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An Effective Implementation of the Nonlinear Homotopy Method for MOS Transistor Circuits

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Abstract. Recently, an efficient homotopy method termed the nonlinear homotopy method (NLH) has been proposed for finding DC operating points of MOS transistor circuits. This method is not only efficient but also globally convergent. However, the programming of sophisticated homotopy methods is often difficult for non-experts or beginners. In this paper, an effective method for implementing the MOS NLH method on SPICE is proposed. By this method, we can implement the MOS NLH method from a good initial solution with various efficient techniques and without programming.

Introduction

Circuit-analysis is increasingly dependent on electronic design automation (EDA) program with the development of semiconductor technology [1]. In analog circuit simulation, the first and fundamental task is to find the DC operating point. If it fails, neither the AC analysis nor the transistor analysis can be done. SPICE-like circuit simulators [1], widely utilized for designing LSI's, adopt the Newton-Raphson (NR) method for solving modified nodal (MN) equations [2]. However, due to the local convergence property, the NR method or its variants often fail to converge to a solution unless the initial estimation point is close enough to the solution [3]. On the contrary, the method guaranteeing the convergence with any initial point is globally convergent.

To overcome this non-convergence problem, many computer-aided design (CAD) researchers have long studied the globally convergent homotopy methods from various viewpoints [4-18]. These studies are divided into three categories. The first one is how to construct a homotopy function. For example, Newton homotopy, Fixed-point homotopy, Nonlinear homotopy, Variable gain homotopy and other homotopy methods have been proposed in [4-8]. The second one is how to numerically trace a solution curve. The algorithm using hypersphere (the BDF curve-tracing algorithm) [9] and the algorithm using hyperplane have been proposed in this category. The third one is how to set an initial solution. Recently, the important criteria have been shown extending the region wherein any initial solution satisfies the uniqueness condition [10], and the initial solution algorithm using the criteria has been proposed [11]. Note that, most previous studies for homotopy methods are mainly focused on the bipolar transistor circuits. At present, the MOS/Bi-MOS transistor circuits are becoming more and more popular in the analog circuit designs. Extending homotopy methods to MOS transistor circuits is important and urgent [12-17].

Recently, an efficient homotopy method termed the nonlinear homotopy method (NLH) has been proposed for finding DC operating points of MOS transistor circuits [15-16]. This method is not only more efficient than the conventional MOS ATAN-SH homotopy method [14] but also globally convergent. However, the programming of sophisticated homotopy methods is often difficult for non-experts or beginners. How to implement the MOS NLH method on SPICE effectively has been an open problem. In this paper, we propose an effective method for implementing the MOS NLH method on SPICE. By this method, we can implement the MOS NLH method with various efficient techniques and without programming, although we do not know the homotopy method well.

This paper is organized as follows. In Section II, as preliminaries, homotopy method and MOS NLH method are presented, followed by the proposed effective netlist implementation method for

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MOS NLH method in Section III. Numerical examples are shown in Sect. IV. Finally, the conclusions are summarized.

The NLH Method for MOS Transistor Circuits

MN Equations of Homotopy Method. We first review homotopy methods for solving systems of nonlinear equations of the form

$$f(x) = 0, \quad f(\cdot): R^n \to R^n. \tag{1}$$

In the MN equation, Eq. (1) is rewritten as follows [6]:

$$f_g(v, i) \triangleq D_g g(D_g^T v) + D_E i + J = 0, \tag{2}$$

$$f_E(v) \triangleq D_E^T v - E = 0, \tag{3}$$

where $f = (f_g, f_E)$, $f_g: R^n \to R^N$, $f_E: R^N \to R^M$, $x = (v, i)^T \in R^n$, and $n = N + M$. The variable vector $v \in R^N$ denotes the node voltages to the datum node and the variable vector $i \in R^M$ denotes the branch currents of the independent voltage sources. Also, the continuous function $g: R^K \to R^K$ is a VCCS (voltage-controlled current source) type. In addition, D_g is an $N \times K$ reduced incidence matrix for the g branches and D_E is an $N \times M$ reduced incidence matrix for the independent voltage source branches. Moreover, $J \in R^N$ is the current vector of the independent current sources and $E \in R^M$ is the voltage vector of the independent voltage sources.

In homotopy methods, we consider an auxiliary equation

$$f^0(x) = 0, \quad f^0: R^n \to R^n, \tag{4}$$

with a known solution x^0 and construct a homotopy

$$h(x, t) = tf(x) + (1 - t)f^0(x), \tag{5}$$

where $h: R^{n+1} \to R^n$ and $t \in [0,1]$ is the homotopy parameter. Then the solution curve of the homotopy equation

$$h(x, t) = 0, \tag{6}$$

is traced from the known initial solution $(x^0, 0)$ at $t = 0$. If the solution curve reaches the $t = 1$ hyperplane at $(x^*, 1)$, then a solution x^* to Eq. (1) is obtained [6].

The MOS NLH Method. In [15-16], the nonlinear homotopy method for MOS transistor circuits is proposed, which is constructed using the nonlinear auxiliary function equations. The auxiliary function equation is as follows:

$$f^0(x) = f(x) - f(x^0) + \tilde{f}(x) - \tilde{f}(x^0), \tag{7}$$

where

$$\tilde{f}(x) = \tilde{f}(v, i) = \begin{bmatrix} D_g \tilde{g}(D_g^T v) \\ -R_{dd} i \end{bmatrix}. \tag{8}$$

In Eq. (8), $\tilde{g}: R^K \to R^K$ is a nonlinear function corresponding to the branch $g = (g_1, g_2, \cdots, g_K)^T$ in the original circuit equation. R_{dd} is non-negative linear resistance. The homotopy function is expressed as follows:

$$h(x, t) = f(x) - (1 - t)f(x^0) + (1 - t)\left(\tilde{f}(x) - \tilde{f}(x^0)\right), \tag{9}$$

which is applied to MOS transistor circuits.

In MOS transistor circuits, the branch g is composed of MOS transistors, diodes, resistors and others. Similar with the nonlinear function for BJT circuits in [6], the nonlinear function $\tilde{g} = (\tilde{g}_1, \tilde{g}_2 \cdots \tilde{g}_k)^T$ in the MOS nonlinear homotopy method can be determined as follows.

- If g_i, g_{i+1} and g_{i+2} are the MOS transistor branches, that is $I_b = [I_g, I_s, I_d]^T = [g_i(v_b), g_{i+1}(v_b), g_{i+2}(v_b)]^T$,

$$g_i(v_b) = I_g = 0, \tag{10}$$

$$g_{i+1}(v_b) = I_s = I_{sd} + F_1(v_s), \tag{11}$$

$$g_{i+2}(v_b) = I_d = I_{ds} + F_1(v_d), \tag{12}$$

where $F_1(x)$ is the current function of the two junction diodes of the MOS transistor [16].

Then the corresponding functions $\tilde{g}_i$, $\tilde{g}_{i+1}$ and $\tilde{g}_{i+2}$ are

$$\tilde{g}_i(v_b) = n_{s1}F_4(V_g - V_s) + n_{s1}F_1(V_g) + n_{d1}F_4(V_g - V_d) + n_{d1}F_1(V_g), \tag{13}$$

$$\tilde{g}_{i+1}(v_b) = -n_{s1}F_4(V_g - V_s) + n_{s1}F_1(V_s), \tag{14}$$

$$\tilde{g}_{i+2}(v_b) = -n_{d1}F_4(V_g - V_d) + n_{d1}F_1(V_d), \tag{15}$$

where $F_4(x)$ is the drain current of additional MOS diode in initial homotopy circuit [16].

- If g_i is not a MOS transistor branch, then $\tilde{g}_i = 0$.

The solution curve of the homotopy equation can be traced by the BDF curve-tracing algorithm [7, 9]. In the BDF algorithm, the solution curve is parameterized by its arc-length s and the following system of differential-algebraic equations

$$h(x(s), t(s)) = 0, \tag{16}$$

$$\sum_{i\in I}(\dot{x}_i(s))^2 + \dot{t}(s)^2 = 1 \tag{17}$$

is solved, we can trace the solution curve of (16) and can realize the MOS NLH method [16]. Here, $I \subseteq \{1,2,\cdots,n\}$, $(|I| = m, m \leq n)$ denotes a subset of indices of the components of x, $\dot{x} = dx/ds$ and $\dot{t} = dt/ds$. Equation (16) is the homotopy equation and Eq. (17) describes the relationship between the arc-length and the components of the solution curve projected into an (m+1)-dimensional Euclidean space.

The Proposed Implementation Method of the MOS NLH Method

In this paper, the idea of SPICE-oriented numerical methods [7, 18] is extended to MOS transistor circuits and an effective netlist implementation method for the MOS NLH method is proposed. In this netlist implementation method, the equations (16) and (17) will be described by some circuits. Then the transient analysis of SPICE will be performed to the circuits starting from $(x^0, 0)$, by which numerical integration is applied to (17) and the solution curve of (16) is traced.

Firstly, as shown in [7, 18], (17) can be described by the path following circuits, which includes differential circuit, square and sum circuit and integration circuit. The circuits are shown in Fig. 1. In this figure, $\dot{v}_{gsm}$, t and $\dot{t}$ denote the node voltage.

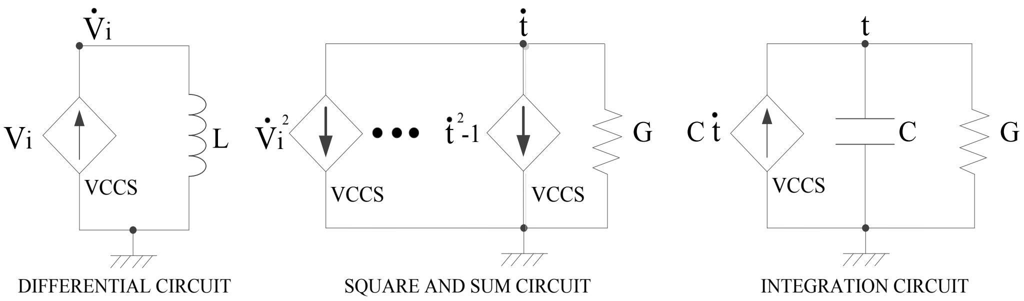


Figure 1 The circuit describing Eq. (17).

Then we consider a circuit to describe the equation (16). However, this is not an easy task due to the following reasons: Firstly, in order to improve the efficiency of finding the DC operating point using the homotopy method, it is natural to choose a good initial point x^0 so that $v_q = (v_{gs}, v_{gd})^T$ becomes a point in the saturation region for all MOS transistors. However, v_q is a branch voltage vector but x is a vector consisting of node voltages and branch currents of the independent voltage sources. Therefore, we have to determine the initial point x^0 such that v_q becomes a point in the saturation region. Secondly, how to determine the constant term $f(x^0) + \tilde{f}(x^0)$ is not an easy task. Since the formulas of $f(x)$ do not appear explicitly in SPICE, this constant term cannot be obtained by substituting x^0 to $f(x)$ and $\tilde{f}(x)$. In summary, the proposed implementation method will consist of two phases.

Phase I: Determination of the initial point $\mathbf{x^0}$ and the constant term $\mathbf{f(x^0) + \tilde{f}(x^0)}$. Firstly, v_q of all MOS transistors are set in saturation region (e.g., $v_{qn} = (v_{thn} + 0.3, 0)^T$ for NMOS transistors and $v_{qp} = (v_{thp} - 0.3, 0)^T$ for PMOS transistors). Let such a point be $v_q^0 = (v_{gs}^0, v_{gd}^0)^T$. Then the independent voltage sources v_{gs}^0 and v_{gd}^0 are connected to each MOS transistors as shown in Fig. 2. Moreover, the controlled current sources J_{gs} and J_{gd} (the additional diode branch in the initial homotopy circuit [16]) defined by $\tilde{g}_i$ are also connected to each MOS transistor, which is equivalent to adding $\tilde{f}(x)$ to the left-hand side of the original MN equations $f(x)= 0$. Such a circuit is called the initial circuit where two independent voltage sources and two controlled current sources are connected to each transistor of the original circuit.

Then, the DC analysis of SPICE will be done to this initial circuit. The solution of this initial circuit can be easily obtained, since it is essentially a linear circuit when v_{gs} and v_{gd} of all the MOS transistors are fixed. Let the solution of the initial circuit be x^0. Since $v_q^0 = (v_{gs}^0, v_{gd}^0)^T$ (in the saturation region) holds in x^0, it can be used as a good initial point of the MOS NLH method to enhance efficiency of DC analysis. Note that, considering the Kirchhoff's current law and homotopy function $h(x, 0) = 0$, it is easily seen that the currents of the independent voltage sources v_{gs}^0 and v_{gd}^0 in the initial circuit (denoted as I_{gs} and I_{gd} in Fig. 2) give the constant term $-(f(x^0) + \tilde{f}(x^0))$, since the original circuit is described by $f(x)$ and the controlled sources J_{gs} and J_{gd} are described by $\tilde{f}(x)$. Moreover, since at the node n_k where transistors are not connected, $f_k(x^0) = 0$ and $\tilde{f}_k(x^0) = 0$ hold, consider the constant term only at the nodes where MOS transistors are connected is sufficient.

Therefore, by solving the initial circuit, the initial point x^0 and the constant term $-(f(x^0) + \tilde{f}(x^0))$ are obtained.

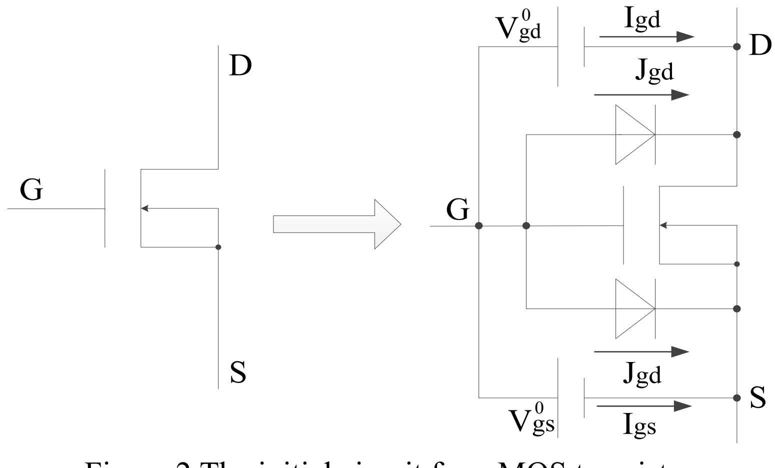


Figure 2 The initial circuit for a MOS transistor.

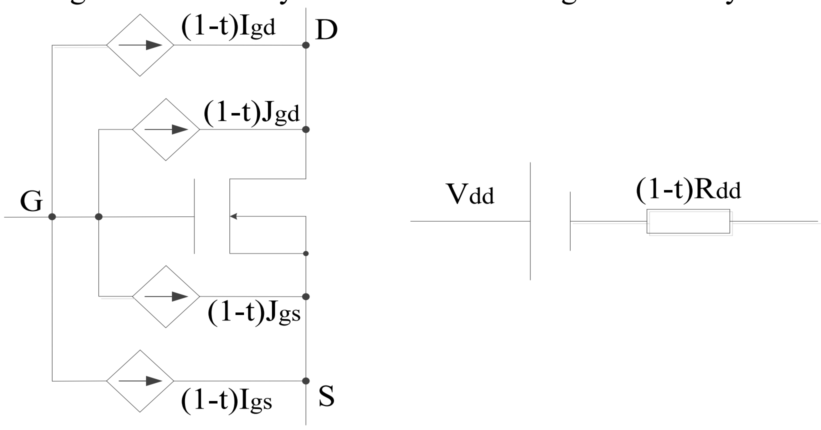
Phase II: Solving circuits that describe equation (16). Firstly, equation (16) can be described by a circuit as shown in Fig. 3. In this Fig. 3a, for each MOS transistor four controlled current sources are connected. In the homotopy function (Eq. (9)), the term $(1 - t)\tilde{f}(x)$ is described by adding $(1 - t)J_{gs}$ and $(1 - t)J_{gd}$. In addition, the term $-(1 - t)(f(x^0) + \tilde{f}(x^0))$ can be described by adding $(1 - t)I_{gs}$ and $(1 - t)I_{gd}$. In this Fig. 3b, for the independent voltage source V_{dd}, additional resistance $(1 - t)R_{dd}$ is employed according to Eq. (8). In summary, we call this circuit as the homotopy circuit.

Secondly, the transient analysis of SPICE is performed to this homotopy circuit and the path following circuits (shown in Fig. 1) and the solution curve of the MOS NLH method can be traced.

Therefore, the proposed netlist implementation method of the MOS NLH method can be summarized as follows.

- Adopt DC analysis of SPICE for the initial circuit (shown in Fig. 2) with a $v_q^0 = (v_{gs}^0, v_{gd}^0)^T$ held in the saturation region to solve the initial solution x^0 and the constant term $f(x^0) + \tilde{f}(x^0)$ to $h(x, 0) = 0$.
- Perform the transient analysis of SPICE to the homotopy circuits and the path following circuit (shown in Figs. 1 and 3) starting from $(x^0, 0)$ and trace the solution curve of $h(x, t) = 0$ until it reaches the $t = 1$ hyperplane at $(x^*, 1)$. Then a solution x^* of $f(x) = 0$ is obtained.

Note that SPICE contains various efficient techniques such as sparse matrix techniques, variable-step variable-order implicit integration methods, and time-step control algorithms [7]. A highly efficient MOS NLH method can be realized by the proposed netlist implementation method. Moreover, programming is not necessary and the netlists of Figs. 1-3 is easy in the proposed method.



(a) (b)

Figure 3 The homotopy circuit that describe (16).

Numerical Examples

In this work, two example circuits including the differential pair circuit and bandgap circuit [13] are considered and they are practically used in analog circuit design. The proposed netlist implementation method of the MOS NLH method is tested to find the DC operating points of the two example analog circuits. The SPICE3F5 running on Windows 7 operating system (CPU: 2.5GHz, Memory: 4GB, Compiler: Visual C .NET) is applied in our implementations. We select two node voltages in every example circuit and show the solution curves traced by the proposed netlist implementation method of the MOS NLH method in Fig. 4, respectively. From this figure, the solution curves reach the $t = 1$ hyperplane, where the DC operating points are found.

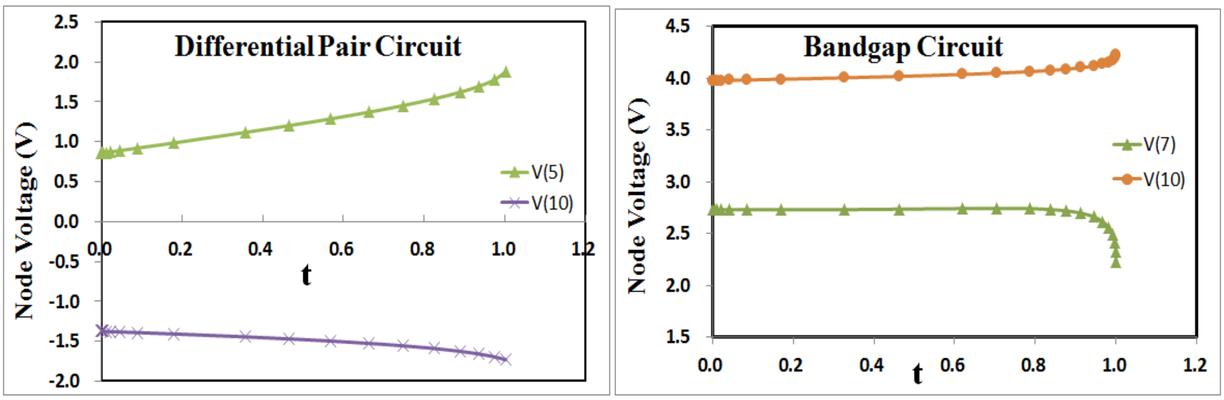


Figure 4 Solution curves of example circuits.

Summary

In this paper, an effective netlist method has been proposed for implementing the MOS NLH method on SPICE. The proposed method has the following advantages: 1) we can implement a sophisticated MOS NLH method easily without programming. Moreover, all can be done using SPICE only; 2) since the proposed method can be started from a good initial point, the path following tends to become smooth and efficient. Moreover, SPICE includes various techniques and know-hows accumulated for 30 years. Therefore, the proposed method will be very effective and useful in practical circuit simulation.

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