbol{H}^{-1}(\mathbf{0})$ and $\lambda = 1$ provides a measure for the unsolvability.

4.3. Example 3: 43-bus ill-conditioned power system

The 43-bus power system has been studied in [3,28].The system data wherein the literature are in the form of bus admittance matrix and power injection. The elements of the bus admittance matrix are directly adopted by our program with no loss of precision. It can be verified that Newton's method and optimal multiplier method will converge using the original system data. Fig. 9 shows the computational result of the proposed method. The two solutions occur between $5 \sim 6$th and $8 \sim 9$th ANM steps, respectively. Part of the power flow solutions ($V_2 \sim V_{10}$) are listed in Table 2.

The 43-bus power system is ill-conditioned because of the existing high r/x values. The power flow solutions are quite sensitive to small changes of the system parameters. This can be verified by the following fact. We change the admittance of line 30–38

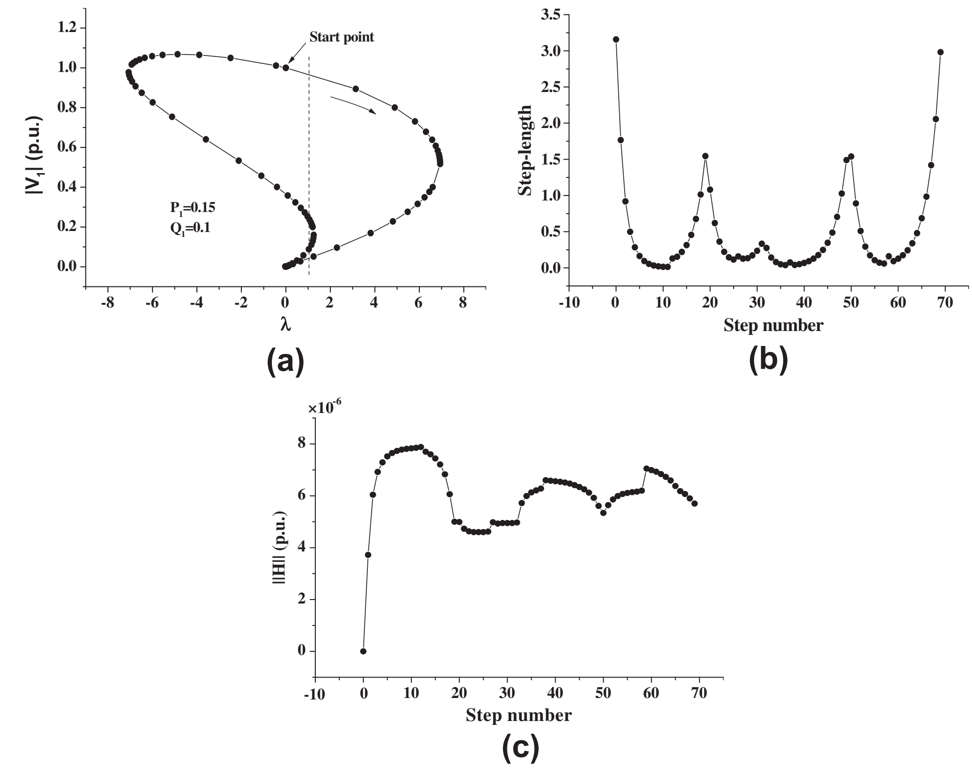

Fig. 7. Computing $\boldsymbol{H}^{-1}(\boldsymbol{0})$ with ANM for the 3-bus power system. (a) $|V_1|$ vs. λ. (b) Step-length curve. (c) Residue curve.

Table 1
Multiple power solutions of the 3-bus system.

Variables (p.u.)	Solution 1	Solution 2	Solution 3	Solution 4
V_1	0.97165 + *j*0.02909	0.02918 − *j*0.02939	−0.00003 − *j*0.08316	0.00894 − *j*0.24853
V_2	0.99550 + *j*0.09481	0.93167 − *j*0.36332	−0.22706 − *j*0.97388	−0.53455 − *j*0.84514

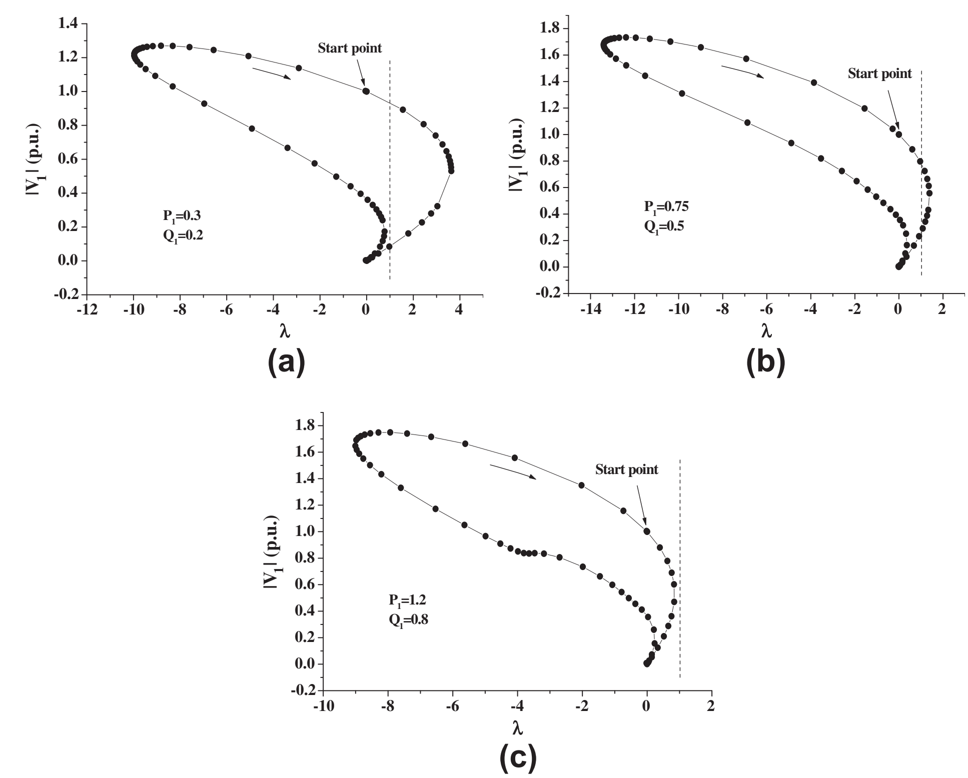

Fig. 8. Evolution of $\boldsymbol{H}^{-1}(\boldsymbol{0})$ at different load levels for the 3-bus power system. (a) 2 Solutions. (b) 2 Solutions. (c) No solution.

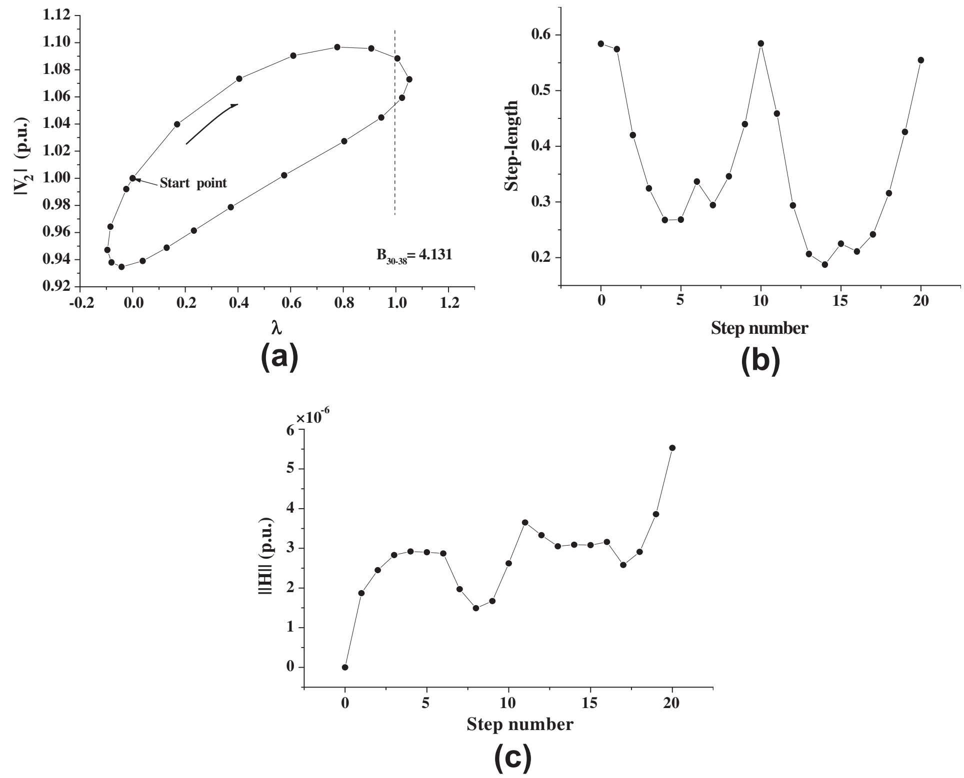

Fig. 9. Computing $\boldsymbol{H}^{-1}(\mathbf{0})$ with ANM for the 43-bus power system. (a) $|V_2|$ vs. λ. (b) Step-length curve. (c) Residue curve.

Table 2
Multiple power solutions of the 43-bus system (part).

Variables (p.u.)	Solution 1	Solution 2
V_2	1.07039 – j0.20129	1.03456 – j0.20153
V_3	1.09853 – j0.28089	1.01597 – j0.27097
V_4	1.06642 – j0.24847	0.98998 – j0.23786
V_5	1.06773 – j0.20317	1.03110 – j0.20307
V_6	1.06868 – j0.20774	1.03229 – j0.20792
V_7	1.06497 – j0.27307	1.02262 – j0.27433
V_8	1.08404 – j0.24152	1.01270 – j0.23621
V_9	1.08245 – j0.24164	1.01096 – j0.23625
V_{10}	1.08449 – j0.24477	1.01283 – j0.23920

from 4.131 p.u. to 4.531, 4.431, and 4.031 p.u., respectively. Fig. 10(a)–(c) shows the variations of the process.

For cases Fig. 10(a) and (b), the Newton-like methods fail to converge, but the proposed method successfully solves the power flow problem. For case Fig. 10(c), both the Newton-like methods and the proposed method fail to find a solution.

4.4. Example 4: 3375-bus power system

The 3375-bus system is the Polish power system. The system data is provided by MATPOWER [26]. The generator reactive power limits constraints have been considered in this example. Firstly, the ANM continuation power flow [22] is used to generate the savecases of the power system at different load levels. This is fulfilled by starting from the base case and increasing all the buses loads of area 1 until the SNBP of the power flow equations is reached (Fig. 11). Based on these savecases, we compare the performance of the proposed method with the Newton-like methods. For the proposed method, only the first power flow solution is computed. Table 3 shows the computational result.

In Table 3, case 1 is the base case, and case 6 is very close to the SNBP of the power flow equations (case 7). The computational effort for the ANM is basically due to the number of Jacobian matrix's factorization and forward/backward substitutions of the sequences of linear systems. With the chosen ANM parameters, the influence of forward/backward substitutions is limited to a certain degree. Compared with the Newton-like methods, the proposed method requires a relatively smaller number of computational steps at different load levels, and computational time is also less than that of the Newton-like methods. The Newton-like methods fail to converge at the SNBP of the power flow equations (case 7), but the proposed method may still find the power flow solution due to the high-order predictor of ANM.

We remark that for heavy load systems, it may take more ANM steps to find a power flow solution. A special strategy may be applied here. First, a large ANM parameter (e.g., $\varepsilon = 0.01$) is used at the beginning. As the solution point (i.e., $\lambda = 1$) is approached, a correction procedure is adopted and subsequently ε is set to the normal value (e.g., $\varepsilon = 10^{-5}$). In this way, the computation efficiency will be improved.

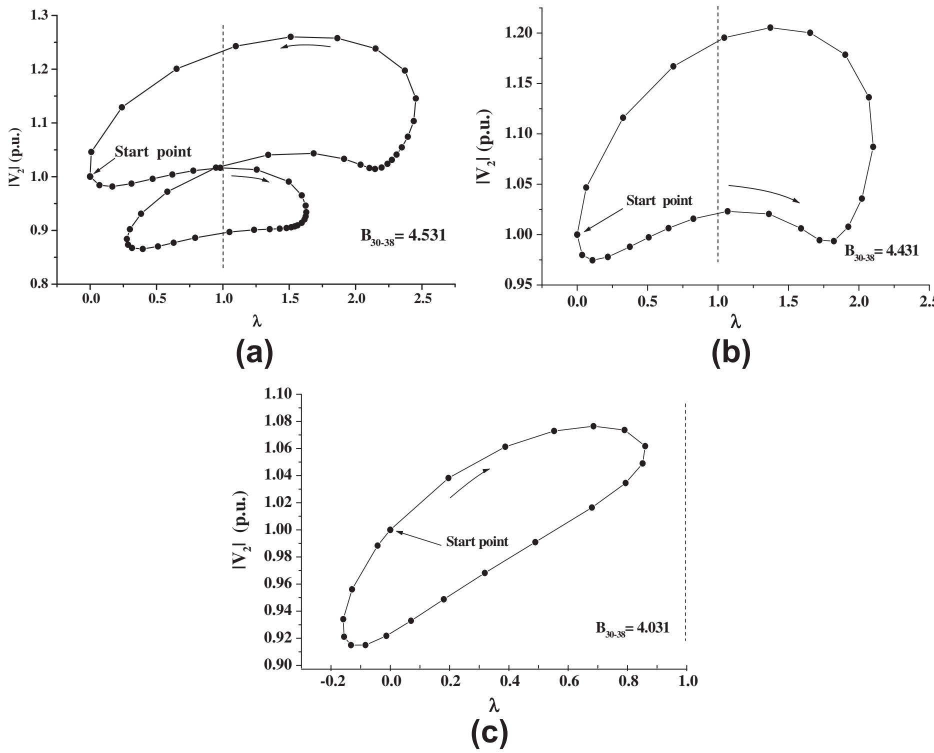

Fig. 10. Evolution of $\boldsymbol{H}^{-1}(\mathbf{0})$ of different system parameters for the 43-bus power system. (a) 4 Solutions. (b) 2 Solutions. (c) No solution.

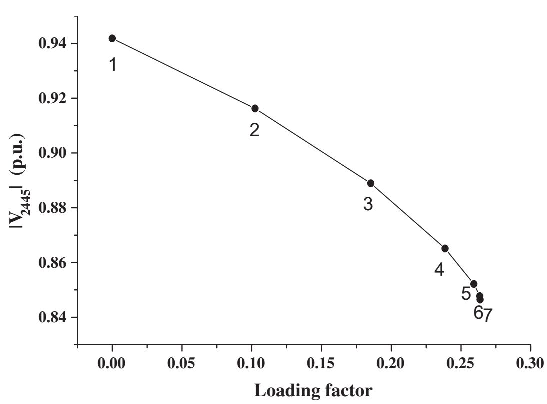

Fig. 11. Power flow cases at different load levels computed by ANM of the 3375-bus power system.

Table 3
Comparison between the proposed method and Newton-like methods at different load levels of the 3375-bus system.

Power flow cases	Newton's method		Iwamoto's method		ANM(ε=10^{-5}, K=10)	
	Iter. number	Time(s)	Iter. number	Time(s)	ANM steps	Time(s)
1	6	4.45	6	4.51	4	4.26
2	6	4.33	6	4.36	4	4.24
3	7	5.06	7	5.07	4	4.28
4	8	5.78	8	5.80	5	5.11
5	9	6.46	9	6.49	6	5.96
6	11	7.87	11	7.92	7	6.75
7	–	–	–	–	9	8.33

5. Conclusions

In this study, we have presented a novel methodology which couples the Newton homotopy transformation with ANM to solve the power flow equations. Numerical examples show that the method is reliable to solve the ill-conditioned power flow. It may even compute the singular point (e.g., the SNBP) of the power flow equations with no difficulty (examples 1 and 4). By tracing the homotopy path continuously, the proposed method may also solve the multiple power flow solutions efficiently.

The sensitivities of λ_{SNBP} on $\boldsymbol{H}^{-1}(\mathbf{0})$ with respect to the control parameters (e.g., bus active power, reactive power, etc.) can be used to restore the power flow solvability when the power flow equations have no real solutions. This work is currently under investigation.

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