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Path Following Circuits——SPICE-Oriented Numerical Methods Where Formulas are Described by Circuits——

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SUMMARY Path following circuits (PFC's) are circuits for solving nonlinear problems on the circuit simulator SPICE. In the method of PFC's, formulas of numerical methods are described by circuits, which are solved by SPICE. Using PFC's, numerical analysis without programming is possible, and various techniques implemented in SPICE will make the numerical analysis very efficient. In this paper, we apply the PFC's of the homotopy method to various nonlinear problems (excluding circuit analysis) where the homotopy method is proven to be globally convergent; namely, we apply the method to fixed-point problems, linear programming problems, and nonlinear programming problems. This approach may give a new possibility to the fields of applied mathematics and operations research. Moreover, this approach makes SPICE applicable to a broader class of scientific problems.

key words: *circuit simulator, SPICE, nonlinear equation, homotopy method, optimization problem*

1. Introduction

In the computer-aided design of electronic circuits, circuit simulation is one of the most central tasks. In circuit simulation, circuits are described by nonlinear equations and then they are solved by numerical methods implemented in circuit simulators. As is well-known, SPICE is the most widely used circuit simulator. In SPICE, various refined techniques such as sparse matrix techniques, LU decomposition (for solving linear equations), modified Newton's method (for solving nonlinear equations), and stiffly stable numerical integration methods with efficient time-step control algorithms (for solving ordinary differential equations) are implemented. Moreover, there are many variations of SPICE, and most of them include various know-hows accumulated over many years. Hence, it is a great loss to apply such an excellent software only to electronic circuits.

In the numerical analysis of nonlinear systems, systems (such as circuits) are described by equations, to which numerical methods are applied. In that case, programming of the numerical methods or using ready-made softwares such as Mathematica and MATLAB is necessary. However, the programming of sophisticated numerical methods (such as the homotopy method) is often difficult for non-experts or beginners.

In this paper, we discuss a new approach of numerical analysis using *path following circuits*, which is based on the counter idea: *formulas of numerical methods are described by circuits, and then they are solved by SPICE.*

Path following circuits are also called solution curve-tracing circuits (STC's). STC's were first developed for finding multi-valued characteristic curves easily using SPICE [1], [2]. Then, this idea was extended to the homotopy method for finding DC operating points of nonlinear circuits with the theoretical guarantee of global convergence [3], [4]. Recently, this idea was further extended so that general scientific problems (not necessarily circuit analysis) can be solved using SPICE [5]–[7]. Such methods are also called *SPICE-oriented numerical methods*. As a related study, see [8], where optimization problems are solved by analog Hopfield neural networks realized on SPICE.

In this paper, we restrict our approach to the homotopy method, which is known to be a very effective method for solving nonlinear equations that cannot be solved by Newton's method. We also restrict the application fields only to those where the homotopy method is proven to be globally convergent. Namely, we apply the STC's describing the formulas of the homotopy method to various problems such as fixed-point problems, linear programming problems, and nonlinear programming problems. These problems are typical application problems of the homotopy method and have been studied for many years in the fields of applied mathematics and operations research. Hence, it is expected that this approach gives a new possibility to the fields of applied mathematics and operations research. Moreover, for SPICE users, the proposed method will be useful because they can easily solve these problems using SPICE that is familiar to them.

In the fields of applied mathematics and operations research, *tracing solution curves* are generally called *path following*. Hence, in this paper we call solution curve-tracing circuit (STC) as path following circuit (PFC).

2. Examples of SPICE-Oriented Numerical Methods

In this section, we first show simple examples of the SPICE-oriented numerical methods [5]–[7].

Consider a system of n nonlinear equations:

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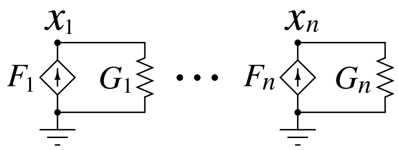


Fig. 1 Circuits describing (1).

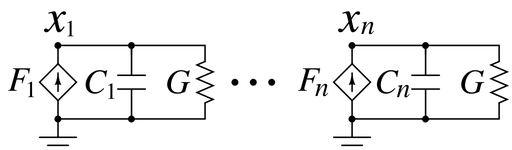


Fig. 2 Circuits describing (2).

$$f(x) = 0 \tag{1}$$

that does not necessarily describe a nonlinear circuit, where $f = (f_1, f_2, \cdots, f_n)^T : R^n \to R^n$ and $x = (x_1, x_2, \cdots, x_n)^T \in R^n$. If we want to solve (1) using SPICE, then we perform DC analysis to the circuits shown in Fig. 1, where x_i denotes the node voltages and F_i denotes the currents of the controlled sources defined as $F_i(x) = f_i(x) + G_i x_i$. Since the circuit equation describing these circuits is $f(x) = 0$, the modified Newton's method implemented in SPICE is applied to (1) by performing the DC analysis†. Note that SPICE has many built-in functions (for example, exp, log, sin, sqrt, abs, etc.) for describing the currents of controlled sources [9].

Similarly, if we want to solve an initial value problem of a system of n ordinary differential equations:

$$\dot{x} = f(x, t), \qquad x(0) = x_0 \tag{2}$$

using SPICE, then we perform transient analysis to the circuits shown in Fig. 2 where F_i is defined as $F_i(x) = C_i f_i(x, t)$. In Fig. 2, G is a dummy conductance that is necessary only to avoid the topological restriction of SPICE, and a negligible small value $G \approx 0$ is used so that it is regarded as virtually open. Since the circuit equation describing these circuits is $\dot{x} = f(x, t)$, the stiffly stable numerical integration methods using efficient time-step control algorithms implemented in SPICE are applied to (2) by performing the transient analysis. Ordinary differential equations of more general forms:

$$f(x, \dot{x}, \ddot{x}, \cdots, x^{(n)}) = 0 \tag{3}$$

can also be described by circuits [6] and can be solved using SPICE.

3. Path Following Circuits That Describe Homotopy Methods

Consider a system of n nonlinear equations:

$$f(x) = 0 \tag{4}$$

again, where $f : R^n \to R^n$ and $x \in R^n$. Newton's method is commonly used for solving nonlinear equations. However, Newton's method often fails to converge unless the initial point $x^0 \in R^n$ is sufficiently close to the solution $x^* \in R^n$. To overcome this convergence problem, the homotopy method has been studied, and its global convergence property has been proved for various problems such as fixed-point problems, optimization problems, and DC analysis of nonlinear circuits.

In the homotopy method, we consider another equation:

$$f^0(x) = 0 \tag{5}$$

that has a known solution x^0, where $f^0 : R^n \to R^n$, and define a homotopy function $h : R^{n+1} \to R^n$ by

$$h(x, t) = tf(x) + (1 - t)f^0(x) \tag{6}$$

where $t \in [0, 1]$. Then, the solution curve (often called the path) of the homotopy equation:

$$h(x, t) = 0 \tag{7}$$

is followed starting from the trivial solution $(x^0, 0)$ at $t = 0$. If the path reaches the $t = 1$ hyperplane $R^n \times \{1\}$ at $(x^*, 1)$, then we obtain the solution x^* of (4).

In the method of PFC's, the path of (7) is followed as follows [1]–[7]. Let any point (x, t) on the path be represented as $(x(s), t(s))$, where s denotes the arc-length of the path starting from the initial point $(x^0, 0)$. Then, the arc-length s is defined mathematically as:

$$(dx_1)^2 + (dx_2)^2 + \cdots + (dx_n)^2 + (dt)^2 = (ds)^2. \tag{8}$$

Combining (7) with (8), we have a system of the algebraic-differential equations as follows:

$$h(x, t) = 0 \tag{9}$$

$$\left(\frac{dx_1}{ds}\right)^2 + \left(\frac{dx_2}{ds}\right)^2 + \cdots + \left(\frac{dx_n}{ds}\right)^2 + \left(\frac{dt}{ds}\right)^2 = 1. \tag{10}$$

Hence, by integrating (9) and (10) starting from $(x^0, 0)$, we can follow the path of (7). This method is called the arc-length method [6].

In order to realize the above method on SPICE, we first describe (9) by the circuits shown in Fig. 3, where H_i is

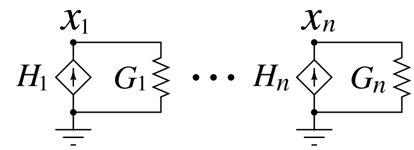


Fig. 3 Circuits describing (9).

†In [6] and [7], $G_i = 1$ is used, which is simple and easy to understand. However, in practical application, using the values of G_i with appropriate order according to the problem often makes the numerical analysis more stable [5]. For example, if $f(x) = 0$ is a circuit equation, then using $G_i = 10^{-3}$ will make the algorithm more stable because actual conductances take the values of order around 10^{-3}.

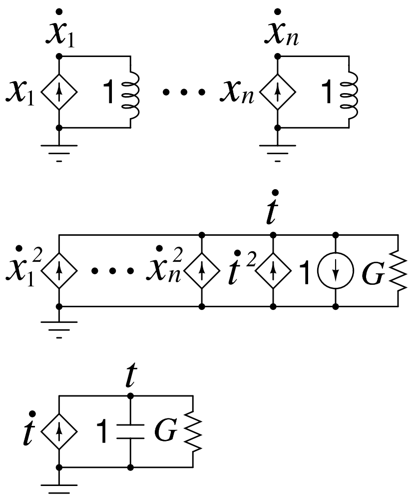


Fig. 4 Circuits describing (10).

defined as:

$$H_i(x,t) = h_i(x,t) + G_i x_i. \tag{11}$$

Then, we describe (10) by the circuits shown in Fig. 4 where G is a dummy conductance with a negligible small value $G \approx 0$. (In our numerical experiments, we used $G = 10^{-12}$.) In Fig. 4, $\dot{x}_i$ or $\dot{t}$ denotes a node voltage that is independent of x_i or t but is equal to dx_i/ds or dt/ds as a result.

From the first circuits in Fig. 4, $\dot{x}_i = dx_i/ds$ holds. From the second circuit in Fig. 4, $(\dot{x}_1)^2 + (\dot{x}_2)^2 + \cdots + (\dot{x}_n)^2 + \dot{t}^2 = 1$ holds. From the third circuit in Fig. 4, $\dot{t} = dt/ds$ holds. Hence, the circuits shown in Fig. 4 are described by (10). Therefore, we can follow the path of (7) by performing transient analysis to the circuits in Figs. 3 and 4 starting from $(x^0, 0)^{\dagger}$. Note that the arc-length s corresponds to the time in the transient analysis. The circuits shown in Fig. 4 are called the path following circuits for the homotopy method.

In this method, if we once write the netlist of the circuits shown in Figs. 3 and 4 (such a netlist can be easily written), then we only need to write $H_i(x,t)$ of the controlled sources for each problem. Thus, this method can be easily realized using SPICE.

4. Application to Fixed-Point Problems

From this section, we apply the method described in the previous section (that will be called the PFC method) to various problems where the homotopy method is proven to be globally convergent, and discuss its effectiveness. In our numerical experiments, we used a free software SPICE3f5 and a Sun Ultra 60 workstation (CPU: UltraSPARC-II 450 MHz).

In this section, we first consider fixed-point problems. The homotopy method has been applied to various fixed-point problems that occur in the filelds of economic equilibrium, game theory, and optimization theory for many years [10]–[12]. Let us consider the problem of finding fixed-points of the following functions [10], [11]:

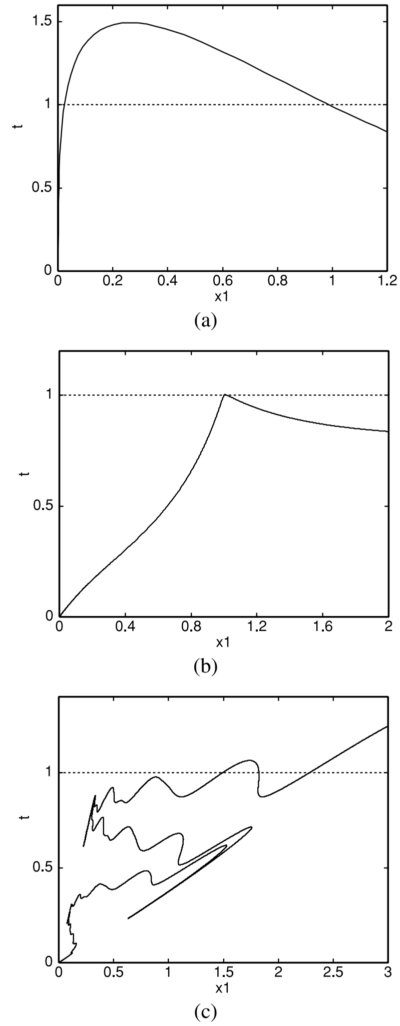


Fig. 5 Result of computation (obtained paths) for the fixed-point problems.

Problem 1: $$g_k(x) = \frac{1}{200}\left(\sum_{i=1}^{100} x_i^3 + k\right)$$
$$k = 1, 2, \cdots, 100$$

Problem 2: $$g_1(x) = x_1 - \left(\prod_{i=1}^{100} x_i - 1\right)$$
$$g_k(x) = x_k - \left(\sum_{i=1}^{100} x_i + x_k - 101\right)$$
$$k = 2, 3, \cdots, 100$$

Problem 3: $$g_k(x) = \exp\left(\cos\left(k \sum_{i=1}^{10} x_i\right)\right)$$
$$k = 1, 2, \cdots, 10$$

[†]When we determine the initial values of the transient analysis by .IC command, we set the initial value of $\dot{t}$ at $s = 0$ as $\dot{t} = 1$ so that the path proceeds to the direction of $t > 0$.

by the PFC method. Note that these problems are often used as test problems. Figures 5(a)–(c) show the result of computation (obtained paths) where the horizontal line is denoted as the x_1-axis and the vertical line as the t-axis. In Fig. 5(b), the path turns sharply at $t = 1$. This is because the solution of this problem is nonsingular. In other words, the PFC method worked well for such a nonsingular problem. In Fig. 5(c), a very complicated path is obtained. Namely, since SPICE includes various efficient techniques for solving stiff circuit equations [9] (such as the variable-step variable-order implicit integration methods and the time-step control algorithms), the PFC method is robust for strongly nonlinear problems where paths become very complicated.

5. Application to Linear Programming Problems

As numerical methods for solving linear programming problems, the simplex method and the interior-point method are well-known. Especially, the interior-point method has been studied very actively since the work of Karmarkar in 1984. The most efficient and widely used interior-point method is the infeasible primal-dual interior-point method. Consider a linear programming problem [13]:

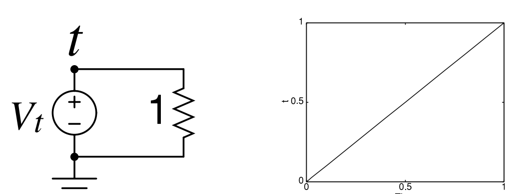
$$\begin{aligned} &\min \quad c^T x \\ &\text{subject to } Ax = b, \quad x \geqq 0 \end{aligned} \tag{12}$$

where $x \in R^n$ is a variable vector, A is an $m \times n$ matrix, and $b \in R^n$ and $c \in R^n$ are constant vectors. Let the dual problem of (12) be

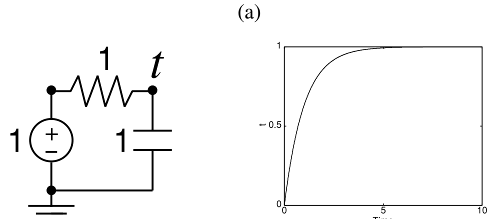
$$\begin{aligned} &\max \quad b^T y \\ &\text{subject to } A^T y + s = c, \quad s \geqq 0 \end{aligned} \tag{13}$$

where $y \in R^m$ and $s \in R^n$ are variable vectors. In [13], it is shown that the infeasible primal-dual interior-point method to (12) is equivalent to the homotopy method applied to the homotopy equation:

$$\begin{aligned} &Ax - b - (1-t)(Ax^0 - b) = 0 \\ &A^T y + s - c - (1-t)(A^T y^0 + s^0 - c) = 0 \\ &Xs - (1-t)X^0 s^0 = 0 \end{aligned} \tag{14}$$



(a)

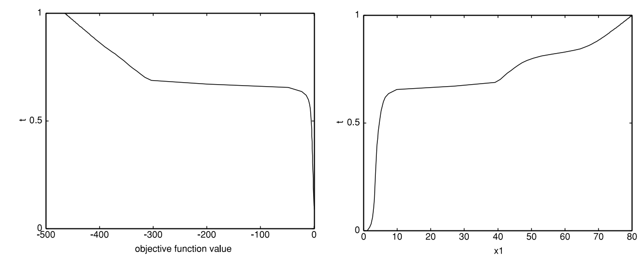


(b)

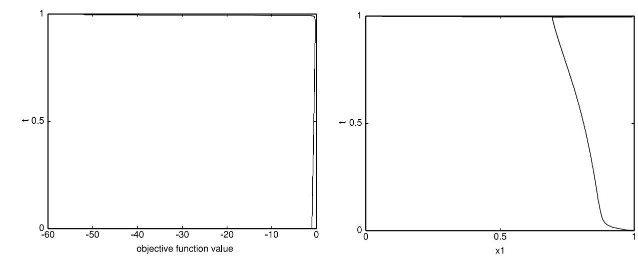
Fig. 6 Circuits for generating t that changes from 0 to 1.

where X is an $n \times n$ diagonal matrix with the (i, i)th element x_i. Detailed explanation of (14) is omitted here. Notice that (14) is a system of $m + 2n$ equations in $m + 2n + 1$ variables.

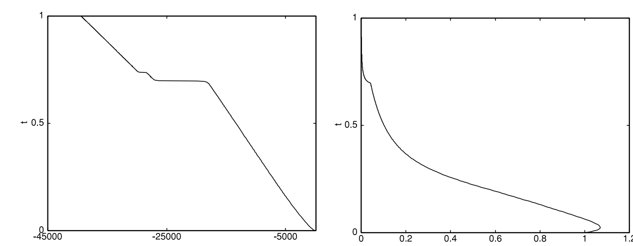
Linear programming problems can be solved by following the paths of (14) using the circuits shown in Figs. 3 and 4. However, since it is also proved [13] that the path of



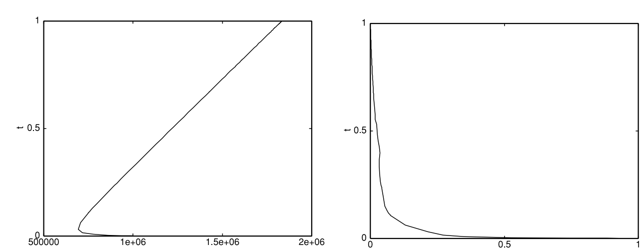
(a)



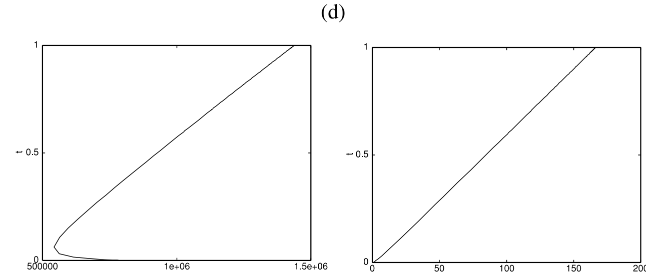
(b)



(c)



(d)



(e)

Fig. 7 Result of computation (obtained paths) for the linear programming problems.

(14) is monotone in t [namely, the solution of (14) is unique for t fixed], we can consider another possibility where the concept of arc-length s is not used. Namely, we can follow the path of (14) by considering t as the node voltage of the circuit shown in Fig. 6(a), where the (piecewise-linear) independent voltage source V_t is defined by a code:

Vt t 0 PWL(0 0 1 1)

[see the right-hand figure of Fig. 6(a)], and performing transient analysis to the circuits shown in Figs. 3 and 6(a). It is also possible to follow the path by considering t as the node voltage of the circuit shown in Fig. 6(b) and performing transient analysis to the circuits shown in Figs. 3 and 6(b). In both methods, we let t play the role of s because the solution of (14) can be written as $(x(t), t)$.

In the numerical experiments, we chose more than 30 problems from the standard bench mark problems repository Netlib [14] (http://www.netlib.org/lp) and solved them by the PFC method using the initial points $(x^0, y^0, s^0) = (1, \cdots, 1, 0, \cdots, 0, 1, \cdots, 1)$.

Figures 7(a)–(e) show the result of computation for AFIRO (28 constraints, 32 variables), SC105 (106 constraints, 103 variables), STOCFOR1 (118 constraints, 111 variables), SHIP08S (779 constraints, 2,387 variables), and SHIP12L (1,152 constraints, 5,427 variables), respectively, where the left-hand figures show the movement of the objective functions and the right-hand figures show the movement of x_1. Although there are several sharp turning points on the paths, the PFC method worked very well. The computation time was 3 s, 18 s, 54 s, 617 s, and 2,908 s, respectively. Note that the system of nonlinear equations (14) that describes SHIP12L consists of 12,108 variables. Thus, linear programming problems of more than five thousand variables (where the system of nonlinear equations consists of more than ten thousand variables) could be solved in reasonable computation time using SPICE. Since SPICE includes efficient time-step control algorithms, it can follow paths with both *almost linear parts* and *sharply bending parts* very efficiently.

6. Application to Nonlinear Programming Problems

Consider a nonlinear programming problem:

$$\begin{aligned} &\max \quad f(x) \\ &\text{subject to} \\ &\quad g_j(x) \geqq 0, \quad j = 1, 2, \cdots, r \\ &\quad h_j(x) = 0, \quad j = 1, 2, \cdots, s \end{aligned} \tag{15}$$

where f, g_j, and h_j are concave functions. It is shown [12] that the optimal solution of (15) can be obtained by following the path of the homotopy equation:

$$\begin{aligned} &-(1-t)(x - x^0) + t\nabla f(x)^T \\ &\quad + \sum_{j=1}^{r} \alpha_j^+ \nabla g_j(x)^T + \sum_{j=1}^{s} \mu_j \nabla h_j(x)^T = 0 \\ &\alpha_j^- - g_j(x) = 0, \quad j = 1, 2, \cdots, r \\ &h_j(x) = 0, \qquad\quad j = 1, 2, \cdots, s \end{aligned} \tag{16}$$

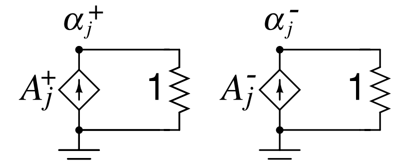


Fig. 8 Circuits for generating α_j^+ and α_j^-.

where α_j^+ and α_j^- are defined by

$$\begin{aligned} \alpha_j^+ &= \max\{0, \alpha_j\} \\ \alpha_j^- &= \max\{0, -\alpha_j\}, \quad j = 1, 2, \cdots, r \end{aligned} \tag{17}$$

for given $\alpha \in R^1$. It is also proved that this homotopy method is globally convergent [12]. Hence, the nonlinear programming problem (15) can be solved by the PFC method. Note that in this case α_j and μ_j are defined as node voltages of the circuits describing $\alpha_j^- - g_j(x) = 0$ and $h_j(x) = 0$, respectively.

However, in order to apply the PFC method, it is necessary to represent α_j^+ and α_j^- by circuits because SPICE does not include the function $\max\{\cdot, \cdot\}$. Since (17) is equivalent to

$$\begin{aligned} \alpha_j^+ &= \frac{1}{2}(|\alpha_j| + \alpha_j) \\ \alpha_j^- &= \frac{1}{2}(|\alpha_j| - \alpha_j), \quad j = 1, 2, \cdots, r, \end{aligned} \tag{18}$$

it is sufficient to consider circuits shown in Fig. 8, where the currents of the controlled sources are defined by

$$\begin{aligned} A_j^+ &= \frac{1}{2}(|\alpha_j| + \alpha_j) \\ A_j^- &= \frac{1}{2}(|\alpha_j| - \alpha_j), \quad j = 1, 2, \cdots, r. \end{aligned} \tag{19}$$

We now consider the following example problems.

Garcia-Zangwill problem [12]:

$$\begin{aligned} f(x) &= x_1 - (x_2)^2 - x_3 \\ g_1(x) &= -x_1 + (x_2)^2 - 2x_2x_3 + (x_3)^2 - 3 \leqq 0 \\ g_2(x) &= -x_1 \leqq 0 \\ h_1(x) &= 2x_1 - x_2 + x_3 = 0 \end{aligned}$$

Rosen-Suzuki problem [15]:

$$\begin{aligned} f(x) &= -(x_1)^2 - (x_2)^2 - 2(x_3)^2 - (x_4)^2 + 5x_1 \\ &\quad +5x_2 + 21x_3 - 7x_4 \\ g_1(x) &= (x_1)^2 + (x_2)^2 + (x_3)^2 + (x_4)^2 + x_1 \\ &\quad -x_2 + x_3 - x_4 - 8 \leqq 0 \\ g_2(x) &= (x_1)^2 + 2(x_2)^2 + (x_3)^2 + 2(x_4)^2 - x_1 \\ &\quad -x_4 - 10 \leqq 0 \\ g_3(x) &= 2(x_1)^2 + (x_2)^2 + (x_3)^2 + 2x_1 - x_2 \\ &\quad -x_4 - 5 \leqq 0 \end{aligned}$$

Figures 9(a) and (b) show the result of computation for these problems, respectively. It is seen that the PFC method could

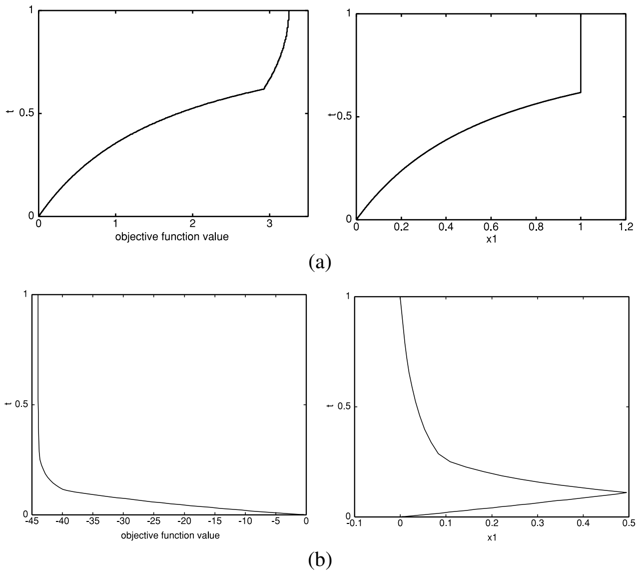


Fig. 9 Result of computation (obtained paths) for the nonlinear programming problems.

follow paths with sharp turning points. The computation time was 0.2 s and 0.3 s, respectively.

We also solved the Bracken-McCormick problem [15], which is a more difficult problem than the above problems, by the PFC method. Then, we found the optimal solution in 0.6 s.

We have also applied the PFC method to various problems such as nonlinear boundary value problems and the computation of channel capacity in information theory, and have obtained similar good results [16].

7. Conclusion

In this paper, we have discussed the PFC method that is based on the counter idea: *formulas of numerical methods are described by circuits, and then they are solved by SPICE*, and have applied it to the problems where the homotopy method is proven to be globally convergent in order to examine its effectiveness. Since SPICE is an excellent software that includes various effective techniques and know-hows accumulated over many years, this approach will give a new possibility to the fields of applied mathematics and operations research, especially for stiff problems. Moreover, for SPICE users (including those who have learned SPICE at universities or elsewhere), the PFC method will be useful because they can easily solve a broad class of problems by the homotopy method without programming, although they do not know the homotopy method well.

The next subject we should consider is to describe many numerical methods by circuits and makes SPICE applicable to a broader class of scientific problems. It will also be interesting to apply the proposed method to various problems in circuit simulation that have not been solved by SPICE, which will open a new possibility in circuit simulation.

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