A Simple Approach to the Solution of the ac–dc Power Flow Problem

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***Abstract*—The advent of inexpensive and reliable high-power SCR's has led to the reemergence of dc power systems for industrial and utility applications. Calculations of the voltage profile for these systems require that the ac–dc power flow problem be solved. While many texts treat the ac power flow, none to date have incorporated the ac–dc power flow. Presented in this paper is an approach for simultaneously solving the ac and dc systems equations which constitute the ac–dc power flow problem. The approach is valid for multiterminal HV dc links as well as industrial dc networks. The use of Newton's method in this approach allows the ac–dc power flow to be presented as a generalization of the ac power flow.**

INTRODUCTION

THE number of power electronic applications is growing at a rapid pace due to the developments in solid-state technology which have made reliable and inexpensive SCR's readily available. The theoretical basis for many of these applications has been established for years and is covered by many textbooks in the area [1]. Many applications, however, are new, and reliable analytical methods for treating these problems have only recently been established. The analysis of converter-fed industrial and utility dc power systems is one such area. Today, industrial dc power systems often consist of interconnected dc grids which are fed throughout by many, often uncontrolled, converters. These systems are growing in size and number. Utility use of HV dc transmission links for connecting remote generation facilities and long distance power transmission is also increasing. Today, 25 HV dc links are in operation throughout the world with many more in the planning stages. Fig. 1 shows the growth rate of HV dc installations [2]. At this pace, it is essential that our power engineers be familiar with dc system design and planning if they are to cope with this trend.

One of the principal design tools used in power system planning is the power flow study. Of the many books that treat the ac power flow problem [3]–[12], few consider ac–dc systems at all and none addresses the ac–dc power flow problem. Of the books which deal with ac–dc systems [13]–[16], none treat the ac–dc power flow problem. The reasons for this lack of treatment may be that only recently have methods for treating this problem been proven in practice to work satisfactorily and reliably.

Two separate iterative techniques have been used to solve the nonlinear equations which constitute the ac–dc power flow problem. The sequential method involves a two-step procedure at each iteration [17]–[20]. In the first step, the ac power flow problem is solved while the dc system is modeled as a constant load on the ac system. The second step involves solving the dc system equations while all ac system quantities are held constant. The simultaneous method involves solving the ac and dc power flow equations simultaneously at each iteration [21]–[27]. The simultaneous method is attractive because of the more elegant way in which the Newton–Raphson method is applied to the problem. In addition, convergence properties of this method are slightly better than those of the sequential method. The sequential method has the advantage that it can easily be interfaced to an existing ac power flow program without restructuring the ac algorithm.

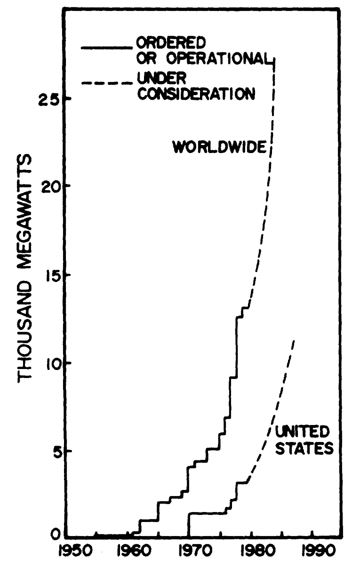

Fig. 1. Direct current transmission capacity in the United States and worldwide.

The goal of this treatment is to use the simultaneous method to introduce the ac–dc power flow solution procedure to the student in such a way that it becomes a logical extension of the ac power flow solution. It is assumed that the student is familiar with the ac per-unit system, the Newton–Raphson (NR) method and the equations which describe the terminal behavior of the three-phase bridge converter under normal operating conditions [13].

PER-UNIT SYSTEM

The use of an ac per-unit system reduces all three-phase power system transformer turns-ratios to 1 : 1. When balanced operation is assumed and single-phase analysis used, the equivalent single-phase 1 : 1 turns-ratio transformer

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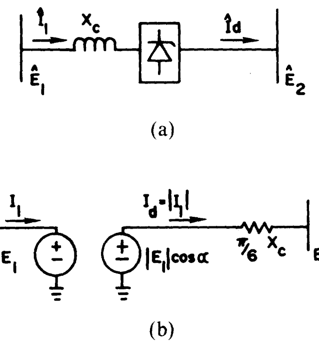

Fig. 2. Bridge converter: (a) schematic diagram; (b) per-unit equivalent circuit.

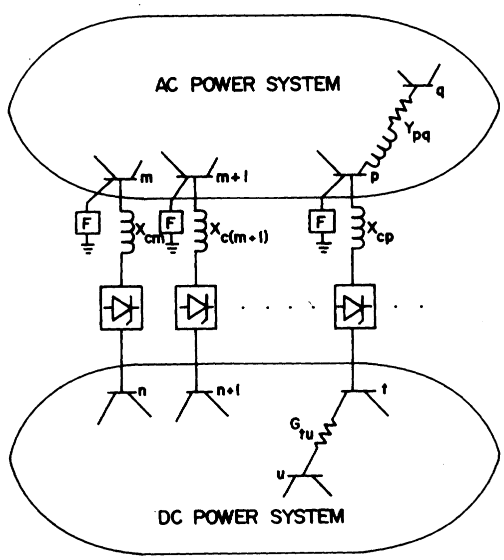

Fig. 3. Generalized ac–dc power system.

model may be replaced by a circuit node. The three-phase bridge converter of Fig. 2(a) may also be viewed as a transformer which transforms ac to dc. In this figure, the symbol (^) is used to indicate actual rather than per-unit quantities and

E_1 = ac side phasor voltage

E_2 = dc side voltage

I_1 = fundamental-component phasor current

I_d = direct current

X_c = commutation reactance.

While the ideal ac power transformer is a linear device, the ac-to-dc transformation characteristics of a bridge converter are nonlinear. Hence, the effective turns-ratio of the "ac-to-dc transformer" changes as the current through the "ac-to-dc transformer" changes, and no one ac–dc per-unit system can be chosen which will allow the bridge converter to be replaced with a circuit node. The selection of an ac–dc per-unit system may therefore be made arbitrarily. For convenience it is usually chosen so that per-unit current and power are equal on both sides of the converter. Use of such an ac–dc per-unit system allows the per-unit equations which describe the terminal relationships of the bridge converter of Fig. 2(a) to be given by

$$E_2 = |E_1| \cos \alpha - \frac{\pi}{6} X_c I_d \tag{1}$$

$$I_d = |I_1| \tag{2}$$

where α is the commutation delay angle.

An additional per-unit equation which is useful in ac–dc system analysis is obtained by equating the real power at the ac and dc terminals of the converter. In per-unit, this equation is

$$I_d E_2 = |I_1| \, |E_1| \cos \phi \tag{3}$$

where ϕ = angle by which I_1 lags E_1 and $\cos \phi$ is the displacement factor. Substituting (2) into (3) and cancelling I_d from both sides gives the result

$$E_2 = |E_1| \cos \phi. \tag{4}$$

For a detailed discussion of this per-unit system and a derivation of these equations see the Appendix.

An electrical circuit analog for (1) and (2) is conceptually helpful when using these equations for network analysis purposes. Kirchhoff's voltage law can be used to verify that Fig. 2(b) is the per-unit electrical analog for (1) and (2).

Problem Specification

A generalized model for an ac–dc power system is shown in Fig. 3. Each bridge converter symbolically shown may represent a rectifier or inverter, depending on the control strategy. The ideal harmonic filters shown serve to limit the commutation reactance to the series reactance measured between the filtered bus and the ac terminals of the converter.

The ac-dc power flow problem may be broken into two special cases: the utility power flow problem and the industrial power flow problem. These cases differ because of the different power system configurations encountered in utility and industrial networks. A utility ac-dc power system is usually comprised of a large ac network and a multiterminal dc link for high voltage power transmission. Today only two-terminal links are in operation, but true multiterminal operation can be expected in the near future. The dc portion of the utility dc system is characterized by controlled converters which appear as real and reactive power sources and loads to the ac system.

Like a utility ac-dc power system, an industrial ac-dc power system is comprised of an interconnected ac system. This ac system is most often radial. The dc portion of the industrial power system differs from a utility system in two important aspects. First, the dc system is usually a grid with many interconnected, and often uncontrolled, rectifiers which act as sinks of ac electrical power. Second, the loads on the industrial dc system are often dc motor loads rather than inverter loads.

For convenience, let the term "industrial ac-dc power system" refer to the case where all rectifiers are uncontrolled. Similarly, "utility ac-dc power system," as used here, will refer to the case where all converters are controlled and all dc loads are inverter loads.

Application of the Newton-Raphson Method

It is convenient to consider the industrial ac-dc power system first since it represents a somewhat simplified case in which no converter control is required. The utility case will be considered second and will be shown to be a generalization of the first case. In the subsequent analysis it is assumed that all equations describing the system are expressed using the per-unit system developed.

The Industrial Power Flow Problem

The converters of Fig. 3, as they represent an industrial ac-dc power system, are uncontrolled. Formulation of the power flow problem may be accomplished by initially ignoring (i.e., removing) the converter connections between the ac and dc power system. The bus-power-equilibrium (BPE) equations which are needed to solve the separated ac-system power flow problem are well known to be

$$P_p - jQ_p = E_p^* \sum_{q=1}^{N} Y_{pq} E_q, \qquad p = 1 \cdots N \tag{5}$$

where

$$Y_{pq} = G_{pq} + jB_{pq} \tag{6}$$

N = number of ac system buses

and Y_{pq} is the (p, q) element of the nodal admittance matrix.

The separated dc system is also governed by (5) where all voltages and power flows are real. The dc system BPE equations are

$$P_t = E_t \sum_{q=N+1}^{M} G_{tq} E_q, \qquad t = N+1, \cdots M \tag{7}$$

where

$M - N$ = number of dc system buses

P_t = dc motor load at bus t

and G_{tq} is the (t, q) element of the nodal conductance matrix. Notice that for each ac-system bus added, two equations [i.e., the real and imaginary parts of (5)] and two unknowns, $|E_p|$ and θ_p, are added. Similarly, for each dc system bus added, one real equation (7), and one unknown E_t, are added to list. Application of the polar form of the Newton-Raphson (NR) method to the BPE equations of (5) is well known to give a form which may be written symbolically as

$$\left[\frac{\Delta Q}{\Delta P}\right] = \left[\frac{H \mid N}{J \mid L}\right]\left[\frac{\Delta\theta}{\Delta E}\right] \tag{8}$$

where

$$H_{pq} = \frac{\partial P_p}{\partial \theta_q} = \begin{cases} -|E_p|^2 B_{pp} - Q_p & p = q \quad (9a) \\ -\operatorname{Im}(E_p^* E_q Y_{pq}) & p \neq q \quad (9b) \end{cases}$$

$$N_{pq} = \frac{\partial P_p}{\partial |E_q|} = \begin{cases} |E_p| G_{pp} + \dfrac{P_p}{|E_p|} & p = q \quad (10a) \\ \dfrac{\operatorname{Re}(E_p^* E_q Y_{pq})}{|E_q|} & p \neq q \quad (10b) \end{cases}$$

$$J_{pq} = \frac{\partial Q_p}{\partial \theta_q} = \begin{cases} -|E_p|^2 G_{pp} + P_p & p = q \quad (11a) \\ -\operatorname{Re}(E_p^* E_q Y_{pq}) & p \neq q \quad (11b) \end{cases}$$

$$L_{pq} = \frac{\partial Q_p}{\partial |E_q|} = \begin{cases} -|E_p| B_{pp} + \dfrac{Q_p}{|E_p|} & p = q \quad (12a) \\ \dfrac{-\operatorname{Im}(E_p^* E_q Y_{pq})}{|E_q|} & p \neq q \quad (12b) \end{cases}$$

and P_p and Q_p are the calculated real and reactive bus powers based on the assumed system voltage profile.

Similarly, application of either form of the NR method to the BPE of (7) gives a form which may be written symbolically as

$$[\Delta P] = [N][\Delta E] \tag{13}$$

where

$$N_{tu} = \frac{\partial P_t}{\partial E_u} = \begin{cases} E_t G_{tt} + \dfrac{P_t}{E_t}, & t = u \quad (14a) \\ E_u G_{tu}, & t \neq u \quad (14b) \end{cases}$$

and P_t is the calculated real bus power based on the assumed voltage profile. Note that matrix equations (13) and (8) may be combined while still retaining the symbolic form of (8). This type of equation will be termed the bus-power-mismatch (BPM) equation.

The definitions of (9)–(12) and (14) show that the coefficient matrix of form (8) is simply the Jacobian of the ac and dc BPE equations with respect to the selected bus variables (i.e., $|E|$ and θ for polar form and $\operatorname{Re}\{E\}$, $\operatorname{Im}\{E\}$ for rectangular form). A prerequisite for a unique solution of (28) is that the Jacobian be square. A square BPM equation is guaranteed by the correspondence of unknowns and equations. Thus for the case of the two independent ac and dc systems, the BPM equation for ac bus p connected to ac bus q and dc bus t connected to dc bus u is given by

$$\begin{bmatrix} \vdots \\ \Delta P_p \\ \Delta Q_p \\ \Delta P_t \\ \vdots \end{bmatrix} = \begin{bmatrix} & \vdots & & & \vdots & & \vdots \\ & H_{pp}\; N_{pp}\; 0 & & & H_{pq}\; N_{pq} & & 0 \\ \cdots & J_{pp}\; L_{pp}\; 0 & \cdots & & J_{pq}\; L_{pq} & \cdots & 0 \\ & 0 \quad 0 \quad N_{tt} & & & 0 \quad 0 & & N_{tu} \\ & \vdots & & & \vdots & & \vdots \end{bmatrix} \begin{bmatrix} \vdots \\ \Delta\theta_p \\ \Delta|E_p| \\ \Delta E_t \\ \vdots \\ \Delta\theta_q \\ \Delta|E_q| \\ \vdots \\ \Delta E_u \\ \vdots \end{bmatrix} \tag{15}$$

where it has been assumed without loss of generality that bus t is ordered directly after bus p in construction of the Jacobian.

Now consider the effects of including the bridge converter connections between the ac and dc systems. It may be assumed that there are any number of such connections. Rather than consider all interconnections, however, it is best to consider one such connection since the process is identical for all connections.

Assume that a rectifier connects ac bus p to dc bus t. The BPE equations which characterize these connected buses must be modified to account for power flow between the ac and dc systems. Thus, for buses p and t the BPE equations are

$$\tilde{P}_p - j\tilde{Q}_p = E_p^* \sum_{q=1}^{N} Y_{pq} E_q + |E_p| I_{dp} \cos\phi_p - j|E_p| I_{dp} \sin\phi_p, \tag{16}$$

$$\tilde{P}_t = E_t \sum_{q=N+1}^{M} G_{tq} E_q - E_t I_{dp}, \tag{17}$$

where

I_{dp} = magnitude of the ac per-unit fundamental-component current flowing from ac bus p to the rectifier
= dc per-unit current (I_d) flowing from the rectifier to bus t

$\cos\phi_p$ = displacement factor associated with ac bus voltage E_p and ac fundamental-component current I_{dp}.

Notice that one of the three terminal relationships previously derived, namely (2), has been used in (16). This allows the terms associated with the real and reactive power delivered to the converter to be expressed as a function of the dc per-unit current rather than the magnitude of the ac per-unit fundamental-component current. Notice also that the variables I_d and ϕ are assumed to be associated with ac bus p in (16) and (17). This assignment is arbitrary.

The added real terms in (16) and (17) account for real power flow from ac bus p to dc bus t. The imaginary term accounts for reactive power flow from the ac bus to the rectifier. From its ac terminals the rectifier looks like a lagging power factor load.

The modification of the BPE equations requires that the partial derivative entries in the Jacobian be reevaluated. If ($\sim$) is used to indicate the entries which need to be modified in (15), then the modified entries are given by

$$\tilde{N}_{pp} = \frac{\partial \tilde{P}_p}{\partial |E_p|} = |E_p| G_{pp} + \frac{P_p}{|E_p|} + I_{dp}\cos\phi_p = N_{pp} + I_{dp}\cos\phi_p \tag{18}$$

$$\tilde{L}_{pp} = \frac{\partial \tilde{Q}_p}{\partial |E_p|} = -|E_p| B_{pp} + \frac{Q_p}{|E_p|} + I_{dp}\sin\phi_p = L_{pp} + I_{dp}\sin\phi_p \tag{19}$$

$$\tilde{N}_{tt} = \frac{\partial \tilde{P}_t}{\partial E_t} = E_t G_{tt} + \frac{P_t}{E_t} - I_{dp} = N_{tt} - I_{dp}. \tag{20}$$

In addition to the modification of existing Jacobian entries, applying Taylor's theorem to (16) and (17) shows that additional terms, which correspond to the added unknowns I_d and ϕ, must be accounted for in the BPM equation. These terms, which result after truncating higher order terms in the Taylor series expansion, are shown in (21)–(23).

$$\Delta\tilde{P}_p = \cdots + \frac{\partial \tilde{P}_p}{\partial I_{dp}} \Delta I_{dp} + \frac{\partial \tilde{P}_p}{\partial \phi_p} \Delta\phi_p + \cdots \tag{21}$$

$$\Delta\tilde{Q}_p = \cdots + \frac{\partial \tilde{Q}_p}{\partial I_{dp}} \Delta I_{dp} + \frac{\partial \tilde{Q}_p}{\partial \phi_p} \Delta\phi_p + \cdots \tag{22}$$

$$\Delta\tilde{P}_t = \cdots + \frac{\partial \tilde{P}_t}{\partial I_{dp}} \Delta I_{dp} + \cdots. \tag{23}$$

Thus, these additional terms along with the corresponding variable increments must be incorporated into the Jacobian and column vector of variable increments, respectively. Notice that this will lead to a nonsquare Jacobian with a nonunique solution. This, however, should have been expected since new variables have been added by the modified BPE equations [i.e., I_{dp} and ϕ_p in (16) and (17)] while adding no new constraints. In general, a unique solution of the BPM equation may be obtained by adding additional independent constraining relationships between these added variables. In practice, the input–output characteristics of the converter constrain the independent variation of I_d and ϕ. This constraining relationship must be accounted for if a realistic solution of the ac–dc power flow problem is to be obtained. The terminal relationships necessary are taken from (1) and (4) as

$$R_{1p} = E_t - |E_p| \cos \phi_p \tag{24}$$

$$R_{2p} = E_t - |E_p| \cos \alpha_t + (\pi/6) X_{cp} I_{dp} \tag{25}$$

where $\alpha = 0$ for an uncontrolled rectifier and X_{cp} is the commutation reactance associated with the rectifier transformer at ac bus p. Equations R_1 and R_2 are termed residual equations because when all of the variables are known, each equals zero. Like the BPE equations, R_1 and R_2 are nonlinear equations. The handiest way to simultaneously solve the BPE and residual equations is to use the NR method on them.

Expanding R_1 and R_2 about an assumed solution point in a Taylor series and truncating higher order terms gives

$$\Delta R_1 = 0 - R_1 = \frac{\partial R_1}{\partial E_t} \Delta E_t + \frac{\partial R_1}{\partial |E_p|} \Delta |E_p| + \frac{\partial R_1}{\partial \phi_p} \Delta \phi_p \tag{26}$$

$$\Delta R_2 = -R_2 = \frac{\partial R_2}{\partial E_t} \Delta E_t + \frac{\partial R_2}{\partial |E_p|} \Delta |E_p| + \frac{\partial R_2}{\partial I_{dp}} \Delta I_{dp}. \tag{27}$$

Incorporating the modifications of the original Jacobian entries along with the addition of new variables gives the revised bus power and residual mismatch (BPRM) equation as

$$\begin{bmatrix} \vdots \\ \Delta \tilde{P}_p \\ \Delta \tilde{Q}_p \\ \Delta R_{1p} \\ \Delta R_{2p} \\ \Delta \tilde{P}_t \\ \vdots \end{bmatrix} = \begin{bmatrix} & \vdots & & \vdots & & \vdots & \\ \cdots & H_{pp}\ \tilde{N}_{pp}\ A_{pp}\ B_{pp}\ 0 & & H_{pq}\ N_{pq} & & 0 & \\ & J_{pp}\ \tilde{L}_{pp}\ C_{pp}\ D_{pp}\ 0 & & J_{pq}\ L_{pq} & & 0 & \\ & 0\ F_{pp}\ 0\ K_{pp}\ F_{pt} & \cdots & 0\ 0 & \cdots & 0 & \cdots \\ & 0\ M_{pp}\ R_{pp}\ 0\ M_{pt} & & 0\ 0 & & 0 & \\ & 0\ 0\ A_{tp}\ 0\ \tilde{N}_{tt} & & 0\ 0 & & N_{tu} & \\ & \vdots & & \vdots & & \vdots & \end{bmatrix} \begin{bmatrix} \vdots \\ \Delta \theta_p \\ \Delta |E_p| \\ \Delta I_{dp} \\ \Delta \phi_p \\ \Delta E_t \\ \vdots \\ \Delta \theta_p \\ \Delta |E_q| \\ \vdots \\ \Delta E_u \\ \vdots \end{bmatrix} \tag{28}$$

where

$$A_{pp} = \frac{\partial \tilde{P}_p}{\partial I_{dp}} = |E_p| \cos \phi_p \tag{29}$$

$$B_{pp} = \frac{\partial \tilde{P}_p}{\partial \phi_p} = -|E_p|\, I_{dp} \sin \phi_p \tag{30}$$

$$C_{pp} = \frac{\partial \tilde{Q}_p}{\partial I_{dp}} = |E_p| \sin \phi_p \tag{31}$$

$$D_{pp} = \frac{\partial \tilde{Q}_p}{\partial \phi_p} = |E_p|\, I_{dp} \cos \phi_p \tag{32}$$

$$F_{pp} = \frac{\partial R_{1p}}{\partial |E_p|} = -\cos \phi_p \tag{33}$$

$$K_{pp} = \frac{\partial R_{1p}}{\partial \phi_p} = |E_p| \sin \phi_p \tag{34}$$

$$M_{pp} = \frac{\partial R_{2p}}{\partial |E_p|} = -1 \tag{35}$$

$$R_{pp} = \frac{\partial R_{2p}}{\partial I_{dp}} = -(\pi/6) X_{cp} \tag{36}$$

$$F_{pt} = \frac{\partial R_{1p}}{\partial E_t} = 1 \tag{37}$$

$$M_{pt} = \frac{\partial R_{2t}}{\partial E_t} = 1 \tag{38}$$

$$A_{tp} = \frac{\partial \tilde{P}_t}{\partial I_{dp}} = -E_t. \tag{39}$$

The residual portion of the BPRM equation is usually included as the last rows of the Jacobian to preserve sparsity and minimize fill in. For the sake of compactness, this is not the case shown in (28).

Once the BPRM equation is defined by applying the method presented to every bus in the system, the solution of the power flow problem is straightforward. An initial estimate of the system unknowns is used to evaluate the Jacobian entries and mismatches. Next, the variable increments are obtained using the desired solution procedure. These increments are added to the initial estimates of the system unknowns to obtain better estimates. The procedure continues until the mismatches are sufficiently small.

Normally, the initial estimate consists of choosing all voltage magnitudes and angles as 1.0 and 0.0, respectively. A good starting point for ϕ is 0.0. A good starting point for I_d depends on the specific industrial power system being studied. Without previous experience, any small value for I_d (i.e., 0.1–0.5) may be assumed and convergence will usually be obtained.

The selection of system variables used in the Taylor series expansion of the BPE and residual equations is somewhat arbitrary. For instance, an equally valid choice of variables is I_d and $\cos\phi$.

The Utility Power Flow Problem

The use of converters as terminals of a multiterminal dc link requires that all converters be controlled via the delay angle α. Four types of converter control are common in HV dc systems. The extinction advance angle, dc voltage, dc current, or dc power may be scheduled at each dc converter. In practice, one dc bus is under constant extinction advance angle or dc voltage control while others are either current or power controlled.

From an analytical point of view, the difference between the utility and industrial power flow problem is that α is a variable which must be identified in the solution process. Since an additional variable is to be added to the system of unknowns, either an existing variable must be eliminated or an additional constraining relationship added. The emphasis in the rest of this section is on showing how this may be accomplished.

Before considering how α is identified for each type of control, it is important to determine if the addition of α affects the portion of the BPRM equations already developed. Inspection indicates that the variable α is found only in (25). Since $\cos\alpha$ is the functional form in which α appears, use of $\cos\alpha$ rather than α as a variable in the expansion of (25) will improve the linearity of the expansion. Using $\cos\alpha$ as a variable, the truncated Taylor series expansion of (25) which is necessary for the case of controlled converters is given by

$$\Delta R_{2p} = \frac{\partial R_{2p}}{\partial E_t}\Delta E_t + \frac{\partial R_{2p}}{\partial |E_p|}\Delta|E_p| + \frac{\partial R_{2p}}{\partial(\cos\alpha_t)}\Delta(\cos\alpha_t) + \frac{\partial R_{2p}}{\partial I_{dp}}\Delta I_{dp}. \tag{40}$$

Comparing (40) to (27) will show two important differences in the expansion of R_{2p} in the utility power flow problem. First, the partial of R_{2p} with respect to $|E_p|$ will have a different form. This term corresponds to the M_{pp} entry in the Jacobian of (28) and is given by

$$M_{pp} = \frac{\partial R_{2p}}{\partial |E_p|} = -\cos\alpha_t. \tag{41}$$

The second difference is that an additional term which corresponds to the variable $\cos\alpha$ appears in (40) but not (27). This is the additional variable which must be identified in the solution process. The way in which this may be handled for each type of control is discussed in the remainder of this section.

Extinction Advance Angle Control: When a converter is operated as an inverter with a constant extinction advance angle, α is fixed. Therefore, $\Delta(\cos\alpha_t) = 0$ in (40) and need not be included as a variable in the BPRM equation. Thus, the BPRM equation which is characteristic of this type of control is given by (28) with M_{pp} defined by (41).

Voltage Control: Under voltage control, E_t is fixed. Thus, $\Delta E_t = 0$ and need not be included in the BPRM equation. This eliminates one variable as required. Inclusion of $\Delta(\cos\alpha)$ as a variable from (40) and elimination of ΔE_t as a variable gives the BPRM equation from (28) as

$$\begin{bmatrix} \vdots \\ \Delta\tilde{P}_p \\ \Delta\tilde{Q}_p \\ \Delta R_{1p} \\ \Delta R_{2p} \\ \Delta\tilde{P}_t \\ \vdots \end{bmatrix} = \begin{bmatrix} & & & \vdots & & & & \vdots & & & \vdots & \\ \cdots & H_{pp} & \tilde{N}_{pp} & A_{pp} & B_{pp} & 0 & \cdots & H_{pq} & N_{pq} & \cdots & 0 & 0 & \cdots \\ \cdots & J_{pp} & \tilde{L}_{pp} & C_{pp} & D_{pp} & 0 & \cdots & J_{pq} & L_{pq} & \cdots & 0 & 0 & \cdots \\ \cdots & 0 & F_{pp} & 0 & K_{pp} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots \\ \cdots & 0 & M_{pp} & R_{pp} & 0 & S_{pt} & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots \\ \cdots & 0 & 0 & A_{tp} & 0 & 0 & \cdots & 0 & 0 & \cdots & N_{tu} & 0 & \cdots \\ & & & \vdots & & & & \vdots & & & \vdots & \end{bmatrix} \begin{bmatrix} \vdots \\ \Delta\theta_p \\ \Delta|E_p| \\ \Delta I_{dp} \\ \Delta\phi_p \\ \Delta(\cos\alpha_t) \\ \vdots \\ \Delta\theta_q \\ \Delta|E_q| \\ \vdots \\ \Delta E_u \\ \Delta(\cos\alpha_u) \\ \vdots \end{bmatrix} \tag{42}$$

where

$$S_{pt} = \frac{\partial R_{2p}}{\partial(\cos\alpha_t)} = -|E_p| \tag{43}$$

and the other variables are defined by previous results.

Current Control: Under dc current control I_d is fixed. Thus, $\Delta I_d = 0$ and need not be included in the BPRM equation. The inclusion of $\Delta(\cos\alpha)$ by (40) and elimination of ΔI_d allows the revised BPRM equation to be written as

$$
\begin{bmatrix} \vdots \\ \Delta\tilde{P}_p \\ \Delta\tilde{Q}_p \\ \Delta R_{1p} \\ \Delta R_{2p} \\ \Delta\tilde{P}_t \\ \vdots \end{bmatrix}
=
\begin{bmatrix}
 & \vdots & & & & & & \vdots & & & \vdots & & \\
 & H_{pp} & \tilde{N}_{pp} & B_{pp} & 0 & 0 & & H_{pq} & N_{pq} & & 0 & 0 & \\
 & J_{pp} & \tilde{L}_{pp} & D_{pp} & 0 & 0 & & J_{pq} & L_{pq} & & 0 & 0 & \\
\cdots & 0 & F_{pp} & K_{pp} & F_{pt} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots \\
 & 0 & M_{pp} & 0 & M_{pt} & S_{pt} & & 0 & 0 & & 0 & 0 & \\
 & 0 & 0 & 0 & \tilde{N}_{tt} & 0 & & 0 & 0 & & N_{tu} & 0 & \\
 & \vdots & & & & & & \vdots & & & \vdots & &
\end{bmatrix}
\begin{bmatrix} \vdots \\ \Delta\theta_p \\ \Delta|E_p| \\ \Delta\phi_p \\ \Delta E_t \\ \Delta(\cos\alpha_t) \\ \vdots \\ \Delta\theta_q \\ \Delta E_q \\ \vdots \\ \Delta E_u \\ \Delta(\cos\alpha_u) \end{bmatrix}
\tag{44}
$$

where all variables in the Jacobian are defined by previous results.

Power Control: Under dc power control $P_{dt} = E_t I_{dp}$ is controlled to be constant at a scheduled value P_{st}. Since ΔP_d is not already a variable, it cannot be eliminated from the BPRM equation. The Jacobian can be made square after the addition of $\Delta(\cos\alpha)$ by adding an additional residual equation which incorporates no new variables. This addition residual is

$$R_{3p} = P_{st} - E_t I_{dp}. \tag{45}$$

A truncated Taylor series expansion of this residual gives

$$\Delta R_{3p} = \frac{\partial R_{3p}}{\partial E_t}\Delta E_t + \frac{\partial R_{3p}}{\partial I_{dp}}\Delta I_{dp}. \tag{46}$$

Incorporation of this residual in to the BPRM equation gives

$$
\begin{bmatrix} \vdots \\ \Delta\tilde{P}_p \\ \Delta\tilde{Q}_p \\ \Delta R_{1p} \\ \Delta R_{2p} \\ \Delta R_{3p} \\ \Delta\tilde{P}_t \\ \vdots \end{bmatrix}
=
\begin{bmatrix}
 & & & \vdots & & & & & \vdots & & & \vdots & & \\
 & H_{pp} & \tilde{N}_{pp} & A_{pp} & B_{pp} & 0 & 0 & & H_{pq} & N_{pq} & & 0 & 0 & \\
 & J_{pp} & \tilde{L}_{pp} & C_{pp} & D_{pp} & 0 & 0 & & J_{pq} & L_{pq} & & 0 & 0 & \\
 & 0 & F_{pp} & 0 & K_{pp} & F_{pt} & 0 & & 0 & 0 & & 0 & 0 & \\
\cdots & 0 & M_{pp} & R_{pp} & 0 & M_{pt} & S_{pt} & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots \\
 & 0 & 0 & T_{pp} & 0 & U_{pt} & 0 & & 0 & 0 & & 0 & 0 & \\
 & 0 & 0 & A_{tp} & 0 & \tilde{N}_{tt} & 0 & & 0 & 0 & & N_{tu} & 0 & \\
 & & & \vdots & & & & & \vdots & & & \vdots & &
\end{bmatrix}
\begin{bmatrix} \vdots \\ \Delta\theta_p \\ \Delta|E_p| \\ \Delta I_{dp} \\ \Delta\phi_p \\ \Delta E_t \\ \Delta(\cos\alpha_t) \\ \vdots \\ \Delta\theta_q \\ \Delta E_q \\ \vdots \\ \Delta E_u \\ \Delta(\cos\alpha_u) \\ \vdots \end{bmatrix}
\tag{47}
$$

where

$$T_{pp} = \frac{\partial R_{3p}}{\partial I_{dp}} = -E_t \tag{48}$$

$$U_{pt} = \frac{\partial R_{3p}}{\partial E_t} = -I_{dp}. \tag{49}$$

The preceding discussion on the form that the BPRM equation takes under dc voltage, current, or power control, focused mainly on the changes in the BPRM equation. Comparing (28), (42), (44), and (47) shows that regardless of the control strategy, or lack thereof, the BPRM equation is very similar in form. It should be noted that the p–q and t–u coupling entries in the Jacobian matrix do not change as the control of the rectifier changes.

Solution of the BPRM equation requires an initial estimate of the system state. This is somewhat easier for the utility ac–dc power flow problem than for the industrial problem. Generally assuming all bus voltage magnitudes and bus voltage angles as 1.0 and 0.0, respectively, is a good initial estimate. Inverters typically operate with a constant extinction angle of $\gamma_0 \cong 15°$ (i.e., $\alpha \cong 165°$), while rectifiers under current or power control operate with $\alpha_0 \cong 15°$. Under voltage control, rectifiers operate with $\alpha_{\min} \cong 5°$. By knowing the scheduled bus power a good initial current estimate may be obtained. In addition, ϕ may be assumed to be equal to 0.0.

Pedagogic Considerations

The material contained in this paper is predicated on a good understanding by the student on the NR ac power flow formulation. The extension then of ac power flow equations to cover the general dc power flow, as presented here, is straightforward and follows naturally. The coupling of these two systems of equations is usually the critical point in the development. It is important to emphasize that the constraints which couple the behavior of ac and dc converter buses must be taken into account if a realistic solution of the ac–dc power flow problem is to be achieved. To show that these constraints are the converter terminal relationships requires only a verbal argument. Consider the following argument in which real power control is assumed at the converter. The BPE equations of (16) and (17) guarantee conservation of power at the ac and dc rectifier buses. If (24) is multiplied by I_{dp}, then it represents a real power equilibrium equation. This equation shows that since no real power is consumed by the converter, the power delivered to the converter equals that converted. This set of equations taken together guarantees power conservation and the fact that real power delivered by one system equals that consumed by the other. Equation (25) guarantees that this power balance occurs for voltages at the ac and dc rectifier buses which are consistent with the converter terminal relationships. Now, for a given real ac power transfer the dc power is constrained and the voltage at which it is delivered is constrained; hence, the direct current is constrained. Since the per-unit magnitudes of the dc and ac currents are identical, the ac current magnitude is constrained. This fact along with the known ac real power transfer and ac–dc voltage relationship gives the phase angle of the current relative to the voltage. Hence, for a given ac voltage and power transfer all equations are satisfied and only one solution is possible. The exact ac converter bus voltage is, of course, a function of the system admittances and system loads. Similar arguments can be made for converters at which other modes of control are used.

The next critical point in the development is the inclusion of the nonlinear terminal relationships in the NR power flow. Students familiar with the ac NR power flow solution often forget its origin. Thus, it is important to quickly review the fact that it is a general method for solving nonlinear equations. Once this is reviewed, the inclusion of the converter residual equations via proper partial derivative entries in the coefficient matrix falls out directly.

If the approach described is used, then the extension from the uncontrolled case to the controlled rectifier case via the method described is relatively easy since it only requires that one extra variable be accounted for in the Taylor series expansion of (25). The controlled case is somewhat more complex than presented here since tap-changing transformers are used at the ac terminals of the converter. Control of the transformer taps is used to maintain the ac terminal voltage while allowing the firing angle to remain within specified limits. It is felt that at the introductory stage, the inclusion of tap-changing transformers adds an unnecessary profusion of "extra" variables and obscures the underlying simplicity of this approach. Once this material is understood, then other complexities, such as tap-changing transformers, can be added without undue confusion.

Two assignments are usually made which help reinforce the material presented. In the first assignment, the students are asked to come up with other sets of residuals which may be used in place of (24) and (25). Since a hint is usually needed, an example is usually given. Equation (25) is solved for I_{dp}, and this result is substituted into (16) and (17) to eliminate one residual and one variable. It is quite surprising how many different residual sets can be found which can be used in the ac–dc power flow solution formulation.

In the second assignment, the students are asked to derive the BPRM equation which corresponds to the residual and BPE equations obtained in the first assignment. In addition, α_t rather than $\cos \alpha_t$ is suggested as the variable to be used in the Taylor series expansion. This assignment reinforces the methods used in constructing the Jacobian of the system of equations and shows that the system variables are not unique.

Conclusions

The approach presented has the advantage of starting with the ac power flow problem which is understood well by the student. Three simple steps are then taken to solve what appears to be a more complex problem. The first step involves showing that the form of the dc BPE equation is very similar to, and in fact can be derived from, the ac BPE equation. In the second step, the converter terminal relationships are introduced and a verbal argument presented to show why these equations are necessary and sufficient for obtaining a power flow solution. The third step involves applying the Newton–Raphson method to solve these equations, a method which is well understood by the student. Taken one at a time the steps are extremely simple and lead to a solution of the more complex ac–dc power flow problem.

Appendix
The ac–dc Per-Unit System

The usual ac-system base quantities chosen in an ac per-unit system are

$$VA_{\text{ac-BASE}} = VA_{3\,\text{Phase}} \tag{A.1}$$

$$V_{\text{ac-BASE}} = V_{LL} \tag{A.2}$$

$$I_{\text{ac-BASE}} = \frac{VA_{\text{ac-BASE}}}{\sqrt{3}\; V_{\text{ac-BASE}}} \tag{A.3}$$

$$Z_{\text{ac-BASE}} = \frac{V_{\text{ac-BASE}}}{\sqrt{3}\; I_{\text{ac-BASE}}} = \frac{(V_{\text{ac-BASE}})^2}{VA_{\text{ac-BASE}}} \tag{A.4}$$

where V_{LL} represents the ac line-to-line voltage and the symbol $VA_{3\text{Phase}}$ represents the three-phase voltampere rating chosen. Denoting quantities with ($\wedge$) as actual phasor values, the current and voltage transformations associated with a per-phase equivalent of a three-phase transformer are

$$\frac{\hat{E}_1}{N_1} = \frac{\hat{E}_2}{N_2} \tag{A.5}$$

$$N_1 \hat{I}_1 = N_2 \hat{I}_2. \tag{A.6}$$

Implicit in the development of these equations is the assumption that the turns-ratio is unaffected by the magnitude of the applied transformer voltages and currents (i.e., the assumption of linearity). The per-unit equivalents of (A.5) and (A.6) may be written as

$$E_1 = E_2 \tag{A.7}$$

$$I_1 = I_2. \tag{A.8}$$

Equations (A.7) and (A.8) show that the advantage of using the ac per-unit system over using actual system values lies in the fact that the per-unit system scales all quantities so that phasor currents and voltages on both sides of the transformer are the same in magnitude and phase. Thus, all transformer turns-ratios are scaled so that each is effectively 1:1. For the purposes of calculating system voltages and currents under normal operating conditions, the 1:1 transformer may be replaced by a circuit node.

Now consider the three-phase full-wave bridge rectifier, shown in Fig. 2(a), as a network element that, as will be shown, has some similarities to a transformer. The terminal relationships for this circuit in actual units are

$$\hat{E}_{dc} = \hat{E}_2 = \frac{3\sqrt{2}}{\pi} |\hat{E}_{1(LL)}| \cos \alpha - \frac{3}{\pi} \hat{X}_c \hat{I}_d \tag{A.9}$$

$$|\hat{I}_1| = \frac{\sqrt{6}}{\pi} \hat{I}_d. \tag{A.10}$$

Note that (A.10) is only an approximate relationship which is accurate to within 0.8 percent under normal conditions. A third equation which is also useful in defining the behavior of the bridge converter comes from equating the real power on the ac and dc sides of the converter. This expression is

$$3|\hat{E}_{1(LL)}| \, |\hat{I}_1| \cos \phi = \hat{E}_2 \hat{I}_2. \tag{A.11}$$

Substituting (A.10) into (A.11) gives

$$\frac{3\sqrt{6}}{\pi} |\hat{E}_{1(LL)}| \, \hat{I}_d \cos \phi = \hat{E}_2 \hat{I}_2. \tag{A.12}$$

Cancelling I_d from both sides yields

$$\hat{E}_2 = \frac{3\sqrt{6}}{\pi} |\hat{E}_{1(LL)}| \cos \phi. \tag{A.13}$$

If the bridge rectifier is viewed as an "ac-to-dc transformer," then comparison of (A.10) and (A.6) shows that the transformer has a $1:\sqrt{6}/\pi$ turns-ratio with a complete loss of phase information. If (A.13) and (A.5) are compared, then the voltage turns-ratio appears to be $1:\pi/(3\sqrt{6}\cos\phi)$ with a complete loss of phase information. While the current turns-ratio is fixed (i.e., a linear transformation), the voltage turns-ratio varies as ϕ varies. Thus, the voltage turns-ratio represents a nonlinear transformation and no one per-unit system can be chosen which will allow Fig. 2(a) to be reduced to a circuit node.

If (A.9) and (A.6) are compared, then the "ac-to-dc transformer" appears as a lossy transformer with voltage turns-ratio $1:\pi/(3\sqrt{2}\cos\alpha)$ and dc side resistance $3/\pi \, \hat{X}_c$. It must be emphasized that this dc side resistance is a fictitious resistance which absorbs no real power. This resistance is simply the circuit analog for the voltage reducing role played by the commutation reactance. When viewed in this way, the voltage turns-ratio is linear but variable since the firing angle may be changed. Hence, no fixed per-unit system can be used to reduce Fig. 2(a) to a circuit node and series resistance. Since no per-unit system is capable of achieving the desired reduction in circuit complexity, the dc system per-unit base quantities may be chosen arbitrarily provided they are internally consistent. A convenient choice which permits the per-unit current and real power to be equal on both sides of the converter is

$$P_{\text{dc-BASE}} = VA_{\text{ac-BASE}} \tag{A.14}$$

$$I_{\text{dc-BASE}} = \pi/\sqrt{6} \, I_{\text{ac-BASE}}. \tag{A.15}$$

Equation (A.14) permits per-unit real power on both sides of the rectifier to be compared directly since the base scaling factors are identical. Comparing (A.15) and (A.10) also shows that a 1.0 per-unit ac current equals a 1.0 per-unit dc current. For example, if

$$\frac{\hat{I}_1}{I_{\text{ac-BASE}}} = I_1 = 1.0 \angle \xi \tag{A.16}$$

then

$$I_d = \frac{\hat{I}_d}{I_{\text{dc-BASE}}} = \frac{\pi/\sqrt{6}\,|\hat{I}_1|}{\pi/\sqrt{6}\, I_{\text{ac-BASE}}} = |I_1|. \tag{A.17}$$

Selection of $P_{\text{dc-BASE}}$ and $I_{\text{dc-BASE}}$ automatically determines $V_{\text{dc-BASE}}$ and $R_{\text{dc-BASE}}$ as

$$V_{\text{dc-BASE}} = \frac{P_{\text{dc-BASE}}}{I_{\text{dc-BASE}}} = \frac{VA_{\text{ac-BASE}}}{\pi/\sqrt{6}\, I_{\text{ac-BASE}}} = \frac{3\sqrt{2}}{\pi} V_{\text{ac-BASE}} \tag{A.18}$$

$$R_{\text{dc-BASE}} = \frac{V_{\text{dc-BASE}}}{I_{\text{dc-BASE}}} = \frac{\frac{3\sqrt{2}}{\pi} V_{\text{ac-BASE}}}{\frac{\pi}{\sqrt{6}} I_{\text{ac-BASE}}} = \frac{18}{\pi^2} Z_{\text{ac-BASE}}. \tag{A.19}$$

Use of this per-unit system is best exemplified by finding the per-unit equivalent of (A.9). The first step is to divide both sides of (A.9) by $V_{\text{dc-BASE}}$ to get

$$\frac{\hat{E}_2}{V_{\text{dc-BASE}}} = E_2 = \frac{3\sqrt{2}\,|\hat{E}_{1(LL)}|}{\pi\,(V_{\text{dc-BASE}})} \cos\alpha - \frac{(3/\pi)\,\hat{X}_c \hat{I}_d}{V_{\text{dc-BASE}}}. \tag{A.20}$$

Substituting (A.18) and (A.19) into (A.20) gives

$$E_2 = \frac{3\sqrt{2}}{\pi} \frac{|\hat{E}_{1(LL)}|}{\frac{3\sqrt{2}}{\pi} V_{\text{ac-BASE}}} \cos\alpha - \frac{\frac{3}{\pi}\hat{X}_c}{R_{\text{dc-BASE}}} \frac{\hat{I}_d}{I_{\text{dc-BASE}}}$$

$$= |E_1| \cos\alpha - I_d \frac{\frac{3}{\pi}\hat{X}_c}{\frac{18}{\pi^2} Z_{\text{ac-BASE}}}$$

$$= |E_1| \cos\alpha - \frac{\pi}{6} X_c I_d. \tag{A.21}$$

Note that in going from step 1 to step 2 in (A.21), it is assumed that the X_c per-unit value will be expressed using the ac-base impedance rather than the dc-base resistance. The choice of which base quantity to use in this case is purely arbitrary. Similar operations on (A.10) and (A.13) give

$$|I_1| = I_d \tag{A.22}$$

$$E_2 = |E_1| \cos\phi. \tag{A.23}$$

Equations (A.21)–(A.23) are the per-unit equations which characterize the bridge converter. Kirchhoff's voltage law can be used to verify that the electrical analog for per-unit equations (A.21) and (A.22) is Fig. 2(b).

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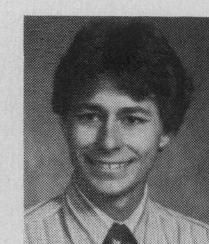

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