

# Unified Newton Framework For The Steady State Simulation Of Networks With Multiple Ac-Dc Converters

J.Arrillaga\*, N.R.Watson\* and G.N.Bathurst\*\*

University of Canterbury, Christchurch, New Zealand\*

Phone: +64-3-3667-001, Fax: +64-3-3642-761

E-Mail: n.watson@elec.canterbury.ac.nz

UMIST, Sackville street, Manchester, U.K.\*\*

## Abstract

*As the numbers of converters and other power electronic devices of large rating increase, the search for the 'ultimate Jacobian' capable of representing the overall system behaviour continues. This paper describes the structure of a flexible Newton framework capable of incorporating the three-phase power flow and harmonic analysis in networks containing multiple ac-dc converters and, potentially, any other types of non-linearities.*

**Key words:** power system simulation, ac-dc converters, harmonics

## 1 Introduction

The increasing presence of large power converters in a modern power system adds controllability at the expense of waveform distortion. The derivation of the current and voltage harmonic content is normally achieved in three stages. First a fundamental frequency power flow (preferably of the three-phase type) is carried out to determine the voltages at the converter terminals. Then using the non-linear characteristics of the converter ac-dc transformation, the terminal voltages permit the derivation of the harmonic (and inter-harmonic) currents. Finally, the latter are injected into the (linear) network to determine the harmonic flows.

In general, however, the power flow, the harmonic current sources and the harmonic penetration study use three independent programs and a direct solution. This approach ignores the harmonic interaction between the various converters present in the system and between the fundamental frequency power flow and the harmonic frequencies. Thus, neither the harmonic sources nor the power flow will be assessed accurately. Such interaction is likely to be of little significance to the power flow solution but can have serious consequences on the harmonic content. Newton-based hybrid (i.e. time and harmonic domain) algorithms with varying degrees of complexity have been proposed for this purpose [1][2]. However, the sequential nature of the hybrid methods causes convergence problems in difficult cases. A more reliable solution is achieved with a full Newton method, referred to as the Harmonic Domain. Moreover, the accuracy of the Harmonic Domain is enhanced by its ability to represent the harmonic modulation produced by the control system on the switching instants [3].

As the numbers of converters and other power electronic devices of large rating increase, the search

for the 'ultimate Jacobian' capable of representing the overall system behaviour continues. This paper describes the structure of a flexible Newton framework capable of incorporating the three-phase power flow and harmonic analysis in networks containing multiple ac-dc converters and, potentially, any other types of non-linearities.

## 2 Common Framework For Harmonics And Power Flow

The conventional Newton method solves the system of non-linear equations, by calculating updates iteratively, as follows

$$\begin{aligned} J\Delta X^N &= F(X^N) \\ X^{N+1} &= X^N - \Delta X^N \end{aligned} \quad (1)$$

where  $J$  is the Jacobian matrix of partial derivatives of  $F$ , i.e.

$$J_{ij} = \frac{\partial F(X^N)_i}{\partial X_j^N} \quad (2)$$

In practice the system of linear equations (1) is solved by LU decomposition, i.e. is solved for  $y$  followed by a solution of  $x$ . The Jacobian sparsity is exploited by using a sparse bifactorisation (e.g. the package <http://netlib.att.com/netlib/y12m>) for the solution of the linear system at each iteration. Despite the use of sparsity, the bifactorisation still represents the bulk of the calculation in each iteration. Overall computation time is greatly reduced by holding the Jacobian constant in the vicinity of the solution, where the equation system is more linear.

### 2.1 Harmonics Jacobian

In the frequency domain, if the analysis takes only one cycle of the ac voltage as the fundamental frequency, the solution provides only harmonic frequencies and is termed the Harmonic Domain [4]. However, techniques are becoming available to extend the method to interharmonic frequencies [5]. In the Harmonic Domain formulation the change from time to frequency domain is achieved by convolving periodic sampled quantities of voltage and current with their sampling functions. The latter are defined in terms of the exact switching instants, which are obtained as part of the overall iterative procedure that accurately models the effect of ac voltage and dc current distortion on the valve conducting periods.

A six-pulse converter bridge passes through 12 states per cycle. Six of these are commutation states

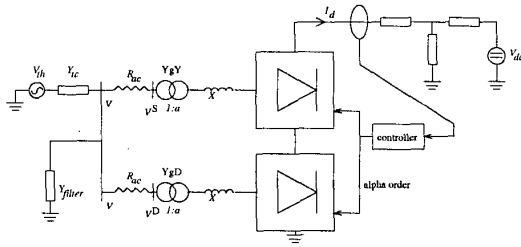


Figure 1. 12-pulse converter model test system

Figure 2. Sparsity structure of the converter Jacobian for 13 harmonics

and six are 'direct' conduction states. Each state is represented by a simple linear circuit, the particular configuration of each circuit depending upon the conduction pattern of the valves in the bridge. Expressions for the commutating currents are easily derived from the commutation circuit, taking into account the transformer connection. The sampling instants, on the other hand, require very complex analysis[8] that takes into account the modulating effect of the ac voltage and dc current distortion on the firing and end of commutation instants.

The outcome of each steady state solution is a harmonic spectrum which, when transformed into the time domain, matches the dc voltage (or ac current) during the appropriate conducting interval only. The final objective is a single spectrum that represents a complete cycle of the dc voltage (or ac current), NOT the 12 spectra of the individual states.

#### Newton's Solution for a Single Converter System

This section describes a Newton solution for the steady state interaction of a controlled converter with the ac and dc systems. The basic 12-pulse system is shown in Figure 1. The ac network is represented by a three-phase Thevenin equivalent and the dc system by a T-circuit equivalent and a dc source.

The variables in the system of Figure 1 are the terminal voltage harmonic phasors,  $V^S$  and  $V^D$ , the ac current phasors on the transformer primaries,  $I_P^S$  and  $I_P^D$ , and secondaries  $I_S^S$  and  $I_S^D$ , the dc current harmonics,  $I_d$ , the firing instants,  $\theta_i$ , the end of commutation instants,  $\phi_i$ , the average delay angle,  $\alpha_0$  and, in the case of constant power control, the average dc current,  $I_{d0}$ .

Eight functional relationships are used in reference [8] to describe the system of Figure 1, representing 426 equations in as many unknowns for a solution up to the 50th harmonic, i.e. 300 ac voltages [3 (phases)  $\times$  2 (real and imaginary components)  $\times$  50 (harmonics)], 100 dc currents [50 (harmonics)  $\times$  2 (real and imaginary)], 12 firing instants [6 for the star and 6 for the delta connected bridges], 12 end of commutation instants [again for the star and delta bridges], 1 constant power control and 1 average delay angle.

The Jacobian elements can be derived either

numerically (by partial differentiation) or analytically (obtaining the partial derivatives in analytical form). As shown in Figure 2 (for the case of 13 harmonics), the elements of the Jacobian are ordered in blocks corresponding to the three phases of terminal voltage, the dc current, the end of commutation angles, the firing angles, and the average delay angle.

For the configuration of Figure 1, the blocks associated with interactions between the dc current harmonics and the ac voltage harmonics comprise the 'ac/dc partition', which is 104 elements square [13(ac side harmonics)  $\times$  3(phases)  $\times$  2(real and imaginary) + 13 (dc side harmonics)  $\times$  2 (real and imaginary)]. All the other parts of the Jacobian contain the 'switching terms'. The Jacobian is made extremely sparse by removing the terms which are approximately zero.

## 2.2 Power Flow Requirements

Three-phase simulation is essential when analysing the interaction between static converters and ac systems, as the presence of any type of asymmetry results in uncharacteristic harmonics and possible waveform instability [6].

Unification of the three-phase power flow and harmonic interaction at the converter terminals imposes a different set of requirements on the power flow implementation than those that led to the development of the fast-decoupled power flow. First, the converter harmonic model is necessarily in Cartesian co-ordinates[3]. If the power flow equations are in polar co-ordinates, as required by the decoupling concept, non-linear polar transforms must be carried out at each iteration to interface with the converter ac terminal (at fundamental frequency). This is likely to increase the number of iterations to convergence substantially, as well as complicating the power flow implementation. An additional factor to consider is that the converter equations take longer to calculate than the solution of the prefactorised Jacobian system. It is therefore desirable to reduce the number of converter mismatch equation evaluations by reducing the number of iterations to convergence. This can be achieved by using the full Jacobian matrix, with no decoupling. Taken together these points indicate that a unified power flow and harmonic solution in Cartesian co-ordinates will be more efficient than one in polar co-ordinates.

## 2.3 Combined Jacobian

The nine busbar network of Figure 3 is used to illustrate the structure of the combined power flow and harmonic solution. The slack busbar is at ROXBURGH011, and loads are placed at ROXBUR220, INVERCARG011 and TIWAI220. Also, the rectifier end of the CIGRE benchmark model [7] is connected at the TIWAI busbar.

The combined Jacobian is illustrated in Figure 4. In this figure, the busbars and specifications of the power flow solution can be observed on the top left hand side. Of course the structural sparsity would be much greater for a realistically sized system, and is clearly not symmetric. The lower right hand

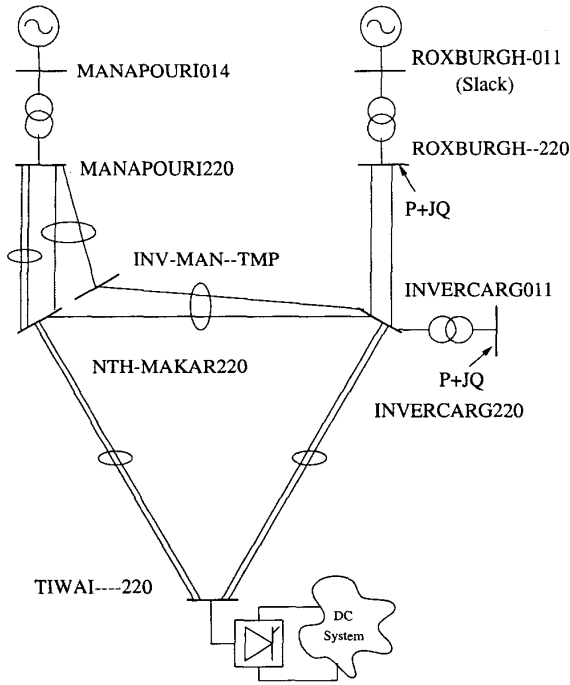


Figure 3. Test system for three-phase power flow

side matrix block represents the converter harmonics solution, which is seen to be coupled only to the fundamental frequency voltages at the converter terminals.

The combined Jacobian contains no more terms than the separate Jacobians of the load flow and converter, and yet convergence has been found to be faster and more robust than a fixed point iteration between separate load flow and converter model updates, which usually diverges.

The Thevenin equivalent of the ac system is retained at all harmonic frequencies except the fundamental, which is the point of coupling to the power flow equations. Similarly, three-phase power flow equations are retained at every bus except the converter bus. It is assumed that were it not for the converter, the converter bus would be PQ. Since the power flow is three-phase, six mismatch equations (real and imaginary parts) are required at the converter bus at fundamental frequency.

Because the processing required at every iteration is dominated by the converter equations, it is essential that convergence of the load flow part of the system be faster than that of the converter model, which typically converges in seven iterations. The decoupled load flow method is not suitable as it requires more iterations for convergence. As explained in Section 2.2, a decoupled load flow, being framed in polar co-ordinates, is also incompatible with the converter model in Cartesian components, unless a non-linear polar transform is applied at every iteration at the converter bus. This additional non-linearity would be likely to degrade convergence, especially if the Jacobian is held constant. Integration of the

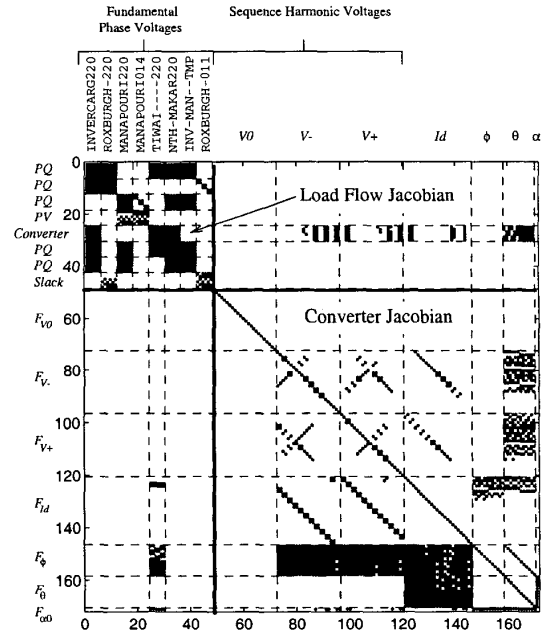


Figure 4. Jacobian matrix for an integrated load flow and harmonic solution

converter model with a load flow therefore requires that the load flow be reformulated in Cartesian components, with no decoupling in the Jacobian matrix.

### 3 General Framework For Systems With Multiple Converters

The method and components of the Newton solution have already been discussed in previous sections with reference to a single 12-pulse converter connected to an ac power system. This section describes the structure of a General Jacobian capable of accommodating, as well as a three-phase power flow, multiple converters of different pulse numbers and dc configurations placed in separate locations.

While all the system busbars must be explicitly represented at the power frequency, due to the non-linear nature of the power flow specifications, only the converter busbars are non-linear at the harmonic frequencies. Thus, in the latter case, the linear network can be reduced to an equivalent system that links the non-linear device busbars. The linear reduction used also produces equivalent current injections for remote constant voltage or current harmonic sources.

#### 3.1 Test Case

Figure 5 shows the structure of the Jacobian representing the power and harmonic flows in the New Zealand South Island transmission system. This system consists of 110 busbars, of which 14 are generation busbars, 24 PQ loads and four are large rectifier busbars. The total system loading is about 2500 MW of which over half is rectified.

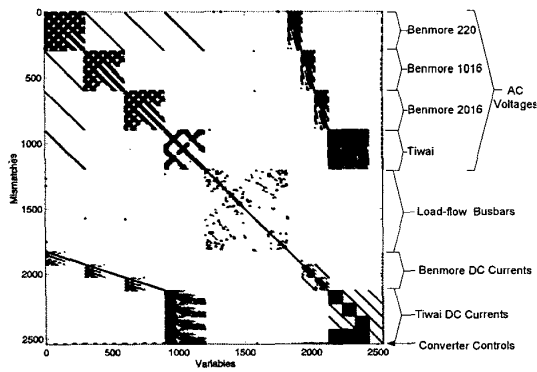


Figure 5. Jacobian structure of the New Zealand South Island system)

Consequently there has been a question mark over the degree of interaction between the four rectifier busbars (shown in Figure 6).

Three of the busbars are grouped together at Benmore in the middle of the South Island and form one end of the 1000 MW bipolar HVdc link. The fourth is located at Tiwai, at the southern end of the S.Island, and is a 500 MW aluminium smelter.

As explained in the appendix 426 mismatches are needed at each converter bus, consisting of 300 ac voltages, 100 dc currents, the remaining 26 being switching and control terms. These three components are separated in the generalised Jacobian, as shown in Figure. 5, where all the switching terms are placed last.

The test system is used to assess the ability of the unified algorithm to determine the degree of interaction between the multiple converters. At light load the Benmore-220 filters are normally out of service to minimise excessive VAR generation. Under these conditions, the results of the unified algorithm, shown by the black trace in Figure 7, are considerably different than those of the simplified model; this indicates that in this case there is a noticeable harmonic interaction between the Tiwai-033 and Benmore converters.

#### 4 Conclusions

A full Newton method, based on the so called Harmonic Domain, has been developed to provide accurate and reliable steady state solutions in networks containing multiple ac-dc converters. The unique reliability property of the proposed algorithm, as compared with earlier methods which often diverge, is related to the use of a unified full Newton solution. This solution incorporates, in a single Jacobian matrix, the non-linear (three-phase) power flow at fundamental frequency, a full individual representation of the linear network at any required harmonic (or inter-harmonic) frequency and a detailed harmonic representation of each present ac-dc converter.

Although the dimensions of the resulting Jacobians are extremely large, their mostly block diagonal and

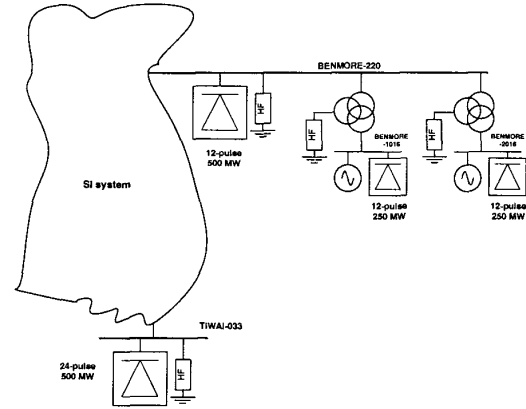


Figure 6. Multiconverter system (New Zealand South Island)

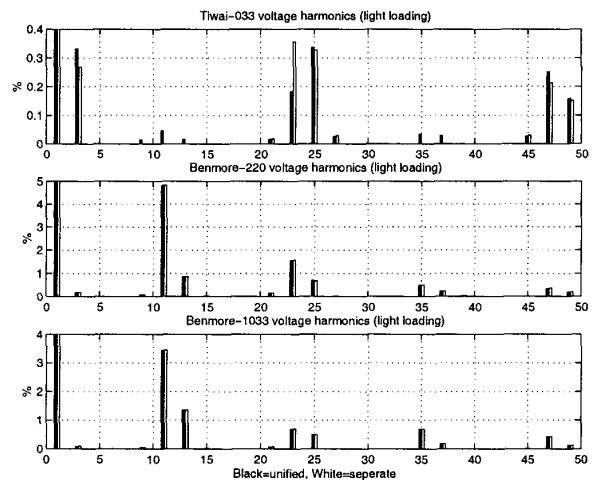


Figure 7. Comparison of harmonic voltages with the Benmore filters out of service

sparse nature has been fully exploited to ensure efficient solutions.

The new algorithm has been used to investigate the steady state frequency interactions in the New Zealand South Island power system. It has been found that the effect of 'normal' harmonic distortion on the power flow results is not significant. However the reverse, i.e. the effect of including the power flow solution, particularly in the presence of transmission unbalance, can produce considerable interaction at non-characteristic harmonic frequencies.

## References

- [1] M.Valcarcel and J.G.Mayordomo, 'Harmonic power flow for unbalanced systems', IEEE PES, Winter meeting 93, WM 061-2 PWRD, 1993.
- [2] W.Xu, J.R.Marti and H.W.Dommel, 'A multi-phase harmonic load flow solution technique', IEEE PES, Winter Meeting 90, WM 098-4, PWRS, 1990.
- [3] B.C.Smith, A harmonic domain model for the interaction of the HVdc converter with ac and dc systems', Ph.D.Thesis, University of Canterbury, New Zealand, 1996
- [4] B.C. Smith, N.R. Watson, A.R. Wood, and J. Arrillaga, 'Steady state model of the ac/dc converter in the harmonic domain', Proc. IEE on Gener., Transm. Distrib., 1995, 142, (2), pp.109-118.
- [5] G.N. Bathurst, N.R.Watson and J.Arrillaga, 'Adaptive frequency selection method for a Newton solution of harmonics and inter-harmonics', Proc.IEE Gener.Transm.Distr., 2000, 147(2), pp. 126-130.
- [6] J.D.Ainsworth, 'Harmonic instability between controlled static converters and ac networks', Proc.IEE,1967,114(7) :949-957.
- [7] M. Szechtman, T.Wess and C.V.Thio, 'First benchmark model for HVDC control studies', ELECTRA, 135, pp. 55-75.
- [8] J.Arrillaga, B.C.Smith, N.R.Watson and A.R.Wood, 'Power System Harmonic Analysis', John Wiley and Sons, London, 1997.