

# A harmonic domain solution for systems with multiple high-power AC/DC converters

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**Abstract:** The unified Newton solution used to derive converter/system interactions in the harmonic domain is extended in the paper to systems with multiple AC/DC converters in different locations. The extended solution is used to investigate potential interactions between the HVDC link and aluminium smelters in the New Zealand system. The inaccuracies of conventional direct harmonic solutions are highlighted by comparison with the results obtained with the proposed general iterative solution.

## 1 Introduction

The AC waveforms at AC/DC converters and their characteristic harmonic components are usually derived from information obtained from single-phase AC/DC loadflows. By ignoring system and converter asymmetries, this approach does not predict noncharacteristic harmonic behaviour.

Accurate derivation of the noncharacteristic harmonic currents requires an assessment of the unbalance at the converter terminal, due both to the AC system and the converter itself. The components and simultaneous solution of both the AC system and converters have already been described, i.e. the three-phase load flow [1], three-phase harmonic representation of the AC system [2], and the three-phase harmonic converter model [3]. The principles of a hybrid time/frequency domain solution have also been established [4]. In the solution models proposed so far, however, the problem is partitioned into several modules which are then solved separately. For example, the solution is iterated between separate three-phase load flows and harmonic assessments [5, 6]. Convergence of the decoupled methods depends directly on the degree of decoupling between the component modules; even a relatively small degree of coupling can cause divergence. This being the case, a unified Newton solution of all the unknowns is preferred.

Moreover, the design of large converter plant (in hundreds of megawatts) is often carried out in the absence of other similar distorting sources in the power system. This approach is not recommended due to the multiplicity of high power electronic devices, such as back-to-back HVDC, aluminium smelters etc. which are often geographically close to each other.

This paper describes a general Newton algorithm of the harmonic power flow that can incorporate the various nonlinear components present in the system in a single

Jacobian. The primary motivation for the investigation was the need to assess the potential for harmonic interaction between the HVDC link and aluminium smelters directly connected to the New Zealand primary transmission system.

With this aim, the harmonic voltages solved by the unified Newton method of the multiconverter system are compared with those where only individual converters are represented. A comparison is also made between the solved harmonic voltages of the full system and those derived from a three-phase direct harmonic penetration method.

## 2 Modified AC/DC three-phase loadflow

In fundamental frequency load flows, the voltages are normally described in polar co-ordinates to take advantage of the decoupling existing between voltage magnitude and real power, and phase angle and reactive power.

However, specifying the harmonic voltages and currents in polar co-ordinates leads to ill-conditioning because of the large disparity in the size of the angles and magnitudes for small harmonics (the phase angles can vary between 0 and  $2\pi$  radians); therefore Cartesian co-ordinates are used instead. The latter also avoids the need for nonlinear polar transforms when the load flow is interfaced to harmonic models of nonlinear components framed in Cartesian co-ordinates.

In Cartesian co-ordinates the Jacobian matrix contains twice as many terms, which means that the calculation of the voltage updates at each iteration of Newton's method takes approximately twice as long. However, this stage represents only a small proportion of the overall execution time of the combined load flow and nonlinear component model.

In the absence of large power converters, although the decoupled load flow takes more iterations to converge than the full Newton load flow in Cartesian co-ordinates, the former executes in less time because each iteration is faster. The reverse is true when the load flow is interfaced to a converter model, because each iteration of the converter solution is comparatively slow due to the model complexity; it is therefore important, in this case, to reduce the number of iterations and the full Newton solution is favoured.

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In a three-phase load flow, the real and reactive power in the three-phases ( $a, b, c$ ) at  $PQ$  (loads) busbars are independently specified, and may be unbalanced. The Newton solution mismatches required for the three-phase loadflow solution are as follows

$$\begin{aligned} M_{P_a} &= P_a + \text{Real}\{V_a I_a^*\} \\ M_{Q_a} &= Q_a + \text{Imag}\{V_a I_a^*\} \\ M_{P_b} &= P_b + \text{Real}\{V_b I_b^*\} \\ M_{Q_b} &= Q_b + \text{Imag}\{V_b I_b^*\} \\ M_{P_c} &= P_c + \text{Real}\{V_c I_c^*\} \\ M_{Q_c} &= Q_c + \text{Imag}\{V_c I_c^*\} \end{aligned} \quad (1)$$

$V$  and  $I$  are related by the nodal admittance matrix for the system:

$$I = YV \quad (2)$$

and, consequently,  $I$  is a function of all the nodal busbar voltages to which busbar 1 is connected.

The admittance matrix, being three-phase, models the unbalance effect of transmission line, transformer and shunt components. Asymmetry in the load-flow solution can, therefore, arise from either the admittance matrix, or unbalanced load specifications at  $PQ$  busbars. However, synchronous machines at  $PV$  busbars can be assumed to be symmetric. The three-phase formulation also takes care of harmonic phase shifting across transformers.

At generator ( $PV$ ) busbars, only the positive-sequence real power and voltage are specified, but four extra equations are required to represent the negative and zero sequence currents that flow into the generator in the presence of voltage distortion (the subtransient reactance is used for this calculation):

$$\begin{aligned} M_{P_+} &= P_{set} - \text{Real}\{V_+(I_+ + I_{load+})^*\} \\ M_{V_2} &= V_{set}^2 - \text{Real}\{V_+ V_+^*\} \\ M_{Rn} &= \text{Real}\{I_- + V_- Y_{gen-} + I_{load-}\} \\ M_{In} &= \text{Imag}\{I_- + V_- Y_{gen-} + I_{load-}\} \\ M_{R0} &= \text{Real}\{I_0 + V_0 Y_{gen0} + I_{load0}\} \\ M_{I0} &= \text{Imag}\{I_0 + V_0 Y_{gen0} + I_{load0}\} \end{aligned} \quad (3)$$

At the slack busbar, the generated power set point  $P_{set}$  is maintained as a controlled variable, which allows this busbar to be of a more general form, such as an HVDC inverter.

### 3 Harmonic domain modelling of AC/DC converters

A six-pulse thyristor bridge converter can be viewed as a four-port circuit, i.e. with two inputs and two outputs [3]. The inputs in this case are the AC phase voltage spectra and the DC current spectra. The outputs are the AC phase current and the DC voltage spectra, respectively. The convolution technique approximates the transferred waveshapes (DC voltage and AC phase currents) to piecewise waveshapes consisting of twelve distinct periods of conduction. These periods of conduction are defined explicitly by the switching angles of the converter. In the case of the DC voltage transfer, twelve analytic frequency-domain expressions are derived from nodal analysis of the twelve conduction circuits that determine the steady-state waveshapes corresponding to those circuits. The spectra calculated using these expressions are then convolved with those of band-limited rectangular windowing functions, and the resultant sum of the convolved spectra is that of the total

waveshape. For instance, Fig. 1 illustrates one convolution for the DC voltage of a six-pulse converter as seen in the time domain.

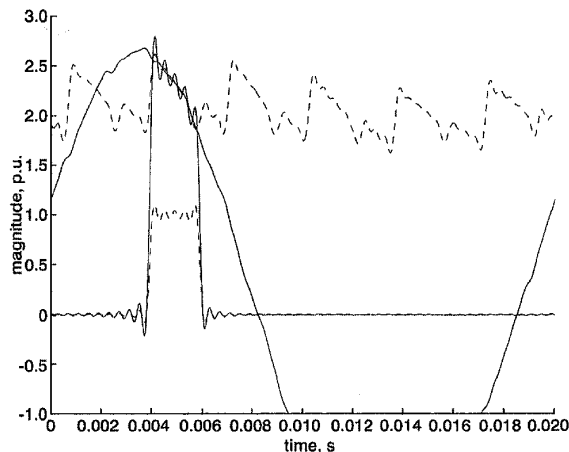


Fig. 1 Convolution sampling for a 6-pulse bridge

A similar process is followed to obtain the harmonic spectra of the AC phase currents. These transfers are general for both inverter and rectifier, with only the direction of the DC current flow and the resultant sign of the DC voltage across the bridge being different.

Higher-pulse configurations are normally achieved by using transformer phase-shifting to provide harmonic cancellation. Therefore, to achieve the standard 12-pulse configuration, two convolution models are used, one star/star and the other star/delta. The harmonic transfers from each model are combined to yield the full transfer characteristic. For higher-pulse converters, some new extra nodal equations must be used for the required transformer configurations [7].

The harmonic transfers across a converter largely depend on the modelling accuracy of the switching instants, particularly the end of commutation angle, defined as the crossover point of the commutation currents with the DC current (harmonics included). For the convolution model these angles are obtained from 12 single-variable Newton-Raphson steps. The firing angles are dependant on the control strategy, but can also be solved using single-variable iterative steps. As the full solution Jacobian still includes the effect of modulating these switching instances, the main solution is still of a full Newton nature.

### 4 Structure of the multiconverter harmonic and power flow solution

The basis of the algorithm is a unified Newton solution that incorporates all the system components, linear and nonlinear, in a single Jacobian. However, the Jacobian only needs to be approximate, and so only the significant terms need to be retained. Thus, the full harmonic Jacobian can be about 96% sparse without affecting convergence. The Jacobian elements are best calculated analytically, even though they require significant effort to derive them, because they increase the solution speed by a factor of 50 with respect to the use of the numerically obtained alternative.

#### 4.1 Variable selection and initialisation

To minimise initialisation complexity, the least-distorted system electrical quantities are chosen as variables. These are normally the AC voltage and DC current harmonics, respectively. The calculation of the mismatch equations

required by Newton's method is also simplified by using AC current mismatches and DC voltage mismatches. This allows the mismatch calculations to be simply the difference between the linear-system calculated quantities and the transferred quantities from the converter models. While the converter could use  $PQ$  mismatches at the power frequency, for connection to the load flow, it is preferable to use current mismatches for consistency. While not globally convergent, the Newton method will converge quadratically, provided that the variable initialisation is sufficiently close to the solution. Good variable initialisation is important for reliable Newton solution convergence. For the full harmonic solution this is achieved in a two stage process. The first initialisation stage is achieved using either a positive-sequence power flow and classical converter equations, or a similar approximate method [8]. This then initialises a three-phase power-flow solution with the fundamental frequency and control variables. Finally, the solution of this three-phase power flow is used to initialise the full harmonic solution. The result is a fast and robust algorithm.

#### 4.2 Reduction of the linear system at harmonic frequencies

While all busbars must be explicitly represented at the power frequency, due to the nonlinear nature of the load-flow specifications, only the converter busbars are nonlinear at the harmonic frequencies. Thus, in the latter case, the linear network can be reduced to an equivalent system that links the nonlinear device busbars. The linear reduction used also produces equivalent current injections for remote constant voltage or current harmonic sources [9].

The system transmission lines are calculated from their physical geometry and are represented in the solution as equivalent  $PI$  models at each harmonic. When lacking detailed information on the passive loads composition, a simple series  $RL$  equivalent, based on the real and reactive loading information, is the default option. A similar approach is used for the generator and transformer components at harmonic frequencies.

Unlike the direct method, where the harmonic variables are calculated directly, in the unified Newton approach these variables are adjusted at every iteration.

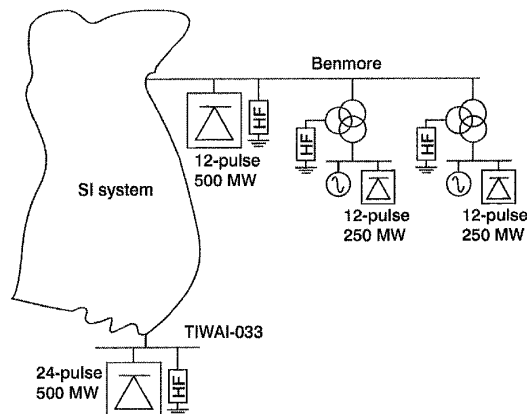


Fig. 2 Transmission system of South Island, New Zealand

#### 4.3 The generalised Jacobian

The structure of the Jacobian needed to represent a multi-converter system is described here with reference to the New Zealand South Island transmission system. This system consists of 120 busbars, 10 six-pulse converters, 71 transmission lines, 24  $PQ$  specified loads, 14 PV specified generators, 21 filter branches and 58 transformers. The

total system loading is about 2500 MW, of which over half is rectified. Three busbars are grouped together at Benmore (as shown in Fig. 2) and form one end of the 1000 MW bipolar DC link; a fourth busbar, located at Tiwai, connects a 500 MW aluminium smelter.

Fig. 3 illustrates the position in the Jacobian of the various components involved. The load flow block is shown in the middle of the matrix and involves 720 variables (120 nodes  $\times$  3 phases  $\times$  2 for real and imaginary components). At the harmonic frequencies, the AC side of each converter is represented by 300 variables, i.e. 50 (harmonics)  $\times$  3 (phases)  $\times$  2 (real and imaginary); these are placed on the top left of the Jacobian. The DC side of each series-connected converter requires 100 (50  $\times$  2) variables; those are placed on the top right and bottom left parts of the Jacobian. The DC sides of the 24-pulse parallel connected converter requires 300 variables, i.e. 50 (harmonics)  $\times$  3 ( $n - 1$  branches)  $\times$  2 (real and imaginary), placed immediately below the direct currents of the series converters. The set is completed by 8 control variables which are placed at the bottom right of the Jacobian matrix.

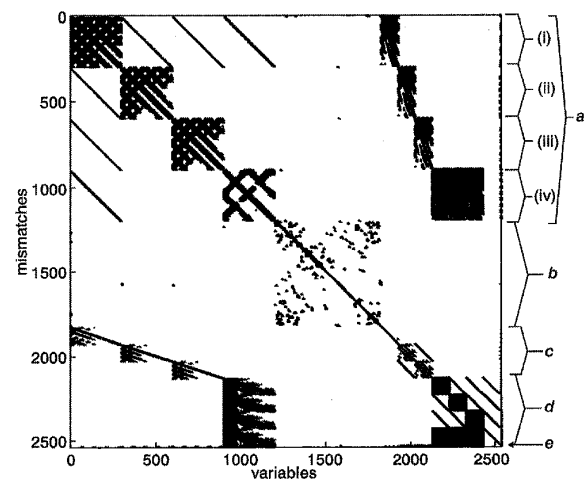
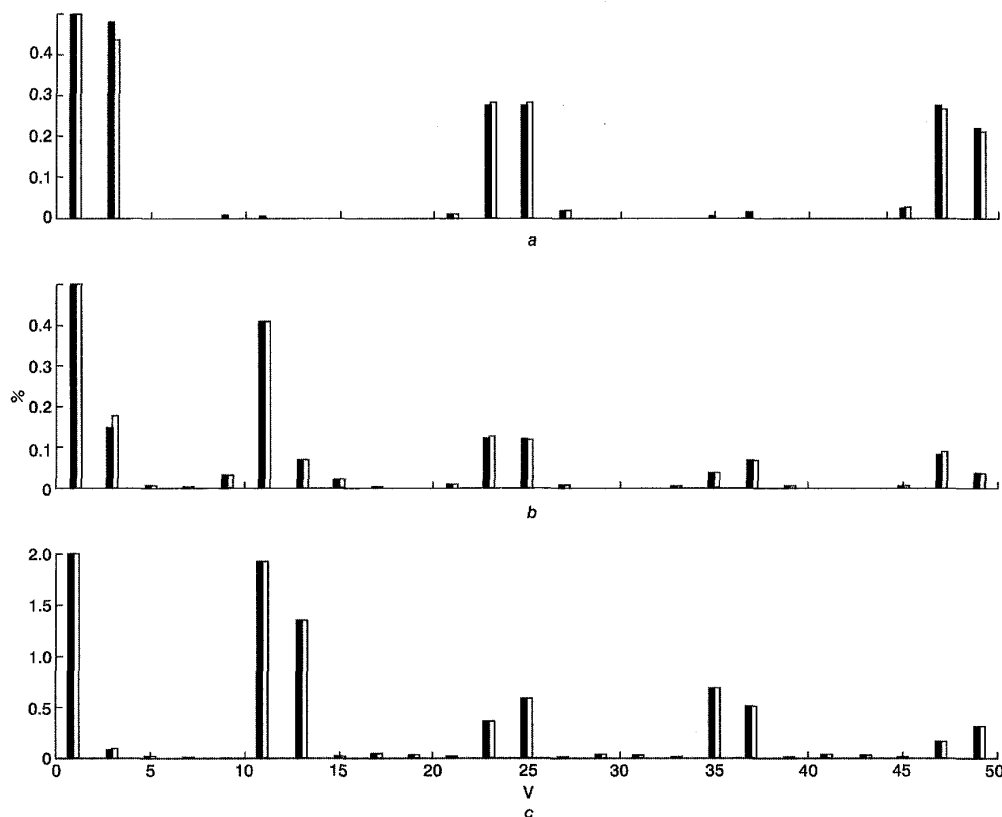


Fig. 3 Structure of the sparse multiconverter Jacobian  
a AC voltages: (i) Benmore 220, (ii) Benmore 1016, (iii) Benmore 2016, (iv) Tiwai  
b Load-flow busbars  
c Benmore DCs  
d Tiwai DCs  
e Converter controls

#### 5 Test system results

Ever since the installation of the aluminium smelter at Tiwai (in the New Zealand South Island system), there has been a question mark over the degree of interaction that exists between the four rectifier busbars shown in Fig. 2.

There are two main interactions of interest, that between the Benmore group and Tiwai, and between the Benmore converters themselves. While a fully conclusive study would require more detailed information of the loads under different operating conditions, a substantive indication of the interaction can be obtained using the two extreme operating modes, i.e. heavy and light-load operation. A key difference between the two cases is the degree of damping provided by the passive system loading. This will change the system impedance and consequently the degree of interaction, if any. However, as the aluminium production is primarily a 24h operation, it will remain at full power in both configurations. The HVDC link, on the other hand, is subject to market restrictions (in the new deregulated environment); it is therefore time-dependent like the passive loads. The light-load case is taken as 10% of the normal loading condition.



**Fig. 4** Harmonic voltages with normal loading  
*a* Tiwai-033 voltage harmonics with normal loading  
*b* Benmore-220 voltage harmonics with normal loading  
*c* Benmore-1016 voltage harmonics with normal loading  
 Black = unified; white = separated

As well as the loading changes, it is assumed that the filter banks at both Benmore and Tiwai may be either in or out of service. During the light-loading configuration it is likely that either some or all of the filter banks will be switched out to minimise excessive VAr generation.

Also in order to assess the need for the iterative solution, the results of the unified Newton solution are compared with those derived from a straight three-phase harmonic penetration method.

### 5.1 Interaction between Benmore and Tiwai

Fig. 4 shows the harmonic voltages at each of the converter busbars, i.e. Tiwai-033, Benmore-220 and Benmore-1016, respectively, for the normal load operating condition. The black voltage levels correspond to the rigorous solution, where the three converters are represented in the unified Newton solution. The white voltage levels relate to the simplified solution where only one converter group is represented in turn, with the remaining converter approximated by an equivalent passive load (i.e. without their harmonic contribution). These results indicate that, in this case, there is very little interaction between Benmore and Tiwai converters. Although not shown in the Figure, corresponding solutions with the filters switched off again showed very little difference between the two solutions.

Turning to the light-load case, with the filters kept in service the two solutions produced very similar results, thus indicating again the absence of practical interaction. However, when the reduced DC link loading was accompanied by filters disconnection at Benmore, comparison with Fig. 5 shows noticeable differences in the harmonic voltages at the Tiwai-033 converter busbar. On the other hand,

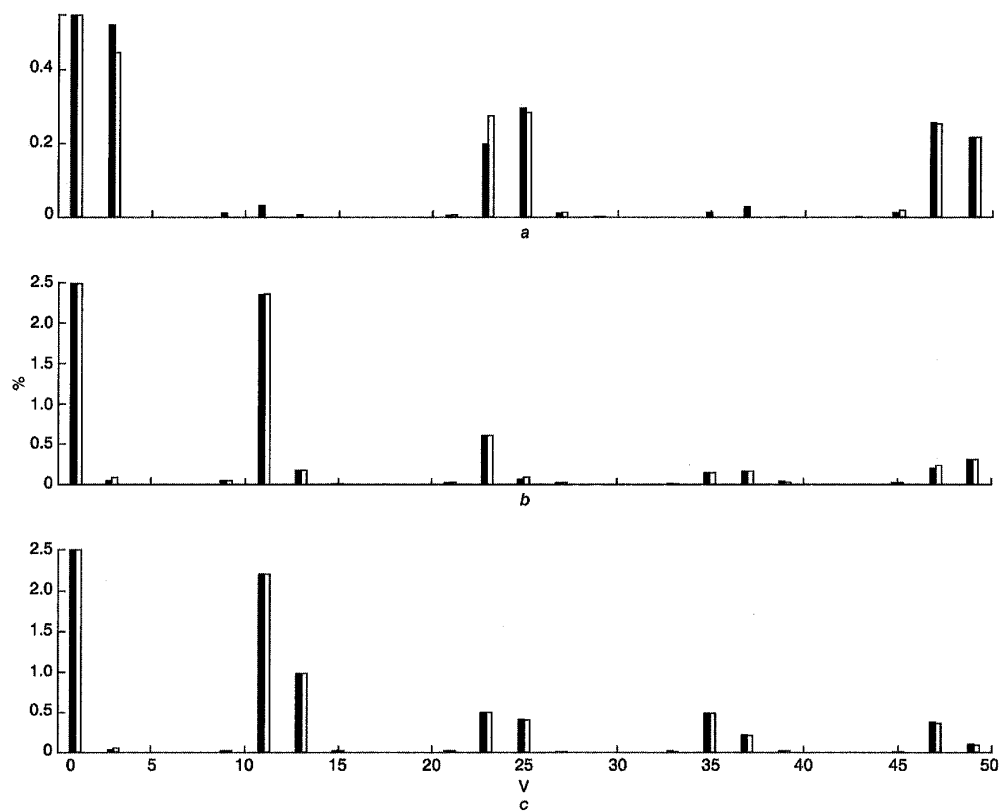
results (not shown) for the presence of the Tiwai-033 converter in the solution indicate that the latter had practically no effect on the harmonic voltages at the two Benmore converter busbars. The degree of interaction was also found to be sensitive to the DC system representation of pole 2, which is directly connected to the 220kV busbar.

### 5.2 Interaction at Benmore between pole 1 and pole 2

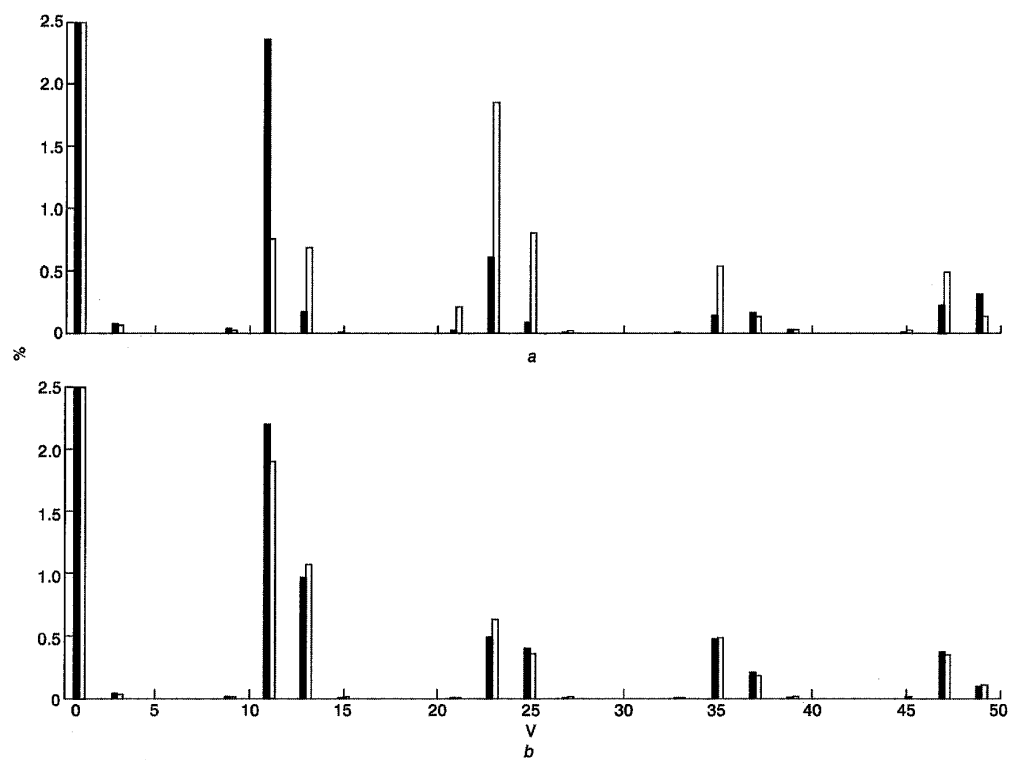
The Benmore HVDC link contains two poles. Pole 1 consists of two identical half-poles formed from the original mercury-arc bipolar scheme as shown in Fig. 2. Each half-pole has a 12-pulse converter and is connected to one of the two 16kV generation busbars. There is an interconnecting three-winding transformer between each generation busbar and the 220kV system and the 33kV tertiary winding only has harmonic filters attached to it. Pole 2 is a new 12-pulse thyristor group with filters directly connected to the 220kV busbar.

Again, to assess the potential interaction, simulations were performed with the whole bipolar link represented, and with only the individual poles; in the latter cases the second pole is represented as a passive load.

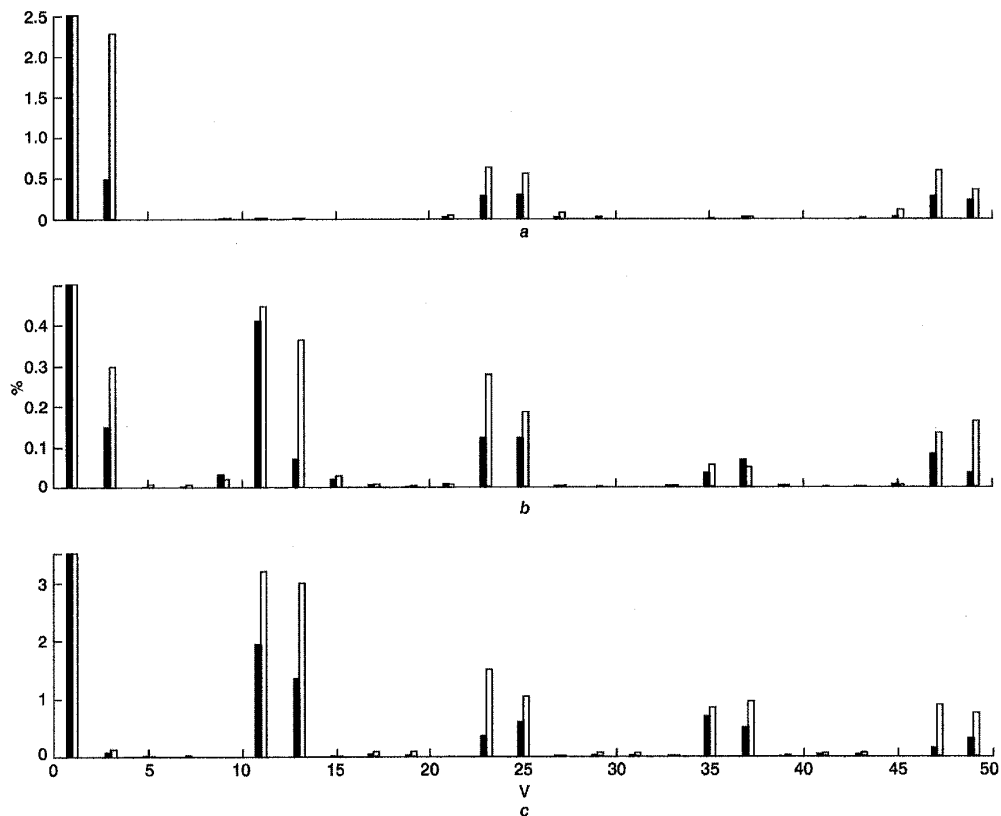
For the full-load case with all the filter banks in service, the results showed very little interaction between the pole rectifiers. Similarly, for the lightly loaded case with the filters in service, there is little sign of interaction. However, as Fig. 6 shows, when the Benmore-220 busbar filters are out of service there is a very noticeable effect on the harmonic voltages between the different solutions. This indicates that there is a reasonable degree of coupling between the converters in this operating condition.



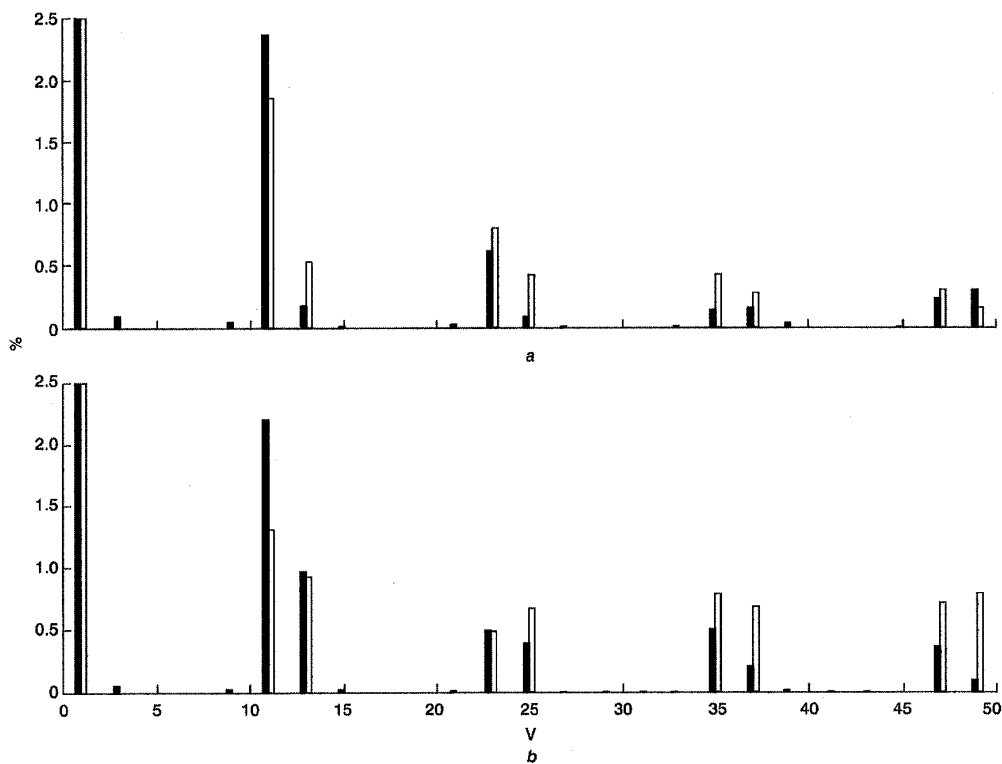
**Fig.5** Harmonic voltages with 10% loading and Benmore-220 filters out of service  
 Black = unified; white = separated  
 a Tiwai-033 voltage harmonics  
 b Benmore-220 voltage harmonics  
 c Benmore-1016 voltage harmonics



**Fig.6** Harmonic voltages resulting from the interaction between the pole 1 and pole 2 converters at Benmore, with 10% loading and Benmore-220 filters out of service  
 Black = unified; white = separated  
 a Benmore-220 voltage harmonics  
 b Benmore-1016 voltage harmonics



**Fig. 7** Comparison between iterative and direct solutions with normal system loading, all filters in service and all converters represented  
 Black = iterative solutions; white = direct solutions  
 a Tiwai-033 voltage harmonics  
 b Benmore-220 voltage harmonics  
 c Benmore-1016 voltage harmonics



**Fig. 8** Comparison between iterative and direct solutions with light system loading, Benmore-220 filters out of service and only Benmore converters represented  
 Black = iterative solutions; white = direct solutions  
 a Benmore-220 voltage harmonics  
 b Benmore-1016 voltage harmonics

Another case requiring investigation is when the Benmore generators are out of service. This can happen under light-load conditions and also requires the filters to be switched out to avoid excessive VAR generation.

### 5.3 Accuracy of the direct penetration method

The majority of harmonic studies performed at present use the direct penetration method. It involves calculating the system harmonic impedance matrix and injecting harmonic currents to obtain the harmonic voltages at each busbar. The current injections are either taken from manufacturers specified tables or calculated from classical converter equations. While accepted that it is an inaccurate solution, it is traditionally assumed that this approach is conservative, i.e. that it predicts higher than expected levels of distortion. This argument was tested by using a direct harmonic solution to the cases shown in Figs. 4 and 6 and comparing them with the full harmonic solution. Figs. 7 and 8 show that, in general, the traditional argument holds, although, of course, it can lead to more expensive filter designs. Moreover, the direct analysis can underpredict at some harmonic frequencies, particularly in the light-load case. Also, the converter DC ripple affects the characteristic harmonics. If this is not taken into account, it can lead to substantial over- and underpredictions.

A possible solution to reduce these effects could be achieved by representing the converter harmonic impedance, so that the direct penetration method uses Norton injections rather than ideal current sources. Some work has shown that this method can improve the results [7], but the converter impedance varies with the operating conditions and its derivation is a very complex exercise.

## 6 Conclusions

A general unified Newton algorithm has been developed that is capable of assessing the load flow and harmonic interactions in a system containing multiple AC/DC converters in separate locations. The proposed solution has been used to try and elucidate a longstanding argument in the New Zealand power system, i.e. the potential interaction between the HVDC converters at Benmore and the

aluminium smelters at Tiwai. The limited study cases carried out so far indicate that, under nominal operation, there is little interaction between the Benmore and Tiwai converters. Interaction can occur, however, under light-load conditions, where some or all of the harmonic filters at Benmore are discontinued. In these cases, it is only the pole-2 converter that has an effect on Tiwai, while there appears to be no significant effect in the other direction. At Benmore, the local interactions between the 220kV and 16kV busbar converters appeared to be much more important than those caused by the Tiwai converters. It has also been shown that a direct penetration approach can produce inaccurate results. Therefore, the proposed unified iterative analysis is required for a reliable, as well as accurate, prediction of harmonic interactions in systems with multiple AC/DC converters.

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