Part A:

Part A

1) Define the events in the problem.

let {C = carrying} , {NC = not carrying}

in the case Beep → let B = Beep

{B|C = Beep Carrying} , {B|NC = Beep not carrying}

2) State known probabilities

$Pr(C) = 0.01$

alarm beeps 98% of the time

$Pr(B|C) = 0.98$

$Pr(NC) = 0.99$

alarm beeps 5% of the time

$Pr(B|NC) = 0.05$

3) Probability that alarm activates for a random person

$Pr(B)$ → Beeps random person

$$Pr(B) = P(B \cap C) + P(B \cap NC)$$

$$= P(B|C)P(C) + P(B|NC)P(NC)$$

$$= (0.98 \times 0.01) + (0.99 \times 0.05)$$

$$= 9.8 \times 10^{-3} + 0.0495$$

$$Pr(B) = 0.0593$$

↳ The probability that it will beep for a random person is 5.93%.

4) Probability that a person is carrying an illegal item given it activates

$Pr(B) = 2.578 \times 10^{-3}$ → using Bayes Rule

$$Pr(C|B) = \frac{P(B|C)\,P(C)}{P(B)}$$

$$= \frac{0.98 \times 0.01}{0.0593} = 0.165 = 16.5\%$$

→ There is a 16.5% chance that when activated → the alarm is the person is carrying an illegal item.

5)

$$\Pr(C \mid B^c) = \frac{(1 - P(B \mid C))\, P(C)}{1 - P(B^c)}$$

$$= \frac{(1 - 0.98) \times 0.01}{1 - 0.0593}$$

$$= 0.000213$$

there is a small chane to 0.0213%
that a person is carrying an illegal item
given that the alarm does not acticate.

Part B:

Question 1 & 2 – using csvread, mean and std functions.

Part B

Initial information

- Cars pass by at an average rate of $\boxed{300 \text{ cars per hour}}$
- Speed of cars is normally distributed

avg speed 58 km/h

Variance 2 km^2/h^2

1) Probability that there is less than 10 seconds time differe between cars

exponential distribution

300 cars per hour

$\frac{300}{60} = 5$ cars per min

$\boxed{\frac{5}{60} = \text{cars per second}}$

$$P(X < x) = 1 - e^{-\lambda x}$$

time diff < 10

$$\lambda = \frac{5}{60} = \frac{1}{12}$$

$$P(X < 10) = 1 - e^{(-\frac{1}{12} \times 10)}$$

$$= 0.57$$

There is a 57% chance that there is less than 10 seconds time difference between one car and the next.

Poisson Distribution

2) Probability that >3 cars pass in a min

$$\frac{300}{60} = 5 \text{ cars per min}$$

$\underline{X > 3}$ $\rightarrow$ $X \leq 4$

Poisson Distribution

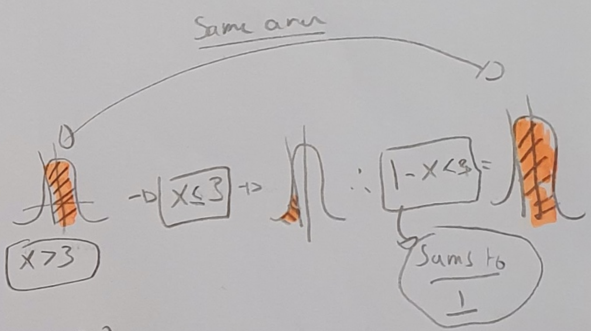
$$f(x) = \frac{\lambda^{x} e^{-\lambda}}{x!}$$

rate = 5 cars per min

$$\lambda = 5 \times 1 = \underline{5} = E(X)$$

$$f(x) = \frac{5^{x} e^{-5}}{x!}$$

$$P(X>3) = 1 - P(X \leq 3) \longrightarrow$$



$$= 1 - (P(X=0) + P(X=1) + P(X=2) + P(X=3))$$

$$= 1 - \left(\frac{5^0 \times e^{-5}}{0!} + \frac{5^1 \times e^{-5}}{1!} + \frac{5^2 \times e^{-5}}{2!} + \frac{5^3 \times e^{-5}}{3!} \right)$$

$$= 1 - (6.74 \times 10^{-3} + 0.034 + 0.084 + 0.14)$$

$$= 1 - 0.26$$

$$= 0.74$$

There is a 74% chance that more than 3 cars pass by in a min.

3) Normal distribution

Speed Lim = 60 km/h

Avg Speed = 58 km/h

Variance = 2 km²/h²

$X = 60 \quad \mu = 58 \quad \sigma = \sqrt{2}$

$$Z = \frac{X - \mu}{\sigma} = \frac{60 - 58}{\sqrt{2}}$$

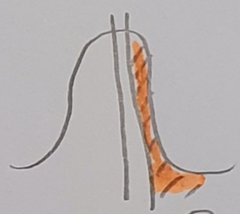
$Z = 1.41$

$P(x > 60) = P(Z > 1.41)$

$= 0.9207$ →

$= 1 - 0.9207$

$= 0.0793$ →



Speedy region

There is a 7.93 % chance that a random car is speeding. (going above the speed limit of the road. 60 km/h).

4) Poisson Distribution

300 cars per hour

$300 \times 0.0793 = 23.79$ cars speeding per hour

$\frac{23.79}{60}$ = Per min = 0.3965 cars per min speeding

$0.3965 \times 10 = 3.965$ per 10 mins speeding (try)

$\lambda = E(x) = 3.965$

$\boxed{\text{0 speeding in 10mins}}$

$$P(X=0) = \frac{3.965^{x} \times e^{-3.965}}{x!}$$

$$= \frac{3.965^{0} \times e^{-3.965}}{0!}$$

$$= \frac{1 \times 0.01897}{1}$$

$$= \underline{0.01897}$$

There is a 1.897% chance that there will be no speeding cars within a 10 min period.

MATLAB code for part B

1) – Same answer as hand working.

```
expcdf(10,(1/(1/12)))
```

ans =

0.5654

2) – Same answer as hand working.

```
poisscdf(3,5,'upper')
```

ans =

0.7350

3) – Same answer, however, due to MATLABS's table being more accurate the number is slightly different due to a higher accuracy.

```
normcdf(60,58,sqrt(2),'upper')
```

ans =

0.0786

4) – Same answer as hand working.

```
poisscdf(0,3.965)
```

ans =

0.0190

Part C:

```
% Using csvread function to read the file (Question 1)
M = csvread('YieldStrengths.csv',0,0);


% Question 2
% Sample Mean using the mean function
a = mean(M)

% Sample Standard Deviation using the std function
b = std(M)
```

Outputs:

```
a =

  246.9210


b =

    3.3444
```

Part C

Q3) Evaluate at 95% confidence interval

$\alpha \rightarrow$ for 95% CI $= \boxed{\alpha = 0.05}$

$t_{N-1}\,, 1-\alpha/2$

$t_{(20-1),(1-(0.05/2))}$

$t_{19}\,, 0.975 \longrightarrow$ From table $\boxed{t = 2.093}$

$$CI_{95\%} = \overline{X} \pm t_{N-1,\,1-\alpha/2} \times \frac{S}{\sqrt{N}}$$

$$= 246.92 \pm \left(2.093 \times \frac{3.34}{\sqrt{20}}\right)$$

$$= 246.92 \pm 1.56$$

$$= (248.48\,, 245.36)$$

Q4) Hypothesis test

Published Yield Stregnth A36 Steel → 250 MPa

$\alpha = 0.05$ → 95% confidence

Hypothesis test:

null - $H_0: \mu = 250$ MPa
alternative - $H_A: \mu \neq 250$ MPa

Test Statistic: $T_{test} = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{N}}}$

$$= \frac{246.92 - 250}{\frac{3.34}{\sqrt{20}}} = \frac{246.92 - 250}{0.75} = -4.107$$

→ reject H_0 if $abs|-4.107| > t_{value}$

$4.107 > 2.093$

True therefore reject null hypothesis.

Since the abs(-4.107) > 2.093 the null hypothesis can be rejected. As there is insufficient evidence to prove that it is true.

Question 5 – Using MATLABS ttest function

```
% Using csvread function to read the file
M = csvread('YieldStrengths.csv',0,0);

xbar = mean(M)
std=sqrt(var(M))

% Conducts the ttest
ttest=(xbar-50)/(std/sqrt(length(M)))

% Calculates pvalue
pvalue = 2*(1-tcdf(ttest,length(M)-1))
```

Outputs:

xbar =

246.9210

std =

3.3444

ttest =

263.3193

pvalue =

0

The P-value is 0 which is less than the chosen significance of 0.05. Hence there is strong evidence against the null hypothesis. Therefore the population mean of 246.9210 is not a good representation of the population.

Looking back at questions 3 & 4. In question 4 through the conduction of a hypothesis test, with the z value found in q3. It was apparent that the null hypothesis should be rejected as abs(-4.107) > 2.093. These cases both match as they provide the same answer of rejecting the null hypothesis due to insufficient evidence.