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A Two-dimensional Steady Convection-Diffusion Equation: the Smith-Hutton problem

by CTTC

1. Exercise

- Write a computer program (according to the specifications given in section 4) to solve the convection-diffusion equation in the situation described in section 2.
- Ensure that the code is correct
- Choose a suitable mesh.
- Run the simulation and submit us the files/information requested in section 3.
- We will check the code and the results:
  - If the code and results are correct, we will ask you to write a short report about the work made and then you passed the test.
  - If the code looks good but there are problems, such as not enough accuracy, wrong programming style, etc, we will help you to enhance it and we will give you more opportunities.
  - We will not accept candidates who have used software not developed by themselves OR have given their own codes to other persons.

Comments:

- You must write your own code, not use already available software.
- A normal PC is enough to do this exercise.
- This is a personal problem, you can not ask for help of other persons.
- Don't give this problem to other persons.
- If you have questions about how to do this exercise, we suggest you to read the chapters 1-4 of the book "Numerical Heat Transfer and Fluid Flow" [1].
- If you have a question about the exercise please ask us.

2. Problem definition

We are interested in the steady state solution of the Smith-Hutton problem, described in [2]. To do so, a two-dimensional convection-diffusion equation

$$\frac{\partial (\rho\phi)}{\partial t} + \nabla \cdot (\rho \mathbf{v} \phi) = \nabla \cdot (\Gamma \nabla \phi) + S \tag{1}$$

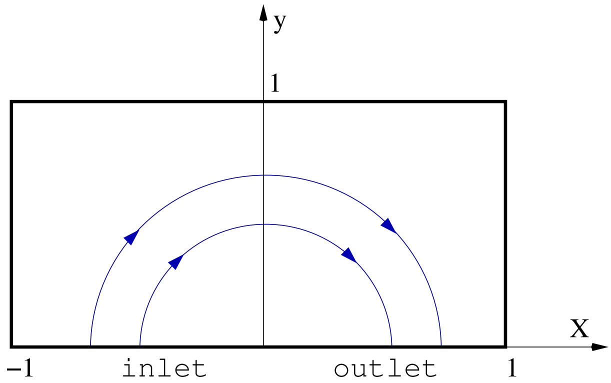

Figure 1: General schema of the proposed problem

must be solved numerically in a rectangular domain (see figure 1) with the prescribed velocity field given by

$$u(x,y) = 2y\left(1-x^2\right) \quad (2)$$
$$v(x,y) = -2x\left(1-y^2\right) \quad (3)$$

and the following boundary conditions for the variable ϕ

$$\phi = 1 + tanh\left(\alpha\left(2x+1\right)\right) \qquad y=0\ ;\ x \in (-1,0) \quad \text{(inlet)} \quad (4)$$
$$\frac{\partial \phi}{\partial y} = 0 \qquad y=0\ ;\ x \in (0,1) \quad \text{(outlet)} \quad (5)$$
$$\phi = 1 - tanh\left(\alpha\right) \qquad \text{(elsewhere)} \quad (6)$$

where $\alpha = 10$.

3. Information requested

The student shall carry out all the simulations and present the plots that he/she considers necessary in order to describe the numerical behaviour for different numerical schemes (CDS, Upwind, Hybrid ...) and mesh sizes. See [1] for details about the numerical schemes.

The plots should be presented with a short written description of the most relevant features. Moreover, the student must find the mesh size and time step appropiated to obtain a good approximation of the correct solution (see table 1).

Moreover, you must submit us the following.

- Your code (see section 4, for details), named "BNAME.c" where BNAME stands for your surname.
- A plot of the ϕ variable at the output for $\rho/\Gamma = 10$, 10^3 and 10^6.

4. Code requirements

- You must write a C or FORTRAN code that can be compiled in a Linux environment.
- You can not use libraries (such as linear algebra libraries, PDE solvers, etc) not developed by you.

x-position	$\rho/\Gamma = 10$	$\rho/\Gamma = 10^3$	$\rho/\Gamma = 10^6$
0.0	1.989	2.0000	2.000
0.1	1.402	1.9990	2.000
0.2	1.146	1.9997	2.000
0.3	0.946	1.9850	1.999
0.4	0.775	1.8410	1.964
0.5	0.621	0.9510	1.000
0.6	0.480	0.1540	0.036
0.7	0.349	0.0010	0.001
0.8	0.227	0.0000	0.000
0.9	0.111	0.0000	0.000
1.0	0.000	0.0000	0.000

Table 1: Numerical results at the outlet for different ρ/Γ numbers.

- The code should be in a single file and compile with no errors.
- The code must run without any input parameter and produce the output file BNAME.dat, that must contain three columns with the following information:
  - Column 1: x-coordinate (see table 1).
  - Column 2: Value of ϕ for $\rho/\Gamma = 10$ at the position specified in column 1.
  - Column 2: Value of ϕ for $\rho/\Gamma = 10^3$ at the position specified in column 1.
  - Column 2: Value of ϕ for $\rho/\Gamma = 10^6$ at the position specified in column 1.

5. Hints

The problem should be treated as two-dimensional. All physical properties remain constant. The student must find the mesh size and timestep appropiated to obtain a good enough approximation.

References

1. Suhas V. Patankar. *Numerical Heat Transfer and Fluid Flow*. Hemisphere Publishing Corporation, McGraw-Hill Book Company, 1980.
2. R. M. Smith and A. G. Hutton. The numerical treatment of advection: a performance comparison of current methods. *Numerical Heat Transfer*, 5:439–461, 1982.