# Portfolio 3 Submission Instructions

Please follow the submission instructions carefully. A failure to do so will result in mark deductions. Make sure you attempt all questions in Parts A, B and C.

1. Solutions to all questions must be presented within a single pdf document and submitted via blackboard.
2. The MATLAB grader will not be used to mark your MATLAB code for this portfolio. All code will be inspected by your tutor when reading your pdf submission.
3. The submitted pdf should include your student number in its same (eg. n#######Portfolio3.pdf)
4. Code and working should be simple for your marker to understand.
5. Portfolio 2 is **due at 11:59pm Friday 27th September** and should be submitted through Blackboard. Late submissions will receive a mark of zero. If you make multiple submissions, the most recent on-time submission will be graded.

## Part A - 2nd Order ODEs (10 Marks)

The suspension system in a car can be described using the 2nd order ODE:

$$\frac{d^2y}{dt^2} + \frac{c}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{F(t)}{m}$$

where $y$ is vertical poition, $c$ is the damping coefficient, $k$ is the spring constant, $m$ is mass and $F(t)$ is the external forcing function.

Consider that $m = 1000$ kg, $c = 4000\ \mathrm{N\,m^{-1}\,s^{-1}}$, $k = 40000\ \mathrm{N\,m^{-1}}$ and $F(t) = -200t$ N.

1. Find the homogeneous solution to the ODE.

   (2 marks)

2. Find the particular solution to the ODE.

   (3 marks)

3. Solve the ODE given $y(0) = 0$ and $y'(0) = 0$.

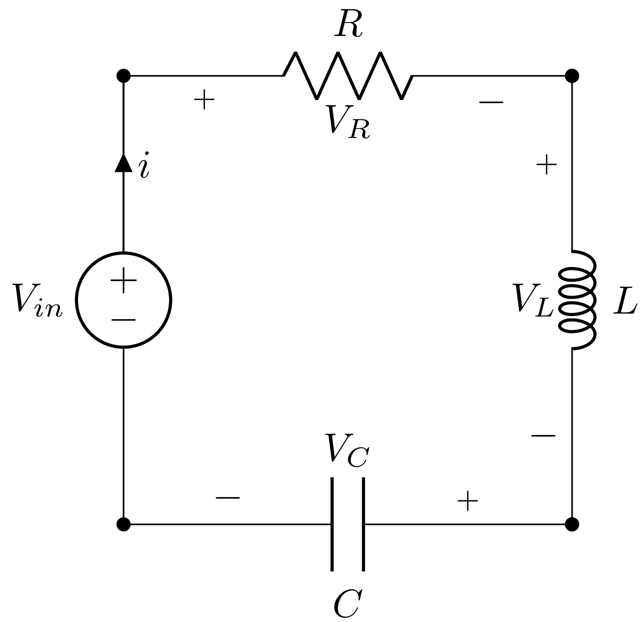
   (3 marks)

4. Use you solution from Q3 to solve the net force acting on the car, $F_{net}(t)$ (**Hint:** Apply Newton's 2nd Law).

   (2 marks)

## Part B - Systems of ODEs (10 Marks)

Consider the series RLC Circuit where $R = 1\ \Omega$, $L = \frac{1}{4}$ H, $C = \frac{4}{3}$ F and $V_{in} = 10$ V.




1. Show that the RLC circuit can be modelled with the system of ODEs:

$$\begin{bmatrix} i' \\ V_L' \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} i \\ V_L \end{bmatrix} + \begin{bmatrix} 0 \\ V_{in}' \end{bmatrix}$$

(2 marks)

2. Solve the eigenvalues of the coefficient matrix.

(2 marks)

3. Solve the eigenvectors of the coefficient matrix.

(2 marks)

4. Solve the eigenvalues and eigenvectors in MATLAB using the `eig` function. Comment on whether your MATLAB and hand-worked solutions agree.

(2 marks)

5. Solve for $i(t)$ and $V_L(t)$ given $i(0) = 10$ A and $V_L(0) = 0$.

(2 marks)

## Part C - Numerical Methods (10 Marks)

Charlie the cyclist (from portfolio 2) would now like to analyse an ODE of his velocity while pedalling on his bicycle. This will lead to a non-linear, non-separabe ODE, so he would like for the velocity to be solved numerically.

For any forcing function, his bike can be described with the ODE

$$\frac{dv}{dt} + \frac{d}{m}v^2 = \frac{F(t)}{m}.$$

Charlie approximates that his pedalling results in a forcing function of $F(t) = 100 + 100\sin(8t)$ N. He has a mass of $m = 70$ kg, his drag coefficient is $d = 1$ kg/m and his initial velocity is $v(0) = 0$.

1. Use MATLAB's `ode45` function to approximate Charlie's velocity while riding his bike. Plot your solution from $t = 0$ s to $t = 30$ s using MATLAB.
   (2 mark)
2. Use the `euler` function on Blackboard to approximate Charlie's velocity using $h = 2$. Plot the solution in the same figure as your `ode45` solution, and comment on the accuracy of the `euler` solution.
   (2 marks)
3. Recommend to Charlie what step size should be used in Euler's method in order to obtain a sufficiently accurate solution for velocity without wasting computational time. In your solution, clearly state how you defined "sufficiently accurate" and provide reasoning for your decision (including any relevant code, plots and/or metrics).
   (3 marks)
4. Charlie is interested in modelling both his velocity and position while riding his bike. Create a system of ODEs with velocity and position as the two dependant variables, and use MATLAB's `ode45` function to solve (assume initial position is 0). Plot the position from $t = 0$ s to $t = 30$ s using MATLAB.
   (3 marks)

## Marking Scheme

### Part A

1. Correct roots to characteristic equation (1 mark), correct homogenous solution (1 mark).
2. Correct 'guess' for $y_p$ (1 mark), correct substitution (1 mark), correct solving of coefficients (1 mark).
3. General solution (0.5 marks), initial conditions (1 mark each), final answer (0.5 marks).
4. Correct application of NSL (1 mark), correct differentiation (1 mark).

### Part B

1. Component laws (0.5 marks), KVL (0.5 marks), algebra (0.5 marks), conversion to matrix-vector form (0.5 marks).
2. Correct process (1 mark), determinant (0.5 marks), solving roots (0.5 marks).
3. 1 mark for each eigenvector.
4. Correct code (1 mark), correct comment (1 mark).
5. Applying initial conditions correctly (1 mark), solving correctly (1 mark).

### Part C

**Note:** For full marks, all plots must have a title, axis labels and a legend where appropriate.

1. Correct `ode45` code (1 mark), correct plot (1 mark).
2. Correct code (0.5 marks), correct plot (1 mark), correct comment (0.5 marks)
3. (3/3) - solution is reasonable and full working is provided to explain why the solution has been chosen.

   (2/3) - full working is provided to explain why the solution has been chosen, but there are minior errors in the analysis.

   (1/3) - a reasonable solution is provided, but working is minimal.
4. Correctly modelled system (1 mark), correct code (1 mark), correct plot (1 mark).