Theoretical Documentation

In this section a theoretical explanation of the phenomena analysed by this library is explained. It is composed by a description of the components of a photovoltaic generator, including their purpose in the facility and their electrical parameters, and an explained mathematical development, which is closely related to the main functions of the library.

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Overview

A photovoltaic generator is the result of hierarchically structuring different functional entities with specific rules. In the most basic level, a group of PV cells (considered identical in this library) are combined in series to form a string. To enhance the performance of the generator, the string is combined with an antiparallel diode known as bypass diode. Then, several strings are grouped together in series to from a panel.

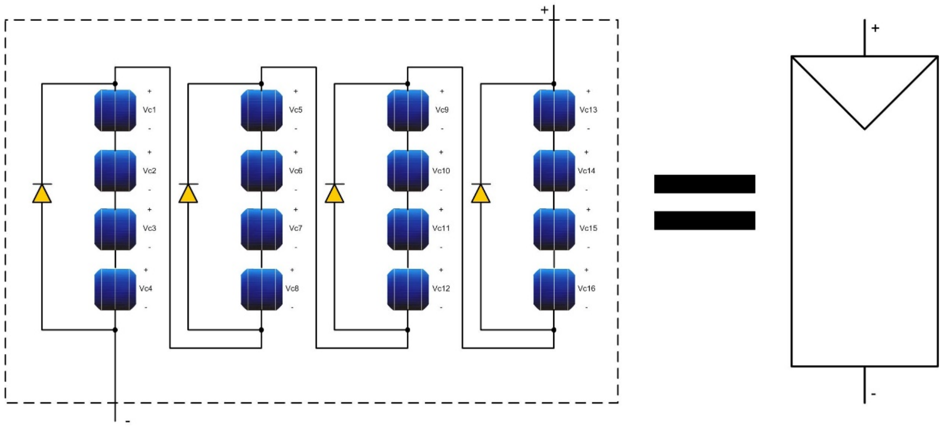

Fig. 1. Photovoltaic panel as a combination of cells and bypass diodes

Different panels can be combined in series or in parallel to create bigger photovoltaic generators. Each branch of panels can terminate in a blocking diode to ensure a single direction of current. However, this step is out of the scope of this library.

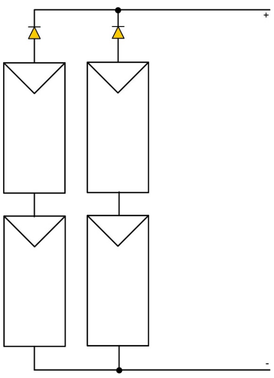

Fig. 2. Photovoltaic generator as a combination of panels and blocking diodes

When a photovoltaic generator has a dispersion of characteristics such as a difference in the working temperatures or a full or partial shading or dirt in the PV panels, we say that the installation is working under mismatched conditions. This cases can entail an important impact on the energy production. This library is able to provide simulations of photovoltaic generators working under mismatched situations. To achieve it, optimization techniques will be applied using object oriented programming (OOP). Starting from the most basic devices that conform a photovoltaic generator, its components will be represented by classes, such as **PV cells** or **bypass diodes**. These components can be combined in series or parallel following the respective electric laws (Kirchhoff's laws).

Solar Cell unit

Introduction

The solar cell is one of the elementary components of a photovoltaic generator. Its structure is the same of a PN junction diode but with a large and transparent surface where solar rays can have an impact on. These, when acting on the union, are absorbed by the outermost electrons of the crystalline structure and are released from their bonds with the atoms, leaving them free to conduct current. Broken bonds act as moving positive charges and are called holes. Both electrons and photogenerated holes contribute to the electric current if the cell undergoes a certain potential difference. In this text the following representation of a PV cell will be used in electrical circuits and simple figures.

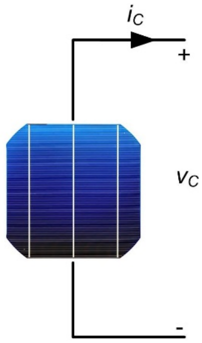

Fig. 3. Current and voltage in the solar cell

Main parameters

The development of this library uses the following equivalent electrical circuit of a solar cell. An ideal representation of a solar cell would not include the series and shunt resistances. Even though there are models that represent a solar cell more precisely, this model requires a simpler mathematical development and a reliable enough solution.

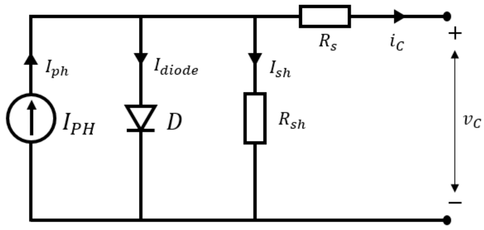

Fig. 4. Equivalent electrical circuit of a solar cell

The basic equation that governs the electrical operation of the cell, with i_C and v_C being the current and voltage between terminals of the cell, as defined in Figure 3, is as follows:

$$i_C = I_{ph} - I_O \cdot \left(e^{\frac{v_C + i_C \cdot R_S}{V_T}} - 1 \right) - \frac{v_C + i_C \cdot R_S}{R_{SH}}$$

Where:

- I_{ph} is the photogenerated current when iluminating the cell (photovoltaic effect).
- $I_O \times (e^{\frac{v_C + i_C \times R_S}{V_T}} - 1)$ is the equation of the diode. This is due to the fact that the cell is intrinsically a diode.
  - I_O is the reverse saturation current.
  - V_T is the thermal voltage.
  - R_S is the cell series resistance (base material + contacts + conductors).
- R_{SH} is the parallel resistance or cell shunt (leaks in the base material).

The terms of the equation belong to every parallel branch in the equivalent circuit:

- The first term belongs to the photogenerated current.
- The second term belongs to the diode current.
- The third term belongs to the shunt current.

To define the solar cell class, it is necessary to have more information about all the previous parameters. Above all, what are its dependencies in terms of solar radiation, G, and the working temperature of the cell, T (not to be confused with the ambient temperature, Fig. 4).

1) Thermal voltage

This parameter is used in the Shockley diode equation to represent the current generated due to the electrostatic potential. Its only variables are the absolute temperature and the ideality factor.

$$V_T = m \cdot \frac{k \cdot T}{q}$$

Where:

- **m**: Ideality factor. Without dimensions. Values between 1 and 2.
- **k**: Boltzmann constant (1.38×10^{-23} J/ºK).
- **T**: Cell working temperature in ºK.
- **q**: Charge of the electron (1.602×10^{-19} C).

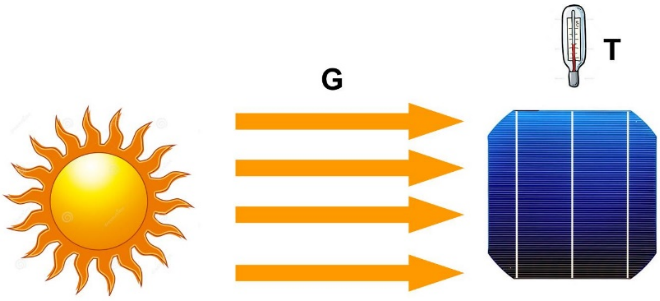

Fig. 4. Factors involved in the operation of the photovoltaic cell

2) Reverse saturation current, Io

When a PN junction is used as a diode, it will always have “leak” when it is in non-conducting state. The direction of this little current is opposed (reversed) to the operational direction of the diode. In the PV cell case, this current goes in opposite direction than the photogenerated current.

$$I_O = I_{O,ref}\left(\frac{T}{T_{ref}}\right)^3 e^{\left[\frac{q\cdot E_g}{m\cdot k}\left(\frac{1}{T_{ref}}-\frac{1}{T}\right)\right]}$$

Where:

- T_{ref}: reference temperature. Usually 298 ºK.
- $I_{O,ref}$: reverse saturation current at the reference temperature. This is a parameter supplied by the manufacturer.
- E_g: Bandgap energy in eV. This value depends on the type of base material and the temperature. In the case of Silicon:

$$E_g(eV) = 1.16 - \frac{7.02\times10^{-4}\,T^2}{T+1108}$$

3) Photogenerated current, Iph

The current generated due to the photogenerated gaps and movement of electrons. Under common operation, in a PV cell this current is the bigger term.

$$I_{ph} = I_{ph,ref}\cdot[1+\alpha\cdot(T-T_{ref})]\cdot SF\cdot\frac{G}{G_{ref}}$$

Where:

- G_{ref}: Reference irradiance. Usually $1000\,W/m^2$.
- α: Temperature coefficient. Depends on the base material. In Silicon this is typically 0.0004 A/ºC.
- $I_{ph,ref}$: Photogenerated current under reference conditions (G_{ref} and T_{ref}). This is a parameter supplied by the manufacturer.
- SF: Cell dirt factor (between 0 and 1).

In addition to the above parameters, there are two more that are very important to characterize a photovoltaic cell: the short-circuit current, I_{SC}, and the open-circuit voltage, V_{OC}. The equation that governs the electrical operation of the cell can be plotted on an $i_C - v_C$ axis:

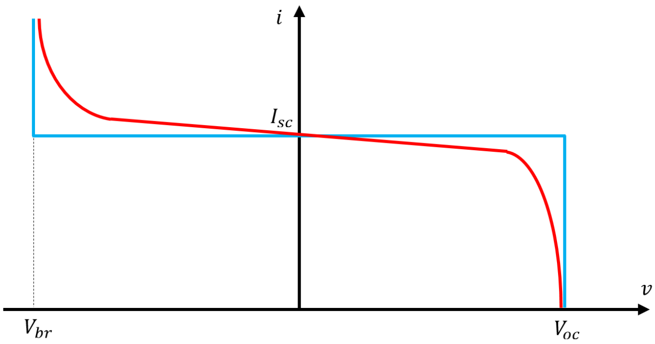

Fig. 5. Example of I-V graph of a photovoltaic solar cell

The blue line represents an ideal PV cell. This cell would be represented by removing the series and shunt resistances in the scheme shown in Figure 3. The red line represents the behaviour of a more real PV cell. This exact behaviour depends on every cell, since many different technologies exist to manufacture a photovoltaic cell.

4) Short-circuit current, Isc

The short-circuit current is the one that is extracted from the cell when its voltage at terminals is 0 V. Notice that there is a big range of voltages where the value of the current is close to the short-circuit current. And, when it gets close to the open circuit voltage, the current drops quickly.

$$I_{SC} = I_{SC,ref}\cdot[1+\alpha\cdot(T-T_{ref})]\cdot SF\cdot\frac{G}{G_{ref}}$$

5) Open circuit voltage, Voc

The open circuit voltage is the one that is measured at the terminals of the cell when there is no current at its terminals, when the circuit is opened. At this voltage, for an ideal PV cell, the current is the short-circuit current.

$$V_{OC} = V_{OC,ref} + \beta \cdot (T - T_{ref}) + V_T \cdot ln\left(SF \cdot \frac{G}{G_{ref}}\right)$$

Where:

- $V_{OC,ref}$ and $I_{SC,ref}$ values are supplied by the manufacturer and refer to standard conditions (T_{ref} and G_{ref}).
- β is the temperature coefficient of the voltage, in V/ºC and depends on the base material of the cell. In the case of Silicon β = -2.3 mV/ºC.

Bypass Diode

Introduction

When several strings of solar cells are placed in series their behaviour can be understood by comparing them to current sources. This fact implies that whenever a single string in the generator has operational problems (because of shades, dirt, rupture of drivers, etc.), the total current of the generator is equal to this decreased current. To avoid these terrible loses, the by-pass diode of a string allows the current of the rest of strings when it is compromised.

Figure 6 shows a possible scenario where two out of four cells in a string are affected by a partial shadow. In this case the respective by-pass diode drives the rest of current that the string cannot handle.

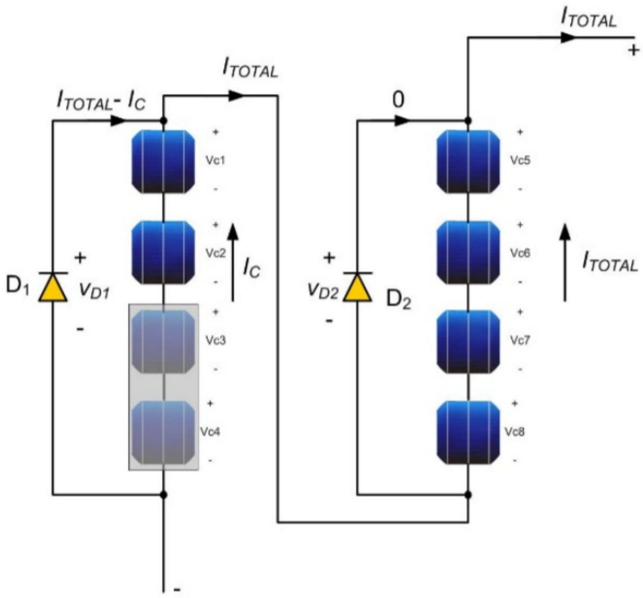

Fig. 6. By-pass diode operation

- V_F: Forward (or knee) voltage of the by-pass diode.
- $V_{D1} = -V_F = V_{C1} + V_{C2} + V_{C3} + V_{C4}$
- $V_{D2} = V_{C5} + V_{C6} + V_{C7} + V_{C8} \geq -V_F$

For example, imagine a PV generator created by 2 strings. Figure 7 represents the current-voltage I-V characteristic in three different scenarios. The first one would be the ideal case of equal high irradiance in both strings. The second one could be the case of partial shading in one of the strings with no bypass diodes. Since the current generated in one of the strings is lower, the rest of the strings are forced to work with that same current. The third case would be the same partial shading but with bypass diodes. For some range the non-shaded string is forced to work with a different current until the bypass diode of the shaded string starts conducting.

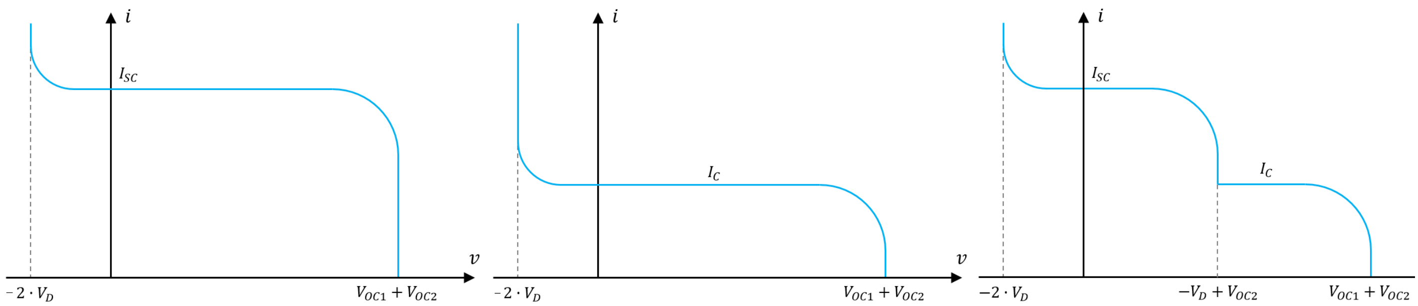

Fig. 7. I-V characteristics of three different cases

Where V_{OCi} is the open circuit voltage of the string i and V_D is the knee voltage of the diode.

Main Parameters

The by-pass diode associated equation is very similar to the cell's one. Both are based on a PN junction. However, being a diode a device with much smaller physical size, its parasitic resistances are much smaller and can be considered negligible. Also, since the device is well shed there is no photo-generation.

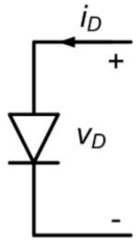

Fig. 8. Diode scheme

$$i_D = I_O \cdot \left(e^{\frac{v_D}{V_T}} - 1\right)$$

1) Thermal voltage

$$V_T = m \cdot \frac{k \cdot T}{q}$$

Where:

- **m**: Ideality factor. Without dimensions, between 1 and 2.
- **k**: Boltzmann constant ($1.38 \times 10^{-23}\ \mathrm{J/°K}$).
- **T**: Diode working temperature in $°\mathrm{K}$.
- **q**: Charge of the electron ($1.602 \times 10^{-19}\ \mathrm{C}$).

2) Reverse saturation current, Io

$$I_O = I_{O,ref}\left(\frac{T}{T_{ref}}\right)^3 e^{\left[\frac{q \cdot E_g}{m \cdot k}\left(\frac{1}{T_{ref}} - \frac{1}{T}\right)\right]}$$

Where:

- T_{ref}: reference temperature. Usually 298 ºK.
- $I_{O,ref}$: reverse saturation current at the reference temperature. This is a parameter supplied by the manufacturer.
- E_g: Bandgap energy in eV. This value depends on the type of base material and the temperature. In the case of Silicon:

$$E_g(eV) = 1.16 - \frac{7.02 \times 10^{-4}\, T^2}{T + 1108}$$

Solar string

Introduction

A group of **solar cells** connected in series and assembled under the same **bypass diode** (although the bypass diode is optional) is considered a string. The physical properties of all the cells in the same string, which depend on the manufacturer, should be identical in order to obtain the maximum profit. Nevertheless, the external factors that affect a PV cell performance, irradiance and temperature, can be different between cells of the same string (mismatched conditions). This leads to groups of cells working under different current and voltage values. A PV cell can be electrically understood as a current source, therefore under mismatched conditions the cells with lower short-circuit current impose the total intensity through the string. Consequently, a partial shadow can trigger the bypass diode or, in certain cases, get a cell or cells into breakdown. Both scenarios can dramatically change the electric point (I,V) where the circuit is working.

As explained in the **mathematical development section**, to find the actual power delivered (I•V) an aproximated point to start the iteration calculus is needed. The better this point is aproximated, the faster and more reliable the solution will be. For this reason these possible states have to be completely understood.

Main Parameters

Since the string is just a configuration of more basic components already described, it has no intrinsic new parameters. Although, in order to obtain the first value of current I and voltage V for the iteration, some parameters of the conglomerate will be needed. The basic values that define a string are the following:

- Number of PV cells in the string.
- Parameters of the PV cells that compose the string. All these cells inside the same string must be identical.
- The presence of a bypass string (Boolean value).
- Parameters of the bypass diode of the string.

And to solve the non-linear equations in order to know the next state of the string, the current following data must be known:

- State of the bypass diode (conducting / non-conducting / breakdown).
- Potential difference between the terminals of the diode (V_{diode}).
- Current through the diode (I_{diode}).
- The sum of all the open circuit voltage of the cells in the string (S_{Voc}).
- The sum of all the breakdown voltage of the cells in the string (S_{Vbreak}).
- Potential difference between the terminals of the string (V_{string}).
- The sum of all the potential difference between the terminals of every cell in the string (S_{Vcell}). This value can be different from the total voltage in the string.
- Total current through the string, sum of currents of the diode and the cells (I_t).
- Individual parameters of each group of cells in the string working under different conditions (with the same short-circuit current). The following parameters are included in the list of **TableStr** structs **CellsGr**. These parameters are:
  - An index to identify the physical position of the cell in the string.

- short-circuit current of the cells in the group.
- Sum of the breakdown voltage of all the cells in the group.
- Sum of the open circuit voltage of all the cells in the group.
- Sum of the breakdown voltage of the group (S_{Vbrx}). This value is only calculated when the bypass diode is present and forward conducting. It is used to know if the cells in the group can get into breakdown or, otherwise, the bypass diode will enter the conducting state before. The calculations to get this value is the following:

$$S_{Vbrx} = -V_{diode} - (Sum\, of\, V_{oc}\, of\, the\, cells\, with\, higher\, I_{sc}) - (Sum\, of\, V_{br}\, of\, the\, cells\, with\, lower\, I_{sc})$$

If the following comparison is true, then it means that the cells in this group can be in breakdown.

$$S_{Vbr} > S_{brx}$$

For complete understanding of this parameter check the **possible scenarios** section.

Possible scenarios

To comprehend the methodology and the results obtained in the simulation is necessary to understand the different scenarios under where the cells can be found working. For the following examples, consider the following string, where three groups of cells receive different values of irradiance. In addition, in order to simplify it, consider ideal PV cells, as explained in the **solar cell** section.

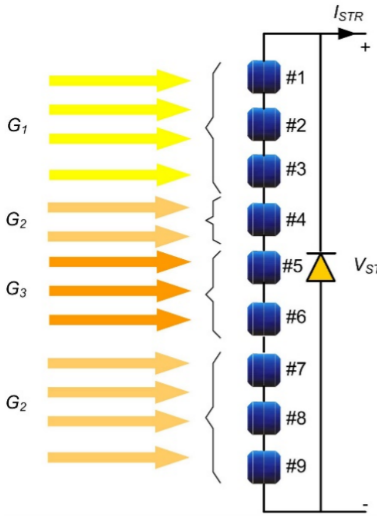

Fig. 9. String of cells with different values of irradiance

The short-circuit current depends on the irradiance and temperature in the cell and it represents the maximum current the cell will be working at. If the ideal model is applied, for any voltage between the breakdown voltage and the open circuit voltage of the cell, it will work at the short-circuit current. In real cases the working current will not be the short-circuit current but a close value. For positive voltage values, it will be a lower value.

Therefore, if the short-circuit current of any group of cells in the string is lower than the current generated by the rest of the panel or a different group of cells of the same string, then the cells will deliver as much power as can be obtained by the one with lower irradiance. For example, considering that $G_1 < G_2 < G_3$ in Figure 9, the lowest short-circuit current in the string will belong to the cells 1, 2 and 3. If this current is also lower than the current generated in the rest of the panel, then the whole panel will work at this current.

In the graph in Figure 10, the cells of the group are working in the greened area:

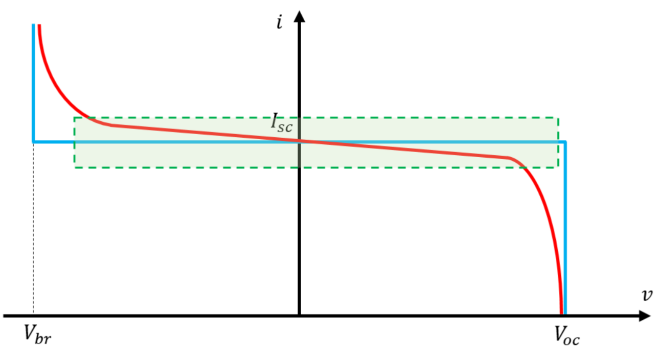

Fig. 10. I-V working zone of the cells with lower Isc than the panel

In this study, the PV cells working in the green area in Figure 10 are called active cells.

If the short-circuit current of a cell in the string is higher than the one in any other cell in the same string or higher than the current in the rest of the panel, then the cell must work at an imposed current, the lowest in the generator, with a voltage close to its V_{oc}. So, the greater the difference between the short-circuit currents the greater the power loss. Under this circumstances the PV cell would be working in the green area in the characteristic showed in Figure 11:

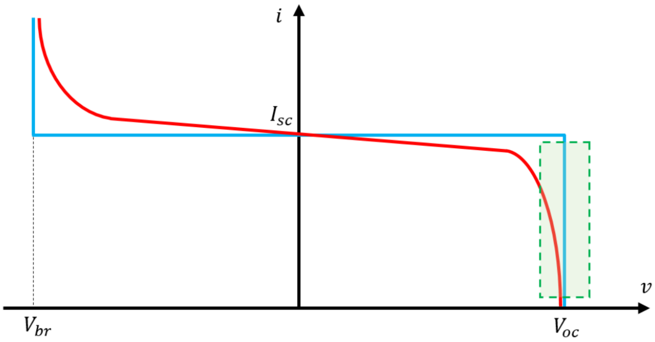

Fig. 11. I-V working zone of the cells with higher Isc than the panel

In this study, the PV cells working in the green area in Figure 11 are called non-active cells.

There is a third scenario where the cell or cells that are imposing their current in the panel are dragged into breakdown. If the voltage of operation of the cell drops below zero until the breakdown point is reached, then the cell is in breakdown. Then the voltage between the terminals of the cell will be its breakdown voltage and the current through the string will be either the next lowest short-circuit current in the string or the current generated by the rest of the panel. The area in the characteristic graph of the cell under this circumstances is the greened one on Figure 12:

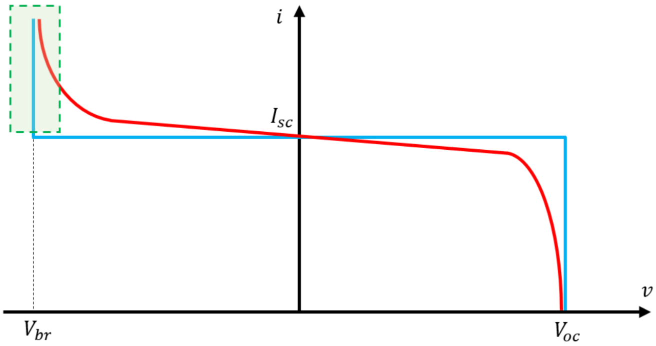

Fig. 12. I-V working zone of the cells in breakdown

In this study, the PV cells working in the green area in Figure 12 are called breakdown cells.

Mathematical development

This section is focused on how the non-linear equations that define the I-V characteristic of every cell that compose the strings are solved. The I-V characteristic of a PV cell is defined by the equation explained in the **solar cell section**. This equation's exponential properties make it susceptible to be solved by means of the Newton-Raphson iterative method. The Newton-Raphson method will be explained in detail in the following section.

Newton-Raphson method

The Newton-Raphson method is a root-finding algorithm. This means that the algorithm is meant to find the root of an equation $f(x) = 0$. To guarantee the convergence of this iterative method the following assumptions must be satisfied:

- There are no local minimums or maximums in the range in between the root and the initial estimation. That is $f'(x) \neq 0$ for $x \in R$ being R the interval $[a - x_0, a + x_0]$ where a is the root and x_0 the initial estimation.
- The second derivative of the function $f''(x)$ is continuous for all $x \in R$
- The initial estimate is close enough to the root.

This last assumption is left open to interpretation, since it is very related to the context where the method is being applied. In our case, the mathematical circumstances when trying to solve the circuit can differ greatly when modifying its parameters. The convergence of this method is empirically ensured since a mathematical proof of convergence can not be obtained in such a dynamic case. Therefore, the initial estimation in every state is crucial to obtain a reliable solution. For more information about the proof of quadratic convergence you can check Wikipedia.

One function and one variable

The Newton-Raphson method is applied in this library to solve the nonlinear systems of equations that define the behaviour of the circuit. However, for the better understanding of this method this section describes the general process to solve a single function with a single variable. The steps followed are represented in the flowchart in Figure 15:

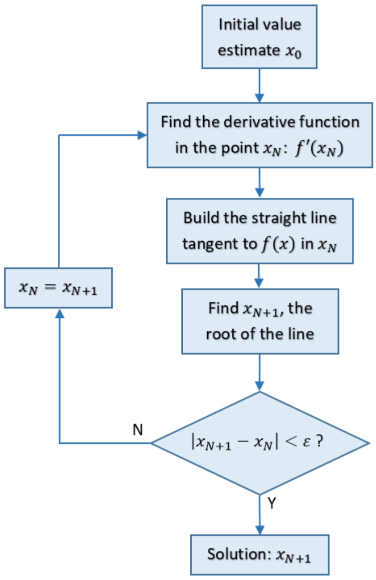

Fig. 15. Iterative flowchart of Newton-Raphson method for a single variable

The steps followed are:

1. Approximate a first value of x_N , where N is the iteration number. That is x_0 since in the first iteration $N = 0$.
2. Calculate the derivative function in the x_N point $f'(x_N)$
3. The stright line tangent to $f(x)$ in x_N is build.

$$y = f'(x_N) \cdot (x - x_N) + f(x_N)$$

4. The next value of the iteration x_{N+1} is found by finding the root of y .

$$x_{N+1} = x_N - \frac{f(x_N)}{f'(x_N)}$$

5. This sequence is repeated by using the new value x_{N+1} as x_N in the point 2. The iterative process is ended when the difference between the values obtained in k consecutive iterations is lesser than a certain real and positive value ε:

$$|x_{N+1} - x_N| < \varepsilon$$

Although sometimes ε can be defined differently. For example, a common technic is using ε as:

$$|\frac{x_{N+1} - x_N}{x_{N+1}}| < \varepsilon$$

This process is represented in the Figure 16.

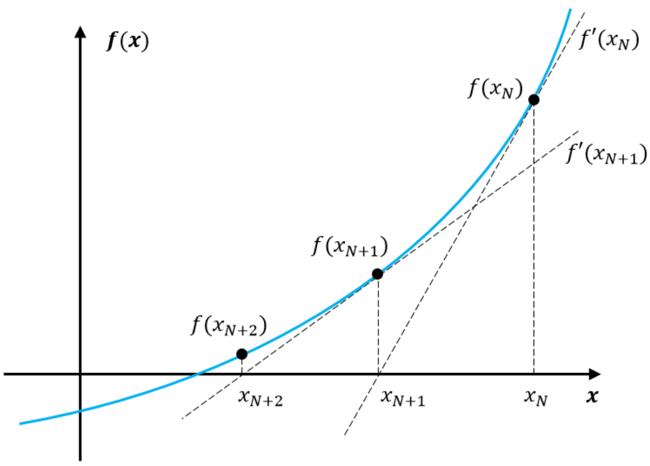

Fig. 16. Graphic representation of Newton-Raphson method for a single variable

M functions and M variables

The iterative method of Newton-Raphson can also be used to solve systems of nonlinear equations. To make it easier to work with systems, the matrix notation will be used. So, the system of M equations faced could be like this:

$$\begin{pmatrix} f_1(x_1, x_2, \dots, x_M) \\ f_2(x_1, x_2, \dots, x_M) \\ \vdots \\ f_M(x_1, x_2, \dots, x_M) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

And the expresion of x_{N+1} is now the following vectorial equation:

$$\overrightarrow{x_{N+1}} = \overrightarrow{x_N} - J(\overrightarrow{x_N})^{-1} \times \vec{f}(\overrightarrow{x_N})$$

However, from the point of view of numerical methods, it is way more computationally expensive to calculate the reverse matrix than to solve the following equation:

$$J(\overrightarrow{x_N})\vec{\Delta} = \vec{f}(\overrightarrow{x_N})$$

$$\vec{\Delta} = \left(\overrightarrow{x_{N+1}} - \overrightarrow{x_N}\right)$$

$$\overrightarrow{x_{N+1}} = \overrightarrow{x_N} + \vec{\Delta}$$

where:

$$\vec{f}(\overrightarrow{x_N}) = \begin{pmatrix} f_1(x_1, x_2, \ldots, x_M) \\ f_2(x_1, x_2, \ldots, x_M) \\ \vdots \\ f_M(x_1, x_2, \ldots, x_M) \end{pmatrix} \quad \overrightarrow{x_N} = \begin{pmatrix} x_{1,N} \\ x_{2,N} \\ \vdots \\ x_{M,N} \end{pmatrix} \quad \overrightarrow{x_{N+1}} = \begin{pmatrix} x_{1,N+1} \\ x_{2,N+1} \\ \vdots \\ x_{M,N+1} \end{pmatrix}$$

And $J(\overrightarrow{x_N})$ is the Jacobian matrix, defined as:

$$J(\overrightarrow{x_N})^{-1} = \begin{pmatrix} \frac{\partial f_1(\vec{x}_N)}{\partial x_1} & \frac{\partial f_1(\vec{x}_N)}{\partial x_2} & \ldots & \frac{\partial f_1(\vec{x}_N)}{\partial x_M} \\ \frac{\partial f_2(\vec{x}_N)}{\partial x_1} & \frac{\partial f_2(\vec{x}_N)}{\partial x_2} & \ldots & \frac{\partial f_2(\vec{x}_N)}{\partial x_M} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_M(\vec{x}_N)}{\partial x_1} & \frac{\partial f_M(\vec{x}_N)}{\partial x_2} & \ldots & \frac{\partial f_M(\vec{x}_N)}{\partial x_M} \end{pmatrix}$$

Analogously, the condition to end the iteration process, although other methods to determine ε exist, the most common is the one below.

$$||\overrightarrow{x_{N+1}} - \overrightarrow{x_N}|| < \varepsilon\,, \quad \varepsilon \geq 0$$

The following flowchart in Figure 17 represents the iterative method of Newton-Raphson for M equations with M variables:

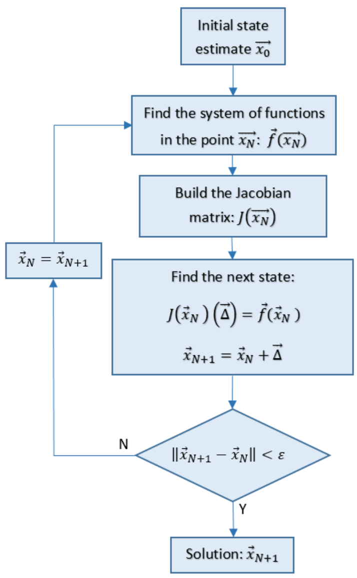

Fig. 17. Iterative flowchart of Newton-Raphson method for M equations and M variables

Applied mathematical development

In this section the Newton-Raphson method explained above will be applied to the components of a solar panel.

Single component mathematical development

Firstly, the case of a single solar cell will be contemplated. Assuming that the voltage between the terminals of the cell v_C is known, the current generted through it i_C will be found using an iterative method. As seen in the previous section about the **solar cell**, the characteristic equation that defines the behaviour of a single solar cell is the one below.

$$i_C = I_{ph} - I_O \cdot \left(e^{\frac{v_C + i_C \cdot R_S}{V_T}} - 1\right) - \frac{v_C + i_C \cdot R_S}{R_{SH}}$$

From this electrical formula we can obtain the function $f(x)$ described in the definition of the **Newton-Raphson method**. The formula obtained for a certain tension in the cell $v_C = V_C$ would be the following:

$$f_C(i_C) = i_C - I_{ph} + I_O \cdot \left(e^{\frac{V_C + i_C \cdot R_S}{V_T}} - 1\right) + \frac{V_C + i_C \cdot R_S}{R_{SH}}$$

When calculating the I-V characteristic of the cell knowing the voltage, the current i_C is the variable, so the function $f_C(i_C)$ shall be differentiated respect it:

$$\frac{df_C(i_C)}{di_C} = 1 + \frac{I_O \cdot R_S}{V_T}\left(e^{\frac{V_C + i_C \cdot R_S}{V_T}} - 1\right) + \frac{R_S}{R_{SH}}$$

With this result, if we directly find the root of the tangent line obtained through f'_C, the expression of the next point for every iteration is:

$$i_{C,N+1} = i_{C,N} - \frac{i_{C,N} - I_{ph} + I_O \cdot \left(e^{\frac{V_C + i_C \cdot R_S}{V_T}} - 1\right) + \frac{V_C + i_C \cdot R_S}{R_{SH}}}{1 + \frac{I_O \cdot R_S}{V_T}\left(e^{\frac{V_C + i_C \cdot R_S}{V_T}} - 1\right) + \frac{R_S}{R_{SH}}}$$

Solar string mathematical development

In the case of a **solar string**, it is composed by M PV cells and a bypass diode. Therefore, to find the total voltage or current through it a system of equations shall be solved. To solve such a system, the theory explained in the **M functions and M variables** section will be applied. In this case both voltage and current are needed for every cell. Including a bypass diode in the string, the variables in this system of equations are:

- Since the string is composed by M cells, there are M voltage variables.
- The current in each cell is the same, so the string current adds 1 variable.
- The current through the diode adds another variable.

That makes a total of M+2 variables. However, because the I-V characteristic is built by analysing the total current for each value of current, the total voltage between the terminals of the string is known. And since the sum all the cell voltages equals the voltage in the string, one of these voltage variables can be eliminated. Hence, having M+1 variables.

Notice that, in the case of a single string, it does not makes any difference, but when the state of several strings has to be solved, if instead of looking for the current in every diode, the total current is the variable, no matter how many strings are considered, it only adds 1 variable, since the current in the diodes can be found using the current through the cells in every string:

$$I_{diode} = I_{total} - I_{string}$$

Regarding the equations:

- M equations of the PV cells, taking into account that one of them shall be replaced with the difference between the total voltage and the sum of every cell's voltage.
- The equation of the diode.

That makes M+1 equations too.

Also notice that, when solving more than one string, the number of variables and equations is still the same. For every string, M+1 variables, corresponding to M cell voltages and the string current, are added. However, M+1 equations are also added, corresponding to M cell's equations and a diode's equation.

The variables and equations expressed as vectors for the single string case are:

$$\overrightarrow{x_N} = \begin{pmatrix} v_{1_N} \\ v_{2_N} \\ \vdots \\ v_{M-1_N} \\ i_{T_N} \\ i_{STR_N} \end{pmatrix} \quad \overrightarrow{x_{N+1}} = \begin{pmatrix} v_{1_{N+1}} \\ v_{2_{N+1}} \\ \vdots \\ v_{M-1_{N+1}} \\ i_{T_{N+1}} \\ i_{STR_{N+1}} \end{pmatrix} \quad \vec{f}(\overrightarrow{x_N}) = \begin{pmatrix} f_1(v_1, v_2, \ldots, v_{M-1}, i_T, i_{STR}) \\ f_2(v_1, v_2, \ldots, v_{M-1}, i_T, i_{STR}) \\ \vdots \\ f_M(v_1, v_2, \ldots, v_{M-1}, i_T, i_{STR}) \\ f_D(v_1, v_2, \ldots, v_{M-1}, i_T, i_{STR}) \end{pmatrix}$$

where:

$$f_M(v_1, v_2, \ldots, v_{M-1}, i_T, i_{STR}) = V_{TOTAL} - \sum_{k=1}^{M-1} v_k$$

Then, taking this into account, the resulting Jacobian matrix would be like below:

$$J(\overrightarrow{x_N}) = \begin{pmatrix} \frac{\partial f_1(\vec{x}_N)}{\partial v_1} & 0 & \ldots & 0 & 0 & \frac{\partial f_1(\vec{x}_N)}{\partial i_{STR}} \\ 0 & \frac{\partial f_2(\vec{x}_N)}{\partial v_2} & \ldots & 0 & 0 & \frac{\partial f_2(\vec{x}_N)}{\partial i_{STR}} \\ \vdots & \vdots & \ddots & & & \vdots \\ 0 & 0 & & \frac{\partial f_{M-1}(\vec{x}_N)}{\partial v_{M-1}} & 0 & \frac{\partial f_M(\vec{x}_N)}{\partial i_S} \\ \frac{\partial f_M(\vec{x}_N)}{\partial v_1} & \frac{\partial f_M(\vec{x}_N)}{\partial v_2} & \ldots & \frac{\partial f_M(\vec{x}_N)}{\partial v_{M-1}} & 0 & \frac{\partial f_M(\vec{x}_N)}{\partial i_{STR}} \\ 0 & 0 & \ldots & 0 & 1 & -1 \end{pmatrix}$$

The process now would be identical as the one described in the **M functions and M variables** section.

Solar string initial estimate

The initial estimate of the voltage and current values of every cell in the panel is done by assuming ideal solar PV cells. The assignation of the current straightforward. It is assumed that, since the cells are ideal, the active cells (as described in section **Solar string**) are always working with their short-circuit current. Therefore, all the cells in a string will be working either at the lowest short-circuit current in the string or at a lower current, imposed by the rest of the panel. The current for breakdown cells is the same as the rest of the cells in the string. It does not correspond to the cell's I-V characteristic since at that point, the cell is permanently damaged, and is no longer represented by its model.

The assignation of voltages is more complex. Non-active cells voltage is supposed to be their open circuit voltage. That is because, as explained in section **Solar string**, for ideal PV cells, when working under their short-circuit current, the value of the voltage will always be the open circuit voltage.

For breakdown cells the voltage assigned is the breakdown voltage of the cells, since this is the lowest voltage value that an ideal cell can acquire.

And finally, in order to find the voltage of the active cells, the second law of Kirchhoff is applied. The mesh considered for the calculations depends on the state of the diode in the string:

- **Conducting diode**. In this case the cells in the string and the diode conform the mesh. Since the number and voltages of the non-active and breakdown cells are already known, the voltage of active cells can be found by computing:

$$V_{active} = \left(-V_{diode} - \sum_{j=1}^{N_{non-active}} V_{oc_j} - \sum_{k=1}^{N_{br}} V_{br_k}\right) / N_{active}$$

where:

> - V_{active} is the voltage of a single active cell.
> - V_{diode} is the knee voltage of the bypass diode.
> - V_{oc_j} is the open circuit voltage of the non-active cell j.
> - V_{br_k} is the breakdown voltage of the breakdown cell k.
> - $N_{non-active}$ is the number of non-active cells in the string.
> - N_{br} is the number of broken cells in the string.
> - N_{active} is the number of active cells in the string.

- **Non-conducting diode.** In this case the cells in the string and the rest of the generator conform the mesh. Since the number and voltages of the non-active and breakdown cells are already known and the voltage between the terminals of the string too, the voltage of active cells can be found by computing:

$$V_{activecell} = \left(V_{string} - \sum_{j=1}^{N_{non-activecells}} V_{oc_j} - \sum_{k=1}^{N_{breakdowncells}} V_{br_k}\right) / N_{activecells}$$

The resulting values are summarized in the following table:

Scenario	Groups	Current	Voltage
Non-conducting diode	Active cells	I_{sc}	$\left(V_{panel} - \sum_{j=1}^{N_{non-activecells}} V_{oc_j} - \sum_{k=1}^{N_{breakdowncells}} V_{br_k}\right) / N_{activecells}$
	Non-active cells	I_{sc} of the active cells OR current of the panel	V_{OC}
	Broken cells	I_{sc} of the active cells OR current of the panel	V_{BR}
Conducting diode	Active cells	I_{sc}	$\left(-V_{diode} - \sum_{j=1}^{N_{non-activecells}} V_{oc_j} - \sum_{k=1}^{N_{breakdowncells}} V_{br_k}\right) / N_{activecells}$
	Non-active cells	I_{sc} of the active cells	V_{OC}
	Broken cells	I_{sc} of the active cells	V_{BR}

The accuracy of this assumption depends on the material and manufacturing of the panels, but for most cases is good enough to get the iterative method to converge.